# **Gravitational Capture Using a Four-Body Model**

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Abstract: A spacecraft (or any particle with negligible mass) suffers a gravitational capture when its orbit changes from hyperbolic (small positive energy) around a celestial body into elliptic (small negative energy) using only gravitational forces. The force responsible for this modification in the orbit of the spacecraft is the gravitational force of the third and the fourth bodies involved in the dynamics. Those forces are equivalent of a zero cost control applied to the spacecraft, equivalent to a continuous thrust. One of the most important applications of this property is the construction of trajectories to the Moon. The concept of gravitational capture is combines with the principles of the gravity-assisted maneuver and the bi-elliptic transfer orbit, to generate a trajectory that requires a fuel consumption smaller than the one required by the Hohmann transfer. The present paper study the energy required for the ballistic gravitational capture in a dynamical model that has the presence of four bodies. Those bodies are assumed to follow the assumptions of the bi-circular model. In particular, the Earth-Moon-Sun-Spacecraft system is considered.

Keywords: Astrodynamics, Celestial Mechanics, Space Trajectories, Gravitational Capture, Space Missions.

# **1.Introduction**

The bi-circular problem is a particular case of the problem of four bodies, where one of the masses, let us say  $m_4$ , is supposed to be infinitely smaller than the other three masses. With that hypothesis,  $m_4$  moves under the gravitational forces of  $m_1$ ,  $m_2$  and  $m_3$ , but it doesn't disturb the motion of the three bodies with significant mass. In the bicircular problem, the motion of  $m_1$ ,  $m_2$  and  $m_3$ around the center of mass is considered as formed by circular orbits and the motion of  $m_4$  has to be a certain function of the initial conditions. We can consider the bi-circular problem as a disturbance of the restricted problem of three bodies. This problem can be used as a model for the motion of a space vehicle in the Sun-Earth-Moon system.

In the first part of the paper we supplied the equations of motion of the model and we defined gravitational capture. The second part is used for the calculation of some numerical results for the bicircular problem, such as direct orbits, retrograde orbits, capture orbits, etc.

# 2.Mathematical models

The problem of four bodies with the two hypotheses shown below is called bi-circular problem.

**First hypothesis**: It is considered two bodies with significant mass moving in circular orbits around the mutual center of mass. Those two bodies are called primaries.

**Second hypothesis**: The third body with significant mass is in a circular orbit around the center of mass of the system formed by the two first primaries and its orbit is coplanar with the orbits of those primaries.

Figure 1 shows the motion of the three primaries in the fixed system of coordinates, also called sidereal system.



Figure 1 – The three primaries.

We will study the motion of a fourth body, with negligible mass, moving under the gravitational attractions of the three bodies with significant mass. We will calculate the planar equations of motion of the space vehicle in the sidereal and synodical systems. We will use the canonical system of units by dividing all the distances by the distance between the two primaries and dividing the masses by the total mass of the two primaries. It will also be defined that the angular speed of the system is unitary. The masses and distances of the Earth, Moon and Sun are: Mass of the Earth.  $M_T = 5.98 \times 10^{24} kg$ ; Mass of the Moon.  $M_L = 7.35 x 10^{22} kg$ ; Mass of the Sun.  $M_{\rm s} = 1.99 \times 10^{30} \, kg$ . Earth-Moon distance  $d_1 = 3.844 \times 10^5 \, km$ ; Earth-Sun distance  $d_2 = 1.496 \times 10^8 \, km$ .

Then, the masses of the Earth, Moon and Sun in the canonical system are:

Mass of the Earth =  $\mu_E = \frac{M_T}{M_L + M_T} = 0.9878715$ ; Mass of the Moon

$$=\mu_M = \frac{M_L}{M_T + M_L} = 0.0121506683$$
; Mass of the

Sun 
$$\mu_s = \frac{M_s}{M_T + M_L} = 328900.48$$
.

The circumferences described by the Moon and the Earth has radius  $\mu_E$  and  $\mu_M$ , respectively.  $(x, y), (x_E, y_E), (x_M, y_M)$  and  $(x_S, y_S)$  are the coordinates of the space vehicle, the Earth, the Moon and the Sun, respectively. Below are the equations of motion of the Earth, Moon and Sun:  $x_E = -\mu_M \cos(t), y_E = -\mu_M \sin(t),$   $x_M = \mu_E \cos(t), y_M = \mu_E \sin(t),$  $x_S = R_S \cos(\psi), y_S = R_S \sin(\psi)$  and  $\psi = \psi_0 + \omega_S t$ . Alexandre Lacerda Machuy, Terezinha De Jesus Stuchi, Antonio Fernando Bertachini De Almeida Prado

Where Rs = 389.1723985 is the distance between the Sun and the center of the system and  $\omega_s = 0.07480133$  is the angular speed of the Sun. We observed that the positions of the Moon, Earth and Sun are:  $(\mu_E, 0)$ ,  $(\mu_M, 0)$  and  $(R_s \cos(\psi_0), R_s \sin(\psi_0))$ .

The distance of the space vehicle to the Earth is  $r_1 = \sqrt{(x - x_E)^2 + (y - y_E)^2}$ ; to the Moon is  $r_2 = \sqrt{(x - x_M)^2 + (y - y_M)^2}$ ; to the Sun is  $r_3 = \sqrt{(x - x_S)^2 + (y - y_S)^2}$ .

Therefore, we have the equations of motion of the space vehicle in the inertial system:

$$\ddot{x} = -\mu_E \frac{(x - x_E)}{r_1^3} - \mu_M \frac{(x - x_M)}{r_2^3} - \mu_S \frac{(x - x_S)}{r_3^3}, (1)$$
$$\ddot{y} = -\mu_E \frac{(y - y_E)}{r_1^3} - \mu_M \frac{(y - y_M)}{r_2^3} - \mu_S \frac{(y - y_S)}{r_3^3}. (2)$$



Figure 2. Initial position

The fixed system of coordinates is called sidereal. In Figure 2 we have the initial position.

We will introduce a system of rotating coordinates on the center of mass of the Earth-Moon system with the same angular speed of the primaries. Be  $(\xi, \eta)$ the coordinates of the particle in this synodical system. The equations that convert the coordinates of the fixed system to the rotating system are:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix}$$
(3)

If we now differentiate each component in equation (3) twice we obtain

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix} \begin{pmatrix} \dot{\xi} - \eta \\ \xi + \dot{\eta} \end{pmatrix},$$
(4)

and

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) \\ \sin(t) & \cos(t) \end{pmatrix} \begin{pmatrix} \ddot{\xi} - 2\dot{\eta} - \xi \\ \ddot{\eta} + 2\dot{\xi} - \eta \end{pmatrix}$$
(5).

The positions of the four bodies are: Moon  $(\xi_M, \eta_M) = (\mu_E, 0)$ , Earth  $(\xi_E, \eta_E) = (-\mu_M, 0)$ , Space Vehicle  $(\xi, \eta)$ , Sun  $(\xi_s, \eta_s) = (R_s[\cos((1-\omega_s)t - \psi_0)]), -R_s[\sin((1-\omega_s)t - \psi_0)])$ . It is clear that  $1 - \omega_s$  is the angular speed of the Sun in the synodical system. The coordinates  $(\xi, \eta)$  are called synodical and the coordinates (x, y) are called sidereal. The three distances in the synodical system are shown below. From the space vehicle to the Earth is  $r_1 = \sqrt{(\xi + \mu_M)^2 + \eta^2}$ ; from the space vehicle to the Moon is  $r_2 = \sqrt{(\xi - \mu_E)^2 + \eta^2}$  and from the space vehicle to the Sun is  $r_3 = \sqrt{(\xi - \xi_s)^2 + (\eta - \eta_s)^2}$ .

The equations of motion of the space vehicle in the new system are:

$$\ddot{\xi} - 2\dot{\eta} - \xi - \frac{\mu_s}{R_s^3} \xi_s = -\mu_E \frac{\xi + \mu_M}{r_1^3} - \mu_M \frac{\xi - \mu_E}{r_2^3} - \mu_S \frac{\xi - \xi_s}{r_3^3}$$
(6)  
$$\ddot{\eta} + 2\dot{\xi} - \eta + \frac{\mu_S}{R_s^3} \eta_S = -\mu_E \frac{\eta}{r_1^3} - \mu_M \frac{\eta}{r_2^3} - \mu_S \frac{\eta - \eta_S}{r_3^3}$$
(7)







In Figure 3 the axes (x,y) are of the fixed system, and the axes  $(\xi,\eta)$  are of the rotating system.

#### **3.** Gravitational Capture

Figure 4 shows a trajectory that ends in gravitational capture. To define gravitational capture it is necessary to use some basic concepts of the problem of two bodies. We will call  $C_3$  the double of the sum of the kinetic and potential energy of the problem of two bodies, the space vehicle and the Moon, that is given by:  $C_3 = V^2 - \frac{2\mu_M}{r}$ , where r and V are, respectively, the distance and the

velocity of the space vehicle with respect to the Moon, and  $\mu_M$  is the gravitational parameter of the Moon.

If we consider only two bodies (the Moon and the space vehicle),  $C_3$  is constant, if only gravitational forces are considered. We will describe the orbits of the space vehicle for values of  $C_3$  according to the classification: *i*) If  $C_3 > 0$ , we have hyperbolic orbits, *ii*) If  $C_3 = 0$ , we have parabolic orbits, *iii*) If  $C_3 < 0$ , we have elliptic orbits.

We defined  $C_3$  as being the double of the energy of the system Moon-vehicle. Unlike what happens in the problem of two bodies,  $C_3$  is not constant in the bi-circular problem. Then, for some initial conditions, the space vehicle can alter the sign of the energy from positive to negative or from negative to positive. When the variation is from positive to negative it is called a gravitational capture orbit. The opposite situation, when the energy changes from negative to positive, is called gravitational escape. We describe the numerical methodology below.

1) A Runge-Kutta of fourth order integrator was used, programmed in the FORTRAN language.

2) We integrated the equations of motion of the space vehicle in the sidereal system.

3) The initial conditions are obtained in the following way. We consider the Moon in the origin of the *XY* system and the Earth with coordinates (-1,0). The starting point of each trajectory is at a distance of 100 km from the surface of the Moon  $(r_p = 1838 \text{ km}, \text{ starting from the center of the Moon})$ . To specify the initial position completely it is necessary to know the value of one more variable. The variable used is the angle  $\alpha$  that represents the

position of the Moon. This angle is measured starting from the Earth-moon line, in the counterclockwise sense, starting from the opposite side of the Earth. The magnitude of the initial velocity V is calculated from the initial value of  $C_3 = V^2 - \frac{2\mu_M}{2}$ . The direction of the velocity

vector of the vehicle is chosen as being perpendicular to the line that links the space vehicle to the center of the Moon, appearing in the counterclockwise direction for the direct orbits and in the clockwise direction for the retrograde orbits.

The orbit is considered of capture when the particle reaches the distance of 100000 km (0.26 canonical units) from the center of the moon in a time smaller than 50 days (approximately 12 canonical units). The sphere with radius 100000 km centered in the Moon is defined as the sphere of influence of the Moon. Figure 3 shows the point P, where the space vehicle escapes from the sphere of influence. The angle that defines this point is called the angle of the entrance position and the Greek letter describes it  $\beta$ . During the numeric integration the step of time is negative, therefore the initial conditions are really the final conditions of the orbit after the capture.



Figure 4 – Initial conditions.

### **4.** Effects of the Angle $\alpha$ .

We now show some results obtained. Figure 5 shows direct orbits and figure 6 shows retrograde orbits. In both situations the angle  $\psi$  is constant and equal to  $0^{\circ}$ , and  $C_3 = -0.15$ . The angle  $\alpha$  assumes the values:  $30^{\circ}$  (6),  $60^{\circ}$  (5),  $90^{\circ}$  (4),  $120^{\circ}$  (3),  $150^{\circ}$ (2),  $180^{\circ}$  (1). The coordinates of the position vector for the case of direct or retrograde orbits are:  $x = r_p \cos(\alpha) + \mu_E$  and  $y = r_p \sin(\alpha)$ . The Alexandre Lacerda Machuy, Terezinha De Jesus Stuchi, Antonio Fernando Bertachini De Almeida Prado

coordinates of the velocity vector for the case of direct orbits are:  $x_v = -Vsin(\alpha)$  and  $y_v = V \cos(\alpha) + \mu_E$ . For the case of retrograde orbits the coordinates of the velocity vector are:  $x_v = Vsin(\alpha)$  and  $y_v = -V \cos(\alpha) + \mu_E$ . We can clearly see the effect of the Sun pushing the trajectories to the right. The direct orbits has a direct path, while the retrograde orbits start their motion to the left and then feel the effects of the Sun and turn to the right.



Figure 5 - Direct orbits.



# 5. Variation of the Energy

We now turn our attention to study the effects of the initial value of the energy  $C_3$ . We consider  $\alpha = \psi = 30^{\circ}$  and make  $C_3$  to assume the values - 0.1 (1), -0.2 (2), -0.3 (3) and -0.4 (4). In figure 7 we have direct orbits and in figure 8 we have retrograde orbits. We see that the reduction of the energy increase the length of the trajectories and, in some cases, it includes loops. Trajectories with energy close to zero escape faster from the primary.



Figure 7 – Direct trajectories varying  $C_3$ 



Figure 8 - Retrograde trajectories varying  $C_3$ 

#### **6.** Variation of the Angle $\psi$

We now study the effects of the position of the Sun. We considered  $\alpha = 90^{\circ}$  and  $C_3 = -0.3$  in figures 9 and 10. The angle  $\psi$  will assume the values  $0^{\circ}$  (5),  $30^{\circ}$  (4),  $45^{\circ}$  (3),  $60^{\circ}$  (2) and  $90^{\circ}$  (1). In figure 9 we have direct orbits and in figure 10 we have retrograde orbits. It is clear that the Sun attracts the trajectories. The results show this fact. The retrograde trajectories have this characteristic more visible.



Figure 9 - Direct orbits.

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### **7. Minimum Value of** C<sub>3</sub>

The value  $C_3$  is associated to the amount of fuel consumed to complete the capture, that is, if this value is smaller, smaller is the amount of fuel consumed in the maneuver.

The first objective of this part of the text is to obtain the smallest consumption of fuel to complete the maneuver of the space vehicle when we have a gravitational capture in the bi-circular problem of four bodies. The second objective is to find favorable areas for gravitational capture in the bicircular problem. We called more favorable areas for gravitational capture, areas where there is a minimum energy.

In all the graphs below, it will be made a variation of the angle  $\alpha$  from 0° up to 360°, in steps of 1°. The initial value of  $C_3$  is -0.65, and the final value is -0.01, with variations of -0.01. For the angle of the Sun ( $\psi$ ), we choose the values : 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°, 210°, 225°, 240°, 270°, 300°, 315° e 330°. For each one of these values of the angle of the sun we have a graph, shown below, that shows the minimum time of gravitational capture. Below we have two groups of graphs for the bi-

Below we have two groups of graphs for the bicircular problem, where the energy is a function of the angle of the Sun. The angle of the Sun is shown in degrees, and the energy in canonical units. The first group is for direct orbits and the second for retrograde orbits.



Figure 11 – Minimum energy for  $\psi = 0^0$  (red),  $30^0$  (green),  $45^0$  (blue) and  $60^0$  (pink) for direct orbits.



Figure 12 – Minimum energy for  $\psi = 90^{\circ}$  (red), 120° (green), 135° (blue) and 150° (pink) for direct orbits.



Figure 13 – Minimum energy for  $\psi = 180^{\circ}$  (red), 210° (green), 225° (blue) and 240° (pink) for direct orbits.

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Figure 14 – Minimum energy for  $\psi = 270^{\circ}$  (red), 300° (green), 315° (blue) and 330° (pink) for direct orbits.

The minimum value of the energy happens in all the graphs above when  $C_3 = -0.65$ . In all the graphs there is an area where gravitational capture doesn't happen. In that area we have collisions and the motion is dominated predominantly by the effect of the Sun.

For example, when  $\psi = 0^{\circ}$  there are no gravitational capture in the interval  $350^{\circ} \le \alpha \le 360^{\circ}$ . The minimum value of the energy happens when  $160^{\circ} \le \alpha \le 180^{\circ}$ .

Several similar results can be obtained just by looking at the plots.

This analyzes are now repeated for the group of retrograde orbits.



Figure 15 – Minimum energy for  $\psi = 0^{\circ}$  (red),  $30^{\circ}$  (green),  $45^{\circ}$  (blue) and  $60^{\circ}$  (pink) for retrograde orbits.



Figure 16 – Minimum energy for  $\psi = 90^{\circ}$  (red), 120<sup>°</sup> (green), 135<sup>°</sup> (blue) and 150<sup>°</sup> (pink) for retrograde orbits.



Figure 17 – Minimum energy for  $_{W} = 180^{\circ}$  (red), 210<sup>°</sup> (green), 225<sup>°</sup> (blue) and 240<sup>°</sup> (pink) for retrograde orbits.



Figure 18 – Minimum energy for  $\psi=270^{\circ}$  (red), **300**<sup>0</sup> (green), **315**<sup>0</sup> (blue) and **330**<sup>0</sup> (pink) for retrograde orbits.

Again, as an example, we see that for  $w = 0^0$  there are no gravitational capture in the interval  $1^0 \le \alpha \le 19^0$ . The minimum value of the energy happens when  $164^{\circ} \le \alpha \le 196^{\circ}$ .

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Several similar results can be obtained by examining the figures.

# 8 Initial distances different from 100 Km

Now we will consider the initial distance between the space vehicle and the Moon having three different values: 100 km, 500 km and 1000 km. Our objective is to see how this value interferes in the minimum value of the energy of the space vehicle.

In Figure 19 we have direct motion in red (100 km), in green (500 km) and in blue (1000 km). The initial angle of the Sun is  $\psi = 0^0$ .



Figure 19 – Different values for the initial distance from the Moon: 100 km, 500 km and 1000 km, for  $\Psi = \mathbf{0}^0$ .

In Figure 20 we have direct motion in red (100 km), in green (500 km) and in blue (1000 km). The initial angle of the Sun is  $\psi = 60^{\circ}$ .



Figure 20 - Different values for the initial distance from the Moon: 100 km, 500 km and 1000 km, for  $_{\rm W} = 60^{\circ}$ 

In Figure 21 we have direct motion in red (100 km), in green (500 km) and in blue (1000 km). The initial angle of the Sun is  $\psi = 90^{\circ}$ .



Figure 21 - Different values for the initial distance from the Moon: 100 km, 500 km and 1000 km, for  $\psi = 90^{\circ}$ 

In Figure 22 we have retrograde motion in red (100 km) and in green (500 km). The initial angle of the Sun is  $\psi = 0^0$ .



Figure 22 - Different values for the initial distance from the Moon: 100 km and 500 km for  $\psi = 0^{\circ}$ 

In Figure 23 we have retrograde motion in red (100 km) and in green (1000 km). The initial angle of the Sun is  $\psi = 0^0$ .



Figure 23 - Different values for the initial distance from the Moon: 100 km and 1000 km for  $\psi = 0^{\circ}$ 

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In Figure 24 we have retrograde motion in red (100 km) and in green (500 km). The initial angle of the Sun is  $\psi = 90^{\circ}$ .





In Figure 25 we have retrograde motion in red (100 km) and in green (1000 km). The initial angle of the Sun is  $\psi = 90^{\circ}$ .



Figure 25 - Different values for the initial distance from the Moon: 100 km and 1000 km for  $\psi = 90^{\circ}$ 

In Figure 26 we have retrograde motion in red (100 km) and in green (500 km). The initial angle of the Sun is  $\psi = 270^{\circ}$ 



Figure 26 - Different values for the initial distance from the Moon: 100 km and 500 km for  $\psi = 270^{\circ}$ 

In Figure 27 we have retrograde motion in red (100 km) and in green (1000 km). The initial angle of the Sun is  $\psi = 270^{\circ}$ 



Figure 27 - Different values for the initial distance from the Moon: 100 km and 1000 km for  $\psi = 270^{\circ}$ 

We can observe that there are differences among the values of the energy when the initial distance is 1000 km in both graphs. Those differences do not exist when the distance is 100 km or 500 km.

#### **9.** Conclusions

This paper studied the problem of gravitational capture under the bi-circular four-body problem. The approach is to perform numerical simulations, in order to know the main characteristics of the problem. In particular, the effects of the initial position and the energy of the spacecraft are considered, as well as the position of the Sun. The results shown here can help mission designers to get the most of the gravitational forces involved in the problem.

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