The Coupled Method Fuzzy-AHP Applies to Solve Multi-criteria Decision Making Problems

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Abstract: The multi-criteria decision making (MCDM) problems with fuzzy preference information on alternatives are essential problems of the importance of weighting and ranking. In order to solve this problem, Analytical Hierarchy Process (AHP) and fuzzy comprehensive evaluation method are coupled to form a new approach named Fuzzy-AHP. This method is different from the traditional FAHP, which used to facilitate the pairwise comparison process and avoid the complex and unreliable process of comparing fuzzy utilities. It utilizes the advantage of AHP on computing index weight and comparing index in the same row than at ranking and the advantage of fuzzy comprehensive evaluation method on establishing quantitative indexes membership and qualitative indexes membership and classifying level. Finally, a numerical example is presented to clarify the methodology, the model evaluation results showed that the proposed system is able to provide very good solution both in accuracy and speed for the top managers.

Key-Words: MCDM, Fuzzy, AHP, index, weighing, membership degree, matrix

1 Introduction

The typical multi-criteria decision making (MCDM) problem often requires the decision maker to provide qualitative assessments for determining the performance of each alternative with respect to each criterion and the relative importance of the evaluation criteria with respect to the overall objective of the problem [1]. During the past two decades, there has been a steady growth in the number of MCDM methods for assisting decision making with multiple objectives. One of the commonly used methods for multi attribute decision-making is analytic hierarchy process (AHP).

The AHP, which was developed by Professor Saaty in the early 1970s, is a subjective tool with which to analyze, based on a crisp 9-point scale, the qualitative criteria needed to generate alternative priorities and preferences [2]. The AHP enables decision makers to structure complex problems in a simple hierarchical form, and to evaluate a large number of quantitative and qualitative factors in a systematic manner despite the presence of multiple conflicting criteria. The AHP has proven to be a powerful decision analysis technique in the area of MCDM, and has been successfully applied to the tackling of MCDM problems. In general, the AHP has been utilized in various areas such as R&D planning, the selection of the best policy alternatives once a set of alternatives has been identified, the allocation of resources, the determination of requirements, prediction of outcomes, design systems, measurement of performance, and the optimization and resolution of decision conflicts [3].
So it is a simple and practical multi-criteria evaluation method applied in many fields. Fuzzy comprehensive evaluation is a branch of fuzzy mathematic which is created by a well-known electronic engineer and cybernetics expert L.A. Zadeh and dealt with the fuzzy phenomenon with mathematical method [4].

The best approach of accounting for uncertainty and ambiguity is to take advantage of the tools provided by the theory of fuzzy sets [5]. In the real world, precise data pertaining to measurement indicators is very hard to extract from human judgments. This is because human preferences encompass a degree of uncertainty, and decision makers may very well be reluctant or unable to assign crisp numerical values to comparison judgments. Decision makers also prefer natural language expressions over exact numbers when assessing criteria and alternatives. Fuzzy set theory deals with ambiguous or not well-defined situations. By approximating information and uncertainty where the generation of reasonable alternatives to problems needing decisions is concerned, it effectively resembles human thoughts and perceptions. Fuzzy theory includes elements such as fuzzy set, membership function, and the fuzzy numbers used to efficiently change vague information into useful data. Fuzzy set theory uses groups of data with boundaries that feature lower, median, and upper values that are not sharply defined. Because most of the decision-making in the real world takes place amidst situations where pertinent data and the sequences of possible actions are not precisely known, the merit of using the fuzzy approach is that it expresses the relative importance of the alternatives and the criteria with fuzzy numbers rather than crisp ones. A fuzzy set is characterized by a membership function, which assigns a membership range value between 0 and 1 to each criterion and alternative [6]. As a result, the fuzzy mathematic method has been widely used in the field of systematic evaluation.

The AHP is better at computing index weight and comparing index in the same row than at ranking, while fuzzy comprehensive evaluation method can mostly reflect the nature of subjective assessment without limitation of scale [7], but its weight is usually given by the experts based on experiences can not help with subjectivity. The advantages of the two methods are coupling to form a new method: Fuzzy-AHP (FAHP), which combines the qualitative analysis with quantitative analysis to make subjective estimates more objective.

FAHP has grown rapidly, due to be continuously refined and improved, which has advantage on dealing with complex issues of multi-level assessment and problems of decision-making, and has gradually expanded to apply on several fields in recent years. At the same time, the models increasingly enriched and became more and more different as a result of the complexity of various fields. There are many FAHP methods proposed by various authors. The earliest work in FAHP appears in refs [8]; Chang [9] used triangular fuzzy membership value for pair-wise comparison. Ching [10] proposed a new FAHP algorithm for evaluating naval tactical missile systems, while in China more detailed FAHP literature can be found in refs. Li Wanqing et al [11] combined fuzzy set and the theory of AHP to set a fuzzy overall analysis model, which is used in the engineering contracting risk analyses. JIN Jingyu et al [12] combined AHP (Analytic Hierarchy Process) and fuzzy evaluation to get a comprehensive evaluation method for evaluating venture capital environments of big cities in China.

During the issue of multiple criteria decision-making (MDCM), not only the weight value of each criterion should be obtained, but the average intensity of criterion's importance and common consensus of experts also need to be incorporated into the consideration [13]. So we describe an application model that couples Analytical Hierarchy Process (AHP) and fuzzy comprehensive evaluation method, named Fuzzy-AHP. This method does not follow the traditional FAHP, which used to facilitate the pairwise comparison process and avoid the complex and unreliable process of comparing fuzzy utilities. It utilizes the advantage of AHP on computing index weight and comparing index in the same row than at ranking and the advantage of fuzzy comprehensive evaluation method on establishing quantitative indexes membership and qualitative indexes membership and classifying level to make the final comprehensive evaluation. At last it applies on eco-campus assessment and the result shows that it not only solve the multilevel of indexes, but also own a good engineering application.

2 Methodology

In this section, two different MCDM methods are presented. The first one is AHP proposed by Saaty. The second is Fuzzy comprehensive evaluation method. Finally, a coupled method Fuzzy-AHP is introduced.

2.1 Analytic hierarchy process (AHP)
2.1.1 Evaluation Factors Sets Establishment
Establishing an evaluation factor set: $U = \{U_1, U_2, \ldots, U_n\}$

The establishment of indexes should follow four principles: systematic, scientific, comparability, and feasibility. The Delphi method is used to construct a hierarchical structure for the indexes and Delphi questionnaire survey will be repeated through the indexes are divided into several levels according to their attributes.

The Delphi method accumulates and analyzes the results of anonymous experts that communicate in written, discussion and feedback formats on a particular topic. Anonymous experts share knowledge, skills, expertise and opinions until a mutual consensus is achieved[14].

The Delphi method consists of five procedures.
1) select the anonymous experts.
2) conduct the first round of a survey.
3) conduct the second round of a questionnaire survey.
4) conduct the third round of a questionnaire survey.
5) integrate expert opinions and to reach a consensus.

Steps 3) and 4) are normally repeated until a consensus is reached on a particular topic.

The decision-making group probably should not be too large, i.e. a minimum of five to a maximum of about 50 suggested that the modified Delphi method summarize expert opinions on a range from 10 to 30. So, in this study, 20 experts participated in the modified Delphi method decision group[15].

The hierarchical structure of indexes can be roughly divided into three categories[16]:
1) The highest level: There is only one element factor in this level, in general, which is the target analysis, thus, also known as target level.
2) The middle level: This level includes intermediate links used for the realization of target level. It can be composed of a number of levels, including the criteria and the sub-criteria, thus, also known as criteria level.
3) The lowest level: This level includes a variety of measures and decision-making schemes for achieving the target, thus, also known as scheme level.

2.1.2 Weight Sets Determination
After the hierarchical structure has been established, a questionnaire based on the proposed structure should be formulated. The main goal of the questionnaire is to compare pairs of element, or criteria, in each level with respect to every, element in the next higher level. The nine-point scale is recommended.

The weight sets are determined by AHP can be written in the following form:

$$A = \{A_1, A_2, \ldots, A_n\}$$

With:

- $A_n$ —The matrix of criteria level $n$.

The major steps of determination of weight sets:

Step 1 Design Delphi expert questionnaire. The questionnaires aiming at determining the degrees of preference by the help of the pairwise comparisons among the attributes are prepared. The questionnaires facilitate the answering of pairwise comparison questions[17].

Though questionnaires at each level, the structure matrix of pairwise comparison judgments are formulated as follows:

$$A = (a_{ij})_{n \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Suppose there are factors: $X_1, X_2, \ldots, X_n$, and take two factors: $X_i$ and $X_j$ each time. $a_{ij}$ is the ratio of the importance of $X_i$ and $X_j$, determined by the Saaty's 1-9 scale [18] in Table 1.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Compare Factor i and j</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal Importance</td>
</tr>
<tr>
<td>3</td>
<td>Weak Importance</td>
</tr>
<tr>
<td>5</td>
<td>Strong Importance</td>
</tr>
<tr>
<td>7</td>
<td>Very Strong Importance</td>
</tr>
<tr>
<td>9</td>
<td>Extreme Importance</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate Value of the Comparison</td>
</tr>
<tr>
<td></td>
<td>Reciprocal</td>
</tr>
</tbody>
</table>

The judgement matrix ($A$) is a reciprocal matrix with $a_{ij} > 0, a_{ij} = 1/a_{ji}, a_{ii} = 1$. The nature of it determines that we only need to obtain upper (or lower) triangular $n(n-1)/2$ factors of $n \times n$ judgment matrix for analysis. That is to say we only need $n(n-1)/2$ judgments[16].

Step 2 Solve judgment matrix and calculate eigenvector and eigenvalue.
The method of calculating the weights are as follows:

\[ AW = \lambda_{\text{max}} W \]

(3)

With:

- \( \lambda_{\text{max}} \) — The largest eigenvalue about A
- W — Corresponding eigenvector

Table 2 Average Random Index.

<table>
<thead>
<tr>
<th>Matrix order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.I.</td>
<td>0</td>
<td>0.52</td>
<td>0.89</td>
<td>1.12</td>
<td>1.26</td>
<td>1.36</td>
<td>1.41</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix order</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.I.</td>
<td>1.46</td>
<td>1.49</td>
<td>1.52</td>
<td>1.54</td>
<td>1.56</td>
<td>1.58</td>
<td>1.59</td>
</tr>
</tbody>
</table>

We can normalize \( w \) to obtain a new weight vector as follows:

\[ W = (w_1, w_2, \ldots, w_n)^T \]

(4)

Matlab software can be used to calculate the largest eigenvalue and the corresponding eigenvector.

Step 3 Test the consistency of each matrix.

In order to control the result of the method, the consistency ratio need to be calculated. The deviations from consistency are expressed by the following equation[19]:

Consistency Index (C.I.):

\[ C.I. = \frac{\lambda_{\text{max}} - n}{n-1} \]

(5)

The consistency ratio (CR) is used to directly estimate the consistency of pairwise comparisons. The CR is computed by dividing the CI by a value obtained from the table of Random Consistency Index (RI) created by Saaty:

Consistency Ratio (C.R.):

\[ C.R. = \frac{C.I.}{R.I.} \]

(6)

Where \( R.I. \) is the average random consistency index, its value can be obtained by look-up table. As shown in Table 2.

\[
\begin{align*}
C.R. &< 0.1 & \text{acceptable} \\
C.R. &\geq 0.1 & \text{unacceptable}
\end{align*}
\]

(7)

When C.R is unacceptable, you should rectify the matrix appropriately.

Step 4 Calculate complex weight.

The complex weight is calculated from top to bottom, and then test the consistency of the synthesis matrix (The test method is the same as step 3).

2.2 Fuzzy Comprehensive Evaluation Method

2.2.1 Evaluation Set Establishment

The evaluation sets are schemes can be written in the following form:

\[ V = \{V_1, V_2, \ldots, V_p\} \]

(8)

With:

- \( V_p \) — Comments given by experts.

For example, \( V = \) (excellent, good, medium, inferior), each comments are equal to a value to form a vector: \([100 90 80 60]^T\)

2.2.2 Single Index Assessment

Assessment of single index is a set of fuzzy mapping:

\[ f: U \rightarrow V \]

In the process of index system establishment, different methods should be taken to determine membership because of the quantitative and qualitative indexes.

2.2.3 Establishment of Quantitative Indexes Membership

Based on the practical situation of evaluation system, the index system of fuzzy comprehensive evaluation is established on the view of representative, systematic, applicability, etc. Fuzzy evaluation matrix of a single index membership is determined by the sample data. The ultimate goal of Fuzzy comprehensive evaluation is to select a relatively best program through comparing memberships. Suppose \( m \) evaluation programs, including \( n \) indexes each one, make up sample data set as follows[20]:

\[ \{x(i,j) | i = 1 \ldots n, j = 1 \ldots m\} \]

(9)

Where \( x(i,j) \) is the index value and its value is non-negative.

In order to eliminate the effect of evaluation indexes dimensions, determine the membership degree matrixes of fuzzy evaluation, and maintain the information changes with evaluation indexes values, we need deal with \( x(i,j) \) for standardization as follows:

The greater, the more superior type equation:

\[ r(i, j) = x(i,j) / \left[ x_{\text{max}(i)} + x_{\text{min}(i)} \right] \]

(10)

The smaller, the more superior type equation:

\[ r(i, j) = \left[ x_{\text{max}(i)} + x_{\text{min}(i)} - x(i,j) \right] / \left[ x_{\text{max}(i)} + x_{\text{min}(i)} \right] \]

(11)
The more moderate, the more superior type equation:

\[ r(i,j) = \frac{x(u(i,j) - x(v(i,j)))}{x(u(i,j)) - x(v(i,j))} \]  \[ \text{if } x(u(i,j)) \leq x(v(i,j)) \]  \[ r(i,j) = 1 \]  \[ \text{if } x(u(i,j)) > x(v(i,j)) \]  \[ (12) \]

Where, \( x(u(i,j)) \), \( x(v(i,j)) \), \( x(m(id)) \) are the minimum value, the maximum value, the middle of the optimal value and \( r(i,j) \) is evaluation index value after standardization (the ith index of the jth program’ membership).

Then we can get a fuzzy evaluation membership matrix:

\[ R = (r(i,j))_{n \times m} \]  \[ (13) \]

2.2.4 Establishment of Qualitative Indexes Membership

In order to establish evaluation matrix, Specialist Grading is always used. Firstly, experts are chosen as judgement group, in addition, each expert evaluates every of the evaluation factors. Suppose there are \( k \) of \( i \) experts evaluate \( V_{i} \) as \( u_{i} \), finally, we can get a fuzzy evaluation membership matrix: about \( V_{j} \) as follows \[ [11] \]:

\[ R_{i} = \left( \frac{u_{1}}{1}, \frac{u_{2}}{1}, \cdots, \frac{u_{k}}{1} \right) = (V_{1}I, V_{2}I, \cdots, V_{n}I) \]  \[ (14) \]

Where, \( \sum_{k=1}^{K} ; i = 1, 2, \cdots, s ; j = 1, 2, \cdots, n ; \)
\( k = 1, 2, \cdots, m. \)

It is assumed that each expert opinion on the equal treatment. The establishment of single factor evaluation membership matrix can be written as:

\[ R = (r_{jk})_{n \times m} \]  \[ (15) \]

Where: \( r_{jk} \) said that \( u_{ij} \) was rated \( V_{k} \), indexes of membership degree.

2.2.5 Comprehensive Evaluation

As the membership vectors of the lowest level indexes are established, we can make the first grade fuzzy comprehensive evaluation as follows:

\[ B_{k} = A_{r} \cdot R_{i} = [b_{1i}, b_{2j}, \cdots, b_{ni}] (i = 1, 2, \cdots, s) \]  \[ (16) \]

Where, “\( \cdot \)” is synthetical computing operator, which adopts Weighted Average Algorithm such as \( M(\cdot,+) \).

Fuzzy Mathematics has four models such as \( M(\land, \lor) \), \( M(\lor, \land) \), \( M(\land, \land) \), \( M(\lor, \lor) \) for membership degree conversion. Long-term results show that \( M(\land,\lor) \) is widely accepted \[ [21] \]. Then make the second grade fuzzy comprehensive evaluation as follows:

\[ B = A \cdot R = A \cdot [B_{1}, B_{2}, \cdots, B_{n}] \]  \[ (17) \]

With:

\( R \)—The final membership degree vector
\( A \)—The weight of all indexes

The establishment of the fuzzy model degree of membership is converted from bottom to top until the overall goal of the final degree membership vector is obtained.

At last make the final comprehensive evaluation and classification.

2.3 Fuzzy-AHP Comprehensive Evaluation Method

Fuzzy-AHP methodology extends Saaty’s AHP by combining it with the fuzzy set theory \[ [19] \]. FAHP method is a systematic approach to the alternative selection and justification problem by using the concepts of fuzzy set theory and hierarchical structure analysis. The decision maker can specify preferences in the form of natural language or numerical value about the importance of each performance attribute. The system combines these preferences using FAHP with existing data. In the FAHP method, the pair-wise comparisons in the judgment matrix are fuzzy numbers and use fuzzy arithmetic and fuzzy aggregation operators, the procedure calculates a sequence of weight vectors that will be used to choose main attribute. In some situations, the decision maker can specify preferences in the form of AHP numerical pair-wise comparison introduced by Saaty in the form of nine point of scale of importance between two elements \[ [22] \].

In this study, we combined the advantage of LI Wanqing “fuzzy AHP model with JIN Jingyu” fuzzy AHP model, because the steps of this approach are easier and more appropriate than the other fuzzy-AHP approaches and the Steps of fuzzy AHP shown in Fig.1. The proposed approach was described in detail above.

3 Numerical example

Eco-campus Assessment is the typical multi-criteria decision making problem, at present, the theory and construction of eco-campus are still at the exploratory stage in China and abroad and not yet a mature and detailed assessments of eco-campus system for reference \[ [23] \]. Experts and scholars do not appear to have reached a consensus on the method used for eco-campus assessment: Cui Meng.
AHP has been used for ecological evaluation on a campus in the north.

Collect data from literatures

Construction of AHP hierarchy

Design Delphi questionnaires

Are results consistent?

Index hierarchy set

Compute the weights of the index

Design AHP questionnaires

Are results consistent?

Establish evaluation set

Calculate membership matrix

Comprehensive evaluation

Fig. 1 Steps of fuzzy AHP
of China; Zhang Guozhen [25] has used AHP to make evaluation on primary school in Taiwan; Wang Yuan [26] has used fuzzy comprehensive evaluation method to evaluate the old campus of TianJin PolyTechnic University. Therefore, studying in-depth on Eco-campuss evaluation methods will help managers make the right choice and assessment in process of eco-campus construction.

Based on Literature [25], Which targets primary and junior high schools or lower educational institutes, with the support of former study, and coordinates the key factors on environmental consciousness during the planning phases. Its methods are primarily based on experts’ Delphi Technique to converge assessment levels, principles, convert into Analytic Hierarchy Process (AHP) and finally establish an assessment hierarchy system. Expert choice software is used to solve the AHP in the literature. At last the weights and ranks were obtained for each criteria in Table 3, at the same time, the weights and ranks of schemes relevant to each criteria were generated in Tables 4 to 8. And then the complex weight is calculated from top to bottom in Table 9.

Table 3 The weight and rank of criteria level relevant to A

<table>
<thead>
<tr>
<th>Criteria level</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impacts balance of regional environment A1</td>
<td>0.14</td>
<td>4</td>
</tr>
<tr>
<td>Water balance of regional environment A2</td>
<td>0.28</td>
<td>1</td>
</tr>
<tr>
<td>Green balance of regional environment A3</td>
<td>0.24</td>
<td>2</td>
</tr>
<tr>
<td>Comfort ability &amp; energy saving of regional environment A4</td>
<td>0.21</td>
<td>3</td>
</tr>
<tr>
<td>Waste &amp; garbage management balance of regional environment A5</td>
<td>0.13</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5 The weight and rank of scheme level relevant to A1

<table>
<thead>
<tr>
<th>Scheme level</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base condition A11</td>
<td>0.14</td>
<td>4</td>
</tr>
<tr>
<td>Environmental use A12</td>
<td>0.36</td>
<td>1</td>
</tr>
<tr>
<td>Resources sharing and inter growth A13</td>
<td>0.21</td>
<td>3</td>
</tr>
<tr>
<td>Minimum impact on the community environment A14</td>
<td>0.29</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6 The weight and rank of scheme level relevant to A2

<table>
<thead>
<tr>
<th>Scheme level</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water reduction A21</td>
<td>0.16</td>
<td>4</td>
</tr>
<tr>
<td>Water use and storage A22</td>
<td>0.17</td>
<td>3</td>
</tr>
<tr>
<td>Water conservation A23</td>
<td>0.19</td>
<td>2</td>
</tr>
<tr>
<td>Flood penetration A24</td>
<td>0.35</td>
<td>1</td>
</tr>
<tr>
<td>Ecological teaching A25</td>
<td>0.13</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 7 The weight and rank of scheme level relevant to A3

<table>
<thead>
<tr>
<th>Scheme level</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green level A31</td>
<td>0.43</td>
<td>1</td>
</tr>
<tr>
<td>Vegetation Diversity A32</td>
<td>0.35</td>
<td>2</td>
</tr>
<tr>
<td>Ecological Teaching A33</td>
<td>0.22</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 8 The weight and rank of scheme level relevant to A4

<table>
<thead>
<tr>
<th>Scheme level</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air quality A41</td>
<td>0.41</td>
<td>1</td>
</tr>
<tr>
<td>Energy Saving A42</td>
<td>0.21</td>
<td>2</td>
</tr>
<tr>
<td>Illumination quality A43</td>
<td>0.18</td>
<td>4</td>
</tr>
<tr>
<td>Acoustic Environment A44</td>
<td>0.20</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 8: The weight and rank of scheme level relevant to A5

<table>
<thead>
<tr>
<th>Scheme level</th>
<th>Weight</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recycling A51</td>
<td>0.51</td>
<td>1</td>
</tr>
<tr>
<td>Kitchen waste A52</td>
<td>0.15</td>
<td>3</td>
</tr>
<tr>
<td>Reduction A53</td>
<td>0.34</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 9: The complex weight and rank of assessment system of eco-school

<table>
<thead>
<tr>
<th>criteria level</th>
<th>Scheme level</th>
<th>Weight</th>
<th>Rank</th>
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</thead>
<tbody>
<tr>
<td>A1</td>
<td>A11</td>
<td>0.0196</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>A12</td>
<td>0.0504</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>A13</td>
<td>0.0294</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>A14</td>
<td>0.0406</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>A21</td>
<td>0.0448</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>A22</td>
<td>0.0476</td>
<td>9</td>
</tr>
<tr>
<td>A2</td>
<td>A23</td>
<td>0.0532</td>
<td>6</td>
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<tr>
<td></td>
<td>A24</td>
<td>0.0980</td>
<td>2</td>
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<td></td>
<td>A25</td>
<td>0.0364</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>A31</td>
<td>0.1032</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>A32</td>
<td>0.0840</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>A33</td>
<td>0.0528</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>A41</td>
<td>0.0861</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>A42</td>
<td>0.0441</td>
<td>12</td>
</tr>
<tr>
<td>A4</td>
<td>A43</td>
<td>0.0378</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>A44</td>
<td>0.0420</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>A51</td>
<td>0.0663</td>
<td>5</td>
</tr>
<tr>
<td>A5</td>
<td>A52</td>
<td>0.0195</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>A53</td>
<td>0.0442</td>
<td>11</td>
</tr>
</tbody>
</table>

Index weight matrixes that are suitable for primary and junior schools are as follows:

\[ A = [0.14, 0.28, 0.24, 0.21, 0.13] \]
\[ A1 = [0.0196, 0.0504, 0.0294, 0.0406] \]
\[ A2 = [0.0448, 0.0476, 0.0532, 0.0980, 0.0364] \]
\[ A3 = [0.1032, 0.0840, 0.0528] \]
\[ A4 = [0.0861, 0.0441, 0.0378, 0.0420] \]
\[ A5 = [0.0663, 0.0195, 0.0442] \]

The establishment of the evaluation set:

\[ V = (\text{excellent, good, medium, inferior}) \]

Each comment is equal to a value to form a vector:

\[ C = [100, 90, 80, 60]^T \]

The final evaluation is \( S \) in Table 10.

Table 10: Level of eco-campus

<table>
<thead>
<tr>
<th>Eco-campus level</th>
<th>The scope of eco-index</th>
</tr>
</thead>
<tbody>
<tr>
<td>excellent</td>
<td>( 90 \leq S \leq 100 )</td>
</tr>
<tr>
<td>good</td>
<td>( 80 \leq S &lt; 90 )</td>
</tr>
<tr>
<td>medium</td>
<td>( 60 \leq S &lt; 80 )</td>
</tr>
<tr>
<td>inferior</td>
<td>( S &lt; 60 )</td>
</tr>
</tbody>
</table>

Suppose we select 20 experts to assess the content of the school to get fuzzy membership vectors as follows:

\[ R1 = \begin{bmatrix}
0.90 & 0.10 & 0 & 0 \\
0 & 0 & 0.85 & 0.15 \\
0.1 & 0.85 & 0.05 & 0 \\
0.75 & 0.2 & 0.05 & 0 
\end{bmatrix} \]
\[ R2 = \begin{bmatrix}
0 & 0 & 0.8 & 0.2 \\
0 & 0 & 0.05 & 0.95 \\
0.85 & 0.15 & 0 & 0 \\
0 & 0.55 & 0.45 & 0 \\
0 & 0 & 0.75 & 0.25 
\end{bmatrix} \]
\[ R3 = \begin{bmatrix}
0 & 0.15 & 0.8 & 0.05 \\
0 & 0 & 0.1 & 0.9 \\
0 & 0.05 & 0.8 & 0.15 \\
0.75 & 0.25 & 0 & 0 \\
0.05 & 0.75 & 0.2 & 0 \\
0.1 & 0.9 & 0 & 0 \\
0 & 0 & 0.8 & 0.2 
\end{bmatrix} \]
First grade fuzzy comprehensive evaluation
B1 = A1 · R1 = \begin{bmatrix} 0.0510 & 0.0351 & 0.0463 & 0.0076 \end{bmatrix}
Normalized as:
B1 = \begin{bmatrix} 0.3643 & 0.2507 & 0.3307 & 0.0543 \end{bmatrix}
In the same way we can get B2, B3, B4, B5 as follows:
B2 = \begin{bmatrix} 0.1614 & 0.2211 & 0.3914 & 0.2261 \end{bmatrix}
B3 = \begin{bmatrix} 0.00754 & 0.5550 & 0.3696 \end{bmatrix}
B4 = \begin{bmatrix} 0.3362 & 0.4219 & 0.2019 & 0.0400 \end{bmatrix}
B5 = \begin{bmatrix} 0.2043 & 0.2791 & 0.5166 \end{bmatrix}
Make the second grade fuzzy comprehensive evaluation:
B = A · [B1 B2 B3 B4 B5] = \begin{bmatrix} 0.1668 & 0.2302 & 0.3678 & 0.2352 \end{bmatrix}
The final evaluation:
S = B · C = 80.9340
As a result of 80 ≤ S < 90, we can conclude that the level of this school is good.

4 Conclusion
In this paper, we proposed a coupled FAHP method and applied it for eco-campus assessment. The model evaluation results showed that the propose method is able to provide very good solution in accuracy and our research has three advantages applied FAHP to solve the multi-criteria decision making problems.

Firstly, the difficulties of fuzzy and proper weight distribution is overcome, so that it ensures the status and importance of factors and makes the results more actual.

Secondly, when determining the membership degree and weight, it inevitably carries some human factors. Yet our research limits the subjective factors to a single scope, remedies the deficiencies of the old Check List Assess method, and make the evaluation more accurate.

Thirdly, the application of mature FAHP model on MCDM perfectly resolved the measurement of fuzzy factors of the index system, compared each index level, and control the major concern factors.

Though the comprehensive evaluation of the case above, we can draw a conclusion that this method is suitable for MCDM and can be widely used.

References:


