A Mathematical Theory of Psychological Dynamics

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Abstract: The psychological dynamics is governed by the ”observable-unobservable” systemic ratio. The plausible existence of the dynamics of the psychological laws is justified by the presence of a complex psycho-physiological apparatus which changes the informational biofeedback into energy. The psychological insight represented by the (unobservable) psychological soft component obeys the functional models [6] which define the psychological system. The psychological Gordian knot consists in identifying the manner in which the unobservable psychological fact changes into physiology and vice versa. The psychological literature is well represented, especially by the behavior models of the physiological hard component [1], [7], [17], [18]. This paper tries to develop a mathematical theory of psychological insight. The purpose is to create an unifying theory between the functionality of the psychological soft and of the physiological hard – having, from our point of view, the capacity to solve the Gordian knot – based on the results of this paper.

Key–Words: Amplitude of the tensional state, Extinction of the tensional state, Lie group, Metrics, Neuropsychological activation, Psychological experience, Tensional dynamics

1 Introduction

The dynamics of the tensional states [23], [24] dictated by complex informational feedbacks, is a remarkable regulator for the outlets of personality manifested in the behavioral plan.

The evolution mechanisms of the tensional states are responsible for the repartition of the excitation energy, a possible source for the appearance of psychological lack of balance. Their functioning is analogous to the functioning of selectors, influencing the quality of psychological statuses.

The preservation of un-altered psychological statuses involves the permanent activation of certain commands for the rehabilitation of the psychological system, materialized in the inhibition or the excitation of tensional dynamics [12], [13].

The regulation of tensional states in conditions of altered dynamics asks for the employment of certain “optimal control” type programs that should transform the system into a viable one, under conditions of utmost efficiency (time, resources, ...).

Dealing with the dynamics of psychological statuses raises the problem of finding corresponding forms of the representation of tensional states.

The tensional state, modulated by the perception of the tension, is determined by the lack of balance materialized in needs (physiological or superior needs). The association between tensional states and needs offers flexibility in representation, using Maslow’s idea in a form generalized through the imaging of a construct with “c” levels.

The register of any tensional state has three phases: birth-evolution-gratification. If the phase of evolution obeys the laws of dynamics, the birth and the gratification phases are the effect of the strategies of the cognitive system to allow or repress the tensional state. Thus, the tensional state can be defined by a couple of measures (α,β), designating the activation level, respectively the gratification strategy adopted by the psychological system. At each level of activation “c” possible strategies of gratification are associated and a total of “cxc” gratification strategies are also associated to “c” activation levels.

In this frame of representation the dynamic laws of the tensional states should be identified. Amazingly, at the psychological level, things seem to be in contradiction with the reality of the physical world, where bodies of high density attract the flow of matter positioned in a certain area of proximity. At the psychological level, we deal with the flow of attention.

The only way to solve the contradiction, at least at the conceptual level, is to accredit the idea of the existence of certain “germination processes of the tensional states”, an idea that we will be dealing with in the following paragraphs. At the psychological level, the more a tensional state increases in amplitude, the more it tends to attract the flow of attention. The dynamics of the tensional states is limited by the capacity of administration of the cognitive system. The guarantee of the strategy given by the cognitive system to the gratification of the tensional state is doubled by the expected horizon of a gratification (in terms of psychological experience). The amplitude increases in the same time with the approach to the expected limit of tensional extinction and in the same time with the increase of the ratio between the current neuropsychological activation and the minimal one needed for the activation of attention. For the dynamics of the amplitude of the tensional state, we propose the following relation:

$$b_n(t_i) = a^{-(n+1)} \frac{1+\ln m(t_0)}{\ln m(t_0)} \frac{\ln m(t_i)}{1+\ln m(t_i)} \quad (1)$$

where: i – the degree of the tensional state; t_i – the psychological experience – the unit of measure is "second x piece of information" - which is associated with the tensional state i; t_0 – the resident psychological experience which is determined by personality; n – the degree of the failed gratification; a– the number of sources which generate the tensional states; $m\,(t_i)$ – the ratio between the current neuropsychological activation and the minimal one which is necessary for the activation of attention; $m(t_0) \neq \{0, 1\}$. When $t_i = t_0$ then $b_n(t_0) = a^{-(n+1)}$. In the following paragraphs we will analyze the interesting case $t_i \neq t_0$.

What is the meaning of the number of gratifications and what is the connection with the consistency of the tensional state? The reach of the horizon of expectation for the gratification of a tensional state coincides with the exhaustion of the possibilities of administration of a superior dynamics. In the absence of tensional extinction, the cognitive system interferes through the identification of a "derived tensional state" connected to "the original tensional state" and through the evaluation of a new horizon of expectation. The derived tensional state increases the consistency of the original tensional state, tending to quantitatively orientate the flow of attention toward the last one. A new failed horizon of expectation will attract the germination of another derived tensional state from the tensional state previously derived, and so on. The hermetic quality of this point of view is checked through the fact that the germination of the tensional state supposes a tensional amplification, determined by the capacity of the cognitive system to identify the original causes. The passing from the original tensional state to the derived tensional state is followed by an amplifying, at least at the perception level, of the psychological tension. This is an adapted form of Heisenberg's uncertainty principle, which leads to behaviors of the "black hole" type.

In the psychological plan, a tensional state, disposing of a certain degree of "failed gratification", becomes a sufficiently powerful attractor in order to make impossible a psychological de-tensioning. The same effect is produced by the inhibition of the original dynamics of the tensional state, associated with a correct dynamics of the derived tensional state(s). The register of the resetting of a psychological system on wrong formulae should be completed with the situation of a correct dynamics of the original tensional state associated with an incorrect dynamics of the derived tensional state.

Beyond these specific aspects of representation, very important by the perspectives they offer in the research of psychological pathology and, in the same time, in the research of the conditions for the resetting of the psychological system on correct formulae, we believe that interest should be focused on the research of the laws of the administration of tensional states, applied by the psychological system. The enunciation of these laws, provided they are correct, would describe the administration capacities of tensional states.

In the following chapters we present a mathematical point of view starting from the theoretical framework of the representation of tensional states described above and from a group of four differential equations, describing the behavior of certain essential psychological measures which are involved in the tensional dynamics.

2 The Mathematical Model of the Qualitative Dynamics of Tensional States

The approach starts from the following hypothesis:

1. The variation of the psychological experience (t_i) function in physical time ($\bar{t}$) is given by the capacity of the stimulus to attract attention (c). In its turn, the capacity of the stimulus to attract attention is dependent on the capacity of the person to value the stimulus (v_i - apperception):

$$\frac{dt_i}{d\bar{t}} = c(v_i) \quad (2)$$

2. The variation of the ratio between the current neuropsychological activation and the one necessary for the activation of the attention (m_i) function in physical time ($\bar{t}$) is inversely proportional to this ratio (m_i):

$$\frac{dm_i}{d\bar{t}} = -\alpha m_i \quad (3)$$

where α is a positive constant.

3. The variation of apperception (v_i) function in physical time ($\bar{t}$) is inversely proportional to apperception (v_i):

$$\frac{dv_i}{d\bar{t}} = -\beta v_i \quad (4)$$

where β is a positive constant.

4. The variation of the amplitude of the tensional state $b_n(t_i)$ function in physical time ($\bar{t}$), is:

$$\frac{db_n(t_i)}{d\bar{t}} = -\frac{1+\ln m(t_0)}{\ln m(t_0)} \frac{\alpha a^{-(n+1)}}{(1+\ln m(t_i))^2} \quad (5)$$

Both psychological measures - neuropsychological activation and apperception - show decreases over time against the person's activation to the task of habituation to psychological stimuli. This fact is expressed by the equations (3) and (4). It is easy to see that the relation (5) can be obtained by computing the temporary derivation of the relation (1).

Taking into account the fact that the measures {t, m, v, b} are associated with the tensional state "i", in the following paragraphs we will ignore this index. The measures which are associated with the index "0" are constant because they depend on personality and not on a tensional state.

The psychological system presents a tensional capacity which is capable of insuring the adaptability of the person to the enquirer environment. We define the tensional capacity as a set of possible tensional states. The transitions between tensional states are possible if these are equivalent (similar). The presence of a tensional state which is not equivalent to the others should involve the psychological system in the approach of identifying a certain tensional dynamics, capable of overlapping the equivalent and the non-equivalent. This situation indicates a conscious tensional habituation process, which makes possible the transition from unspecific to specific. But this fact brings out the existence of a consciousness of the unconscious state. Much more likely is the existence of a tensional habituation process preserved in inflexible formulae, which would make impossible the intelligible translation of a non-equivalent tensional state as a protection measure of the psychological system.

The existence of the equivalent formulae of the tensional states is determined by the presence of certain common structures, called invariants. These structures represent the psychological mechanisms of the transition between tensional states.

In the study of the tensional habituation process we associate the tensional state with a system of Pfaff forms. Technically speaking, the problem of the nature of the tensional habituation process – seen as a psychological mechanism preserved or un-preserved in inflexible formulae – consists in identifying the relative invariants and, if possible, in identifying the absolute ones for the system (2)-(5). The identification of the constant invariants [2], [8], [14], [19], [20], [21], [22], [26], [27] brings out – taking into account the present theme – a psychological mechanism of the tensional habituation process preserved in inflexible formulae.

The invariants can be determined starting from the system of Pfaff forms associated to the system of equations (2)-(5). In this case, the system of Pfaff forms is:

$$ds^1 = dt - c(v)\, d\bar{t} \quad (6)$$

$$ds^2 = dm + \alpha m d\bar{t} \quad (7)$$

$$ds^3 = dv + \beta v d\bar{t} \quad (8)$$

$$ds^4 = db + \frac{1+ln\,[m\,(t_0)]}{ln\,[m\,(t_0)]} \frac{\alpha a^{-(n+1)}}{[1+ln\,[m\,(t)]]^2} d\bar{t} \quad (9)$$

$$ds^5 = dt \quad (10)$$

Taking into account the relations $ds^i = a^i_k dx^k$ we obtain the following expressions for the coefficients a^i_k:

$$a^1_1 = 1,\ a^1_2 = 0,\ a^1_3 = 0,\ a^1_4 = 0,\ a^1_5 = -c\,(v)$$

$$a^2_1 = 0,\ a^2_2 = 1,\ a^2_3 = 0,\ a^2_4 = 0,\ a^2_5 = \alpha m$$

$$a^3_1 = 0,\ a^3_2 = 0,\ a^3_3 = 1,\ a^3_4 = 0,\ a^3_5 = \beta v$$

$$a^4_1 = 0,\ a^4_2 = 0,\ a^4_3 = 0,\ a^4_4 = 1,$$

$$a^4_5 = \frac{1+ln\,[m\,(t_0)]}{ln\,[m\,(t_0)]} \frac{\alpha a^{-(n+1)}}{(1+ln\,[m\,(t)])^2}$$

$$a^5_1 = 1,\ a^5_2 = 0,\ a^5_3 = 0,\ a^5_4 = 0,\ a^5_5 = 0$$

Since the forms ds^i are independent, the expressions for differentials dx^i result:

$$dx^i = b^i_\alpha ds^\alpha \quad (11)$$

respectively

$$dt = ds^5 \quad (12)$$

$$dm = \frac{\alpha m}{c} ds^1 + ds^2 - \frac{\alpha m}{c} ds^5 \quad (13)$$

$$dv = \frac{\beta v}{c} ds^1 + ds^3 - \frac{\beta v}{c} ds^5 \quad (14)$$

$$db = \frac{1+ln\left[m\left(t_0\right)\right]}{ln\left[m\left(t_0\right)\right]}\frac{\frac{\alpha}{c}a^{-(n+1)}}{\left[1+ln\left[m\left(t\right)\right]\right]^2}ds^1 + ds^4 - \frac{1+ln\left[m\left(t_0\right)\right]}{ln\left[m\left(t_0\right)\right]}\frac{\frac{\alpha}{c}a^{-(n+1)}}{\left[1+ln\left[m\left(t\right)\right]\right]^2}ds^5 \quad (15)$$

$$d\bar{t} = -\frac{1}{c}ds^1 + \frac{1}{c}ds^5 \quad (16)$$

where:

$$b_1^1 = 0,\ b_2^1 = 0,\ b_3^1 = 0,\ b_4^1 = 0,\ b_5^1 = 1$$

$$b_1^2 = \frac{\alpha m}{c},\ b_2^2 = 1,\ b_3^2 = 0,\ b_4^2 = 0,\ b_5^2 = -\frac{\alpha m}{c}$$

$$b_1^3 = \frac{\beta v}{c},\ b_2^3 = 0,\ b_3^3 = 1,\ b_4^3 = 0,\ b_5^3 = -\frac{\beta v}{c}$$

$$b_1^4 = \frac{1+ln\left[m\left(t_0\right)\right]}{ln\left[m\left(t_0\right)\right]}\frac{\frac{\alpha}{c}a^{-(n+1)}}{\left(1+ln\left[m\left(t\right)\right]\right)^2},\quad b_2^4 = 0,\ b_3^4 = 0,\ b_4^4 = 1,\quad b_5^4 = -\frac{1+ln\left[m\left(t_0\right)\right]}{ln\left[m\left(t_0\right)\right]}\frac{\frac{\alpha}{c}a^{-(n+1)}}{\left(1+ln\left[m\left(t\right)\right]\right)^2}$$

$$b_1^5 = -\frac{1}{c},\ b_2^5 = 0,\ b_3^5 = 0,\ b_4^5 = 0,\ b_5^5 = \frac{1}{c}$$

We associate the operators $X_a = b_a^i \frac{\partial f}{\partial x^i}$ with $a = \{1, 2, \ldots.., n\}$ to the forms $ds^a = \mu_k^a dx^k$ if the determinant $D = \left|b_a^i\right|$ is non-zero and the determinant $|\mu_k^a|$ is equal to D^{-1}. We call components on the congruences (a) of a contravariant vector U^i or a covariant vector U_i, the expressions U^a or U_a given by the formulae $U^a = \mu_i^a U^i$ respectively $U_a = b_a^i U_i$. If U_i contain the partial derivations of a function h, then the components U_a coincide with the linear operators $X_a = \frac{\partial f}{\partial s^a} = b_a^i \frac{\partial f}{\partial x^i}$ where $\frac{\partial f}{\partial s^a}$ are derivatives of the function f along the (a) congruences. Using Poisson's paranthesis it results:

$$(X_b, X_c) = \frac{\partial^2 f}{\partial s^b \partial s^c} - \frac{\partial^2 f}{\partial s^c \partial s^b} = \omega_{bc}^a \frac{\partial f}{\partial s^a} \quad (17)$$

If $\omega_{bc}^a = 0$ then s^a can be seen as coordinates. We call the expression $\Delta s^p = \left(\frac{\partial a_i^p}{\partial x^k} - \frac{\partial a_k^p}{\partial x^i}\right) dx^i \delta x^k = \omega_{ef}^p ds^e \delta s^f$ the bilinear covariant of the Pfaff's form $ds^p = a_i^p dx^i$ Taking into account the relations (11) the following expressions result:

$$\omega_{ef}^i = \left(\frac{\partial a_k^i}{\partial x^l} - \frac{\partial a_l^i}{\partial x^k}\right) b\,_e^k b_f^l \quad (18)$$

Using the relations (18) the expressions for the skew-symmetric coefficients ω_{ef}^i result :

$$\omega_{12}^1 = \frac{1}{c}\frac{\beta v}{\alpha m}\frac{\partial c}{\partial v} \quad (19)$$

$$\omega_{13}^1 = \frac{1}{c}\frac{\partial c}{\partial v} \quad (20)$$

$$\omega_{14}^1 = \frac{\beta v}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left(1+ln\left[m\left(t\right)\right]\right)^2}{\alpha a^{-(n+1)}}\frac{\partial c}{\partial v} \quad (21)$$

$$\omega_{15}^1 = -\frac{\beta v}{c^2}\frac{\partial c}{\partial v} \quad (22)$$

$$\omega_{25}^1 = \frac{\beta v}{\alpha m c}\frac{\partial c}{\partial v} \quad (23)$$

$$\omega_{23}^1 = \omega_{24}^1 = \omega_{34}^1 = 0 \quad (24)$$

$$\omega_{35}^1 = \frac{1}{c}\frac{\partial c}{\partial v} \quad (25)$$

$$\omega_{45}^1 = \frac{\beta v}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left(1+ln\left[m\left(t\right)\right]\right)^2}{\alpha a^{-(n+1)}}\frac{\partial c}{\partial v} \quad (26)$$

$$\omega_{12}^2 = -\frac{\alpha}{c} \quad (27)$$

$$\omega_{13}^2 = -\frac{\alpha^2 m}{\beta c v} \quad (28)$$

$$\omega_{14}^2 = -\frac{\alpha^2 m}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left(1+ln\left[m\left(t\right)\right]\right)^2}{\alpha a^{-(n+1)}} \quad (29)$$

$$\omega_{15}^2 = \frac{\alpha^2 m}{c^2} \quad (30)$$

$$\omega_{25}^2 = -\frac{\alpha}{c} \quad (31)$$

$$\omega_{23}^2 = \omega_{24}^2 = \omega_{34}^2 = 0 \quad (32)$$

$$\omega_{35}^2 = -\frac{\alpha^2 m}{c\beta v} \quad (33)$$

$$\omega_{45}^2 = -\frac{\alpha m}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left(1+ln\left[m\left(t\right)\right]\right)^2}{a^{-(n+1)}} \quad (34)$$

$$\omega_{12}^3 = -\frac{\beta^2 v}{\alpha c m} \quad (35)$$

$$\omega_{13}^3 = -\frac{\beta}{c} \quad (36)$$

$$\omega_{14}^3 = -\frac{\beta^2 v}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left(1+ln\left[m\left(t\right)\right]\right)^2}{\alpha a^{-(n+1)}} \quad (37)$$

$$\omega_{15}^3 = \frac{\beta^2 v}{c^2} \quad (38)$$

$$\omega_{23}^3 = \omega_{24}^3 = \omega_{34}^3 = 0 \quad (39)$$

$$\omega_{25}^{3} = -\frac{\beta^2 v}{\alpha c m} \quad (40)$$

$$\omega_{35}^{3} = -\frac{\beta}{c} \quad (41)$$

$$\omega_{45}^{3} = -\frac{\beta^2 v}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left(1+ln\left[m\left(t\right)\right]\right)^2}{\alpha a^{-(n+1)}} \quad (42)$$

$$\omega_{12}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial m} \quad (43)$$

$$\omega_{13}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial v} \quad (44)$$

$$\omega_{14}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial b} \quad (45)$$

$$\omega_{15}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial t} \quad (46)$$

$$\omega_{23}^{4} = \omega_{24}^{4} = \omega_{34}^{4} = 0 \quad (47)$$

$$\omega_{25}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial m} \quad (48)$$

$$\omega_{35}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial v} \quad (49)$$

$$\omega_{45}^{4} = -\frac{1}{c}\frac{\partial a_5^4}{\partial b} \quad (50)$$

$$\omega_{ij}^{5} = 0 \quad (51)$$

The system is determined, making exception for the following transformation:

$$d\overline{s}^1 = A ds^1 \quad (52)$$

$$d\overline{s}^2 = B ds^2 \quad (53)$$

$$d\overline{s}^3 = C ds^3 \quad (54)$$

$$d\overline{s}^4 = D ds^4 \quad (55)$$

$$d\overline{s}^5 = E ds^5 \quad (56)$$

where A, B, C, D, E are arbitrary functions.
We assume that ds^2 and ds^3 are not completely integrable, which imposes the following conditions:

$$\omega_{13}^2 = \omega_{14}^2 = \omega_{15}^2 = \omega_{35}^2 = \omega_{45}^2 = \omega_{12}^3 = \omega_{14}^3 = \\ = \omega_{15}^3 = \omega_{25}^3 = \omega_{45}^3 = 1$$

$$\overline{\omega}_{13}^2 = \overline{\omega}_{14}^2 = \overline{\omega}_{15}^2 = \overline{\omega}_{35}^2 = \overline{\omega}_{45}^2 = \overline{\omega}_{12}^3 = \overline{\omega}_{14}^3 = \\ = \overline{\omega}_{15}^3 = \overline{\omega}_{25}^3 = \overline{\omega}_{45}^3 = 1$$

$$\omega_{ij}^1 = \overline{\omega}_{ij}^1 = \omega_{ij}^4 = \overline{\omega}_{ij}^4 = \omega_{ij}^5 = \overline{\omega}_{ij}^5 = 0$$

where $\{1,\,4,\,5,\,i,\,j\}$ are distinct indices, namely the system:

$$-\frac{\alpha^2 m}{\beta c v} = 1 \quad (57)$$

$$-\frac{\alpha m}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left[1+ln\left[m\left(t\right)\right]\right]^2}{a^{-(n+1)}} = 1 \quad (58)$$

$$\frac{\alpha^2 m}{c^2} = 1 \quad (59)$$

$$-\frac{\beta^2 v}{\alpha c m} = 1 \quad (60)$$

$$-\frac{\beta^2 v}{c}\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}\frac{\left[1+ln\left[m\left(t\right)\right]\right]^2}{\alpha a^{-(n+1)}} = 1 \quad (61)$$

$$\frac{\beta^2 v}{c^2} = 1 \quad (62)$$

$$\frac{\partial c}{\partial v} = 0 \quad (63)$$

$$-\frac{1}{c}\frac{\partial a_5^4}{\partial m} = 0 \quad (64)$$

$$-\frac{1}{c}\frac{\partial a_5^4}{\partial v} = 0 \quad (65)$$

$$-\frac{1}{c}\frac{\partial a_5^4}{\partial t} = 0 \quad (66)$$

The system verifies for:

$$\alpha = \beta = m = v = -c = 1$$

$$\frac{ln\left[m\left(t_0\right)\right]}{1+ln\left[m\left(t_0\right)\right]}a^{n+1} = 1$$

The relations in finite terms for the group (52)-(56)

$$AC - B = 0 \quad (67)$$

$$AD - B = 0 \quad (68)$$

$$AE - B = 0 \quad (69)$$

$$CE - B = 0 \quad (70)$$

$$DE - B = 0 \quad (71)$$

$$AB - C = 0 \quad (72)$$

$$AD - C = 0 \quad (73)$$

$$AE - C = 0 \quad (74)$$

$$BE - C = 0 \quad (75)$$

$$DE - C = 0 \quad (76)$$

lead to the solution $A = B = C = D = E = 1$. Thus the group (52)-(56) changes into the invariant group:

$$d\overline{s}^1 = ds^1 \quad (77)$$

$$d\overline{s}^2 = ds^2 \quad (78)$$

$$d\bar{s}^3 = ds^3 \quad (79)$$

$$d\bar{s}^4 = ds^4 \quad (80)$$

$$d\bar{s}^5 = ds^5 \quad (81)$$

We obtain the following absolute invariants which are equal to +1, -1 and 0:

$$I_1 = \omega^1_{12} \quad (82)$$

$$I_2 = \omega^1_{13} \quad (83)$$

$$I_3 = \omega^1_{14} \quad (84)$$

$$I_4 = \omega^1_{15} \quad (85)$$

$$I_5 = \omega^2_{21} \quad (86)$$

$$I_6 = \omega^2_{23} \quad (87)$$

$$I_7 = \omega^2_{24} \quad (88)$$

$$I_8 = \omega^2_{25} \quad (89)$$

$$I_9 = \omega^3_{31} \quad (90)$$

$$I_{10} = \omega^3_{32} \quad (91)$$

$$I_{11} = \omega^3_{34} \quad (92)$$

$$I_{12} = \omega^3_{35} \quad (93)$$

$$I_{13} = \omega^4_{41} \quad (94)$$

$$I_{14} = \omega^4_{42} \quad (95)$$

$$I_{15} = \omega^4_{43} \quad (96)$$

$$I_{16} = \omega^4_{45} \quad (97)$$

$$I_{17} = \omega^5_{51} \quad (98)$$

$$I_{18} = \omega^5_{52} \quad (99)$$

$$I_{19} = \omega^5_{53} \quad (100)$$

$$I_{20} = \omega^5_{54} \quad (101)$$

Since the invariants are constant, it results that tensional habituation does not exist as a process! The resulted bilinear covariants are:
1. if the absolute invariants are equal to -1, then

$$\Delta s^1 = 0 \quad (102)$$

$$\begin{aligned}\Delta s^2 = &-(ds^2\delta s^1 - ds^1\delta s^2) + (ds^1\delta s^3 - ds^3\delta s^1)\\ &+ (ds^1\delta s^4 - ds^4\delta s^1) + (ds^1\delta s^5 - ds^5\delta s^1)\\ &+ (ds^3\delta s^5 - ds^5\delta s^3) + (ds^4\delta s^5 - ds^5\delta s^4)\end{aligned} \quad (103)$$

$$\begin{aligned}\Delta s^3 = &-(ds^3\delta s^1 - ds^1\delta s^3) + (ds^1\delta s^2 - ds^2\delta s^1)\\ &+ (ds^1\delta s^4 - ds^4\delta s^1) + (ds^1\delta s^5 - ds^5\delta s^1)\\ &+ (ds^2\delta s^5 - ds^5\delta s^2) + (ds^4\delta s^5 - ds^5\delta s^4)\end{aligned} \quad (104)$$

$$\Delta s^4 = 0 \quad (105)$$

$$\Delta s^5 = 0 \quad (106)$$

They can be satisfied by the following forms:

$$ds^1 = dt \quad (107)$$

$$ds^2 = (-cm - cv - cb - ct + k_1)d\bar{t} + dm \quad (108)$$

$$ds^3 = (-cv - cm - cb - ct + k_2)d\bar{t} + dv \quad (109)$$

$$ds^4 = db \quad (110)$$

$$ds^5 = d\bar{t} \quad (111)$$

where k_i are constant. The variables m and v must satisfy the conditions:

$$m = -\frac{c}{\alpha + c}(v + b + t) + \frac{k_1}{\alpha + c} \quad (112)$$

$$v = -\frac{c}{\beta + c}(m + b + t) + \frac{k_2}{\beta + c} \quad (113)$$

2. if the absolute invariants are equal to +1, then

$$\Delta s^1 = 0 \quad (114)$$

$$\begin{aligned}\Delta s^2 = &(ds^2\delta s^5 - ds^5\delta s^2) + (ds^1\delta s^3 - ds^3\delta s^1)\\ &+ (ds^1\delta s^4 - ds^4\delta s^1) + (ds^1\delta s^5 - ds^5\delta s^1)\\ &+ (ds^3\delta s^5 - ds^5\delta s^3) + (ds^4\delta s^5 - ds^5\delta s^4)\end{aligned} \quad (115)$$

$$\begin{aligned}\Delta s^3 = &(ds^3\delta s^5 - ds^5\delta s^3) + (ds^1\delta s^2 - ds^2\delta s^1) +\\ &+ (ds^1\delta s^4 - ds^4\delta s^1) + (ds^1\delta s^5 - ds^5\delta s^1)\\ &+ (ds^2\delta s^5 - ds^5\delta s^2) + (ds^4\delta s^5 - ds^5\delta s^4)\end{aligned} \quad (116)$$

$$\Delta s^4 = 0 \quad (117)$$

$$\Delta s^5 = 0 \quad (118)$$

They can be satisfied by the following forms:

$$ds^1 = dt \quad (119)$$

$$ds^2 = (-cm - cv - cb - ct + k_3)d\bar{t} + dm \quad (120)$$

$$ds^3 = (-cv - cm - cb - ct + k_4)d\bar{t} + dv \quad (121)$$

$$ds^4 = db \quad (122)$$

$$ds^5 = d\bar{t} \quad (123)$$

where k_i are constant. The variables m and v must satisfy the conditions:

$$m = -\frac{c}{\alpha + c}(v + b + t) + \frac{k_3}{\alpha + c} \quad (124)$$

$$v = -\frac{c}{\beta + c}(m + b + t) + \frac{k_4}{\beta + c} \tag{125}$$

3. if the absolute invariants are equal to 0, then

$$\Delta s^1 = 0 \tag{126}$$

$$\Delta s^2 = 0 \tag{127}$$

$$\Delta s^3 = 0 \tag{128}$$

$$\Delta s^4 = 0 \tag{129}$$

$$\Delta s^5 = 0 \tag{130}$$

They can be satisfied by the following forms:

$$ds^1 = dt \tag{131}$$

$$ds^2 = dm \tag{132}$$

$$ds^3 = dv \tag{133}$$

$$ds^4 = db \tag{134}$$

$$ds^5 = d\bar{t} \tag{135}$$

Taking into account the relations $d\bar{s}^i = ds^i$, the equivalences between two systems of forms having the same properties result.

3 The Tensional Adjustment Mechanism as a Defense Mechanism

The defense mechanism theme [3], [4], [5], [9], [10], [11], [15], [16], [25] has made a strong come back in topical preoccupations field, although it was excluded from academic speech for a number of years. Many cognitive psychologists assign a major role to defense mechanisms in the approach to the explanation of psychological functionality.

It's difficult to identify a coherent theory of the functionality of defense mechanisms because they cannot be seen. They constitute unobservable pieces of a puzzle which defines an observable structure – behavior. We have the strong conviction that the reason of the credible existence of defense mechanisms submits to certain laws which define the fundamental formulae of the functionality of the psychological system, specific to the person. In this statement there is a seeming inconsistence generated by the term "fundamental formulae". The term must be understood as an essential structure which must be maintained by another structure or other structures which describe the particular. The essential visible structures in psychology present the characteristics of repeatability, measured by constants or recurrences. The essential structures are strong structures which are also maintained in the specific conditions of psychological lack of balance. It is interesting to treat the case in which the weak particular structures become strong structures in the sense mentioned above. We infer here the activation of certain severe mechanisms of overlapping between homeostasis and entropy with negative consequences in distinguishing between these two natures. The approaches which focus on the essence of defense mechanisms suffer of inconsistency, just by being inefficacious in identifying the essential.

The purpose of this paper consists in identifying the essence of the defense mechanism – involved in tensional adjustment – by pointing out the dynamic laws of the main tensional psychological measures which are influenced by this mechanism.

The particularity of the action of defense mechanisms consists in the breaking of the equivalent evolution tendencies of the psychological measures (constants, linear, exponentials etc.) for the purpose of the preservation of unaltered psychological statuses. The defense mechanisms become active when the meters of the cognitive system detect values which are placed in the tolerance area of the psychological system integrity.

The couple "defense mechanism action – psychological measures (those dynamics)" is governed by invariance laws (laws of the type $M^\alpha M_\alpha \rightarrow invariant$) which assure the dynamics of the psychological system in its functional parameters. The invariance laws, at least in psychology, could be interpreted as dynamic equilibrium laws, which have the capacity of describing the whole functional register of the psychological system, from the right (re)structure formulae to the wrong resetting formulae.

The essence of the defense mechanisms functionality consists in the preservation of certain specific restoring formulae of the psychological system as an effect of annihilating entropic tendencies (internal or external).

The activation frequency of the sources generating tensional states is, to a large extent, determined by the specifics of tensional habituation which restricts the domain of activation of the tensional states selector to those options that are meant to avoid the unpleasant and to conserve the pleasure. Each of the two components – the unpleasant and the pleasurable – amplifies in its dynamics the potential of switching to the opposite state.

There are serious reasons to advance the idea that the action of defense mechanisms can be described by using gradient type measures. These refer to the target of these actions, which consists in the counteracting of non – specific tendencies of dynamics of psychological measures. The effects of the action of the defense mechanisms appear - as purpose - in psychological experience. We could interpret the action of these mechanisms by using a function of psychological experience that should, of course, be a gradient. However, this gradient influences simultaneously the dynamics of apperception, of the ratio between the current neuropsychological activation and the minimal one needed for the activation of attention and of the amplitude of tensional state, which makes it possible to identify it mathematically through a coefficient (a_{ij}) of the metric tensor $ds^2 = a_{ij}dx^i dx^j$. For reasons presented above a_{ij} should not be a constant. The presence of constant coefficients designates a non specific situation, determined by the blocking or the lack of defense mechanisms.

Taking into account the aspects mentioned above, we present a mathematical procedure of the determination of the action of defense mechanism which is involved in the dynamics of the ratio between the current neuropsychological activation and the minimal one needed for the activation of attention, of apperception and of the amplitude of tensional state.

3.1 The mathematical expression of the mechanism of tensional adjustment

The presence of the tensional adjustment mechanism should fit in the metrics formula in the form of a function of psychological experience that should modulate the dynamics of the ratio between the current neuropsychological activation and the minimal one needed for the activation of attention, of apperception and of the amplitude of tensional state. The metrics should have the form:

$$\mathrm{d}s^2 = (i\mathrm{d}\bar{t})^2 + h\,(t)\left[(\mathrm{d}m)^2 + (\mathrm{d}v)^2 + (\mathrm{d}b)^2\right] \tag{136}$$

We will identify the tensional adjustment mechanism "$h\,(t)$", using the three-dimensional metrics:

$$\mathrm{d}s^{(III)2} = h\,(t)\left[(\mathrm{d}m)^2 + (\mathrm{d}v)^2 + (\mathrm{d}b)^2\right] \tag{137}$$

For this metrics, the Christoffel coefficients of the second degree are all zero with the exception of:

$$\left|\begin{array}{c} 2 \\ 12 \end{array}\right| = \left|\begin{array}{c} 3 \\ 13 \end{array}\right| = \left|\begin{array}{c} 4 \\ 14 \end{array}\right| = \frac{1}{2}\frac{h'}{h} \tag{138}$$

The Riemann symbols R^i_{jkl} are zero with the exception of:

$$\begin{aligned} R^2_{121} &= -R^2_{112} = R^3_{131} = -R^3_{113} = R^4_{141} = \\ &= -R^4_{114} = \frac{1}{4}\left(\frac{h'}{h}\right)^2 - \frac{1}{2}\frac{h''}{h} \end{aligned} \tag{139}$$

The curvature tensors of the second degree R_{ij} are zero with the exception of:

$$R_{11} = \frac{3}{4}\left(\frac{h'}{h}\right)^2 - \frac{3}{2}\frac{h''}{h} \tag{140}$$

Let us study the interesting case where the three-dimensional space (m, v, b) is maximally homogeneous. The following differential equation results for the coefficient h(t)":

$$\frac{3}{4}\frac{(h')^2}{h^3} - \frac{6}{4}\frac{h''}{h^2} = -2K \tag{141}$$

where K is a constant which is different from zero. The corresponding expression – which presents psychological significance – is:

$$h(t) = \frac{4}{(\mathrm{o}t + \mathrm{p})^2} \tag{142}$$

where o and p are constant, $K = \frac{3}{4}\mathrm{o}^2$, $\mathrm{o}t + \mathrm{p} < 0$. Thus, the complete metrics for tensional state dynamics is written:

$$\begin{aligned} \mathrm{d}s^{(IV)2} &= (i\mathrm{d}\bar{t})^2 \\ &+ \frac{4}{(\mathrm{o}t + \mathrm{p})^2}\left[(\mathrm{d}m)^2 + (\mathrm{d}v)^2 + (\mathrm{d}b)^2\right] \end{aligned} \tag{143}$$

4 Conclusion

1. The psychological measures implied in tensional dynamics are governed by equivalence laws, which means that they have specific formulae which can be reproduced only by equivalences and not by breaking them! The cases of psychological pathology show up when the equivalences violate the safety limits of the psychological system. In this case the transitions of the equivalences–non-equivalences associated with the lack of flexibility of the safety limits involve the psychological system in a perennial attempt of reconstruction of equivalences which can no longer be reconfigured!

2. Apperception is constant! The neuropsychological activation determined by a stimulus is positioned at the minimal level necessary for the activation of attention, which supports the idea that the psychological system uses minimal energetic resources in the processing of stimuli!

3. Tensional states do not have amplitude! In other words, for the case $t_i \neq t_0$, any tension is felt at the maximum level! This conclusion is valid only in the case of the blocking of defense mechanisms which are involved in tensional adjustment or in the case of their non-existence.

4. There is a serious difference between tensional state amplitude and tensional state amplitude

perception. The theory of tensional state germination – a unifying theory of physics and psychology – shows that in the context of the cognitive solving of a certain tensional state, passing from the general to the detailed is felt by an amplified perception of the tensional effort. The finest detail of a certain unsolved problematic situation would be felt at the maximum level in the conditions of blocking the tensional adjustment mechanism.

5. The action of the tensional adjustment mechanism reduces at the perception level the tensional effort.

6. The tensional adjustment mechanism defends the psychological system by strong stimuli imposing restrictions on the neuropsychological activation and on the apperception.

7. The psychological system collapses when $\mathrm{o}t + \mathrm{p} = 0$. The avoidance of the collapse is possible if the psychological system changes the psychological experience scale through the law $t \to f\left(-\frac{p}{o}\right) \neq 0$.

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