Numerical Simulations for Dynamic Stochastic and Hybrid Models of Internet Networks

G. MIRCEA[1], M. NEAMTU[2], A. L. CIURDARIU[3], D. OPRIS[4]
[1,2]Department of Economic Informatics and Statistics
[3,4]Department of Mathematics
[1,2,4]West University of Timişoara
[3]Politehnica University of Timişoara
[1,2]Pestalozzi Street, nr. 16A, 300115, Timişoara
[3]Piata Victoriei, nr 2, 300004, Timişoara
[4]Bd. V. Parvan, nr. 4, 300223, Timişoara
ROMANIA
[1]gabriela.mircea@feaa.uvt.ro, [2]mihaela.neamtu@feaa.uvt.ro,
[3]cloredana43@yahoo.com, [4]opris@math.uvt.ro

Abstract: - In this paper, we consider Internet models, which respond to a congestion signal from the network described by a stochastic and hybrid differential equation. We consider Internet networks with one source and r access links, as well as with r sources and one access link. We analyze the conditions for the existence of a solution and the algorithms needed to determine the solution. We carry out numerical simulations for certain parameter values.

Key-Words: Internet models, networks, dynamic stochastic models, stochastic delay differential equation, Euler method, numerical simulations

1 Introduction

Congestion control mechanisms and active queue management schemes (AQM) for the Internet have been extensively studied since the work of Kelly et all [5].
In [10], the Hopf bifurcation has been studied for the model of an Internet network with $r\,(r>1)$ link and single source, which can be formulated as:

$$\dot{x}_i(t) = k(v - af(x_i(t-\tau)) - b\sum_{\substack{j=1\\ j\neq i}}^{r} x_j(t-\tau) f(x_i(t-\tau))),\ i=1,\dots,r \tag{1}$$

where $x_i(t)$ is the sending rate of the source i at time t, k is a positive gain parameter, τ is the sum of forward and return delays, v is a target (set – point), a, b are positive real parameters and the congestion indication function $f(x)$ is increasing, nonnegative, which characterizes the congestion. The discretizing model of (1) has been analyzed in [9], and the value of k, for which the Neimark-Sacker bifurcation takes place, has been determined.

The dynamic model with a single link and r sources can be described by:

$$\begin{aligned}\dot{x}_i(t) &= k_i x_i(t-\tau_i)\left(\frac{a_i}{x_i(t)} - b_i x_i(t) p(t)\right)\\ \dot{p}(t) &= k_{r+1} p(t)\left(\sum_{i=1}^{r} x_i(t-\tau_i) - c\right),\ i=1,\dots,r\end{aligned} \tag{2}$$

where, $x_i(t)$ is the rate at which the source i transmits data at the time t, a_i and b_i are positive real numbers, $p(t)$ is the loss probability function, τ_i is round-tripe delay for source i, c is the capacity, k_i, k are gain parameters. The discretized model of (2) has been analyzed in [11].

By employing the dynamic delayed feedback control, we can consider the controlled congestion control system with communication delay as follows:

$$\dot{x}_1(t) = x_1(t-\tau)\left(\frac{1-x_2(t)}{\tau^2 x_1(t)} - a_1 x_1(t) x_2(t)\right)$$
$$\dot{x}_2(t) = \frac{a_2}{a_3} x_2(t)\big(x_1(t-\tau) - a_3\big) + x_3(t) \qquad (3)$$
$$\dot{x}_3(t) = a_4\big(x_1(t) - x_1(t-\tau)\big) + a_5 x_3(t)$$

where $a_5 > 0$ is the washaut filter time constant, which guarantees the stability of controller, $a_4 > 0$ is the feedback gain parameter. This model was analized in [4].
The models (1), (2), and (3) lead to dynamic models described by stochastic delay differential equations, by randomizing one of the parameters. Thus, in section 2 we will consider stochastic delay differential equations, obtained through the randomization of parameter a in the case of the model (1), respectively, of parameters a_i, c, $i = 1,...,r$, in the case of the model (2), and the parameters a_1, a_2, a_4, a_5, in the case of the model (3).

For certain functions which describe the stochastic delay differential equations (SDDE), the existence and uniqueness of the solution is justified. In section 3, we describe the algorithm which approximates the solution of the equations for the stochastic system with $r = 2$ links and a source with a single link and $r = 2$ source, respectively. In section 3 we describe the algorithm numeric for these cases. In section 4, we associate the determinist models (1),(2),(3), hybrid system, by adding a term that comprizes an Ito integral.
For certain parameter values, we carry out numerical simulation with the Maple13 software for the simulation of Wiener and Liu processes.

2 SDDE Models for Internet networks

Let $(\Omega, F, \Pr)$ be a complete probability space with a filtration (F_t) satisfying the usual conditions; that is the filtration $(F_t)_{t\ge 0}$ is right -continuous and each (F_t), where $t \ge 0$ contains all $\Pr$-null sets in F. For general theory we refer to [12]. With $E(x) = \int_\Omega x dx$ we say for $1 \le p \le \infty$ that $x \in L^p = L^p(\Omega, F, \Pr)$ if $E(|x|^p) < \infty$ and we define $\|x\|_p = (E|x|^p)^{\frac{1}{p}}$. Here, E denotes the expectation.
Let $w(t) = (w_1(t),..., w_r(t))^T$ be a r- dimensional Wiener process given on the filtered probability space $(\Omega, F, \Pr)$.
The stochastic delay differential equation (SDDE) with one fixed lag, $(0 < T < \infty)$ is given by:

$$\begin{aligned} dx_i(t) &= f_i(x(t), x(t-\tau))\,dt + \\ &+ g_i(x(t), x(t-\tau))\,dw_i(t), \quad t \in [0,T] \\ x_i(t) &= \Psi_i(t), \ i = 1..r, \ t \in [-\tau, 0] \end{aligned} \qquad (4)$$

where $x_i(t) = x_i(t,\omega), \omega \in \Omega$, $\Psi_i(t)$ is F_{t_0}-measurable with values in $C([-\tau,0], R)$ so that $E(\|\Psi\|^2) < \infty$ and $f_i : R^r \times R^r \to R$, $g_i : R^r \times R^r \to R$, $x_i(t) = x_i(t,\omega)$, $\omega \in \Omega$, $i = 1,...,r$.

The system (4) can then formulated equivalently as:

$$\begin{aligned} x_i(t) &= x_i(0) + \int_0^t f_i(x(s), x(s-\tau))\,ds + \\ &+ \int_0^t g_i(x(s), x(s-\tau))\,dw_i(s), \quad i = 1,..., r \end{aligned} \qquad (5)$$

for $t \in [0,T]$ and with $x_i(t) = \Psi_i(t)$, for $t \in [-\tau, 0]$. The second integral in (5) is a stochastic integral which can be interpreted according to Ito's integral.

For the functions f_i, g_i, $i = 1,...,r$, and $\Psi_i : [-\tau, 0] \to R$, $i = 1,...,r$, we consider the following set of conditions [11]:

1. The functions f_i and g_i are continuous;
2. The functions f_i and g_i satisfy a uniform Lipschitz condition;
3. The functions Ψ_i is Holder - continuous with exponent φ_i;
4. The functions f_i and g_i satisfy a linear growth condition;
5. The partial derivatives of $f_i(\Phi, \Psi)$ exist and are uniformly bounded.

Proposition 1 [12]. Assume that the functions f_i and g_i satisfy the above assumptions 1-3. Then, there exists a unique strong solution of equation (4).

The stochastic model for an Internet network with $r(r > 1)$ link and single source described by SDDE is given by:

$$x_i(t) = x_i(0) + \int_0^t k(v - af(x_i(s-\tau)) - \sum_{\substack{j=1\\ j\neq i}}^{r} x_j(s-\tau))f(x_j(s-\tau)))ds - \int_0^t k_1\alpha_i f(x_i(s-\tau))dw_i(s), \quad t\in[0,T],\ i=1,\dots,r \tag{6}$$

$$x_i(t) = \Psi_i(t), \quad t\in[-\tau,0].$$

The stochastic model for the Internet network with a single link and r sources can be described by:

$$\begin{aligned} x_i(t) &= x_i(0) + \int_0^t k_i x_i(s-\tau)(a_i g(x_i(s)) - b_i p(s)x_i(s))ds + \int_0^t \alpha_i k_i a_i x_i(s-\tau)g(x_i(s))dw_i(s), \\ & t\in[0,T],\ i=1,\dots,r \\ p(t) &= p(0) + k_{r+1}\int_0^t p(s)\left(\sum_{i=1}^{r} x_i(s-\tau) - c\right)ds - k_{r+1}\int_0^t \alpha_{r+1}\mathrm{p(s)dw(s)} \\ x_i(t) &= \Psi_i(t),\ g(x_i) = \frac{1}{x_i}, \quad t\in[-\tau,0]. \end{aligned} \tag{7}$$

The stochastic model for Internet associated to (3) can described by:

$$\begin{aligned} x_1(t) &= x_1(0) + \int_0^t x_1(s-\tau)\left(\frac{1-x_2(s)}{\tau^2 x_1(s)} - a_1x_1(s)x_2(s)\right)ds - \alpha_1\int_0^t x_1(s-\tau)x_1(s)x_2(s)dw_1(s), \\ x_2(t) &= x_2(0) + \int_0^t \frac{a_2}{a_3}x_2(s)(x_1(s-\tau)-a_3)+x_3(s))ds + \frac{\alpha_2}{\alpha_3}\int_0^t x_2(s)(x_1(s-\tau)-a_3)dw_2(s), \\ x_3(t) &= x_3(0) + \int_0^t (a_4(x_1(s)-x_1(s-\tau))-a_5x_3(s))ds - \alpha_3\int_0^t x_3(s)dw_3(s). \end{aligned} \tag{8}$$

If functions f, g and Ψ_i, $i=1,\dots,r$, satisfy conditions 1-5, then SDDE given (6), (7) respectively (8) has a unique solution.

3 Numerical simulation for the SDDE equations (6), (7) and (8)

The problem of solving an SDDE is reduced to one of solving a sequence of systems of SDDE of increasing dimension on successive intervals $[n,(n+1)]$. Using the Euler method of first order from [1], [12], [13] for (6), we obtain:

$$x_i(k+1) = x_i(k) + hk_1(v - af(x_i(k-m)) - b\sum_{\substack{j=1\\ j\neq i}}^{r} x_j(k-m)f(x_j(k-m))) - k_1\alpha_i f((x_i(k-m))G(h), \tag{9}$$

$k,m\in N,\ h\in(0,1),\ \alpha_i>0,\ i=1,\dots,r$

where $G(h) = \mathrm{random[normald}[0,\sqrt{\mathrm{h}}]]$.

Using results from [1], [12], [13] we have that the algorithms given by (9) is convergent.

For $r=2$, $k_1=0.2$, $a=18$, $b=2$, $v=100$, $\alpha_1=0.5$, $\alpha_2=0.5$, $m=3$, $n=1300$, $h=\frac{1}{1300}$, $f(x)=\frac{x^2}{20-3x}$, with Maple 13, we obtain in Fig. 1, the orbit $(k,x_1(k))$, in Fig. 2 the orbit $(k,x_2(k))$ and in Fig. 3 the orbit $(x_1(k),x_2(k))$.

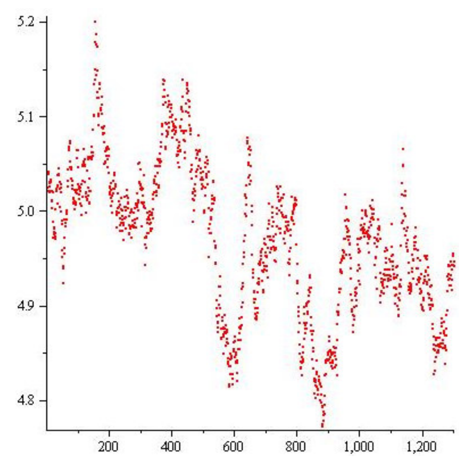

Fig.1 The orbit $(k,x_1(k))$

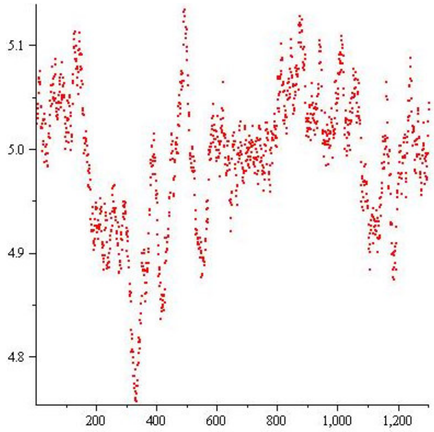

Fig.2 The orbit $(k, x_2(k))$

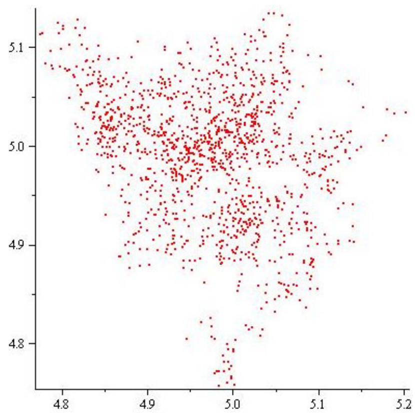

Fig.3 The orbit $(x_1(k), x_2(k))$

Using the Euler method of first order for (7) we obtain:

$$\begin{aligned} x_i(k+1)&=x_i(k)+hk_i x_i(k-m)(a_i g(x_i(k)-b_i p(k)x_i(k))+\\ &+k_i\alpha_i x_i(k-m)g((x_i(k))G(h),\ i=1,\dots r\\ p(k+1)&=p(k)+hk_{r+1}p(k)\left(\sum_{i=1}^{r}x_i(k-m)-c\right)-\\ &-k_{r+1}\alpha_{r+1}p(k)G(h). \end{aligned} \tag{10}$$

For $r=2$, $k=1000$, $h=\frac{1}{1000}$, $m=3$, $k_1=1$, $k_2=2$, $k_3=0.005$, $c=65$, $a_1=\frac{2}{3}$, $a_2=0.9$, $b_1=0.5$, $b_2=0.5$, $\alpha_1=0.2$, $\alpha_2=0.2$, $\alpha_3=0.4$, $g(x)=\frac{1}{x}$, with Maple 13, we obtain the orbits: in Fig. 4, the orbit $(k, x_1(k))$, in Fig. 5 the orbit $(k, x_2(k))$, in Fig. 6 the orbit $(k, p(k))$, in Fig. 7 the orbit $(x_1(k), x_2(k))$, in Fig. 8 the orbit $(x_1(k), p(k))$ and in Fig. 9 the orbit $(x_2(k), p(k))$.

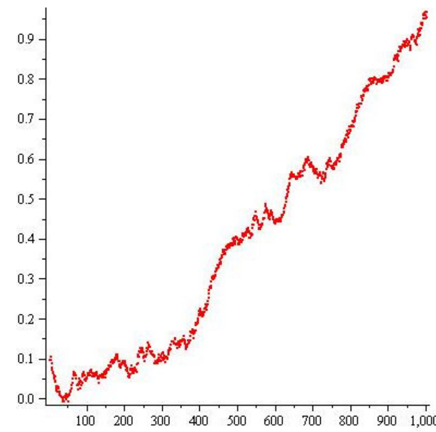

Fig.4 The orbit $(k, x_1(k))$

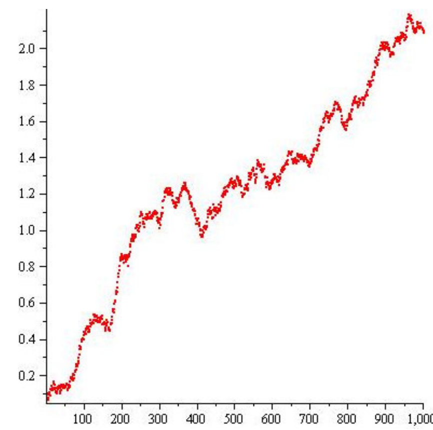

Fig.5 The orbit $(k, x_2(k))$

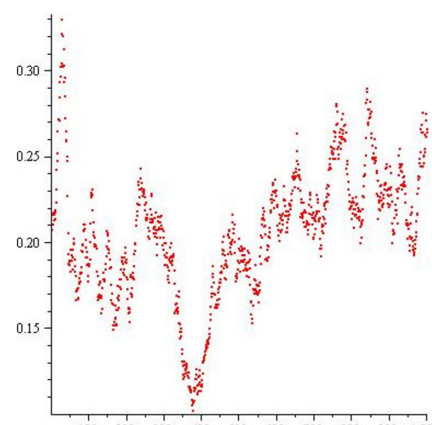

Fig.6 The orbit $(k, p(k))$

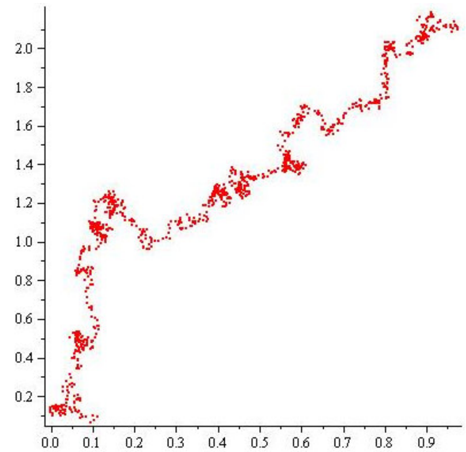

Fig.7 The orbit $(x_1(k), x_2(k))$

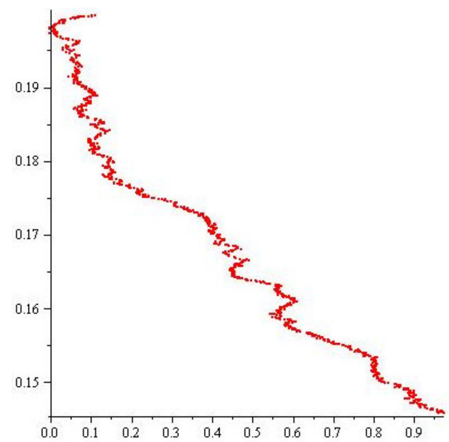

Fig.8 The orbit $(x_1(k), p(k))$

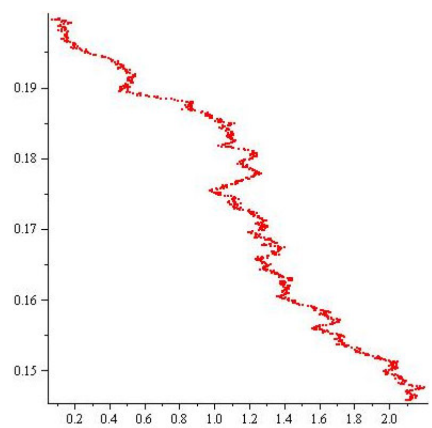

Fig.9 The orbit $(x_2(k), p(k))$

Using the Euler method of first order for (8) we obtain:

$$
\begin{aligned}
x_1(k+1) &= x_1(k) + h x_1(k-m)\left(\frac{1-p(k)}{m^2 x(k)} - a_1 x_1(k) x_3(k)\right) - \\
&\quad - \alpha_1 x_1(k-m) x_1(k) x_3(k) G(h), \\
x_2(k+1) &= x_2(k) + h\left(\frac{a_2}{a_3} x_2(k)(x_1(k-m) - a_3) + x_3(k)\right) + \\
&\quad + \frac{\alpha_2}{\alpha_3} x_2(k)(x_1(k-m) - a_3) G(h), \\
x_3(k+1) &= x_3(k) + h(a_4(x_1(k) - x_1(k-m)) - a_5 x_3(k) - \\
&\quad - \alpha_3 x_3(s) G(h).
\end{aligned}
\tag{11}
$$

For $a_1 = 1$, $a_2 = \frac{1}{2}$, $a_3 = 0.2$, $a_4 = 0.5$, $a_5 = 0.2$, $\alpha_1 = 0.5$, $\alpha_2 = 6$, $\alpha_3 = 0.4$, $m = 4$, $k = 1300$, $h = \frac{1}{1300}$, with Maple 13, we obtain the orbits:

In Fig.10 the orbit $(k, x_1(k))$, in Fig.11 the orbit $(k, x_2(k))$, in Fig.12 the orbit $(k, x_3(k))$, in Fig.13 the orbit $(x_1(k), x_2(k))$, in Fig.14 the orbit $(x_1(k), x_3(k))$ and in Fig.15 the orbit $(x_2(k), x_3(k))$.

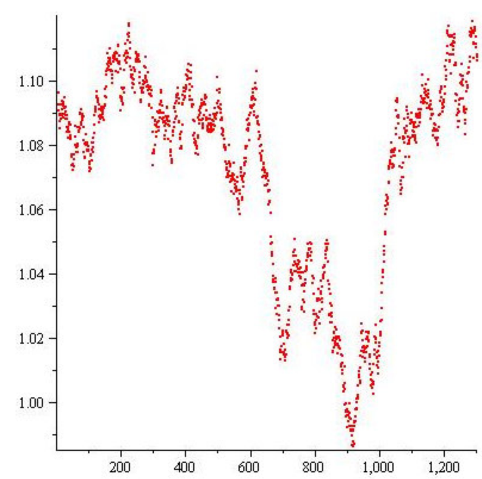

Fig.10 The orbit $(k, x_1(k))$

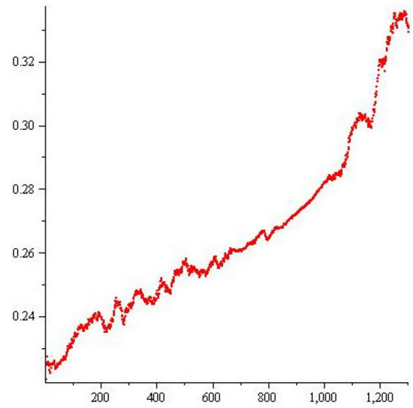

Fig.11 The orbit $(k, x_2(k))$

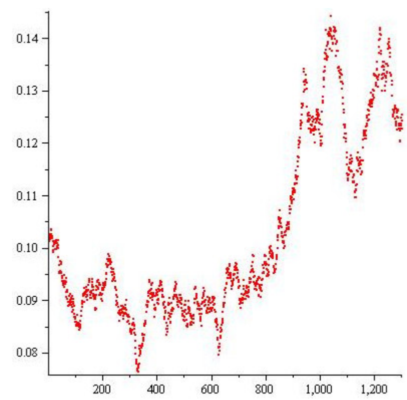

Fig.12 The orbit $(k, x_3(k))$

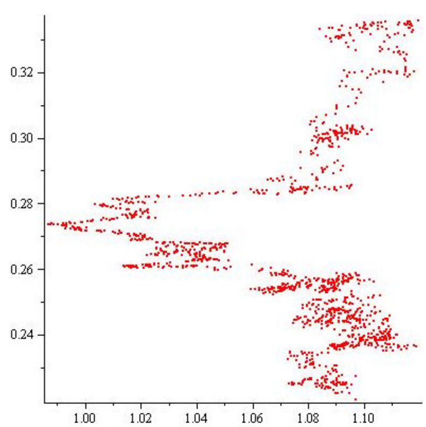

Fig.13 The orbit $(x_1(k), x_2(k))$

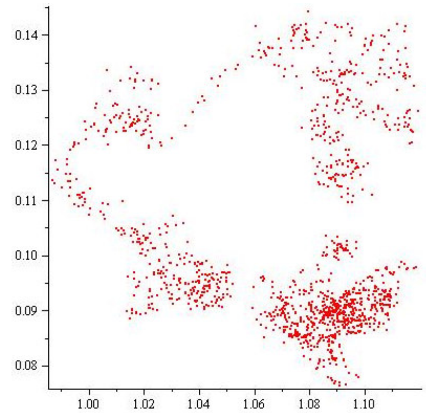

Fig.14 The orbit $(x_1(k), x_3(k))$

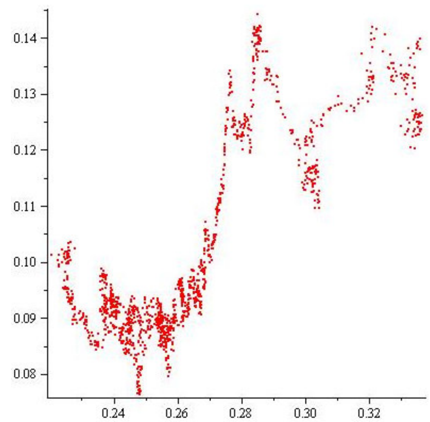

Fig.15 The orbit $(x_2(k), x_3(k))$

4 Hybrid differential equations associated for the SDDE equations (1), (2) and (3)

Randomness in a basic type of objective uncertainly and probability theory is a branch of mathematics for studying the behavior of random phenomena. The concept of fuzzy set was initiated by Zadeh [14] via the membership function in 1965. In order to measure a fuzzy event, Liu B. [6] introduced the concept of credibility measure. The credibility theory was founded as a branch of mathematics for studying the behavior of fuzzy phenomena. Fuzziness and randomness are two basic types of uncertainty. In many cases, fuzziness and randomness simultaneously appear in a system.

Let Θ be a nonempty set, and let P be the power of Θ (i.e. all subsets of Θ). Each element in P is called an event. In order to present an axiomatic definition of credibility, we accept the following four axioms:

1. $Cr(\Theta)=1$;
2. $Cr\{A\}\le Cr\{B\}$, whenever $A<B$;
3. $Cr\{A\}+Cr\{A^c\}=1$, for any $A\in \mathrm{P}$;
4. $Cr\left\{\bigcup_i A_i\right\}=\sup_i Cr\{A_i\}$, for any events $\{A_i\}$ with $\sup_i Cr\{A_i\}<0.5$.

The set function Cr is called a credibility measure and (Θ,P,Cr) is a credibility space. A fuzzy variable is a function from the credibility space (Θ,P,Cr) to the set of real numbers. If a fuzzy variable ξ is defined as a function on a credibility space, then we may get its membership function via:

$$\mu(x)=(2Cr\{\xi=x\})\wedge 1,\ x\in R. \qquad (12)$$

Suppose that (Θ,P,Cr) is a credibility space and $(\Omega,F,\Pr)$ is a probability space. The product $(\Theta,\mathrm{P},Cr)\times(\Omega,F,\Pr)$ is called a chance apace. A hybrid variable is a measurable function, from a chance space $(\Theta,\mathrm{P},Cr)\times(\Omega,F,\Pr)$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set $\{(\theta,\omega)\in\Theta\times\Omega \mid \xi(\theta,\omega)\in B\}$ is an event.

Let T be an index set and let (Θ,P,Cr) be a credibility space. A fuzzy process is a function from $T\times(\Theta,\mathrm{P},Cr)$ to the set of real numbers. A fuzzy process is a C_e is said to be a C process if:

i) $C_0=0$;

ii) C_t has stationary and independent increments;

iii) every increment $C_{s+t}-C_s$ is a normally distributed fuzzy variable with expected value e_1t and variance σ^2t^2, whose membership function is:

$$\mu(x,t)=2\left(1+\exp\left(\frac{\pi(x-e_1t)}{\sqrt{6}\sigma t}\right)\right)^{-1},\ x\in R \qquad (13)$$

Let C process be standard if $e_1=0$ and $\sigma=1$. The C process plays the role of Brownian motion.

Let T be an index set, and $(\Theta,\mathrm{P},Cr)\times(\Omega,F,\Pr)$ a chance space. A hybrid process is a measurable function from $T\times(\Theta,\mathrm{P},Cr)\times(\Omega,F,\Pr)$ to the set of real numbers, i.e., for each $t\in T$ and any Borel set B of real numbers, the set $\{(\theta,\omega)\in\Theta\times\Omega \mid X(t,\theta,\omega)\in B\}$ is an event.

Suppose w_t is a standard Brownian motion, C_t is a standard C process, and f,g,h are some given functions. Then:

$$dx(t)=f(t,x(t))dt+g(t,x(t))dw(t)+h(t,x(t))dC_t \qquad (14)$$

is called a hybrid differential equation. A solution is a hybrid process $x(t)$ that satisfies (14) identically in t.

Hybrid differential equation associated to (1) is:

$$\begin{aligned}dx_i&=k_1(v-af(x_i(t-\tau))-b\sum_{\substack{j=1\\j\neq i}}^{r}x_i(t-\tau)f(x_i(t-\tau)))dt+\\&+\alpha_i(x_i-x_e)dw_i(t)+\beta_i x_i dC_i(t),\ i=1,..r \qquad (15)\\x_i(t)&=\Psi_i(t),\ t\in[-\tau,0],\ \alpha_i>0,\ \beta_i\in\{0,1\},\ i=1,..r\end{aligned}$$

where x_e is the solution of equation $v=af(x)$.

The numerical simulation of (15) is given by:

$$\begin{aligned}x_i(k+1)&=x_i(k)+hk_1(v-af(x_i(k-m))-\\&-b\sum_{\substack{j=1\\j\neq i}}^{r}x_j(k-m)f(x_j(k-m))+\\&+\alpha_i(x_i(k)-x_e)G(h)+\beta_i L(k,z_i), \qquad (16)\end{aligned}$$

$k,m\in N,\ h\in(0,1),\ \alpha_i>0,\ \beta_i\in\{0,1\},\ z_i>0,\ i=1,..r$

$$\begin{aligned}&G(h)=random[normald[0,\sqrt{h}]],\\&L(k,z_i)=2\left(1+\exp\left(\frac{\pi z_i}{h\sqrt{6}\sigma S_i(k)}\right)\right)^{-1}, \qquad (17)\\&S_i(k)=\sum_{j=0}^{k-1}x_i(j).\end{aligned}$$

For $r=2$, $k_1=0.2$, $a=18$, $b=2$, $v=100$, $\alpha_1=0.5$, $\alpha_2=0.5$, $\beta_i=1,\ i=1,2$, $m=3$, $n=1500$, $h=\frac{1}{1500}$, $f(x)=\frac{x^2}{20-3x}$, with Maple 13, we obtain in Fig. 16, the orbit $(k,x_1(k))$, in

Fig.17 the orbit $(k, x_2(k))$ and in Fig.18 the orbit $(x_1(k), x_2(k))$.

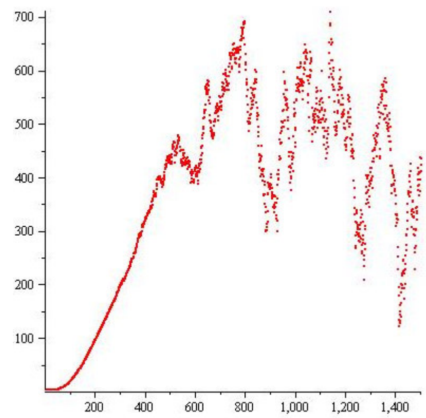

Fig.16 The orbit $(k, x_1(k))$

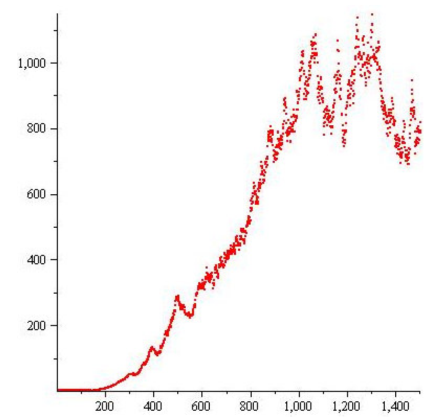

Fig.17 The orbit $(k, x_2(k))$

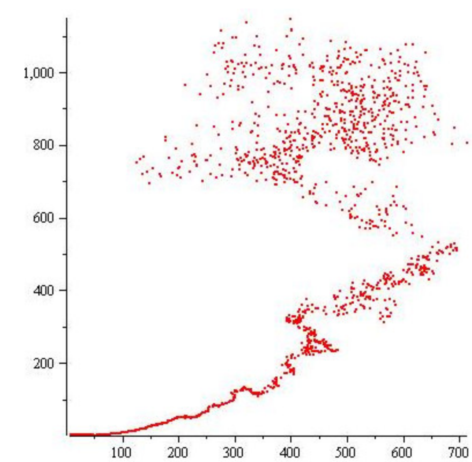

Fig.18 The orbit $(x_1(k), x_2(k))$

The hybrid differential equation associated to (2) is:

$$\begin{aligned} dx_i &= k_i x_i(t-\tau)\left(\frac{a_i}{x_i(t)} - b_i x_i(t)p(t)\right)dt + \\ &+ \alpha_i(x_i(t)-x_e)dw_i(t) + \beta_i x_i(t)dC_i(t),\ i=1,..r \\ dp &= k_{r+1}p(t)\left(\sum_{i=1}^{r} x_i(t-\tau) - c\right)dt + \\ &\alpha_{r+1}(p(t)-p_e)dw(t) + \beta_{r+1}p(t)dC(t) \end{aligned} \tag{18}$$

$\alpha_i > 0,\ \alpha_{r+1} > 0,\ \beta_i, \beta_{r+1} \in \{0,1\},\ i=1,..r$

where (x_{ie}, p_e) is the equilibrium point of the (2).

The numerical simulation of (17) is given by:

$$\begin{aligned} x_i(k+1) &= x_i(k) + hk_i x_i(k-m)\frac{\alpha_i}{x_i(k)} - b_i p(k)x_i(k)) + \\ &+ \alpha_i(x_i(k)-x_{ie})G(h) + \beta_i x_i(k)L(k,z_i),\ i=1,\dots,r \\ p(k+1) &= p(k) + hk_{r+1}p(k)(\sum_{j=1}^{r} x_j(k) - c) + \alpha_{r+1}(p(k)-p_e)G(h) + \\ &+ \beta_{r+1}p(k)L((k, z_{r+1}) \end{aligned} \tag{19}$$

$k, m \in N,\ h \in (0,1),\ \alpha_i > 0,\ \beta_i \in \{0,1\},\ z_i > 0,\ i=1,..r$

where $G(h)$, $L(k, z_i)$ are given by (17).

For $r=2$, $n=1500$, $h=\frac{1}{1500}$, $a_1=\frac{2}{3}$, $a_2=0.9$, $b_1=0.5$, $b_2=0.5$, $c=65$, $k_1=1$, $k_2=1$, $k_3=0.005$, $\alpha_1=0.5$, $\alpha_2=0.3$, $\alpha_3=0.4$, $\beta_i=1$, $i=1,2$, $z_1=0.5$, $z_2=0.3$, $z_3=0.4$, with Maple 13, we obtain in Fig.19 the orbit $(k, x_1(k))$, in Fig.20 the orbit $(k, x_2(k))$ and in Fig.21 the orbit $(k, p(k))$.

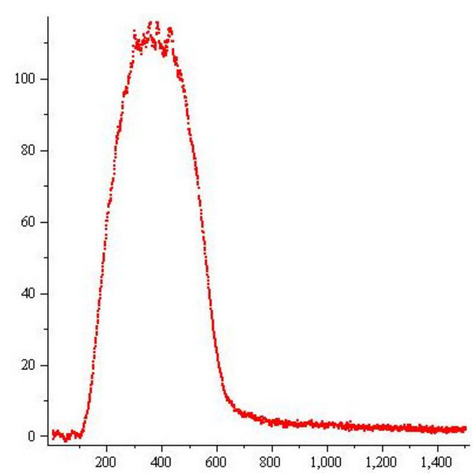

Fig.19 The orbit $(k, x_1(k))$

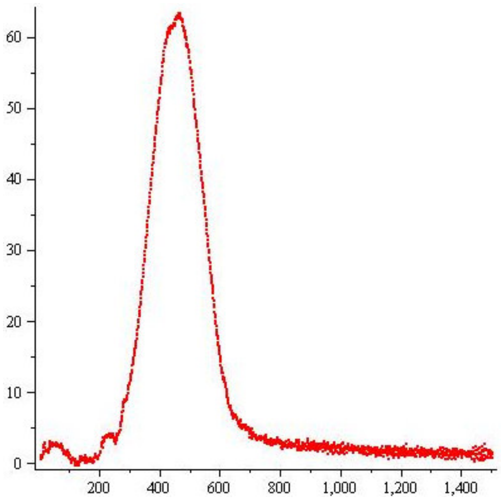

Fig.20 The orbit $(k, x_2(k))$

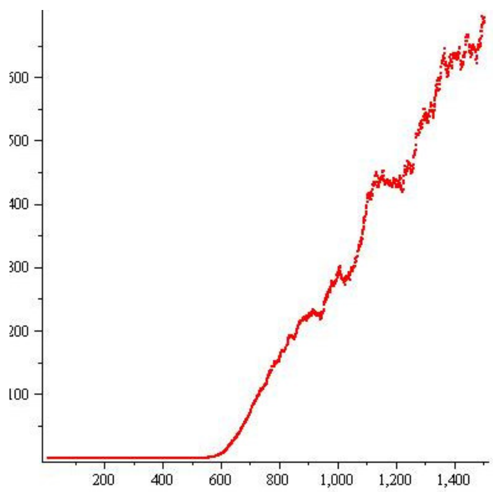

Fig.21 The orbit $(k, p(k))$

The hybrid differential equation associated to (3) is:

$$
\begin{aligned}
dx_1 &= x_1(t-\tau)\left(\frac{1-x_2(t)}{\tau^2 x_1(t)} - a_1 x_1(t) x_2(t)\right)dt + \\
&\quad + \alpha_1(x_1(t) - x_{1e})dw_1(t) + \beta_1 x_1(t) dC_1(t), \\
dx_2 &= \left(\frac{a_2}{a_3} x_2(t)(x_1(t-\tau) - a_3) + x_3(t)\right)dt + \qquad (20) \\
&\quad + \alpha_2(x_2(t) - x_{2e})dw_2(t) + \beta_2 x_2(t) dC_2(t), \\
dx_3 &= (a_4(x_1(t) - x_1(t-\tau)) - a_5 x_3(t))dt + \\
&\quad + \alpha_3(x_3(t) - x_{3e})dw_3(t) + \beta_3 x_3(t) dC_3(t).
\end{aligned}
$$

where x_{1e}, x_{2e}, x_{3e} are the equilibrium points (3) coordinates.

The numerical simulation of (20) is given by:

$$
\begin{aligned}
x_1(k+1) &= x_1(k) + h x_1(k-m)\left(\frac{1-x_2(k)}{m^2 x_1(k)} - a_1 x_1(k) x_2(k)\right) + \\
&\quad + \alpha_1(x_1(k) - x_{ie})G(h) + \beta_1 x_1(k) L(k, z_1), \\
x_2(k+1) &= x_2(k) + h\frac{a_2}{a_3} x_2(k)(x_1(k-m) - a_3) + x_3(k)) + \qquad (21) \\
&\quad + \alpha_2(x_2(k) - x_{2e})G(h) + \beta_2 x_2(k) L(k, z_2), \\
x_3(k+1) &= x_3(k) + h(a_4(x_1(k) - x_1(k-m)) - a_5 x_3(k)) + \\
&\quad + \alpha_3(x_3(k) - x_{3e})G(h) + \beta_3 x_3(k) L(k, z_3).
\end{aligned}
$$

For, $a_1 = 1$, $a_2 = 0.5$, $a_3 = 0.2$, $a_4 = 0.5$, $a_5 = 0.5$, $\alpha_1 = 2$, $\alpha_2 = 3$, $\alpha_3 = 4$, $z_1 = 2$, $z_2 = 8$, $z_3 = 5$, $\beta_i = 1,\ i = 1,2,3,$ $m = 4,$ $h = \frac{1}{1000}$, $n = 1000,$ with Maple 13, we obtain in Fig. 22 the orbit $(k, x_1(k))$, in Fig.23 the orbit $(k, x_2(k))$ and in Fig.24 the orbit $(x_1(k), x_2(k))$.

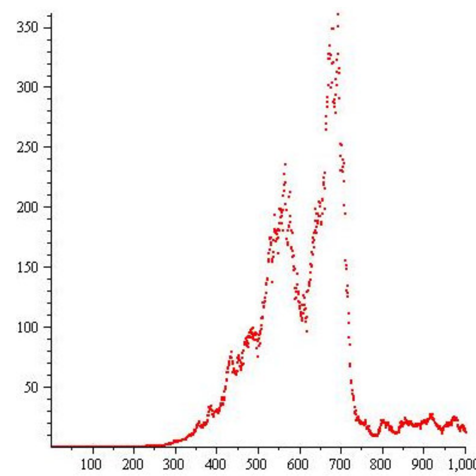

Fig.22 The orbit $(k, x_1(k))$

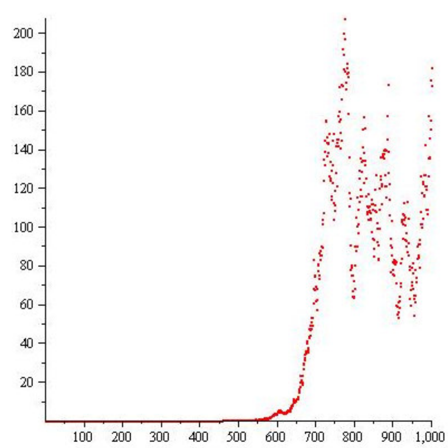

Fig.23 The orbit $(k, x_2(k))$

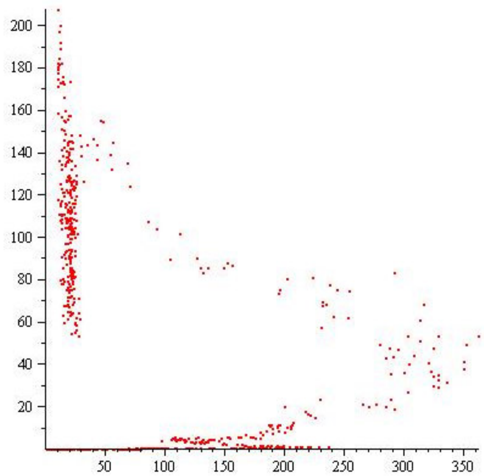

Fig.24 The orbit $(x_1(k), x_2(k))$

5 Conclusion

This paper has introduced SDDE for dynamic stochastic and hybrid models of Internet Networks. The paper has shown that these equations belong to the category of equations that accept a unique solution.

We have described a numerical algorithm in order to determine the approximate solution. The solutions have been visualized with the help of a program in Maple 13, using the Box-Muller method for the simulation of Wiener and Liu processes. A similar study will be conducted for cases in which other confidences will be randomized and fuzziness. Also, we will analyze the stability similarly to what is analyzed in [7]. The models from this paper can be extended considering the fractional integral [3], [14].

Acknowledgement: The research was supported by the Grant with the title "The qualitative analysis and numerical simulation for some economic models which contain evasion and corruption", The National University Research Council from Ministry of Education and Research of Romania (grant No. 520/2009).

References:

[1] Baker C.T.H., Buckwar E., - *Numerical analysis of explicit one step methods for stochastic delay differential equation,* LHS. J.Comput. Math, 3(2000) London, pp. 315-335;

[2] Chunguang L., Guanrong C., Xiaofeng L., Juebang Y., - *Hopf bifurcation in an Internet congestion control model* Chaos, Solitons and Fractals, Vol. 19, 2004, pp. 853-862;

[3] Gastao S. F. F., Delfim F. M. Torres - *Necessary Optimality Conditions for Fractional Action-Like Problems with Intrinsic and Observer Times*, WSEAS Transactions on Mathematics, vol. 7, nr. 1, 2008, pp. 6-11;

[4] Guo S., Feng G., Liao X., Liu Q.- *Hopf bifurcation control in a congestion control model via dynamic delayed feedback*, Chaos 18, 2008, 043104;

[5] Kelly FP., Maulloo A., Tan DKH. - *Rate control in communication networks: shadow prices, proportional fairness, and stability,* J. Oper. Res. Soc. 49, 1998, pp. 237–52;

[6] Liu B., *Uncertainty Theory*, 3rd Edition c 2009 by UTLAB.

[7] Liu J.C., Mackey M.C., -*Stochastic Differential Delay Equation, Moment Stability, And Application To Hematopoietic Stem Cell Regulation System,* SIAM J. APPL. MATH. Society for Industrial and Applied Mathematics Vol. 67, No. 2, 2007, pp. 387–407;

[8] Mircea G., Neamţu M., Opriş D., - *Hopf bifurcations for dynamical systems with time delay and application (in Romanian),* Mirton Publishing House, Timişoara, 2004;

[9] Mircea G., Opris D.- *Internet model with n access link and feedback delay* – Economy Informatics, Vol. III, Number 1/2003, ISSN 1582-7941, Published by Economy Informatics Department and INFOREC Association, with the support of Education and Research Ministry, Economic Publishing House, Bucharest, 2003, pp.78-83;

[10] Mircea G. - *Internet congestion control model,* Proceedings of the 9th WSEAS International Conference on Mathematics and Computers in Business and Economics, Bucharest, Romania, 2008, pp.158-162;

[11] Mircea G., Opris D., - *Neimark-Sacker bifurcation in a discrete time dynamic system,* WSEAS Transactions on Mathematics, Vol. 8, Nr. 2, 2009, pp. 63-72;

[12] Mohamed S.E.A., - *Stochastic schemes for stochastic differential systems with memory. Theory, examples and applications. Progress in Probability,* Birkhauser, 1998;

[13] Tudor M., - *Aproximation schemes for stochastic equation with hereditary argument* Stud. Cerc. Mat. 44, 1992, pp.73-85;

[14] Udriste C., Opris D., Euler-Lagrange-Hamilton Dynamics with Fractional Action, WSEAS Transactions on Mathematics, vol. 7, nr. 1, 2008, pp. 19-30;

[15] Zadeh L.A., *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, 1(1978), pp. 3-28.