

# Elastic-Plastic Analysis of a Thin Rotating Disk of Exponentially Variable Thickness with Inclusion

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**Abstract:** - Transition theory has been used to derive the elastic-plastic and transitional stresses. Results obtained have been discussed numerically and depicted graphically. It has been observed that for disc with exponentially varying thickness ( $k=2$ ), high angular speed is required for initial yielding at internal surface as compared to the flat disc and exponentially varying thickness for  $k=4$  onwards. Thus we can conclude that flat disc ( $C=0.75$ ) is on the safer side of the design as it requires high percentage increase in angular speed to become fully plastic as compared to the flat disc with the compressible factors ( $C=0,0.25$ , etc.).

**Keywords:** elastic, plastic, compressibility, transitional stresses, isotropic, rotating disk.

## Nomenclature of Symbols:-

|  |                                       |                 |                                  |
|--|---------------------------------------|-----------------|----------------------------------|
| a and b                                | : Internal and external radii of disc | u,v,w           | : Displacement components        |
| x, y, z                                | : Cartesian co-ordinates              | r, $\theta$ , z | : Cylindrical polar co-ordinates |
| $e_{ij}$ and $T_{ij}$                  | : Strain and stress tensor            | $e_{ii}$        | : First strain invariant         |
| $\beta$                                | : Function of r only                  | Y               | : Yield stress                   |
| P                                      | : Function of $\beta$ only            | $C_{ij}$        | : Material constants             |
| $\lambda$ and $\mu$                    | : Lamé's constants                    | R               | : Radial distance                |
| $R = (r/b)$ , $R_0 = (a/b)$            |                                       | C               | : Compressibility                |
| $\sigma_r = (T_{rr}/Y)$                | - Radial stress components            |                 |                                  |
| $\sigma_\theta = (T_{\theta\theta}/Y)$ | - Circumferential stress components   |                 |                                  |

## 1 Introduction

This paper is concerned with the analysis of a rotating solid disk made of isotropic material with exponentially varying thickness. There are many applications of such type of rotating disks, such as in turbines, rotors, flywheels and with the advent of computers, disk drives. The use of rotating disk in machinery and structural applications has generated considerable interest in many problems in domain of solid mechanics. The analysis of stress distribution in circular disk rotating at high speed is important for a better understanding of the behavior and optimum design of structures. The analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier [1]. In the classical theory, solutions for such type of discs made of isotropic material can be found in most of standard text books [1-5]. Chakrabarty [2] and Heyman [6] solved the problem for the plastic state by utilizing the solution in the elastic range and considering the plastic state with the help of Tresca's, Von-Mises or any other

classical yield condition. Han [7] has investigated elastic and plastic stresses for isotropic materials with variable thickness. Eraslan [8] has calculated elastic and plastic stresses having variable thickness using Tresca's yield criterion, its associated flow rule and linear strain hardening. Wang [9] has investigated deformation of elastic half rings. Enescu [10] give some numerical method for determining stresses in rolling bearings while Mahri [11] calculated stresses by using finite element method for wind turbine rotors.

Transition is a natural phenomenon and there is hardly any branch of science or technology in which we do not come across transition from one state to another. At transition, the fundamental structure of the medium undergoes a change. The particles constituting a medium re-arrange themselves and give rise to spin, rotation, vorticity and other non-linear effects. This suggests that at transition, non-linear terms are very important and neglect of which may not represent the real physical phenomenon. Therefore transition

WSEAS TRANSACTIONS on MATHEMATICS fields are non-linear, non-conservative and irreversible in nature and should not be treated as superposition of effects. Elasticity-plasticity, visco-elastic, creep, fatigue, relaxation are some of the examples of transition. At present, such problems like elastic-plastic, creep and fatigue are treated by assuming ad-hoc, semi-empirical laws with the result that discontinuities, singular surfaces, non-differentiable regions have to be introduced over which two successive states of a medium are matched together. In a series of papers, Bohra [17] has given an entirely different orientation to this interesting problem of transition. He has discussed about 'transition theory' of elastic-plastic and creep deformation. Transition theory neither requires the yield criterion nor the associated flow rules to derive the transitional and plastic stresses. The transition theory utilizes the concept of generalized principal strain measure and asymptotic solution at critical points or turning points of the differential equation defining the deformed field and has been successfully applied to a large number of problems [14-26]. The generalized principal strain measure [12] is defined as,

$$e_{ii}^A = \int_0^{e_{ii}^A} [1 - 2e_{ii}^A]^{n-1} de_{ij}^A = \frac{1}{n} [1 - (1 - 2e_{ii}^A)^n], \quad (i, j = 1, 2, 3) \quad (1)$$

where  $n$  is the measure and  $e_{ij}^A$  is the principal Almansi finite strain components. For  $n = -2, -1, 0, 1, 2$  it gives Cauchy, Green, Hencky, Swainger and Almansi measures respectively.

In this paper an attempt has been made to study the behavior of isotropic thin rotating disk with exponentially variable thickness and edge load using transition theory. The thickness of the disk is assumed to vary along the radius in the form

$$h = h_0 e^{-\left(\frac{r}{b}\right)^k}$$

where  $h_0$  is the constant thickness,  $k$  is the geometric parameter and  $b$  is the radius of the disk.

## 2 Objective of the Present Study

In order to explain the elastic-plastic deformation, it is first necessary to recognize the transition state as an asymptotic one and in this work; it is our main aim to eliminate the need for assuming semi-empirical laws, yield condition. We also obtain the constitutive equation corresponding to the transition state.

Borah [17] identified the transition state in which the governing differential system shows some criticality. The general yield condition of transition is identified from the vanishing of the Jacobian of transformation,  $\frac{\partial(X, Y, Z)}{\partial(x, y, z)} = 0$ , where  $(X, Y, Z), (x, y, z)$

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## 3 Governing Equations

We consider a thin disk of constant density with central bore of radius 'a' and external radius 'b'. The disc is rotating with angular speed ' $\omega$ ' about an axis perpendicular to its plane and passed through the center of the disc. The thickness of the disc is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, that is, the axial stress  $T_{zz}$  is zero. The disk is assumed to be symmetric with respect to the mid plane, and the geometry of the consideration is presented in figure 1.

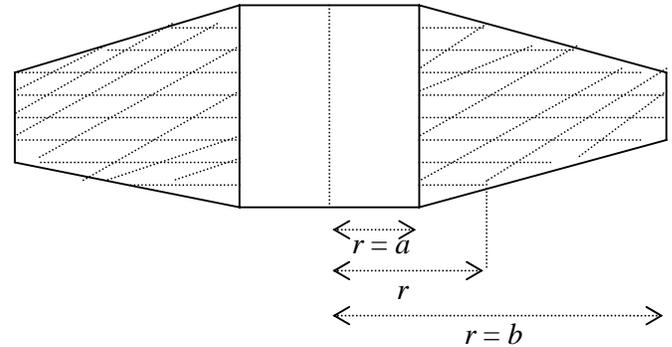


Fig.1 Isotropic disc having exponentially variable thickness.

The displacement components in cylindrical polar coordinates are given by [12].

$$u = r(1 - \beta); \quad v = 0; \quad w = dz \quad (2)$$

where  $\beta$  is a function of  $r = \sqrt{x^2 + y^2}$  only and  $d$  is a constant. The finite strain components are given as,

$$e_{rr}^A = \frac{\partial u}{\partial r} - \frac{1}{2} \left( \frac{\partial u}{\partial r} \right)^2 = \frac{1}{2} [1 - (\beta + r\beta')^2]$$

$$e_{\theta\theta}^A = \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2} (1 - \beta^2)$$

$$e_{zz}^A = \frac{\partial w}{\partial z} - \frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2 = \frac{1}{2} [1 - (1 - d)^2]$$

$$e_{r\theta}^A = e_{\theta z}^A = e_{zr}^A = 0 \quad (3)$$

where  $\beta' = \frac{d\beta}{dr}$ .

On substitution of equation (3) in (1), the generalized components of strain are given as

$$e_{rr} = \frac{1}{n} \left[ 1 - (\beta + r\beta')^n \right]$$

$$e_{\theta\theta} = \frac{1}{n} \left[ 1 - \beta^n \right]$$

$$e_{zz} = \frac{1}{n} \left[ 1 - (1-d)^n \right]$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0$$

(4)

The stress-strain relations for isotropic material are given as,

$$T_{ij} = \lambda \delta_{ij} I_1 + 2\mu e_{ij},$$

$$(i, j = 1, 2, 3)$$

(5)

where  $T_{ij}$  and  $e_{ij}$  are the stress and strain components respectively,  $\lambda$  and  $\mu$  are the Lamé's constants and  $I_k = e_{kk}$  is the first strain invariant,  $\delta_{ij}$  is the Kronecker's delta.

Equation (5) for this problem becomes

$$T_{rr} = \frac{2\lambda\mu}{\lambda + 2\mu} [e_{rr} + e_{\theta\theta}] + 2\mu e_{rr}$$

$$T_{\theta\theta} = \frac{2\lambda\mu}{\lambda + 2\mu} [e_{rr} + e_{\theta\theta}] + 2\mu e_{\theta\theta}$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0$$

(6)

Substituting equation (3) in (5), the strain components in terms of stresses are obtained as

$$e_{rr} = \frac{1}{2} \left[ 1 - (r\beta' + \beta)^2 \right]$$

$$= \frac{1}{E} \left[ T_{rr} - \left( \frac{1-C}{2-C} \right) T_{\theta\theta} \right]$$

$$e_{\theta\theta} = \frac{1}{2} \left[ 1 - \beta^2 \right]$$

$$= \frac{1}{E} \left[ T_{\theta\theta} - \left( \frac{1-C}{2-C} \right) T_{rr} \right]$$

$$e_{zz} = \frac{1}{2} \left[ 1 - (1-d)^2 \right]$$

$$= - \left( \frac{1-C}{2-C} \right) \frac{1}{E} (T_{rr} - T_{\theta\theta})$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0$$

(7)

where E is the Young's modulus and C is the compressibility factor of the material. In terms of Lamé's constant they are given as

$$C = \frac{2\mu}{(\lambda + 2\mu)} \quad \text{and} \quad E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)}$$

Substituting equation (4) in (6), we get the stresses as

$$T_{rr} = \frac{2\mu}{n} \left[ 3 - 2C - \beta^n \left\{ 1 - C + (2-C) \left( \frac{r\beta'}{\beta} + 1 \right)^n \right\} \right]$$

$$T_{\theta\theta} = \frac{2\mu}{n} \left[ 3 - 2C - \beta^n \left\{ 2 - C + (1-C) \left( \frac{r\beta'}{\beta} + 1 \right)^n \right\} \right]$$

$$T_{r\theta} = T_{\theta z} = T_{zr} = T_{zz} = 0$$

(8)

Equations of equilibrium are all satisfied except

$$\frac{d}{dr} (hrT_{rr}) - hT_{\theta\theta} + \rho r^2 \omega^2 h = 0$$

(9)

where  $\rho$  is density of the material and h is the exponentially variable thickness of the disc.

Using equation (8) in (9), we get a non-linear differential equation in  $\beta$  as

$$(2-C)nP\beta^{n+1}(P+1)^{n-1} \frac{dP}{d\beta} = \frac{n\rho\omega^2 r^2}{2\mu} + \beta^n \left\{ k \left( \frac{r}{b} \right)^k - nP \right\}$$

$$\left\{ 1 - C + (2-C)(P+1)^n \right\} + \beta^n \left\{ 1 - (P+1)^n \right\} - k(3-2C) \left( \frac{r}{b} \right)^k$$

(10)

where  $r\beta' = \beta P$  (P is a function of  $\beta$  and  $\beta$  is a function of r). Transition or turning points of P in equation (10) are  $P \rightarrow -1$  and  $P \rightarrow \pm\infty$ . The boundary conditions are:

$$u = 0 \quad \text{at} \quad r = a$$

$$T_{rr} = 0 \quad \text{at} \quad r = b$$

(11)

#### 4 Solution Though the Principal Stress

It has been shown in large number of cases that the asymptotic solution through the principal stress leads from elastic to plastic state at the transition point  $P \rightarrow \pm\infty$ , we define the transition function R as

$$R = \frac{n}{2\mu} T_{\theta\theta}$$

$$= \left[ (3-2C) - \beta^n \left\{ 2 - C + (1-C)(P+1)^n \right\} \right]$$

(12)

Taking the logarithmic differentiation of equation (12) with respect to r and using equation (10), we get

$$\beta^n \left( \frac{1-C}{2-C} \right) \left[ \begin{array}{l} \frac{n\rho\omega^2 r^2}{2\mu\beta^n} + k \left( \frac{r}{b} \right)^k \\ \left\{ 1-C + (2-C)(P+1)^n \right\} \\ -nP(1-C) + \left\{ 1-(P+1)^n \right\} \\ -\frac{k(3-2C)}{\beta^n} \left( \frac{r}{b} \right)^k \end{array} \right]$$

$$\frac{d}{dr} (\log R) = -\frac{+nP\beta^n(2-C)}{r \left[ 3-2C - \beta^n \left\{ 2-C + (1-C)(P+1)^n \right\} \right]} \quad (13)$$

Taking the asymptotic value of equation (13) as  $P \rightarrow \pm\infty$  and integrating, we get

$$R = A_1 r^{-\frac{1}{2-C}} e^{\left( \frac{r}{b} \right)^k} \quad (14)$$

where  $A_1$  is a constant of integration, which can be determined by the boundary condition. From equation (12) and (14), we have

$$T_{\theta\theta} = \frac{2\mu}{n} A_1 r^{-\frac{1}{2-C}} e^{\left( \frac{r}{b} \right)^k} \quad (15)$$

Substituting equation (15) in (9) and integrating, we get

$$T_{rr} = \frac{2\mu A_1 (2-C)}{n(1-C)} r^{-\frac{1}{2-C}} e^{\left( \frac{r}{b} \right)^k} - \frac{\rho\omega^2 f(r)}{r} e^{\left( \frac{r}{b} \right)^k} + \frac{B_1}{rh_0} e^{\left( \frac{r}{b} \right)^k} \quad (16)$$

where  $B_1$  is a constant of integration and

$$f(r) = \int r^2 e^{-\left( \frac{r}{b} \right)^k} dr.$$

Substituting equations (15) and (16) in second equation of (7), we get

$$\beta = \sqrt{1 - \frac{2}{E} \left( \frac{1-C}{2-C} \right) \left\{ \frac{\rho\omega^2}{r} f(r) - \frac{B_1}{rh_0} \right\}} e^{\left( \frac{r}{b} \right)^k} \quad (17)$$

Substituting equation (17) in (2), we get

$$u = r - r \sqrt{1 - \frac{2}{E} \left( \frac{1-C}{2-C} \right) \left\{ \frac{\rho\omega^2}{r} f(r) - \frac{B_1}{rh_0} \right\}} e^{\left( \frac{r}{b} \right)^k} \quad (18)$$

where  $E = \frac{2\mu(3-2C)}{(2-C)}$  is the Young's modulus in terms of compressibility factor. Using boundary condition (11) in equation (16) and (18), we get

$$B_1 = h_0 \rho \omega^2 f(a),$$

$$A_1 = \frac{n(1-C)}{2\mu(2-C)} \left[ \frac{\rho\omega^2 \{f(b) - f(a)\}}{b} \right] b^{\frac{1}{2-C}} \quad (19)$$

Substituting the values of constant of integration  $A_1$  and  $B_1$  from equation (19) in equations (15), (16) and (18) respectively, we get the transitional stresses and displacement as

$$T_{\theta\theta} = \left[ \frac{\rho\omega^2 \{f(b) - f(a)\}}{b} \right] \frac{(1-C)}{(2-C)} \left( \frac{b}{r} \right)^{\frac{1}{2-C}} e^{\left( \frac{r}{b} \right)^k} \quad (20)$$

$$T_{rr} = \left[ \frac{\rho\omega^2 \{f(b) - f(a)\}}{b} \right] \left( \frac{b}{r} \right)^{\frac{1}{2-C}} - \frac{\rho\omega^2 \{f(r) - f(a)\}}{r} e^{\left( \frac{r}{b} \right)^k} \quad (21)$$

$$u = r - r \sqrt{1 - \frac{2}{E} \left( \frac{1-C}{2-C} \right) \frac{\rho\omega^2}{r} \{f(r) - f(a)\}} e^{\left( \frac{r}{b} \right)^k} \quad (22)$$

From equation (20) and (21), we get

$$T_{rr} - T_{\theta\theta} = \left[ \frac{\rho\omega^2 \{f(b) - f(a)\}}{b} \right] \left( \frac{b}{r} \right)^{\frac{1}{2-C}} e^{\left( \frac{r}{b} \right)^k} \frac{1}{2-C} - \frac{\rho\omega^2 \{f(r) - f(a)\}}{r} e^{\left( \frac{r}{b} \right)^k} \quad (23)$$

### 4.1 Initial Yielding

From equation (23), it is seen that  $|T_{rr} - T_{\theta\theta}|$  is maximum at the internal surface (i.e. at  $r = a$ ), therefore yielding will take place at the internal surface of the disc and equation (23) become,

$$|T_{rr} - T_{\theta\theta}|_{r=a} = \left[ \frac{\rho\omega^2 \{f(b) - f(a)\}}{b} \right] \left( \frac{b}{a} \right)^{\frac{1}{2-C}} e^{\left( \frac{a}{b} \right)^k} \cdot \frac{1}{2-C}$$

$\equiv Y$  (say)

and the angular speed necessary for initial yielding is given by

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \left[ \frac{(2-C)}{e^{\left( \frac{a}{b} \right)^k}} \left( \frac{a}{b} \right)^{\frac{1}{2-C}} \right] \frac{b^3}{\{f(b) - f(a)\}} \quad (24)$$

### 4.2 Fully Plastic State

The disc becomes fully plastic ( $C \rightarrow 0$ ) at the external surface (i.e. at  $r = b$ ) and equation (23) become

$$|T_{rr} - T_{\theta\theta}|_{r=b} = \left[ \left[ \left( \frac{\rho \omega^2}{2b} \right) e \{f(b) - f(a)\} \right] \right] = Y^* \tag{25}$$

Angular speed required for the disc to become fully plastic is given by

$$\Omega_f^2 = \frac{\rho \omega_f^2 b^2}{Y^*} = \frac{2}{e} \frac{b^3}{[f(b) - f(a)]} \tag{25}$$

where  $w_f = \frac{\Omega_f}{b} \sqrt{\frac{Y^*}{\rho}}$ .

We introduce the following non-dimensional components as

$$R = \frac{r}{b}, R_0 = \frac{a}{b}, \sigma_r = \frac{T_{rr}}{Y}, \sigma_\theta = \frac{T_{\theta\theta}}{Y}, \text{ and } \bar{u} = \frac{u}{b}$$

Transitional stresses, angular speed and displacement can be obtained from equation (20)-(22) and (24) in non-dimensional form as,

$$\sigma_\theta = \left[ \Omega_i^2 \int_{R_0}^1 R^2 e^{-R^k} dR \right] \frac{(1-C)}{(2-C)} R^{-\frac{1}{2-C}} e^{R^k} \tag{26}$$

$$\sigma_r = \left[ \Omega_i^2 \int_{R_0}^1 R^2 e^{-R^k} dR \right] R^{-\frac{1}{2-C}} e^{R^k} - \frac{\Omega_i^2}{R} e^{R^k} \left[ \int_{R_0}^R R^2 e^{-R^k} dR \right] \tag{27}$$

$$\bar{u} = R - R \sqrt{1 - \frac{2(1-C)Y}{E(2-C)} \frac{\Omega_i^2 e^{R^k}}{R} \int_{R_0}^R R^2 e^{-R^k} dR} \tag{28}$$

$$\Omega_i^2 = \left[ \frac{(2-C)R_0^{\frac{1}{2-C}}}{e^{R_0^k}} \right] \frac{1}{\int_{R_0}^1 R^2 e^{-R^k} dR} \tag{29}$$

Stresses, displacement and angular speed for fully-plastic state ( $C \rightarrow 0$ ) are obtained from equation (26)-(28) and (25) as

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$$\sigma_\theta = \frac{1}{2} \left[ \Omega_f^2 \int_{R_0}^1 R^2 e^{-R^k} dR \right] R^{-\frac{1}{2}} e^{R^k} \tag{30}$$

$$\sigma_r = \left[ \Omega_f^2 \int_{R_0}^1 R^2 e^{-R^k} dR \right] R^{-\frac{1}{2}} e^{R^k} - \frac{\Omega_f^2}{R} e^{R^k} \left[ \int_{R_0}^R R^2 e^{-R^k} dR \right] \tag{31}$$

$$\bar{u} = R - R \sqrt{1 - \frac{Y}{E} \frac{\Omega_f^2}{R} \int_{R_0}^R R^2 e^{-R^k} dR} e^{R^k} \tag{32}$$

$$\Omega_f^2 = \frac{2}{e} \frac{1}{\int_{R_0}^1 R^2 e^{-R^k} dR} \tag{33}$$

### 5 Numerical Illustration and Discussion

In figure 2, curves have been drawn between angular speed ( $\Omega_i^2$ ) and various radii ratios  $R_0 = (a/b)$  for different compressibility factors ( $C = 0, 0.25, 0.5, 0.75$ ). It has been observed that for flat disc ( $k = 0$ ), high angular speed is required for initial yielding at internal surface for incompressible material as compared to compressible material. For compressible material, high percentage increase in angular speed is required for material to become fully plastic. As thickness of the disk varies exponentially (decreasing radially), high angular speed is required for initial yielding at internal surface ( $k = 2$ ) for incompressible material as compared to compressible material. It has been seen that with the increase of  $k$  from 2 onwards, less angular speed is required for fully plasticity at internal surface as compared to the disk with thickness at  $k = 2$ .

It can be seen from table 1, as thickness of disc decreases radially, high angular speed is required for the compressible material ( $C = 0.75$ ) to become fully plastic. As the thickness of disc decreases radially, angular speed required for fully plastic state is much less as compared to the flat disc ( $k = 0$ ).

In figures 3-6, curves have been drawn for transitional stresses and displacement. For flat disc- radial, circumferential transitional stresses are maximum at internal surface. For disk made of incompressible material, radial as well as circumferential stresses, is maximum at internal surface as compared to disk made of compressible material. The displacement is maximum at external surface for incompressible/compressible material. The displacement is large for incompressible material as compared to compressible material. For disk with exponentially varying thickness,

radial stress is maximum at internal surface while circumferential stress and displacement is maximum at external surface. Circumferential stress is going on increasing as the thickness of disc decreasing radially. Circumferential stress is decreasing as compressibility of disc increases i.e.  $C = 0, 0.25, 0.5$ .

For flat disc ( $k = 0$ ), radial and circumferential plastic stresses are maximum at internal surface as can be seen from figure 7. For disc whose thickness varying exponentially (decreasing radially), radial stress is maximum at internal surface while circumferential stress is maximum at external surface. As the thickness of the disk decreases radially, the circumferential stress increases.

Table 1  
Angular speed required for Initial Yielding and Fully Plastic state with Variable thickness

|           | k | C    |              |                 | Percentage increase in angular speed |
|-----------|---|------|--------------|-----------------|--------------------------------------|
|           |   |      | $\Omega_i^2$ | $\Omega_{fi}^2$ |                                      |
| $R_0=0.5$ | 0 | 0    | 4.84         | 6.86            | 41.421                               |
|           |   | 0.75 | 2.46         | 6.86            | 178.58                               |
|           | 2 | 0.75 | 3.64         | 4.79            | 31.59                                |
|           |   | 0.75 | 3.60         | 3.95            | 9.092                                |

**5 Conclusions:**

For disk with exponentially varying thickness ( $k = 2$ ), high angular speed is required for initial yielding at internal surface as compared to the flat disk and exponentially varying thickness for  $k = 4$  onwards. Thus we can conclude that flat disc ( $C=0.75$ ) is on the safer side of the design as it requires high percentage increase in angular speed to become fully plastic as compared to the flat disc with the compressible factors, i.e.  $C = 0, 0.25, 0.5$ , etc. and the disc in which thickness decreases radially (i.e. for  $k = 3, 4, 5$ , etc.).

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Fig.2 Angular speed required for initial yielding at the internal surface of the rotating disc with variable thickness (k=0, 2, 4).

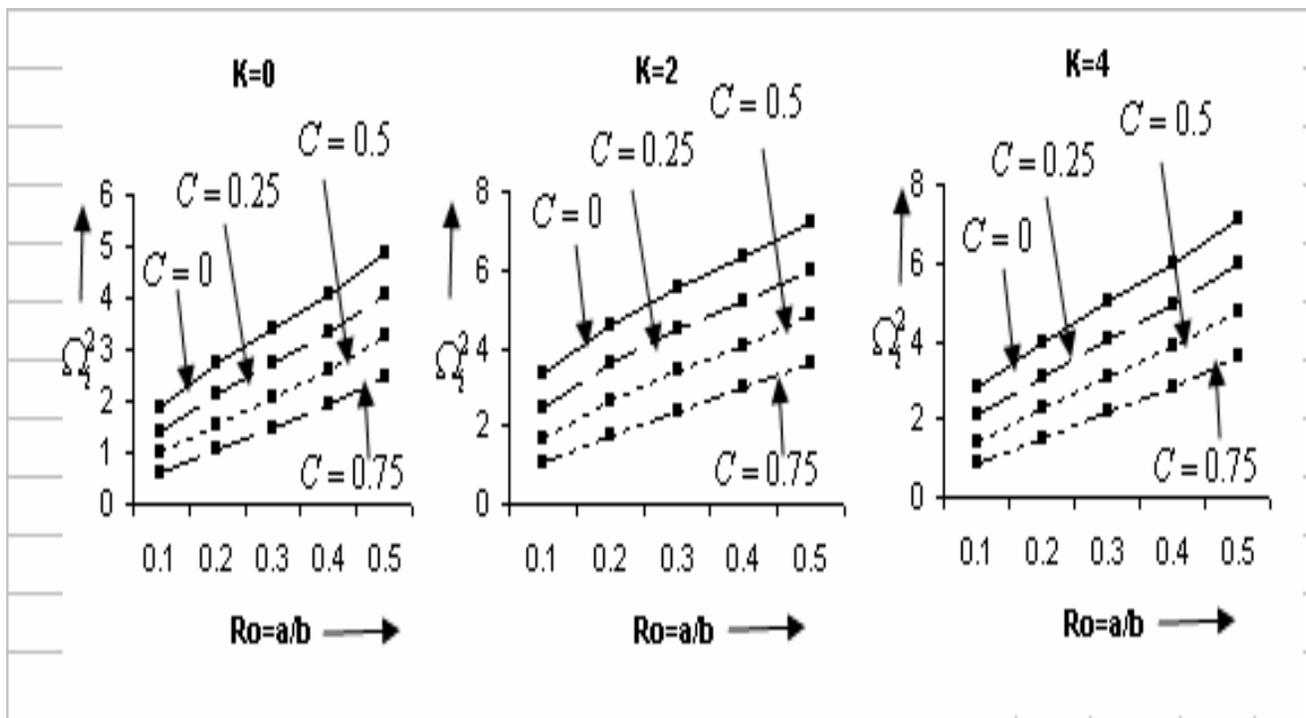


Fig.3 Transitional stresses and displacement in a thin rotating disc along various radii ratio ( $R = r/b$ ) with compressibility ( $C = 0$ ) for variable thickness ( $K=0, 2, 4$ ).

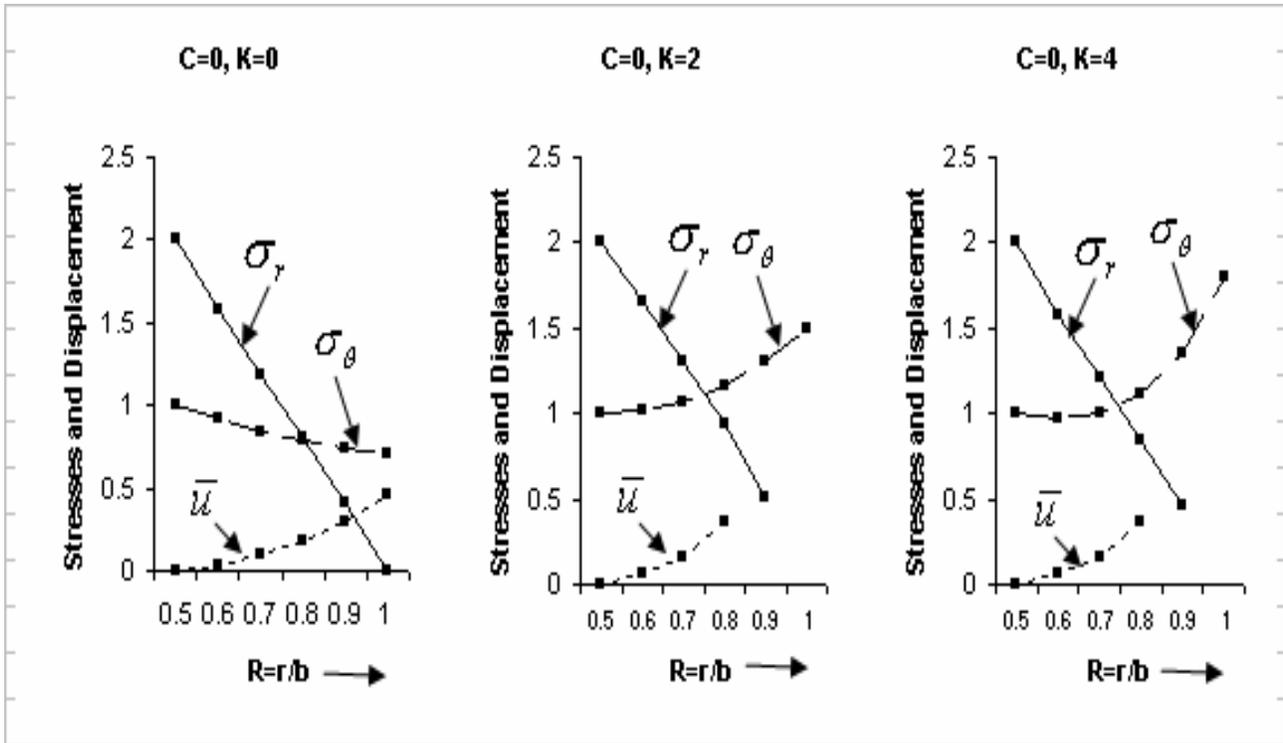


Fig.4 Transitional stresses and displacement in a thin rotating disc along various radii ratio ( $R = r/b$ ) with compressibility ( $C = 0.25$ ) for variable thickness ( $K=0, 2, 4$ ).

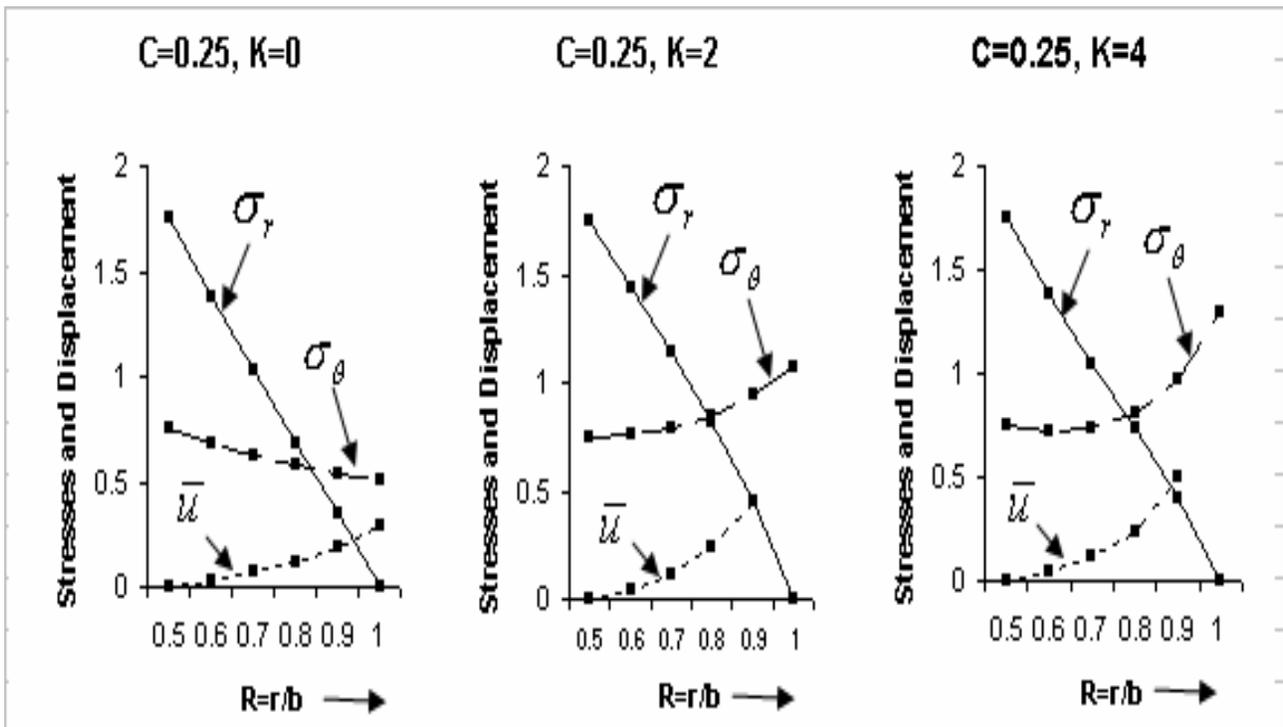


Fig.5 Transitional stresses and displacement in a thin rotating disc along various radii ratio ( $R = r/b$ ) with compressibility ( $C = 0.5$ ) for variable thickness ( $K=0, 2, 4$ ).

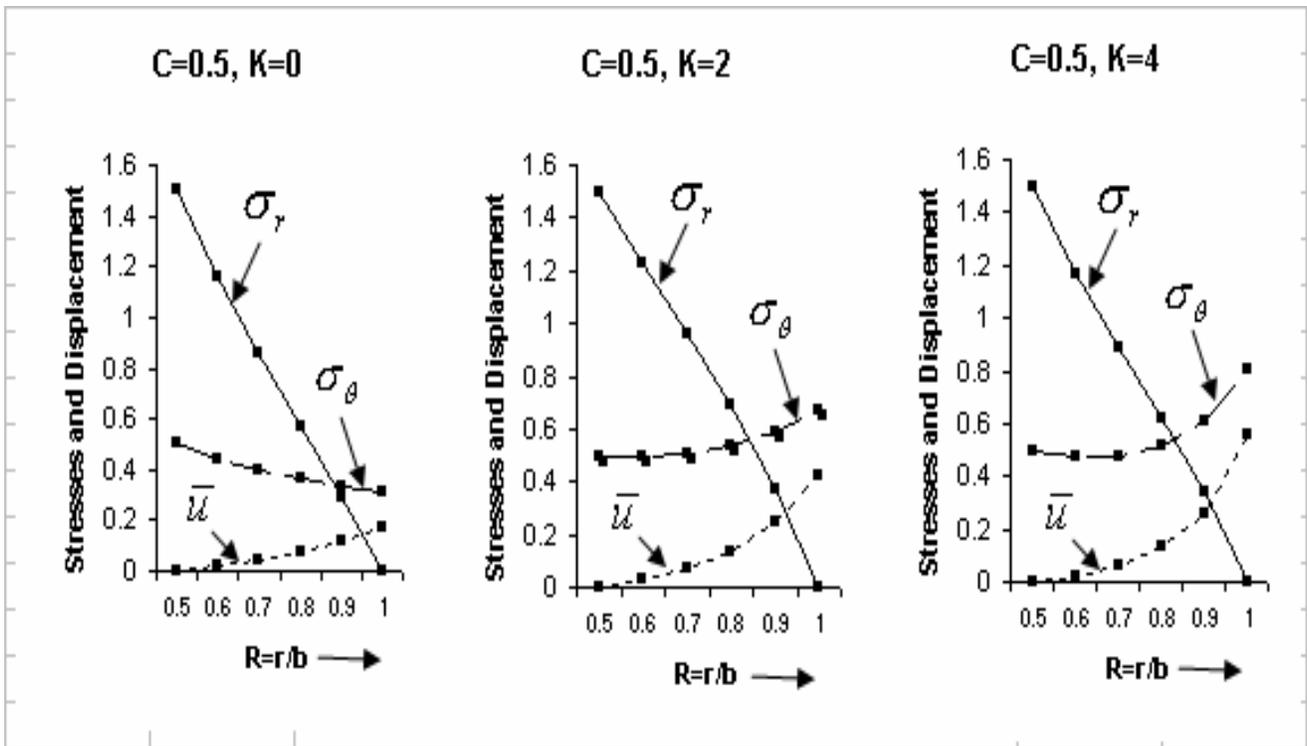


Fig.6 Transitional stresses and displacement in a thin rotating disc along various radii ratio ( $R = r/b$ ) with compressibility ( $C = 0.75$ ) for variable thickness ( $K=0,2,4$ ).

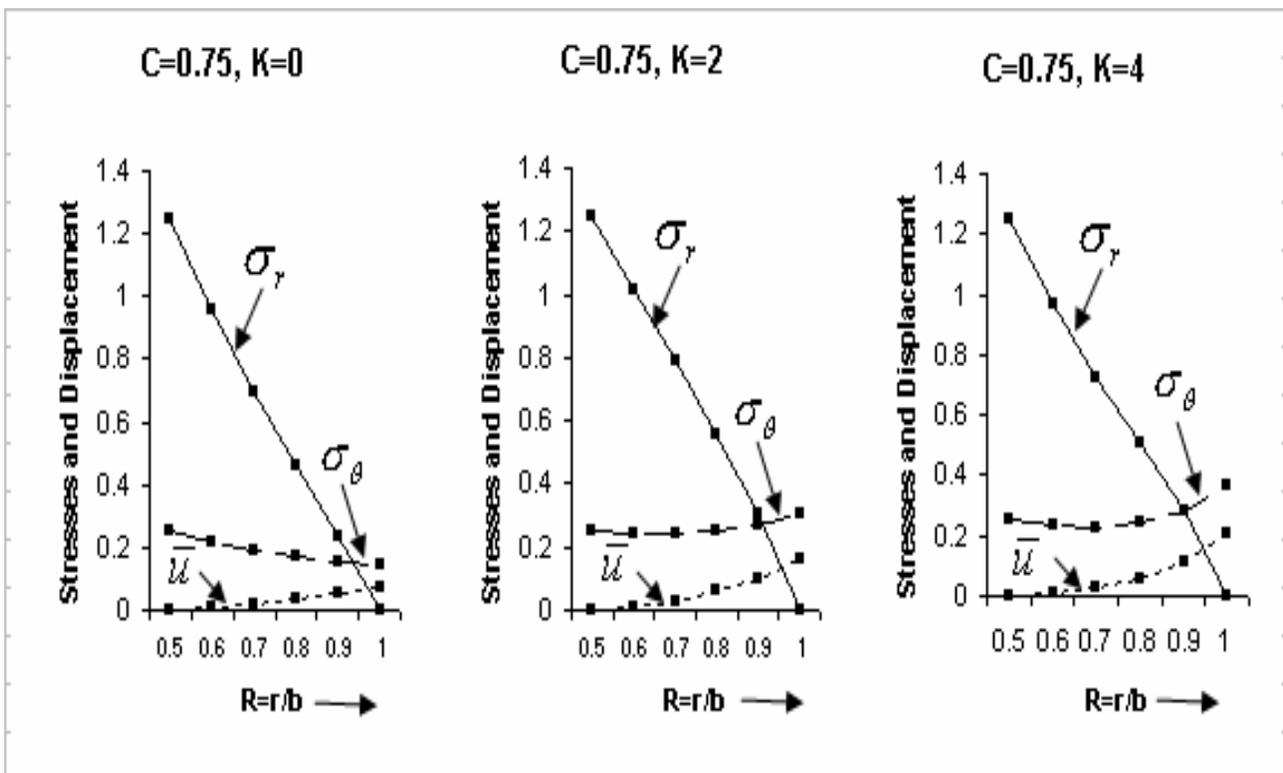


Fig.7 Plastic stresses and displacement in a thin rotating disc along various radii ratio ( $R = r/b$ ) for variable thickness ( $K=0, 2, 4$ ).

