Global Diversification, Hedging Diversification, and Default Risk in Bank Equity: An Option-Pricing Model

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Abstract: Many banks diversify their operations, either across different national markets (global diversification), across different borrowers by offsetting credit risks (hedging diversification), or both. Can multiple diversifications provide greater safety for banks? This paper aims to answer this question by using an option-based pricing model to formulate the default risk in bank equity returns under global and hedging diversifications. In particular, we apply Vassalou and Xing’s (2004) formula, which is a nonlinear option-based function of the default probability of an individual bank’s equity return. This formula is calculated using the contingent claim methodology of Black and Scholes (1973) and Merton (1974). We find that the extent of global diversification may provide greater safety for banks, but also that the extent of hedging diversification may not.

Key-words: Default Risk, International Lending Diversification, Loan Portfolio Swap

1 Introduction

In recent years, international banking activities have reached a historical peak (Focarelli and Pozzolo, 2005). However, Acharya, Hasan, and Saunders (2006) prove that lending across different national markets (global diversification) does not produce greater safety for banks. On a related diversification issue, banks can use conventional methods, such as loan portfolio swaps, to hedge against adverse moves in the credit quality of their loans,
since loan portfolio swaps often appeal to banks whose loans are concentrated in particular industries (hedging diversification). For instance, within a few years (2004-2008), credit default swaps (CDS) are used to hedge mortgage-backed securities and market peaks at $62 trillion. However, the American International Group nears default on $14 billion of CDS policies and is bailed out by the government in 2008 (Philips, 2008). Neal (1996) argues that loan portfolio swaps, if used properly, can reduce a bank’s overall credit risk. Can these two diversifications together further provide greater safety for banks?

The answer to this question is largely dependent on an understanding of the diversification characteristics of bank spread management. In practice, spread management is done through a “cost of goods sold” approach in which deposits are the “material” and loans are the “work in process” (Finn and Frederick, 1992). Diversification is a strand of default risk or volatility modeling issue. A standard theoretical paradigm for modeling default risk is the structural model pioneered by Merton (1974). Much of literature, for example, Duffie and Singleton (1997) and Duffee (1999), follows Merton (1974) by explicitly linking the risk of a firm’s default process to the variability in its value of asset. Galluzio (1999), and Chou and Wang (2007) examine the performance of the default risk model based on the barrier option framework, which is a barrier of the market value of firm asset for triggering default prior to the maturity. The primary difference between our model and these papers is that we use Merton’s (1974) option pricing model to formulate default measure.

The purpose of this paper is to follow this approach by providing an option-based model of bank behavior to study the determination of optimal bank interest margins and default probability in equity returns. The results of this paper show how global and hedging diversifications determine the default probability in the bank’s equity return through the optimal interest margin decision. It is found that an increase in the degree of global diversification may decrease the default risk in the bank’s equity return, whereas an increase in the swapped portion of the bank may not. These results support the view that the extent of global diversification may provide greater safety for banks, whereas the extent of hedging diversification may not.

The remainder of this paper is organized as follows: Section 2 presents the framework for global and hedging diversifications. Section 3 describes the basic model of a banking firm under multiple sources of loan diversification. Section 4 and 5 develop the equilibrium and the comparative static properties of the model. Section 6 presents the conclusions.
2 Global and Hedging Diversifications

Our model is myopic in the sense that all financial decisions are made and values are determined within a one-period horizon, $0 \leq t \leq 1$. In order to get closed-form, tractable solutions, a few simplifying assumptions are made. We shall point out when these assumptions affect the qualitative results derived in this paper.

This paper designs the framework to incorporate two distinct diversifications through loan portfolio diversifying globally and loan portfolio swap hedging. In the domestic (home) market, the bank’s demand function is a function of domestic loan interest rate: $R_M$, $M = M(R_M)$. Following Wong (1997), this paper assumes that the bank has some market power in the home lending business, $\partial M / \partial R_M < 0$. Relatively speaking, in the foreign market, the bank takes the loan rate $R_X$ determined in the market, since foreign banks find themselves facing much stiffer competition from local banks (Damanpour, 1986). An increase in $R_X$ increases the bank’s foreign lending activities $X$ and then decreases its home lending, ceteris paribus. An increase in $R_M$ decreases $M$ and then increases $X$, $X = X(R_M)$. These relations reflect the allocation effect of global diversification.

In loan portfolio swaps, the bank swaps some of its loan repayments for a counterparty bank’s loan repayments. This hedging diversification allows each counterparty to diversify some of its credit risk, and hence, both counterparties are better off since there is little common movement in the default risks between the two counterparty banks. Rather than directly mitigating the credit risk, this model follows Sorensen and Bollier (1994) by seeking to price it; doing so, prices the magnitude of potential default risk and estimates a reasonable adjustment to the fixed rate in an interest rate swap. The risk pricing method will depend on a combination of the two banks’ credit conditions. This research prices the swap default risk so that market frictions are minimal; in this way, the bank and the counterparty bank agree on the parameters of the default risk model. Both banks have option positions that create risk in both directions. The bank estimates the required credit-risk adjustment allocated to its counterparty bank’s risk of default as:

$$ CR_B = P_r RV_{B} - P_B RV_C $$  \hspace{1cm} (1) 

where $P_B$ ($P_r$) is the probability that the bank (the counterparty bank) will default on the single default date; and $RV_B$ ($RV_C$) is the value of the option for the bank (the counterparty bank) to replace the swap. This equation indicates that the second term is offset against the first term. If $CR_B > 0$ is larger, then the bank will either receive a higher fixed coupon or pay a lower fixed
coupon.

At \( t = 0 \), a part of initial funds is invested in domestic and foreign loans maturing at \( t = 1 \). Prior to hedging diversification, the bank’s expected loan repayment at \( t = 1 \) is:

\[
V = (1 + R_M)M + e(\omega)(1 + R_X)X \tag{2}
\]

where the exchange rate at \( t = 0 \) is assumed to 1, and is random, and that it is given by \( e(\omega) \), where \( \omega \) is the state of the world at \( t = 1 \). It is assumed that at the beginning period the bank swaps a portion \( \alpha \) of the expected repayments from its counterparty. It is further assumed that the exchange rate in the swap contract equals the exchange rate at \( t = 0 \). Thus, \( RV_B \) in equation (1) is defined as:

\[
RV_B = \alpha[(1 + R_M)M + (1 + R_X)X] \tag{3}
\]

3 The Model

At \( t = 0 \) the bank has the following balance-sheet constraint:

\[
M + X + B = D + K = K(\frac{1}{q} + 1) \tag{4}
\]

where \( B \) is a composite variable denoting the bank’s net position in the default-free asset market. It is assumed that the bank can borrow and lend the default-free assets at a known market rate \( R \). The bank accepts \( D \) dollars of deposits at \( t = 0 \). Following Zarruk and Madura (1992), this paper assumes that the bank provides its depositors with a market rate of return equal to the risk-free rate \( R_p \). Concerns about bank asset quality have prompted the home regulatory authorities to adopt a risk-based system of capital standards. Regulations require equity capital, held by the bank, be tied to a fixed proportion \( q \) of the bank’s deposits, \( K \geq qD \).

According to Zarruk and Madura (1992), the required ratio of capital-to-deposits \( q \) is assumed to be an increasing function of the amount of the loans held by the bank at \( t = 0 \), \( \partial q / \partial M = \partial q / \partial X = q' > 0 \). The bank is fully insured by the Federal Deposit Insurance Corporation (FDIC). For simplicity, this paper assumes that the bank pays a zero deposit insurance premium.

At any time during the period horizon, the value of the bank’s risky assets with loan portfolio swap is:

\[
[(1 - \alpha)[(1 + R_M)M + e(\omega)(1 + R_X)X] \\
A + CR_B = A^0 \quad \text{if embodied risk}=0 \tag{5}
\]

\[
< A^0 \quad \text{if embodied risk}>0
\]

The value of the bank’s earning-asset portfolio then is:

\[
E = A + (1 + R)[K(\frac{1}{q} + 1) - M - X] + RV_C \tag{6}
\]

The bank’s equity return at \( t = 1 \) may now be stated as:
\[ S = \max\{0, E - Z\}, \]
where \( Z = (1 + R_D)K / q \). In the model, \( S \) represents the residual equity value of the bank after meeting all of its obligations. The total cost \( Z \) is assumed to be only the deposit payment cost at \( t = 1 \).

The bank’s objective is to set \( R_M \) to maximize the market value of the Black-Scholes (1973) valuation function defined in terms of equity return, subject to equation (4). The selection of our model’s objective function follows Mullins and Pyle (1994), Lin, Chang, and Lin (2009), and Lin, Lin, and Jou (2009). Our argument, which is based on Merton (1974), suggests that the equity of a banking firm can be viewed as a call option on the bank’s risky-asset portfolio. The primary reason is that after all other obligations have been met, equity holders are residual claimants on the bank’s risky-asset portfolio. The strike price of the call option is the book value of the bank’s net liabilities. Our approach in expressing the default probability, also using Merton’s (1974) model, is similar to the one adopted by Vassalou and Xing (2004). The market value of a banking firm’s underlying risky assets follows a geometric Brownian motion of the form:
\[ dA = \mu A dt + \sigma A dW, \]
where \( \mu \) is an instantaneous drift, \( \sigma \) is an instantaneous volatility, and \( W \) is a standard Wiener process. The market value of equity return \( S \) can then be given by the Black-Scholes (1973) formula for the call option:
\[ \max_{S_{\text{up}}} = AN(d_1) - Ge^{-\delta}N(d_2) \quad (7) \]
where
\[ G = (1 + R_D)K - (1 + R)[K(1 + 1) - M - X] - RV_C \]
\[ d_1 = \frac{1}{\sigma \sqrt{t}} \left[ \ln \frac{A}{G} + \delta + \frac{1}{2} \sigma^2 \right] \]
\[ d_2 = d_1 - \sigma \]
\[ \sigma^2 = \sigma_r^2 + \sigma_i^2 - 2 \rho_{r,i} \sigma_r \sigma_i \]
\[ \delta = R - R_D \]

\( G \) is the book value of the strike price. \( N(\cdot) \) is the cumulative density function of the standard normal distribution. \( \sigma \) is the variance with \( \sigma_r \) and \( \sigma_i \), which are the instantaneous deviation of the rates of return on the risky and default-free repayments, respectively. \( \rho_{r,i} \) is the instantaneous correlation coefficient of the bank’s earning-asset portfolio. \( \delta \) is the spread, which is defined as the difference between \( R \) and \( R_D \) in this model.

In addition, this paper applies Vassalou and Xing (2004), and models the probability of default or the default risk in the bank’s equity return as:

\[ \text{It is recognized that the probability of default in the bank’s equity return is also influenced by changes in macroeconomic climate. It is the case, structural changes expressed by the statistical fat tails can be incorporated into equation (8) (Asosheha, Bagherpour, and Yahyapour, 2008). For simplicity, this consideration is ignored in this paper.} \]
\( P_S = N(-d_3) \) \hspace{1cm} (8)

where

\[ d_3 = \frac{1}{\sigma} \left[ \ln \frac{A}{G} + \mu - \frac{1}{2} \sigma^2 \right] \]

In equation (8), \( \mu \) is defined as the mean of the change in \( \ln V \). Based on the expression of the bank’s equity return in equation (7), \( d_3 \) demonstrates how many standard deviations the log of \( V \) needs to deviate from its mean in order for default to occur. In equation (8), it is noted that although the value of the call option in equation (7) does not depend on \( \mu \), but on \( \delta \), \( P_S \) does, because \( d_3 \) depends on the future value of the bank’s risky-asset portfolio, which is given in \( d_1 \).

4 Equilibrium of the Model

Partially differentiating equation (7) with respect to \( R_M \), the first-order condition is given by:

\[
\frac{\partial S}{\partial R_M} = \frac{\partial A}{\partial R_M} N(d_1) + A \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1}{\partial R_M} \\
- \frac{\partial G}{\partial R_M} \ e^{-\delta} N(d_2) - G \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2}{\partial R_M} = 0
\]

(9)

where

\[
\frac{\partial A}{\partial R_M} = (1-\alpha) \left[ M + (1+R_M) \frac{\partial M}{\partial R_M} \right] + \epsilon(w)(1+R_M) \frac{\partial X}{\partial R_M} + \frac{\partial CR_M}{\partial R_M}
\]

\[
\frac{\partial N(d_2)}{\partial d_2} = \frac{1}{\sqrt{2\pi}} e^{-d_2^2/2} > 0
\]

It is worth noting that \( d_1 \) and \( d_2 \) are functions of \( A \) and \( G \), which are functions of \( R_M \) in equation (7). A problem in applying equation (7) is in calculating the cumulative normal distribution \( N(\cdot) \). In equation (7), there is

\[ d_2^2 = d_1^2 + \sigma^2 - 2d_1\sigma = d_1^2 - 2(\ln \frac{A}{G} + \delta)
\]

(10)

Following Hull (1993), \( N(d_2) \) can be evaluated directly using numerical procedures. One such approximation is:

\[ N(d_2) = 1 - (a_1 k + a_2 k^2 + a_3 k^3) \frac{\partial N(d_2)}{\partial d_2}
\]

(11)
\[
\frac{\partial N(d_2)}{\partial \delta} = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}(d_2^2-2(d_2^*\delta))} = \frac{\partial N(d_1)}{\partial \delta} A G e^\delta \\
\]  

(12)

Further,

\[
A \frac{\partial N(d_1)}{\partial \delta} \frac{d_1}{\partial R_M} - Ge^{-\delta} \frac{\partial N(d_2)}{\partial \delta} \frac{d_2}{\partial R_M} \\
= (A \frac{\partial N(d_1)}{\partial \delta} \frac{d_1}{\partial \delta} A e^\delta) \frac{d_1}{\partial R_M} \\
= 0 \\
\]

(13)

where

\[
\frac{\partial d_2}{\partial R_M} \neq 0
\]

It is evident that these two terms, \(A(\partial N(d_1)/\partial \delta)(\partial d_1/\partial R_M)\) and \(Ge^{-\delta}(\partial N(d_2)/\partial \delta)(\partial d_2/\partial R_M)\), are equal. Imposing this condition on the first-order condition in equation (9), we have the simplified form of equilibrium condition:

\[
\frac{\partial S}{\partial R_M} = \frac{\partial A}{\partial R_M} N(d_1) - \frac{\partial G}{\partial R_M} e^{-\delta} N(d_2) = 0
\]

(14)

where the second-order condition is required to be satisfied, that is, \(\frac{\partial^2 S}{\partial R_M^2} < 0\).

In equation (14), the term associated with \(N(d_2)\) represents the bank’s risk-adjusted value for its marginal net domestic loan interest rate obligations. This marginal value is negative and can be treated as the allocation effect of the bank’s international lending diversification. The term associated with \(N(d_1)\) in equation (14) is the bank’s risk-adjusted value for its marginal risky-asset repayments with the loan portfolio swap of domestic loan interest rate. Based on the first-order condition of the model, this marginal value is negative as well. Given the equilibrium condition in equation (14), the bank determines the optimal domestic loan rate \(R_M^*\) to maximize the market value of its equity return. To obtain the default probability of the bank’s equity return in equation (8), the optimal domestic loan interest rate \(R_M^*\) is substituted. The default probability evaluated at \(R_M^*\) implies that there is an important link between the bank’s optimal domestic loan interest rate and its default risk of the equity return.

5 Comparative Static Results

This paper asks the first question of whether the extent of global diversification leads to greater safety for the bank. This question is answered by differentiating equation (8) evaluated at \(R_M^*\) with respect to \(R_X\):

\[
\frac{\partial P_S}{\partial R_X} = \frac{\partial P_S}{\partial R_X} + \frac{\partial P_S}{\partial R_M} \frac{\partial R_M}{\partial R_X}
\]

(15)

where

\[
\frac{\partial P_S}{\partial R_X} = -\frac{\partial N(d_1)}{\partial d_1} \frac{1}{\frac{\partial d_1}{\partial d_3}} \frac{\partial A}{\partial R_X} < 0
\]
The first term on the right-hand side of equation (15) can be explained as the direct effect, which captures the change in $P_S$ due to an increase in $R_X$ holding $R_M^*$ constant. This direct effect is negative since an increase in the foreign market loan rate increases the value of the bank’s risky-asset repayments, and hence, lower default risk in equity return.

The second term on the right-hand side of equation (15) can be explained as the indirect effect. This effect represents the impact on the $P_S$ from changes in $R_X$ through $R_M^*$. The sign of this effect is determined by the product of how changes in $R_X$ affect $R_M^*$ as well as how changes in $R_M^*$ affect $P_S$.

First, the impact on $P_S$ from changes in $R_M^*$ is positive ($\partial P_S / \partial R_M^* > 0$) since the loan rate elasticity effect is negative. Second, the impact on $R_M^*$ from changes in $R_X$ (\( \partial R_M^* / \partial R_X \)) is governed by $\partial^2 S / \partial R_M^* \partial R_X$. The term associated with $N(d_1)$ in $\partial^2 S / \partial R_M^* \partial R_X$ is explained as the mean effect on $\partial A / \partial R_M^*$, while the term associated with $\partial d_1 / \partial R_M^*$ is explained as the variance effect. The mean effect is positive since $\partial^2 A / \partial R_M^* \partial R_X > 0$. The variance effect is positive as well since there are $\partial A / \partial R_M^* < 0$ and $\partial d_1 / \partial R_X > 0$. Thus, we have $\partial^2 S / \partial R_M^* \partial R_X > 0$; accordingly, there is $\partial R_M^* / \partial R_X > 0$. In light of previous work, we have the positive indirect effect.
Proposition 1: An increase in the foreign loan market rate decreases the default risk in the bank’s equity return.

As the bank faces an increasing loan rate in the foreign market, it provides a return to a larger foreign loan base. One way the bank may attempt to augment its total returns is by shifting its lending activities to foreign markets and away from the domestic market. If domestic loan demand is relatively rate-elastic, a smaller domestic lending business is possible at an increased domestic loan rate. The default risk in the bank’s equity return is generally higher, the higher the degree of global diversification through an increased loan rate in the domestic market (the indirect effect). As stated earlier, the default risk is directly lower, the higher the degree of global diversification (the direct effect). The sign of this total effect is indeterminate.

However, Proposition 1 provides us with a belief that the total effect will be negative should the direct effect be partially offset by the indirect effect. The rationale is that an increase in the loan rate in the foreign market makes lending in the domestic loan market less attractive relative to that in the foreign market. An increase in foreign lending decreases the default risk in the bank’s equity return while decreasing domestic lending due to an increase in foreign lending, thereby increasing the default risk. As a result, this induces the bank to cut risky lending by increasing global diversification.

Next, the question is whether the extent of the swap transaction leads to greater safety for the bank. This question is answered by differentiating equation (8) evaluated at \( R_M^* \) with respect to \( \alpha \):

\[
\frac{dP_S}{d\alpha} = \frac{\partial P_S}{\partial \alpha} + \frac{\partial P_S}{\partial R_M} \frac{\partial R_M}{\partial \alpha} \tag{16}
\]

where

\[
\frac{\partial P_S}{\partial \alpha} = -\frac{\partial N(d_1)}{\partial \alpha} \frac{1}{\sigma} \frac{\partial A}{\partial \alpha}
\]

\[
\frac{\partial A}{\partial \alpha} = (P_C - 1)(1 + R_M)M + (P_C - e(w))(1 + R_X)X
\]

\[
\frac{\partial R_M}{\partial \alpha} = -\frac{\partial^2 S}{\partial R_M \partial \alpha} - \frac{\partial^2 S}{\partial R_M^2}
\]

\[
+ \frac{\partial A}{\partial R_M} \left( \frac{\partial N(d_1)}{\partial d_1} - \frac{N(d_1)}{N(d_2)} \frac{\partial d_1}{\partial \alpha} \right) \frac{\partial d_1}{\partial \alpha}
\]

\[
\frac{\partial^2 S}{\partial R_M \partial \alpha} = \frac{\partial^2 A}{\partial R_M \partial \alpha} N(d_1)
\]

\[-e(w)(1 + R_X) \frac{\partial X}{\partial R_M}
\]

\[
\frac{\partial^2 A}{\partial R_M \partial \alpha} = (P_C - 1)[M + (1 + R_M) \frac{\partial M}{\partial R_M}]
\]

\[
\frac{\partial d_1}{\partial \alpha} = \frac{1}{\sigma} \left( \frac{\alpha}{A} \frac{\partial A}{\partial \alpha} - \frac{\alpha}{G} \frac{\partial G}{\partial \alpha} \right)
\]

The first term on the right-hand side of equation (16) can be interpreted as the direct effect, while the second term can be interpreted as the indirect effect. The direct effect captures the change in \( P_S \) due to an increase in \( \alpha \), holding \( R_M^* \)
constant. If the home currency is depreciated, the term \((P_c - \epsilon(w))\) in \(\partial A/\partial \alpha\) is negative in sign. Under the circumstances, the direct effect is positive \((\partial P_s / \partial \alpha > 0)\). An increase in \(\alpha\) makes loans more risky to grant, and hence higher default risk in equity return, ceteris paribus.

The indirect effect represents the loan rate effect on \(P_s\) from a change in \(\alpha\). The sign of this term is determined by how changes in \(\alpha\) effect the bank’s optimal loan rate, as well as by the relationship between \(P_s\) and \(R^*_s\). As stated previously, \(\partial P_s / \partial R^*_s\) is positive. In addition, \(\partial^2 S / \partial R^*_s \partial \alpha\) is positive if the home currency is depreciated. Thus, \(\partial R^*_s / \partial \alpha > 0\). The following proposition states the result of equation (16).

**Proposition 2:** When the currency in the home country is depreciated, an increase in the swapped portion of the bank increases its default risk in the equity return.

An increase in the amount of the loan portfolio swap transaction increases the bank’s domestic loan rate. The bank now provides a return to a larger hedging base. To make this adjustment, the bank shifts its investments into foreign lending and away from domestic lending. This adjustment indicates that an increase in international lending diversification increases the bank’s default risk in equity return, since an increase in \(\alpha\) decreases the bank’s risky-asset repayment or makes loans more risky to grant. Thus, the multiple diversifications of international lending associated with the increasing loan portfolio swap transaction may produce less safety for the bank.

**6 Conclusion**

This paper develops an option-based firm-theoretical model of the default risk in bank equity return under global and hedging diversifications. The model is utilized to show how the international lending allocation and loan portfolio swap transaction determine the optimal domestic loan rate, and hence the default risk related to equity return. Specifically, it is found that the default risk is negatively related to the foreign market loan rate, and positively to the swapped portion of the bank’s risky-asset portfolio. An increase in the extent of global diversification, captured by an increase in the foreign market loan rate, decreases the default risk, whereas an increase in the extent of hedging diversification, demonstrated by an increase in the hedging portion, increases the default risk. The former can be motivated based on a global diversification, which may produce greater safety for the bank; while the latter, viewed as a hedging diversification, may not.

In financial markets without strict global and hedging diversifications, other factors can and will affect the lending...
portfolio structure. For example, industrial diversification and competition among banks can play a very important role, as will the more extreme problems of game-play asymmetries. Although such concerns are beyond the scope of this paper, the role played by lending structures in affecting the quality of equity returns, and hence related to the issue of whether or not producing greater safety for banks, is important.

References


