Gravitational Capture Using a Four-Body Model

ALEXANDRE LACERDA MACHUY[1],
ANTÔNIO FERNANDO BERTACHINI DE ALMEIDA PRADO[2], TEREZINHA DE JESUS STUCHI[3]

[1,2] Division of Space Mechanics and Control – INPE
C.P. 515, 12227-310 São José dos Campos - SP, Brazil
machuy@dem.inpe.br ; prado@dem.inpe.br
[3]Department of Mathematical Physics, Federal University of Rio de Janeiro(UFRJ)
P. O. box 68528. 21945-970. Rio de Janeiro, RJ - Brazil

Abstract: A spacecraft (or any particle with negligible mass) suffers a gravitational capture when its orbit changes from hyperbolic (small positive energy) around a celestial body into elliptic (small negative energy) using only gravitational forces. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of the third and the fourth bodies involved in the dynamics. Those forces are equivalent of a zero cost control applied to the spacecraft, equivalent to a continuous thrust. One of the most important applications of this property is the construction of trajectories to the Moon. The concept of gravitational capture is combines with the principles of the gravity-assisted maneuver and the bi-elliptic transfer orbit, to generate a trajectory that requires a fuel consumption smaller than the one required by the Hohmann transfer. The present paper study the energy required for the ballistic gravitational capture in a dynamical model that has the presence of four bodies. Those bodies are assumed to follow the assumptions of the bi-circular model. In particular, the Earth-Moon-Sun-Spacecraft system is considered.

Keywords: Astrodynamics, Celestial Mechanics, Space Trajectories, Gravitational Capture, Space Missions.

1.Introduction

The bi-circular problem is a particular case of the problem of four bodies, where one of the masses, let us say m_4, is supposed to be infinitely smaller than the other three masses. With that hypothesis, m_4 moves under the gravitational forces of m_1, m_2 and m_3, but it doesn't disturb the motion of the three bodies with significant mass. In the bi-circular problem, the motion of m_1, m_2 and m_3 around the center of mass is considered as formed by circular orbits and the motion of m_4 has to be a certain function of the initial conditions. We can consider the bi-circular problem as a disturbance of the restricted problem of three bodies. This problem can be used as a model for the motion of a space vehicle in the Sun-Earth-Moon system.

In the first part of the paper we supplied the equations of motion of the model and we defined gravitational capture. The second part is used for the calculation of some numerical results for the bi-circular problem, such as direct orbits, retrograde orbits, capture orbits, etc.

2.Mathematical models

The problem of four bodies with the two hypotheses shown below is called bi-circular problem.

First hypothesis: It is considered two bodies with significant mass moving in circular orbits around the mutual center of mass. Those two bodies are called primaries.

Second hypothesis: The third body with significant mass is in a circular orbit around the center of mass of the system formed by the two first primaries and its orbit is coplanar with the orbits of those primaries.

Figure 1 shows the motion of the three primaries in the fixed system of coordinates, also called sidereal system.

position of the Moon. This angle is measured starting from the Earth-moon line, in the counterclockwise sense, starting from the opposite side of the Earth. The magnitude of the initial velocity V is calculated from the initial value of $C_3 = V^2 - \frac{2\mu_M}{r}$. The direction of the velocity vector of the vehicle is chosen as being perpendicular to the line that links the space vehicle to the center of the Moon, appearing in the counterclockwise direction for the direct orbits and in the clockwise direction for the retrograde orbits.
The orbit is considered of capture when the particle reaches the distance of 100000 km (0.26 canonical units) from the center of the moon in a time smaller than 50 days (approximately 12 canonical units). The sphere with radius 100000 km centered in the Moon is defined as the sphere of influence of the Moon. Figure 3 shows the point P, where the space vehicle escapes from the sphere of influence. The angle that defines this point is called the angle of the entrance position and the Greek letter describes it β. During the numeric integration the step of time is negative, therefore the initial conditions are really the final conditions of the orbit after the capture.

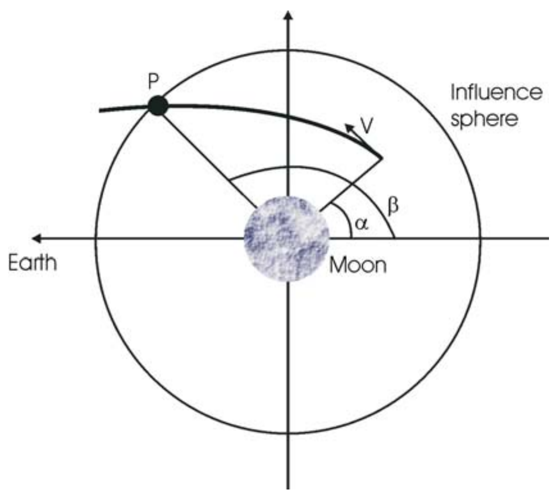

Figure 4 – Initial conditions.

4. Effects of the Angle α .

We now show some results obtained. Figure 5 shows direct orbits and figure 6 shows retrograde orbits. In both situations the angle ψ is constant and equal to 0°, and $C_3 = -0.15$. The angle α assumes the values: 30° (6), 60° (5), 90° (4), 120° (3), 150° (2), 180° (1). The coordinates of the position vector for the case of direct or retrograde orbits are: $x = r_p \cos(\alpha) + \mu_E$ and $y = r_p sin(\alpha)$. The coordinates of the velocity vector for the case of direct orbits are: $x_v = -Vsin(\alpha)$ and $y_v = V\cos(\alpha) + \mu_E$. For the case of retrograde orbits the coordinates of the velocity vector are: $x_v = Vsin(\alpha)$ and $y_v = -V\cos(\alpha) + \mu_E$. We can clearly see the effect of the Sun pushing the trajectories to the right. The direct orbits has a direct path, while the retrograde orbits start their motion to the left and then feel the effects of the Sun and turn to the right.

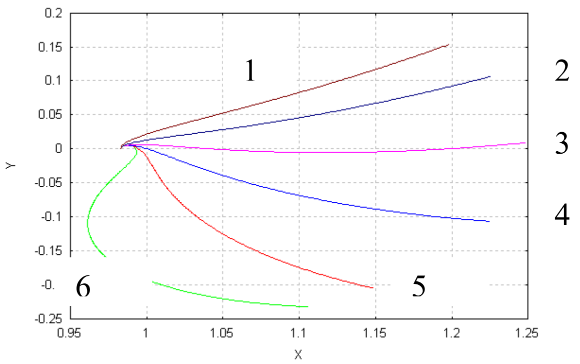

Figure 5 - Direct orbits.

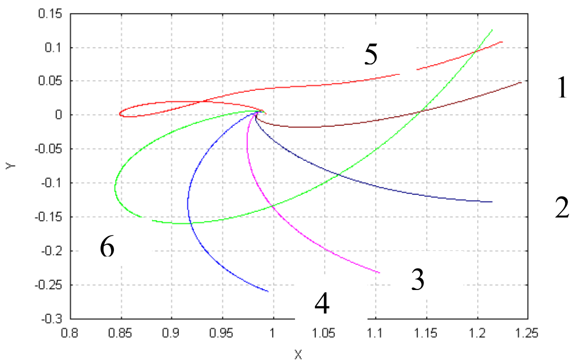

Figure 6 - Retrograde orbits.

5. Variation of the Energy

We now turn our attention to study the effects of the initial value of the energy C_3. We consider $\alpha = \psi = 30^\circ$ and make C_3 to assume the values -0.1 (1), -0.2 (2), -0.3 (3) and -0.4 (4). In figure 7 we have direct orbits and in figure 8 we have retrograde orbits. We see that the reduction of the energy increase the length of the trajectories and, in some cases, it includes loops. Trajectories with energy close to zero escape faster from the primary.

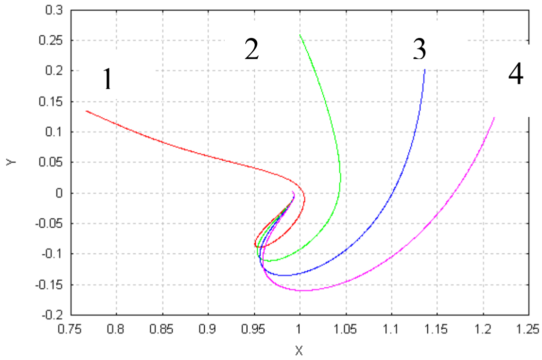

Figure 7 – Direct trajectories varying C_3

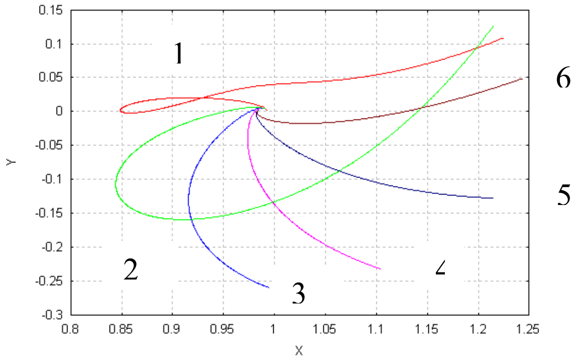

Figure 8 - Retrograde trajectories varying C_3

6. Variation of the Angle ψ

We now study the effects of the position of the Sun. We considered $\alpha = 90^\circ$ and $C_3 = -0.3$ in figures 9 and 10. The angle ψ will assume the values 0° (5), 30° (4), 45° (3), 60° (2) and 90° (1). In figure 9 we have direct orbits and in figure 10 we have retrograde orbits. It is clear that the Sun attracts the trajectories. The results show this fact. The retrograde trajectories have this characteristic more visible.

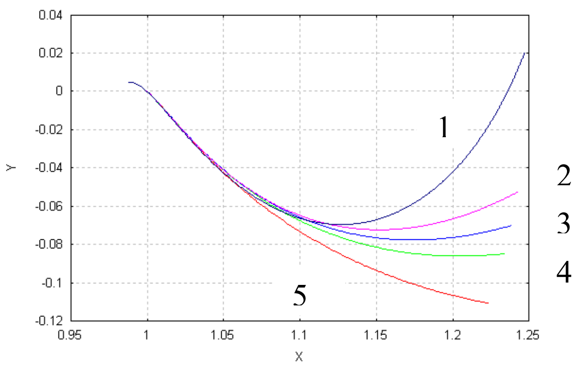

Figure 9 - Direct orbits.

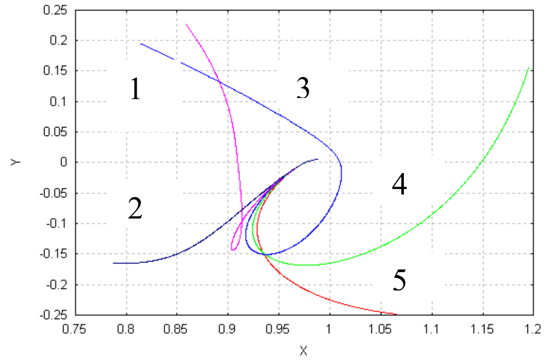

Figure 10 - Retrograde orbits.

7. Minimum Value of C_3

The value C_3 is associated to the amount of fuel consumed to complete the capture, that is, if this value is smaller, smaller is the amount of fuel consumed in the maneuver.

The first objective of this part of the text is to obtain the smallest consumption of fuel to complete the maneuver of the space vehicle when we have a gravitational capture in the bi-circular problem of four bodies. The second objective is to find favorable areas for gravitational capture in the bi-circular problem. We called more favorable areas for gravitational capture, areas where there is a minimum energy.

In all the graphs below, it will be made a variation of the angle α from 0° up to 360°, in steps of 1°. The initial value of C_3 is -0.65, and the final value is -0.01, with variations of -0.01.

For the angle of the Sun (ψ), we choose the values : 0^0, 30^0, 45^0, 60^0, 90^0, 120^0, 135^0, 150^0, 180^0, 210^0, 225^0, 240^0, 270^0, 300^0, 315^0 e 330^0. For each one of these values of the angle of the sun we have a graph, shown below, that shows the minimum time of gravitational capture.

Below we have two groups of graphs for the bi-circular problem, where the energy is a function of the angle of the Sun. The angle of the Sun is shown in degrees, and the energy in canonical units. The first group is for direct orbits and the second for retrograde orbits.

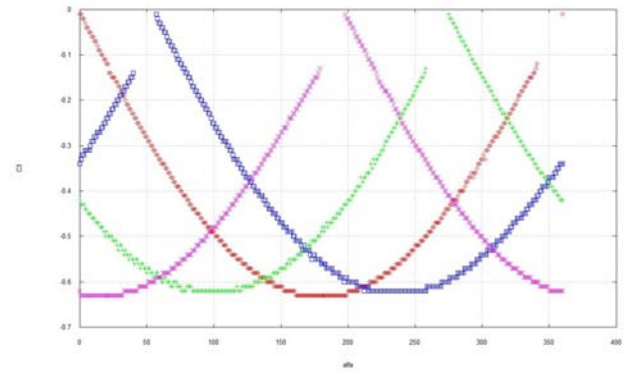

Figure 11 – Minimum energy for $\psi = \mathbf{0^0}$ (red), $\mathbf{30^0}$ (green), $\mathbf{45^0}$ (blue) and $\mathbf{60^0}$ (pink) for direct orbits.

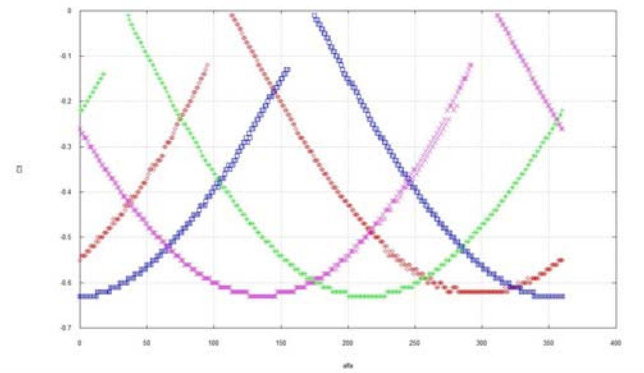

Figure 12 – Minimum energy for $\psi = \mathbf{90^0}$ (red), $\mathbf{120^0}$ (green), $\mathbf{135^0}$ (blue) and $\mathbf{150^0}$ (pink) for direct orbits.

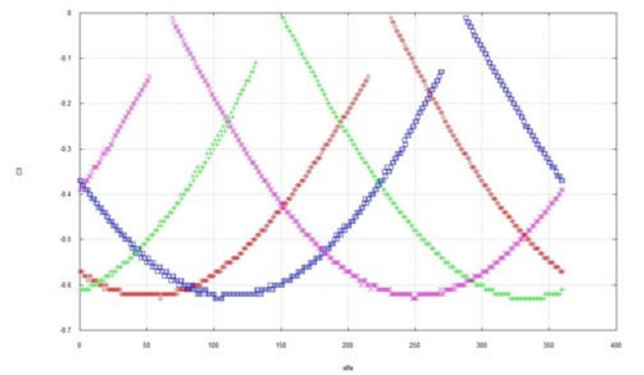

Figure 13 – Minimum energy for $\psi = \mathbf{180^0}$ (red), $\mathbf{210^0}$ (green), $\mathbf{225^0}$ (blue) and $\mathbf{240^0}$ (pink) for direct orbits.

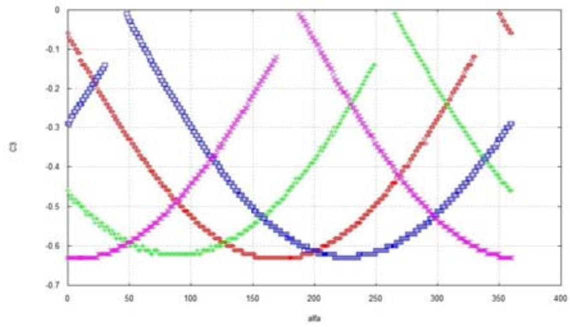

Figure 14 – Minimum energy for $\psi = \mathbf{270^0}$ (red), $\mathbf{300^0}$ (green), $\mathbf{315^0}$ (blue) and $\mathbf{330^0}$ (pink) for direct orbits.

The minimum value of the energy happens in all the graphs above when $\mathbf{C_3 = -0.65}$. In all the graphs there is an area where gravitational capture doesn't happen. In that area we have collisions and the motion is dominated predominantly by the effect of the Sun.

For example, when $\psi = \mathbf{0^0}$ there are no gravitational capture in the interval $\mathbf{350^0} \le \alpha \le \mathbf{360^0}$. The minimum value of the energy happens when $\mathbf{160^0} \le \alpha \le \mathbf{180^0}$.

Several similar results can be obtained just by looking at the plots.

This analyzes are now repeated for the group of retrograde orbits.

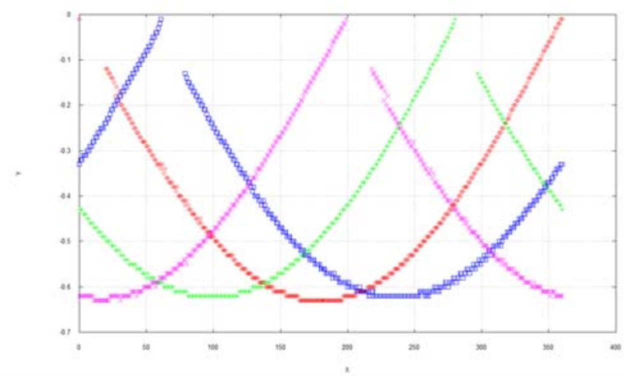

Figure 15 – Minimum energy for $\psi = \mathbf{0^0}$ (red), $\mathbf{30^0}$ (green), $\mathbf{45^0}$ (blue) and $\mathbf{60^0}$ (pink) for retrograde orbits.

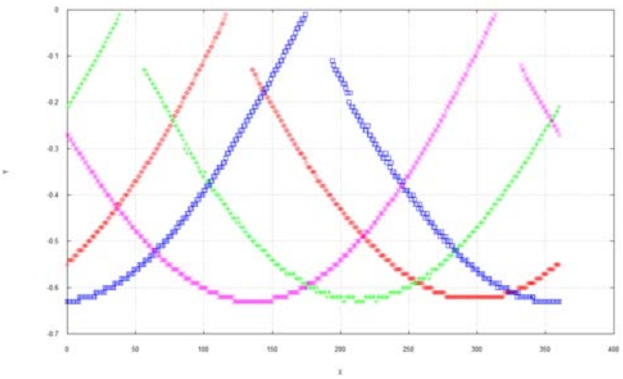

Figure 16 – Minimum energy for $\psi = \mathbf{90^0}$ (red), $\mathbf{120^0}$ (green), $\mathbf{135^0}$ (blue) and $\mathbf{150^0}$ (pink) for retrograde orbits.

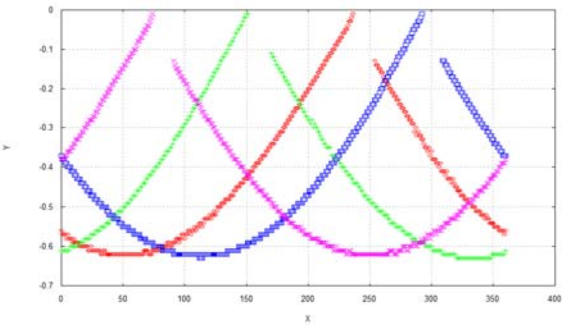

Figure 17 – Minimum energy for $\psi = \mathbf{180^0}$ (red), $\mathbf{210^0}$ (green), $\mathbf{225^0}$ (blue) and $\mathbf{240^0}$ (pink) for retrograde orbits.

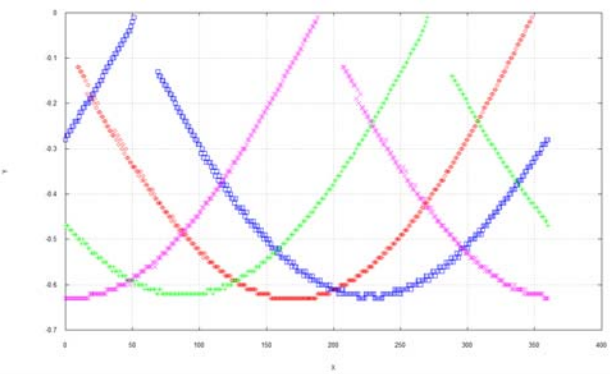

Figure 18 – Minimum energy for $\psi = \mathbf{270^0}$ (red), $\mathbf{300^0}$ (green), $\mathbf{315^0}$ (blue) and $\mathbf{330^0}$ (pink) for retrograde orbits.

Again, as an example, we see that for $\psi = \mathbf{0^0}$ there are no gravitational capture in the interval $\mathbf{1^0} \le \alpha \le \mathbf{19^0}$. The minimum value of the energy happens when $\mathbf{164^0} \le \alpha \le \mathbf{196^0}$.

Several similar results can be obtained by examining the figures.

8 Initial distances different from 100 Km

Now we will consider the initial distance between the space vehicle and the Moon having three different values: 100 km, 500 km and 1000 km. Our objective is to see how this value interferes in the minimum value of the energy of the space vehicle.

In Figure 19 we have direct motion in red (100 km), in green (500 km) and in blue (1000 km). The initial angle of the Sun is $\psi = \mathbf{0^0}$.

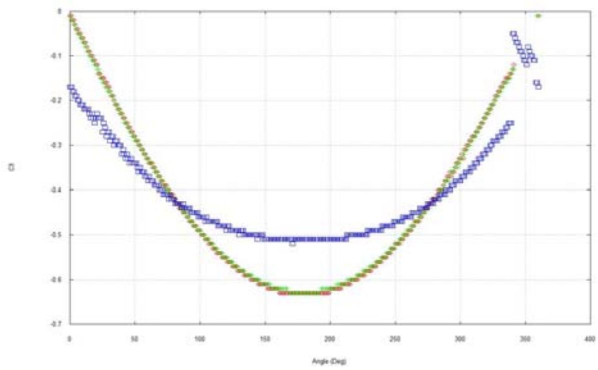

Figure 19 – Different values for the initial distance from the Moon: 100 km, 500 km and 1000 km, for $\psi = \mathbf{0^0}$.

In Figure 20 we have direct motion in red (100 km), in green (500 km) and in blue (1000 km). The initial angle of the Sun is $\psi = \mathbf{60^0}$.

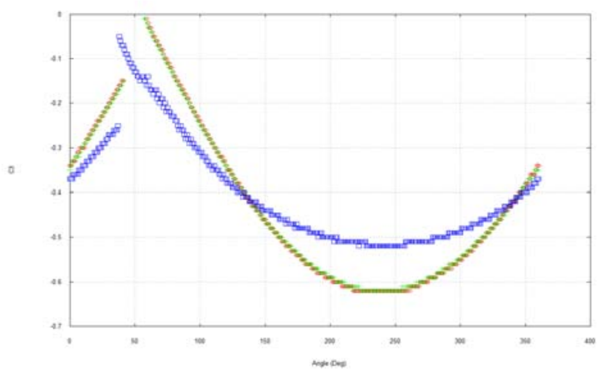

Figure 20 - Different values for the initial distance from the Moon: 100 km, 500 km and 1000 km, for $\psi = \mathbf{60^0}$

In Figure 21 we have direct motion in red (100 km), in green (500 km) and in blue (1000 km). The initial angle of the Sun is $\psi = \mathbf{90^0}$.

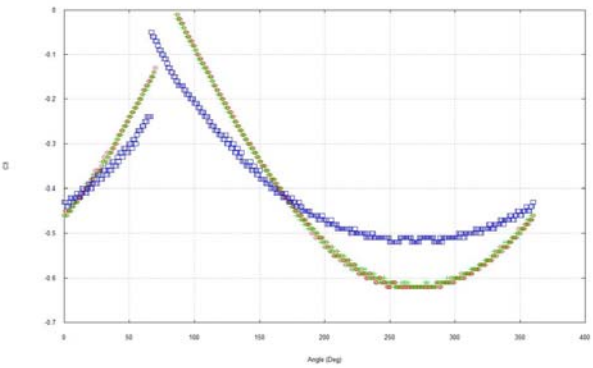

Figure 21 - Different values for the initial distance from the Moon: 100 km, 500 km and 1000 km, for $\psi = \mathbf{90^0}$

In Figure 22 we have retrograde motion in red (100 km) and in green (500 km). The initial angle of the Sun is $\psi = \mathbf{0^0}$.

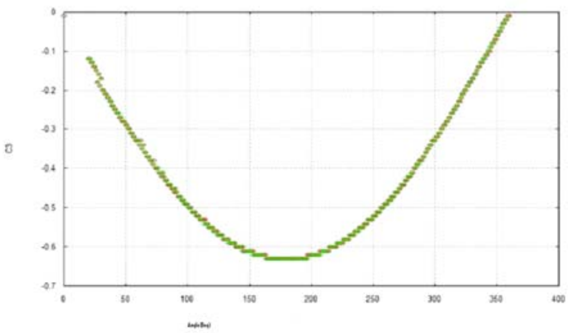

Figure 22 - Different values for the initial distance from the Moon: 100 km and 500 km for $\psi = \mathbf{0^0}$

In Figure 23 we have retrograde motion in red (100 km) and in green (1000 km). The initial angle of the Sun is $\psi = \mathbf{0^0}$.

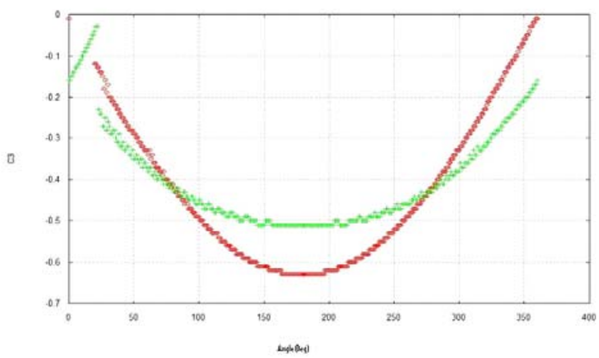

Figure 23 - Different values for the initial distance from the Moon: 100 km and 1000 km for $\psi = \mathbf{0^0}$

In Figure 24 we have retrograde motion in red (100 km) and in green (500 km). The initial angle of the Sun is $\psi = \mathbf{90^0}$.

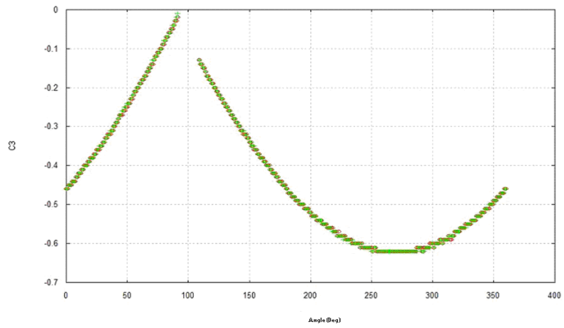

Figure 24 - Different values for the initial distance from the Moon: 100 km and 500 km for $\psi = \mathbf{90^0}$

In Figure 25 we have retrograde motion in red (100 km) and in green (1000 km). The initial angle of the Sun is $\psi = \mathbf{90^0}$.

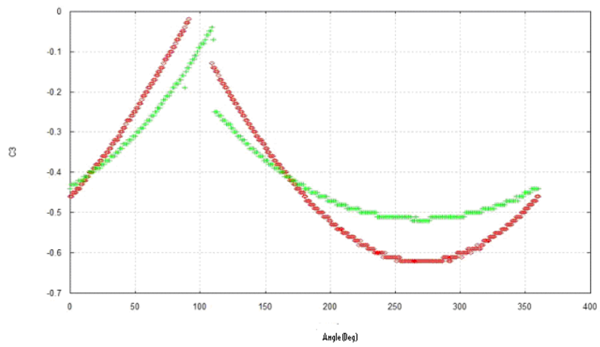

Figure 25 - Different values for the initial distance from the Moon: 100 km and 1000 km for $\psi = \mathbf{90^0}$

In Figure 26 we have retrograde motion in red (100 km) and in green (500 km). The initial angle of the Sun is $\psi = \mathbf{270^0}$

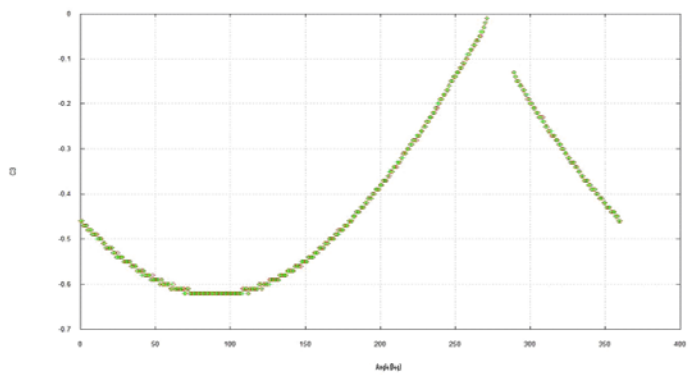

Figure 26 - Different values for the initial distance from the Moon: 100 km and 500 km for $\psi = \mathbf{270^0}$

In Figure 27 we have retrograde motion in red (100 km) and in green (1000 km). The initial angle of the Sun is $\psi = \mathbf{270^0}$

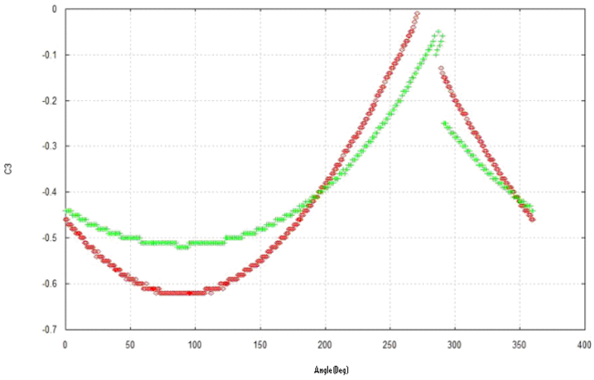

Figure 27 - Different values for the initial distance from the Moon: 100 km and 1000 km for $\psi = \mathbf{270^0}$

We can observe that there are differences among the values of the energy when the initial distance is 1000 km in both graphs. Those differences do not exist when the distance is 100 km or 500 km.

9. Conclusions

This paper studied the problem of gravitational capture under the bi-circular four-body problem. The approach is to perform numerical simulations, in order to know the main characteristics of the problem. In particular, the effects of the initial position and the energy of the spacecraft are considered, as well as the position of the Sun. The results shown here can help mission designers to get the most of the gravitational forces involved in the problem.

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