Introduction to the elliptical trigonometry

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Abstract: - Trigonometry is a branch of mathematics that deals with relations between sides and angles of triangles. It has some relationship to geometry, though there is disagreement on exactly what that relationship is. For some, trigonometry is just a subtopic of geometry. The trigonometric functions are very important in technical subjects like science, engineering, architecture, and even medicine. In this paper, the elliptical trigonometry is introduced in order to be in the future a part of the trigonometry topic. Thus, the definition of this original part is presented. The elliptic trigonometric functions are also defined. The importance of these functions in producing different signals and forms are analyzed and discussed.

Key-Words: - Mathematics, geometry, trigonometry, trigonometric functions, derivative functions, simulation.

1 Introduction

Trigonometry is a branch of mathematics that deals with triangles, particularly those plane triangles in which one angle has 90 degrees (right triangles) [1]. Trigonometry deals with relationships between the sides and the angles of triangles and with the trigonometric functions, which describe those relationships [2],[3]. Trigonometry has an enormous variety of applications. The ones mentioned explicitly in textbooks and courses on trigonometry are its uses in practical endeavors such as navigation, land surveying, building, and the like [4], [5]. It is also used extensively in a number of academic fields, primarily mathematics, science and engineering [6],[7],[8]. The mathematical topics of Fourier series and Fourier transforms rely heavily on knowledge of trigonometric functions and find application in a number of areas, including statistics [9],[10],[11].

The elliptical trigonometry, which is introduced in this paper, is an original study regarding the trigonometry literature. The existed trigonometry is a particular case of the proposed elliptical trigonometry. The last one describes an elliptic curve, but the existed one describes a circular form (like the familiar trigonometry [12]). In fact, the elliptical trigonometry is based on the angular function, the cosine and the sine functions.

The most important functions of the elliptical trigonometry are the elliptic cosine and the elliptic sine. Therefore, the angular function and the elliptic trigonometry functions are defined in the second and the third sections. To illustrate the importance of this new topic, the elliptic cosine function is analyzed in the fourth section. In section five, by varying some parameters included in the elliptic cosine function, different signals and forms can be produced as treated. A conclusion is presented in section six.

2 The angular function

Angular functions are new mathematical functions that produce a rectangular signal, in which period is function of angles. Similar to trigonometric functions, the angular functions have the same properties as the precedent, but the difference is that a rectangular signal is obtained instead of a sinusoidal signal [13],[14],[15] and moreover, one can change the width of each positive and negative alternate in the same period. This is not the case of any other trigonometric function. In other hand, one can change the frequency, the amplitude and the width of any period of the signal by using the general form of the angular function.

- The expression of the angular function related to the (ox) axis is defined, for K ∈ Z, as:

$$\operatorname{ang}_x[\beta(\alpha+\gamma)] = \begin{cases} +1 \quad \text{for} \\ (4K-1)\dfrac{\pi}{2\beta}-\gamma \le \alpha \le (4K+1)\dfrac{\pi}{2\beta}-\gamma \\ \\ -1 \quad \text{for} \\ (4K+1)\dfrac{\pi}{2\beta}-\gamma \le \alpha \le (4K+3)\dfrac{\pi}{2\beta}-\gamma \end{cases} \tag{1}$$

For β = 1 and γ = 0, the expression of the angular function becomes:

$$\text{ang}_x(\alpha) = \begin{cases} +1 & \text{for} \quad \cos(\alpha) \geq 0 \\ -1 & \text{for} \quad \cos(\alpha) \prec 0 \end{cases} \qquad (2)$$

Its waveform is illustrated in figure 1.

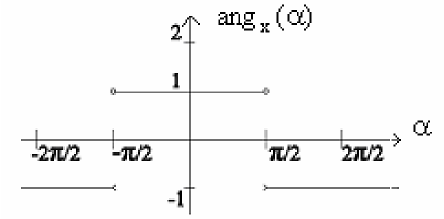


Fig. 1: The $\text{ang}_x(\alpha)$ waveform.

The expression of the angular function related to the (oy) axis is written as:

$$\text{ang}_y[\beta(\alpha+\gamma)] = \begin{cases} +1 \quad \text{for} \\ 2k.\frac{\pi}{\beta} - \gamma \leq \alpha \leq (2k+1).\frac{\pi}{\beta} - \gamma \\ \\ -1 \quad \text{for} \\ (2K+1)\frac{\pi}{\beta} - \gamma \leq \alpha \leq (2K+2).\frac{\pi}{\beta} - \gamma \end{cases} \qquad (3)$$

For $\beta = 1$ and $\gamma = 0$, the simplified expression of the angular function, which is presented in figure 2, is:

$$\text{ang}_y(\alpha) = \begin{cases} +1 & \text{for} \quad \sin(\alpha) \geq 0 \\ -1 & \text{for} \quad \sin(\alpha) \prec 0 \end{cases} \qquad (4)$$

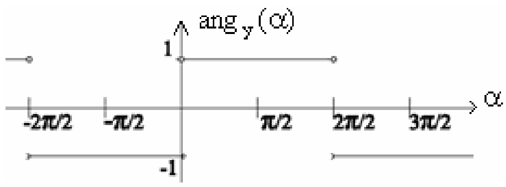


Fig. 2: The $\text{ang}_y(\alpha)$ waveform.

3 The elliptical trigonometry

3.1 The elliptical trigonometry unit

The Elliptical Trigonometry unit is an ellipse with a center O (x = 0, y = 0) and has the equation form:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \qquad (5)$$

with:

'a' is the radius of the ellipse on the (x'ox) axis,
'b' is the radius of the ellipse on the (y'oy) axis.

The Elliptical Trigonometry unit is used to define functions that describe the ellipse (figure 3) as the elliptic cosine and the elliptic sine.

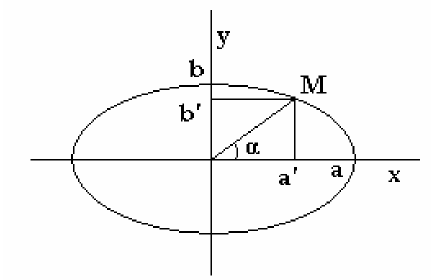


Fig. 3: The elliptical trigonometry unit.

It is essential to note that 'a' and 'b' must be positive. In this paper, 'a' is fixed to 1. One is interested to vary only a single parameter which is 'b'.

3.2 The elliptic trigonometric functions

3.2.1 Definitions

- The elliptic cosine is the function that gives the ratio of $\frac{a'}{a}$ with a' is the projection of the point M (onto the ellipse) on the (x'ox) axis (figure 3):

$$\cos\text{el}_b(\alpha) = \frac{a'}{a} \qquad (6)$$

with α is the angle defined by the position of the point M:

$\alpha = (\widehat{OX;OM})$.

- The elliptic sine is the function that gives the ratio of $\frac{b'}{b}$. Therefore:

$$\sin\text{el}_b(\alpha) = \frac{b'}{b} \qquad (7)$$

- The elliptic tangent function is written:

$$\text{tg}\,\text{el}_b(\alpha) = \frac{\sin\text{el}(\alpha)}{\cos\text{el}(\alpha)} = \frac{a}{b}.\frac{\sin(\alpha)}{\cos(\alpha)} = \frac{a}{b}.\text{tg}(\alpha) \qquad (8)$$

3.2.2 Determination of the elliptic cosine function

Given $\text{tg}(\alpha) = \frac{y}{x} = \frac{b'}{a'}$, to calculate $\cos\text{el}_b(\alpha)$, it is significant to replace the equation $y = \text{tg}(\alpha).x$ in that defined in (5) in order to obtain the elliptic cosine formula. In fact:

$$\frac{x^2}{a^2}+\frac{[tg(\alpha)\cdot x]^2}{b^2}=1 \Rightarrow \frac{x^2}{a^2}\left(1+\frac{[a\cdot tg(\alpha)]^2}{b^2}\right)=1 \Rightarrow$$

$$\cos el_b(\alpha)=\frac{x}{a}=\frac{\pm 1}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}}.$$

Therefore:

$$\cos el_b(\alpha)=\frac{+1}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}} \text{ for } -\frac{\pi}{2}\le\alpha\le\frac{\pi}{2} \text{ and}$$

$\frac{x}{a}\ge 0$, or,

$$\cos el_b(\alpha)=\frac{-1}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}} \text{ for } \frac{\pi}{2}\le\alpha\le\frac{3\pi}{2} \text{ and}$$

$\frac{x}{a}\prec 0$.

By using the angular function expression defined in (2) and that of the elliptical tangent illustrated in (8), the elliptic cosine expression becomes:

$$\cos el_b(\alpha)=\frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}}=\frac{ang_x(\alpha)}{\sqrt{1+[tgel_b(\alpha)]^2}} \quad (9)$$

For specific parameter values, this equation gives the classic cosine expression [12].

3.2.3 The elliptic sine function

The same analysis applied to determine the elliptic cosine expression is proposed to search the elliptic sine formula. Therefore:

$$\sin el_b(\alpha)=\frac{a}{b}tg(\alpha)\frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}tg(\alpha)\right]^2}}=tgel_b(\alpha)\frac{ang_X(\alpha)}{\sqrt{1+[tgel_b(\alpha)]^2}} \quad (10)$$

3.3 Derivative of the elliptic functions

3.3.1 Derivative of the elliptic cosine

The calculation of the elliptic cosine derivative is based on the following formulas:

$$\begin{cases}(ang_x(u))'=0\\ (tg(u))'=u'.\left(1+tg(u)^2\right),\\ \left(\sqrt{f(u)}\right)'=\frac{f(u)'}{2\sqrt{f(u)}}\end{cases} \quad (11)$$

From equation (9) and for a variable angle u, the elliptic cosine function can be written as:

$$Cosel_b(u)=\frac{ang_X(u)}{\sqrt{1+(\frac{a}{b}tg(u))^2}}$$

Its derivative is:

$$(Cosel_b(u))'=-\frac{ang_X(u)\left(\sqrt{1+(\frac{a}{b}tg(u))^2}\right)'}{1+(\frac{a}{b}tg(u))^2}$$

$$(Cosel_b(u))'=-ang_X(u)\cdot\left(\frac{a}{b}\right)^2\frac{tg(u)\cdot u'\left(1+tg^2(u)\right)}{\left(1+(\frac{a}{b}tg(u))^2\right)\left(\sqrt{1+(\frac{a}{b}tg(u))^2}\right)}$$

$$\Rightarrow (Cosel_b(u))'=-\frac{a}{b}Cosel_b^{\,2}(u)Sinel_b(u).u'\left(1+tg^2(u)\right)$$

For the used variable angle (u), equation (8) can be written as:

$$tgel_b(u)=\frac{Sinel_b(u)}{Cosel_b(u)}=\frac{a}{b}\frac{\sin(u)}{\cos(u)}=\frac{a}{b}tg(u)$$

Thus, the derivative of the elliptic cosine function becomes:

$$(Cosel_b(u))'=-\frac{b}{a}u'.Sinel_b(u).\left(\left(\frac{a}{b}\right)^2Cosel_b^{\,2}(u)+Sinel_b^{\,2}(u)\right) \quad (12)$$

- In a particular case, and for a = b = 1:

$Cosel_b(u)=Cos(u)$ and $Sinel_b(u)=Sin(u)$,

Therefore:

$$(Cosel_b(u))'=-Sinel_b(u).u'\left(Cosel_b^{\,2}(u)+Sinel_b^{\,2}(u)\right)=-Sinel_b(u).u'$$

$$\Rightarrow (Cos(u))'=-u'.Sin(u)$$

This is a known formula in the classic trigonometry topic.

3.3.2 Derivative of the elliptic sine

Based on the same analysis as that used for the elliptic cosine, the derivative of the elliptic sine is:

$$(\mathrm{Sinel}_b(u))' = \left(\frac{a}{b}\cdot\frac{\mathrm{ang}_X(u)\cdot \mathrm{tg}(u)}{\sqrt{1+(\frac{a}{b}\mathrm{tg}(u))^2}}\right)'$$

$$= \frac{b}{a}u'\mathrm{Cosel}_b(u)\left(\left(\frac{a}{b}\right)^2 \mathrm{Cosel}_b{}^2(u) + \mathrm{Sinel}_b{}^2(u)\right) \tag{13}$$

3.3.3 Derivative of the elliptic tangent

The elliptic tangent derivative is:

$$(\mathrm{tgel}_b(u))' = \frac{a}{b}(\mathrm{tg}(u))' = \frac{a}{b}u'(1+\mathrm{tg}^2(u))$$

$$= \frac{a}{b}u'\left[1+\left(\frac{b}{a}\right)^2 \mathrm{tgel}_b{}^2(u)\right] \tag{14}$$

3.4 Original formula

To complete the set of formulas that describe the elliptical trigonometric functions, it should be noted that:

$$\mathrm{Cosel}_b{}^2(u) + \mathrm{Sinel}_b{}^2(u) = 1 \tag{15}$$

In fact:

$$\mathrm{Cosel}_b{}^2(u) + \mathrm{Sinel}_b{}^2(u) =$$

$$\left(\frac{\mathrm{ang}_X(u)}{\sqrt{1+(\frac{a}{b}\mathrm{tg}(u))^2}}\right)^2 + \left(\frac{a}{b}\cdot\frac{\mathrm{ang}_X(u)\cdot\mathrm{tg}(u)}{\sqrt{1+(\frac{a}{b}\mathrm{tg}(u))^2}}\right)^2$$

$$= \frac{1}{1+(\frac{a}{b}\mathrm{tg}(u))^2} + \frac{\left(\frac{a}{b}\right)^2\mathrm{tg}^2(u)}{1+(\frac{a}{b}\mathrm{tg}(u))^2} = 1$$

4 A survey on the elliptic cosine function

Consider 'a = 1' and 'b' a variable parameter. This choice is proposed to simplify the study of the elliptic cosine function.

4.1 First case (b = 1)

When a = b = 1, the ellipse equation, defined in (5), becomes:

$$x^2 + y^2 = 1 \tag{16}$$

which is the equation of a circle (figure 4).

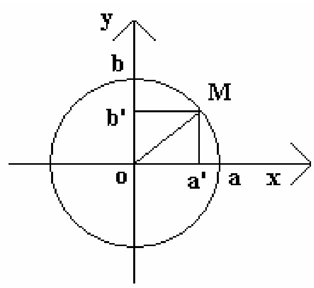
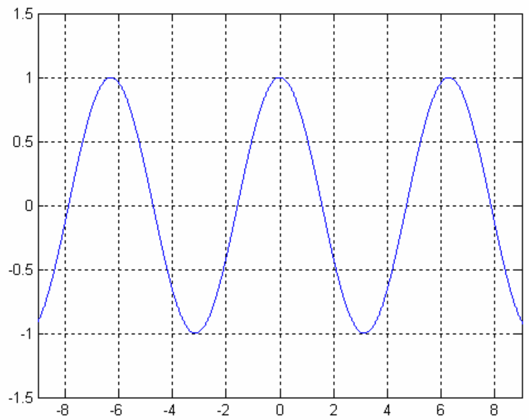


Fig. 4: The circle trigonometry unit.

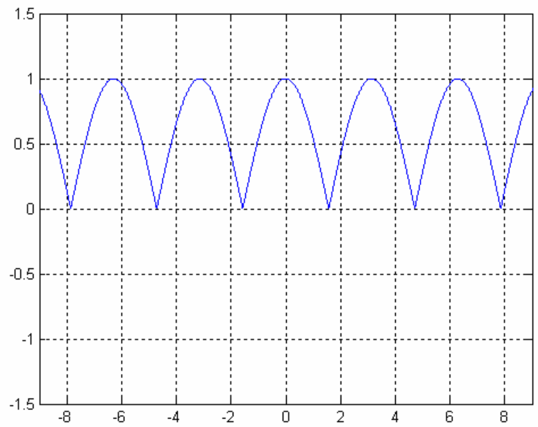
As $\cos\mathrm{el}_b(\alpha) = \frac{a'}{a} = \frac{a'}{1}$ and $\cos(\alpha) = \frac{a'}{OM} = \frac{a'}{1}$, because OM is the radius of the circle and is equal to unity, therefore:

$\cos\mathrm{el}_b(\alpha) = \cos(\alpha)$.

Figures 5.a and 5.b represent the waveforms of the elliptic cosine function and its absolute value for b=1.



a) $\cos\mathrm{el}_b(\alpha)$



b) $|\cos\mathrm{el}_b(\alpha)|$

Fig. 5: Elliptic cosine waveforms for b = 1.

4.2 Second case (b >> 1)

From equation (9), two configurations are studied:

1. Consider $\alpha = \frac{(2.k+1)\pi}{2}$ and $b \prec\prec a.tg(\alpha)$.

In this case, $tg(\alpha) = \pm\infty$. As 'b' is a constant, therefore: $\left[\frac{a}{b}.tg(\alpha)\right]^2 \succ\succ 1$. Thus, the elliptic cosine becomes:

$$\cos el_b(\alpha) = \frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}.tg(\alpha)\right]^2}} = \frac{ang_X(\alpha)}{\sqrt{\left[\frac{a}{b}.tg(\alpha)\right]^2}}$$

$$= \frac{ang_X(\alpha)}{\frac{a}{b}.tg(\alpha)} = \frac{ang_X(\alpha)}{\pm\infty} \approx 0.$$

Thus, the obtained signal is a zero signal.

2. Consider $\alpha \neq \frac{(2.k+1)\pi}{2}$ and $b \succ\succ a.tg(\alpha)$.

In this case, $tg(\alpha) \neq \infty$. As 'b' is a constant, therefore: $\left[\frac{a}{b}.tg(\alpha)\right]^2 \approx \varepsilon$ is very much smaller than unity. Then, the obtained elliptic cosine expression is: $\cos el_b(\alpha) = \frac{ang_X(\alpha)}{\sqrt{1+\varepsilon}} = ang_X(\alpha)\left[(1+\varepsilon)^{-\frac{1}{2}}\right]$.

By using the Taylor's development to the first term only, this expression will be:

$$\cos el_b(\alpha) = ang_X(\alpha).\left(1-\frac{1}{2}.\varepsilon\right)$$

$$= ang_X(\alpha).\left(1-\frac{1}{2}.\left[\frac{a}{b}.tg(\alpha)\right]^2\right)$$

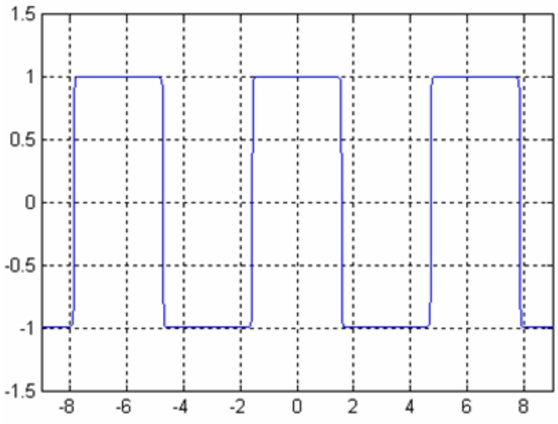
Or $\frac{a}{b}tg(\alpha) << 1$; therefore: $\cos el_b(\alpha) = ang_X(\alpha)$.

Thus, the obtained signal is a square signal.

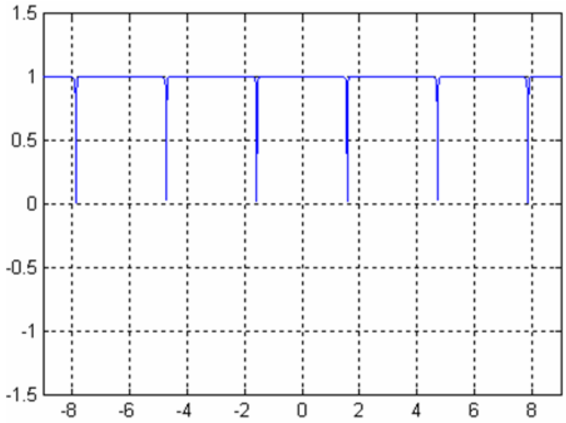
- Consequently:

$$Cosel_b(\alpha) = \begin{cases} \pm 0 & \text{for} \quad \alpha = \frac{(2k+1)\pi}{2} \\ \pm 1 & \text{for} \quad \alpha \neq \frac{(2k+1)\pi}{2} \end{cases} \qquad (17)$$

Thus, a purely square signal can be obtained (figure 6.a). Experimentally, as b is not infinite, it results a small distortions for $\alpha = \frac{(2k+1).\pi}{2}$ which are negligible. By using its absolute value (figure 6.b), the obtained signal is a continuous or dc signal. Theoretically, this signal contains a discontinuity for $\alpha = \frac{(2k+1).\pi}{2}$. This discontinuity does not affect experimentally the signal; therefore, it appears as a perfect dc signal.



a) $\cos el_b(\alpha)$



b) $|\cos el_b(\alpha)|$

Fig. 6: Elliptic cosine waveforms for b = 80 >> 1.

4.3 Third case (b << 1)

From equation (9), two configurations are also studied:

1. Consider $\alpha \neq k.\pi$ and $b \prec\prec a.tg(\alpha)$:

In this case $tg(\alpha) \neq 0$, Therefore $\left[\frac{a}{b}.tg(\alpha)\right]^2 \succ\succ 1$.

Thus:

$$Cosel_b(\alpha) = \frac{ang_X(\alpha)}{\sqrt{1+\left[\frac{a}{b}tg(\alpha)\right]^2}} = \frac{ang_X(\alpha)}{\sqrt{\left[\frac{a}{b}tg(\alpha)\right]^2}}$$

$$= \frac{ang_X(\alpha)}{\frac{a}{b}tg(\alpha)} = \frac{ang_X(\alpha)}{\pm\infty} \approx 0$$

2. Consider $\alpha = k.\pi$ and $b \succ\succ a.tg(\alpha)$.

In this case $tg(\alpha) = 0$, Therefore $\left[\frac{a}{b}.tg(\alpha)\right]^2 = 0$,

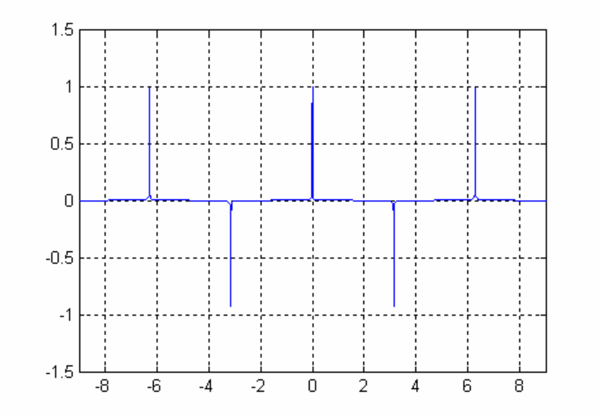
thus:

$$Cosel_b(\alpha) = \frac{ang_X(\alpha)}{\sqrt{1+0}} = ang_X(\alpha).$$

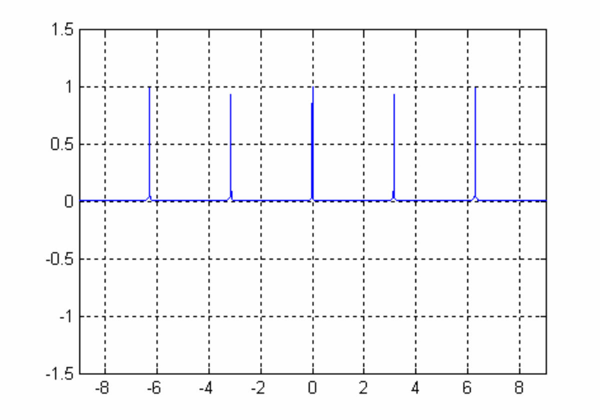
- As conclusion:

$$Cosel_b(\alpha) = \begin{cases} \pm 0 & \text{for} \quad \alpha \neq k.\pi \\ +1 & \text{for} \quad \alpha = 2k\pi \\ -1 & \text{for} \quad \alpha = (2k+1).\pi \end{cases} \qquad (18)$$

For this configuration, a Dirac signal with positive and negative pulses or with only positive pulses can be obtained (figures 7.a and 7.b).



a) $\cos el_b(\alpha)$

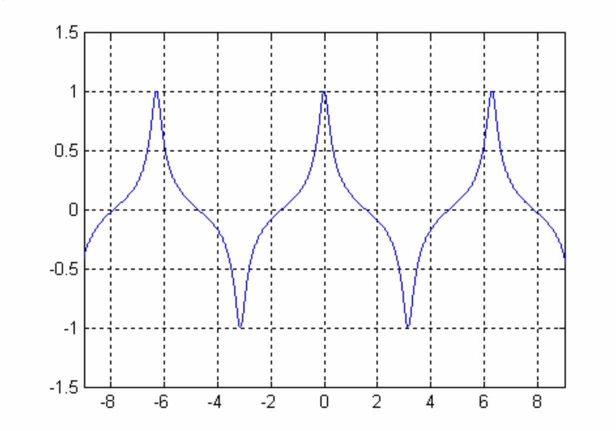


b) $|\cos el_b(\alpha)|$

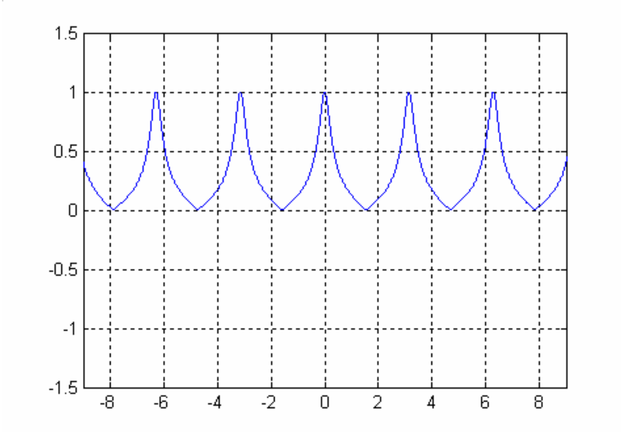
Fig. 7: Elliptic cosine waveforms for b = 0.001 << 1.

4.4 Fourth case (b < 1)

In this case, an elliptic deflated form is obtained (figure 8.a). The absolute value of this signal (figure 8.b) has an average value less than that of an absolute value of the sinusoidal signal. Hence, the advantage is that the average of the signal can be decreased by only varying the value of the parameter 'b' without any use of any other functions.



a) $\cos el_b(\alpha)$

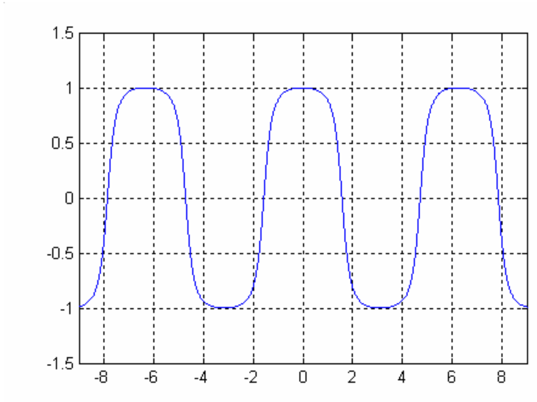


b) $|\cos el_b(\alpha)|$

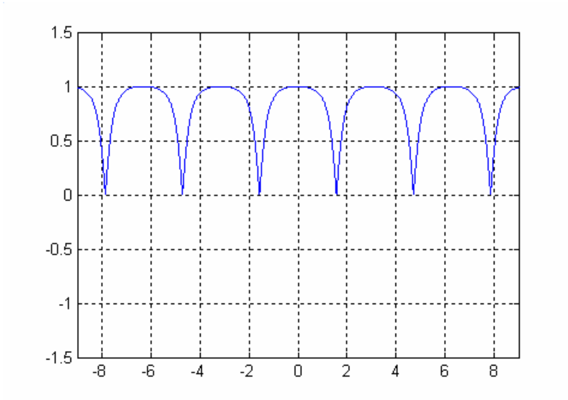
Fig. 8: Elliptic cosine waveforms for b = 0.2 < 1.

4.5 Fifth case (b > 1)

For this configuration, the signal takes the elliptical swollen form (Figure 9.a). The average value of its absolute signal (figure 9.b) is greater than that obtained by the absolute value of the sinusoidal wave. Therefore, increasing the average of a signal by varying only one parameter 'b', is also another advantage of the elliptical trigonometric functions.



a) $\cos el_b(\alpha)$



b) $|\cos\mathrm{el}_b(\alpha)|$

Fig. 9: Elliptic cosine waveforms for b = 3 > 1.

4.6 Determining the value of 'b' for a quasi-triangular signal

The main objective of this part is to obtain a signal closed to a triangular one. Therefore, the following method is proposed in order to calculate the value of 'b' for which the error between the desired signal and the obtained one is minimized. As the symmetric axis of the elliptic cosine is the 'oy' axis, this study is therefore limited to the angular interval $\left[0, \frac{\pi}{2}\right]$.

1. Consider the equation of a straight line:

$$y = c.\alpha + d \tag{19}$$

This line is supposed to contain the following two points: $(\alpha = 0, y = 1)$ and $\left(\alpha = \frac{\pi}{2}, y = 0\right)$.

For $c = -\frac{2}{\pi}$ and d = 1, the expression (19), which is drawn in figure 8, becomes:

$$y = -\frac{2}{\pi}.\alpha + 1 \tag{20}$$

2. For the same interval $\left[0, \frac{\pi}{2}\right]$, the angular function, $\mathrm{ang}_x(\alpha)$, is equal to one, therefore, the elliptical cosine is:

$$\mathrm{Cosel}_b(\alpha) = \frac{+1}{\sqrt{1 + \left(\frac{a}{b}.\mathrm{tg}(\alpha)\right)^2}}$$

The wave form of this function is also illustrated in figure 8.

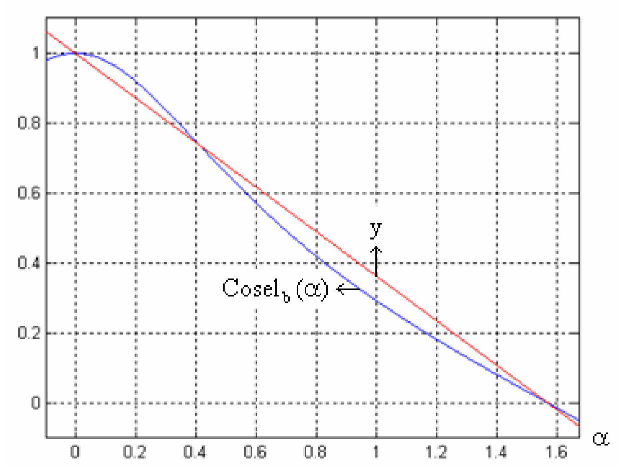


Fig. 10: Obtained signal Cosel_b and desired signal (y) waveforms.

The difference between these two functions, Cosel_b and y, is:

$\varepsilon = \|\mathrm{Cosel}_b(\alpha) - y\|$.

It is considered that, for $\alpha = \frac{\pi}{4}$ (the center of the studied interval $\left[0, \frac{\pi}{2}\right]$), the error ε is equal to zero.

Thus, b takes the value of $\frac{\sqrt{3}}{3}$. To calculate the maximum and the minimum error values, ε_{max} and ε_{min}, the derivative of ε must be equal to zero.

Therefore: $$\begin{cases} \varepsilon_{max} = 0.0737 \approx 7.4\% \\ \varepsilon_{min} = 0.0178 \approx 1.8\% \\ \varepsilon_{avg} = 0.0257 \approx 2.6\% \end{cases}$$

Figure 11 represents the variation of this error in the interval $\left[0, \frac{\pi}{2}\right]$.

For $b = \frac{\sqrt{3}}{3}$, it is difficult to reduce the error to zero. Therefore, the elliptical cosine function takes a quasi-triangular signal waveform as illustrated in figure 12.

In spite of the presence of the error, its amplitude is small and can be considered in some applications as negligible.

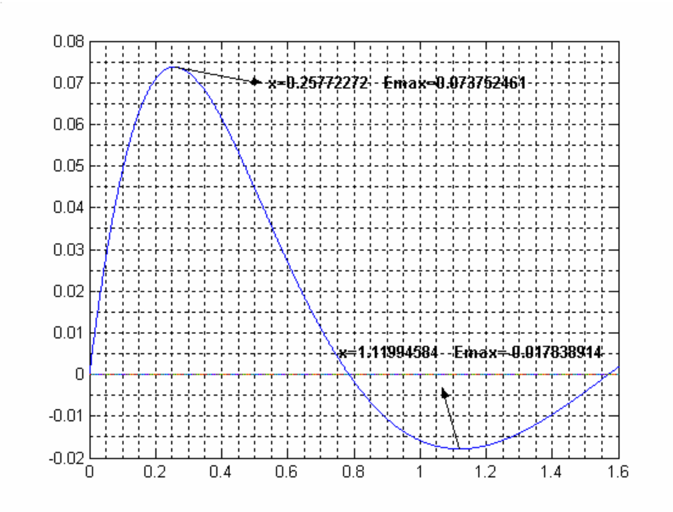
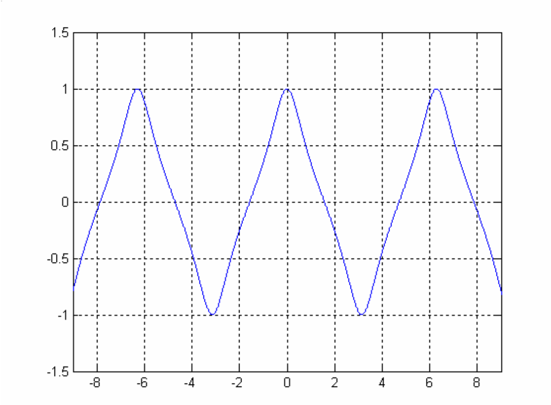
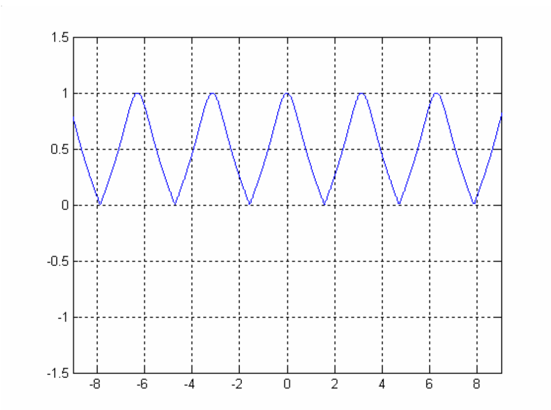


Fig. 11: Error between the desired signal and the obtained one.



a) $\cos el_b(\alpha)$



b) $|\cos el_b(\alpha)|$

Fig. 12: Elliptic cosine waveforms for $b = \sqrt{3}/3$.

Consequently, by varying b between 0 and 1 in the elliptic cosine function, different signal forms can be obtained. Thus, the elliptical trigonometry functions will have important influence in engineering, especially in power electronics [16],[17].

5 Example using the elliptical trigonometry

In this section, an example using the elliptical trigonometry is treated. For this, consider the following function:

$$y^2 = (K.rec_T(x).\cos el_b(\varphi.x))^2 \quad (21)$$

With K is the amplitude of the signal,
'x' is a variable parameter,
K, φ and T are positive constants.

In fact, the rectangular function, $rect_T(x)$, which is illustrated in figure 13 is defined as:

$$rect_T(x - x_0) = \begin{cases} 1 \ \text{for} \ x_0 - \frac{T}{2} \le x \le x_0 + \frac{T}{2} \\ 0 \quad \text{otherwise} \end{cases} \quad (22)$$

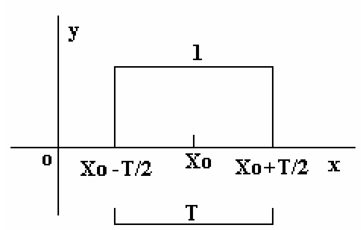


Fig. 13: $rect_T(x)$ wave form

Then: $y = \pm K.rec_T(x).\cos el_b(\varphi.x)$ (23)

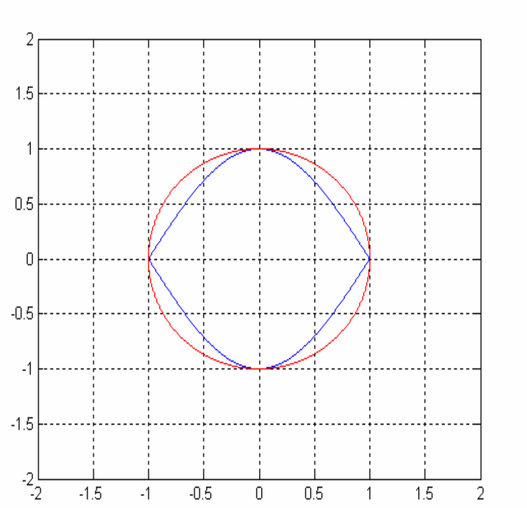
For a particular case, let K = 1, $\varphi = \frac{\pi}{T}$ for T = 2.

Thus, equation (23) becomes:

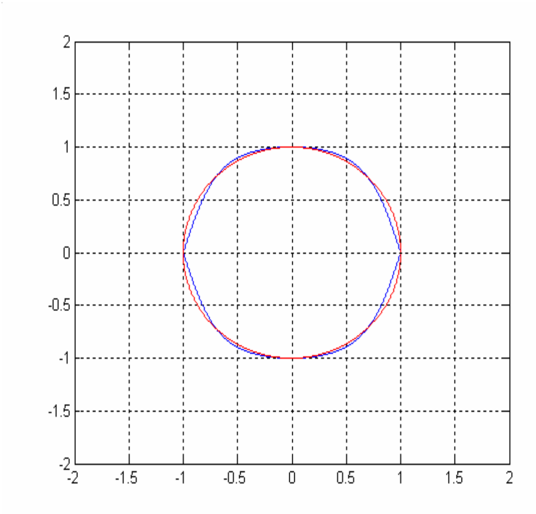
$$y = \pm K.rect_T(x).\cos el_b(\frac{\pi}{T}x) \ \Rightarrow$$

$$y = \pm rect_T(x).\cos el_b(\frac{\pi}{2}x) \quad (24)$$

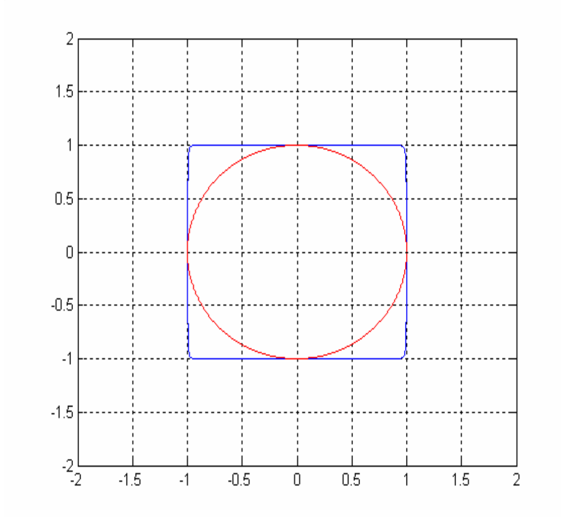
When the parameter b is varied, this new function, (24), can be transformed to many shapes as illustrated in figures 14.a to 14.g.



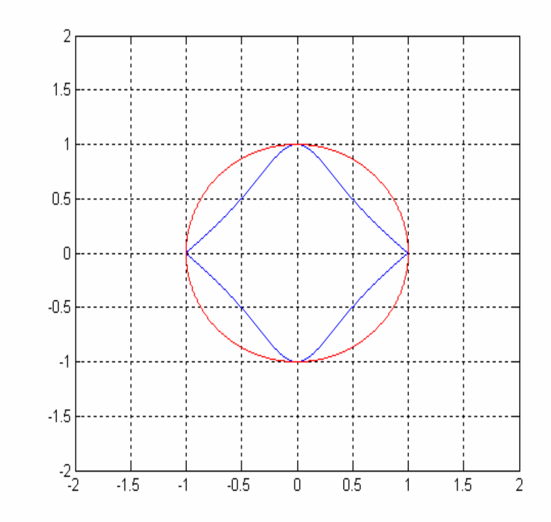
a) b = 1 (eye form)



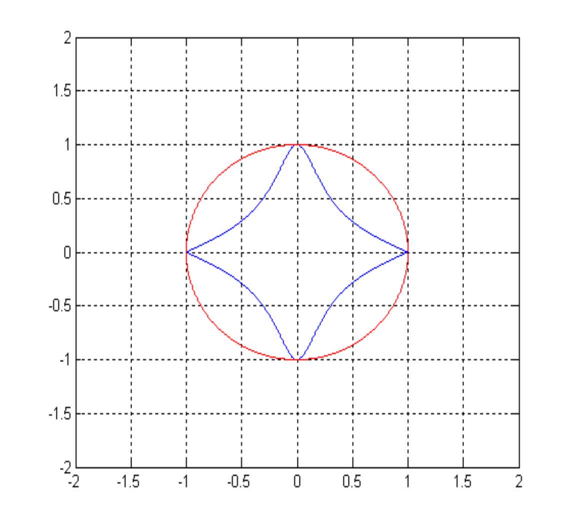
b) b = 2 (quasi-circular form)



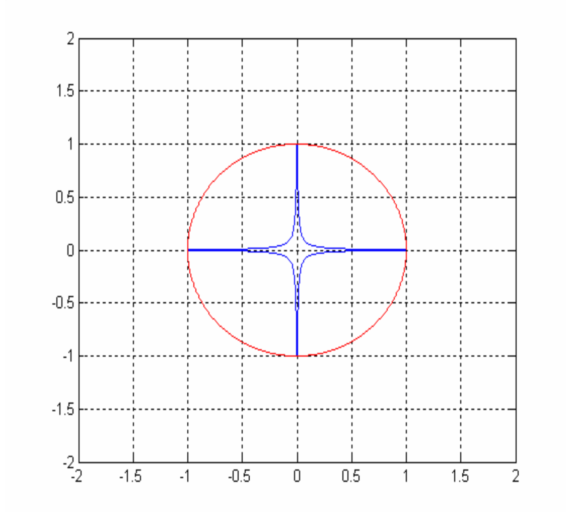
c) b = 120 (Square form)



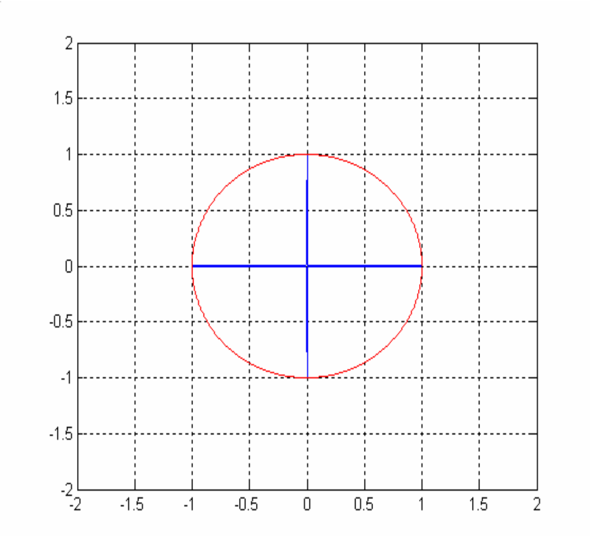
d) $b = \frac{\sqrt{3}}{3}$ (Quasi-rhombus form)



e) b = 0.3 (star form)



f) b = 0.01 (cross form)



g) b = 0.0001 (plus form)

Fig. 14: Different shapes of the y function for different value of b, red color for the circle and blue color for the y function.

Consequently, for the studied function, equation (24), and for different values of b, important forms are obtained:
- Square form for b = 120,
- Quasi-rhombus form for $b = \frac{\sqrt{3}}{3}$,
- quasi-circular form for b = 2,
- eye form for b = 1,
- quasi-ellipse form for b = 0.0001,
- star form for b = 0.3,
- cross, plus form (+), for b = 0.01.

6 Conclusion

In this paper, an original study in trigonometry is introduced. The elliptical unit and its trigonometry functions are presented and analyzed. In fact, the proposed elliptical trigonometry is a new form of trigonometry that permits producing a multiple form of signals by varying some parameters; it can be used in numerous scientific domains and particularly in mathematics and in engineering. For the case treated in this paper, more than six signals are produced from one function by varying one parameter. These signals are: sinusoidal signal, rectangular, square, elliptical swollen, elliptic deflated, quasi-triangular, impulse of Dirac with positive and negative part, impulse of Dirac positive part only, continuous signal …

The elliptical trigonometry functions will be widely used in electronic domain especially in power electronics. Thus, several studied will be improved and developed after introducing the new functions of the elliptic trigonometry. Some mathematical expressions and electronic circuits will be replaced by simplified expressions and reduced circuits. In order to illustrate the importance of these new functions, an example of a proposed function is treated. Consequently, with this single function, the following forms are obtained: rectangle, square, quasi-rhombus (diamond-shaped), quasi-circle, quasi-ellipse, star, cross, plus form (+), and many others…

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