Optimal analysis MAX MIN of the dependency between the energy consumption of electrical actions and their operation safety

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Abstract: -The study sets off the relation between the reliability of an electric drive system and the specific consumption of energy through the efficiency function used as a quantizing analytical element for the medium energy consumption throughout the safe functioning period of the system. The stability moments of the system as well as the collapse moments are determined using the MIN-MAX or MAX-MIN optimizing methods and also elements belonging to the theory of probabilities and differential calculus. An essential obtained result is represented by the dependence relation between the reduction of the specific power consumption which corresponds to a certain increase in the safe operation of the system. Economically, knowing these results leads to considerable saves of materials, human resources and finances. The explanation of these economic consequences is connected primarily with the fact that the results presented are an immediate consequence of correlating fundamental elements of electrical actions, system reliability and economic analysis and calculation (based mainly on minimizing different types of costs). The purpose of this paper is to determine, mainly, the specific electrical energy consumption and the reliability function for extreme properties (minimum for the specific electrical energy consumption and maximum for the reliability function). Secondly, to determine the equilibrium point of the specific electrical energy consumption and of the reliability of the analyzed system. Thirdly, to draw conclusions that will allow the economic analysis.

Key-Words: - reliability, standardized energy consumption, stability moment, efficiency function

1 Inspects on the dependence between the specific electric power consumption and the safe functioning

For the electric drive systems, the dependence between the power consumption and the safe functioning was set off practically, using the relation between data related to the increase of energy in time on one hand, and to the lowering reliability on the other.

The analytical expression shows that the time moments which characterize the energy increase and the lowering reliability of the analyzed system, depend on m and D (medium value and dispersion), which represent main indicators of reliability; normally, m and D are determined from experimental data.

The efficiency function F resembles the one introduced by Ghermeier in the study of safe functioning through certain methods related to the game theory in mixed strategies.

$$F(R,w)=\int_0^{\infty}\left[\int_0^{x} w(t)\,dt+m\cdot w(x)\right]dR(x) \qquad (1)$$

having the initial conditions:

$$R(0)=1,\ \ w(0)=0 \qquad (2)$$

and:

$$\int_0^{\infty} R(t)\,dt=m,\ \ 2\int_0^{\infty} t\cdot R(t)\,dt=m^2+D \qquad (3)$$

In consequence, we shall consider that the efficiency function F represents the medium energy consumption in the safe functioning period, being built due to the system's reliability function R and the specific energy consumption w.

For calculus, it is easier to detail the following equation (1):

$$F(R,w)=\int_0^\infty\left[\int_0^x w(t)\,dt+m\cdot w(t)\right]dR(x)= \\ \int_0^\infty\int_0^x w(t)\,dt\cdot dR(x)+m\int_0^\infty w(x)\cdot dR(x) \tag{4}$$

2 Finding the stability moments and the falling moments

The system's stability point t_e (figure 1) can be determined as the intersection between optimal curves R_{max} and w_{in} (maximum reliability curve and minimum consumption curve).

These extreme curves represent solutions to the following optimizing problem (P) $\max_R \min_w F(R,w)$

The analytical expressions of the extreme curves R_{max} and w_{min} are the following:

$$R_{max}(t)=\begin{cases}1 & ,0\le t<t_1\\ e^{-\frac{t-t_1}{m}} & ,t_1\le t\le t_2\\ 0 & ,t>t_2\end{cases} \tag{5}$$

$$w_{min}(t)=\begin{cases}0 & ,0\le t<t_1\\ \theta\left[1-k\left(e^{\frac{t_2}{m}}-e^{\frac{t_1}{m}}\right)\right]+ \\ +k\left(e^{\frac{t}{m}}-e^{\frac{t_1}{m}}\right) & ,t_1\le t\le t_2\\ 0 & ,t>t_2\end{cases} \tag{6}$$

and $\theta=\dfrac{t-t_1}{t_2-t_1}$.

The solution to the following equation is the stability point we are looking for:

$$e^{-\frac{t-t_1}{m}}=\frac{t-t_1}{t_2-t_1}\left[1-k\left(e^{\frac{t_2}{m}}-e^{\frac{t_1}{m}}\right)\right]+k\left(e^{\frac{t}{m}}-e^{\frac{t_1}{m}}\right) \tag{7}$$

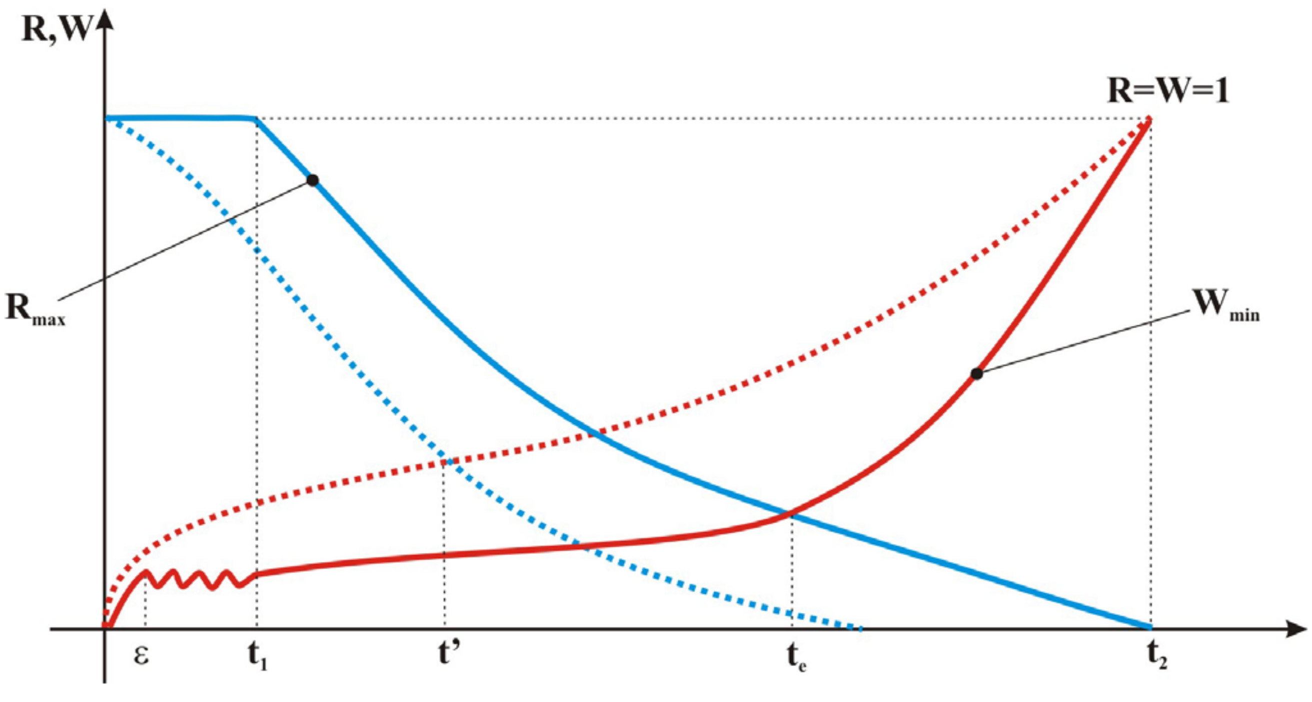

Fig.1 The system's stability point

where $t_1=m\cdot e^{-z}$, $t_2=m\cdot z+m\cdot e^{-z}$, z is the solution to the transcendental equation $e^{-2z}+2\cdot z\cdot e^{-z}+\dfrac{D}{m^2}-1=0$.

An approximate solution to z is $z_0=\ln\dfrac{1}{\sqrt{a}}$

$a=1-\dfrac{D}{m^2}$, with the following miscount $\varepsilon=-\sqrt{a}\cdot\ln a$.

Constant k depends on the analyzed system's technical characteristics, being an integration constant ($k=\frac{z\left(e^{\frac{1-z}{z}}-1\right)}{e^{1-z}(e^{z}-1)\cdot(e^{-z}+z)}\approx\frac{z}{e^{1-z}(e^{z}-1)}$).

For low z_0 (which equals a low dispersion D) the approximate solution to equation (7) is $t_e \approx m$. Having found the stability point, we can determine the energy consumption value as well as the reliability function value in this point.

$$R(t_e)=w(t_e)\approx e^{-\frac{D}{2\cdot m^2-D}} \quad (8)$$

For $t_e \approx m$, results $R(t_e)=w(t_s)\approx 0{,}55$.

Practically, in the moment that the specific energy consumption becomes $e^{-\frac{D}{2\cdot m^2-D}}$, it is obvious that a rapid increase of this consumption will follow, as well as a rapid decrease in the system's reliability.

Time value, t_e marks the moment in which the system has to be improved (improve on the technical performances) or within an immediate period of time (no more than t_2-t_e).

It can be stated that the time moment t_0 (the falling moment of the system on the maximum reliability curve) and t_e (the stability moment) are very close, practically

$$t_e-t_0\approx\sqrt{\frac{-D+\sqrt{D(4m^2-3D)}}{2}}.$$

Moreover, the real moment in which the system falls is t_{real} situated within $[t_0,t_2]$. Having considered that $t_2-t_0\approx\frac{D^2}{2\cdot m\cdot(2\cdot m-D)}$ .it is obvious that the true analysis of the interventions that can be performed on the system, has to consider point t_e because of its economical signification, but also due to the fact that it tolerates a convenient analytical expression.

An important problem related to the analysis of the optimal energy consumption is calculating it within $[0,t_2]$ and $[0,t_e]$ in relation with the optimal curve w_{min}.

The knowledge of these indicators is essential from an economical point of view, because they allow that the interventions on the system (in the sense of improving performances) to be done considering not the time and the cost, but mainly the cost (because the intervention time moments are well known).

In this sense, we are interested in the following ratio:

$$Q=\frac{r_1}{r_2}, \text{ where } r_1=\int_0^{t_e} w_{min}(t)\,dt,\ r_2=\int_0^{t_2} w_{min}(t)\,dt.$$

According to the calculus results $Q=\left(\frac{1-e^{-z}}{z}\right)^2$ and it is obvious that Q's graphic (figure 2) depends on m and D.

For $z_0=\frac{1}{\sqrt{a}}$ results $Q\approx\frac{2m^2-D}{2m^2}=1-\frac{D}{2m^2}$.

If a low dispersion takes place , it can be considered that $Q\approx 1$, therefore the intervention moment approximately equals the system's falling moment.

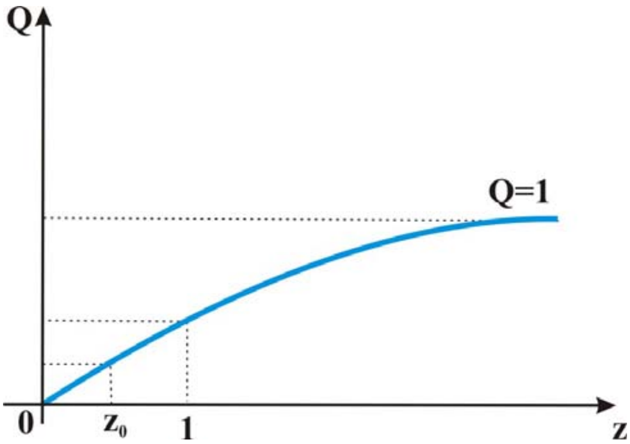

Fig.2 The Q's graphic

3 Determination of the energy consumption reduction and the cost accounting of the interventions on the system

The reduction of the energy consumption and the cost accounting of the interventions on the system are analyzed in relation with the stability points (of the components' and of the system's) but also in the relation with the increasing safety when functioning resulted from the interventions.

Practically, there are the following situations:

a) Interventions on the system's components are done in the stability point (assuming that the stability moments of the components are found around the stability moment of the system).

The reduction of the energy consumption due to the increase in t_e moment (the intervention moment on the system) of the safe functioning having C_s value is represented in figure 3 through the hatched portion (curvilinear trapeze's area $ECt't_e$).

The reduction of the energy consumption S_r can be determined on the grounds of the following equality:

$$S_r = \int_{t'}^{t_e} \left(w_{min}(t) - t' \right) dt \qquad (9)$$

where t' is the solution to the following equation:

$$R_{max}(t) = w_{min}(t_e) + C_s \qquad (10)$$

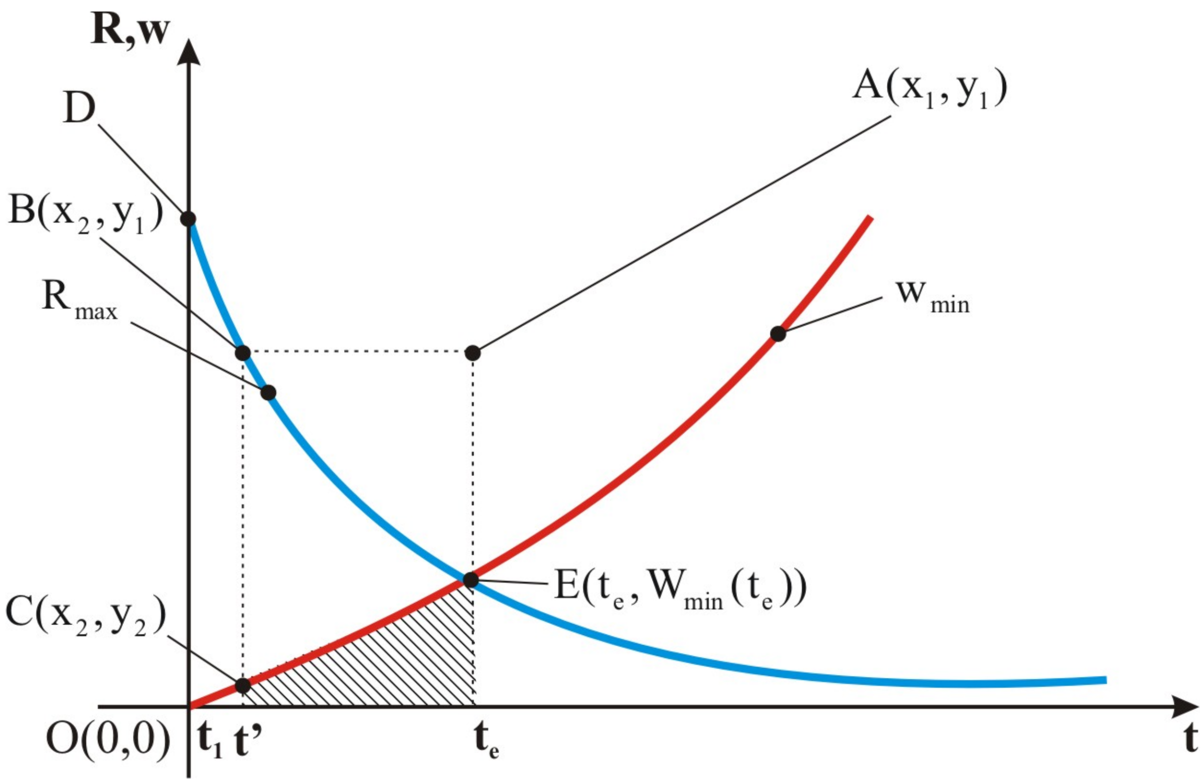

Fig.3 The intervention moment on the system

According to the calculus,

$$t' = t_1 + m \cdot \ln\left(\frac{1}{w_{min}(t_e) + C_s} \right) \qquad (11)$$

The actual determination of is difficult to achieve because of the complex form which the consumption's primitive w_{min} w_{min} has.

Because of this, S_r shall be calculated by approximating the curvilinear trapeze's area $ECt't_e$ through the right angled trapeze's area $ECt't_e$.

The Cartesian coordinates of E and C can be determined considering the OE and DE lines' equations:

$$(D_{OE}) \quad y = \frac{a}{t_e} x \qquad (12)$$

$$(D_{DE}) \quad y = \frac{a-1}{t_e} x + 1 \qquad (13)$$

where $a = w_{min}(t_e) \approx 0,55$ (according to (8) for $t_e \approx m$).

$$x_1 = t_e \approx m, \quad y_1 = a + C_s$$

$$x_2 \approx \frac{m(a + C_s - 1)}{a-1}, \quad y_2 \approx \frac{(a + C_s - 1)}{a-1}$$

$$a = W_{min}(t_e) = e^{-1-e^{-z}}, \quad t' = t_1 + m \cdot \ln\left(\frac{1}{a + C_s} \right)$$

According to the calculus, results:

$$S_r = \frac{C_s}{(a-1)^2}(2-2\cdot a - C_s)\cdot S \qquad (14)$$

where S represents the triangle's area Ot_eE.

Therefore, the energy consumption's reduction coefficient K is:

$$K = \frac{S_r}{S} = \frac{C_s}{(a-1)^2}(2-2\cdot a - C_s) \qquad (15)$$

that is:

$$K \approx 4\cdot C_s \cdot (1-C_s) \qquad (16)$$

The relation $K \approx 4\cdot C_s \cdot (1-C_s)$ represents and important result of this study because it expresses in a precise manner the dependence between the energy consumption reduction and the increase of the safe functioning of the system.

Function $K(C_s) \approx 4\cdot C_s \cdot (1-C_s)$ graph within the interval $\left[0, \frac{1}{2}\right]$ (within this interval the increase in the safe functioning of the system is done only by starting with $R_{max}(t_e) \approx 0,55$ value-figure 4).

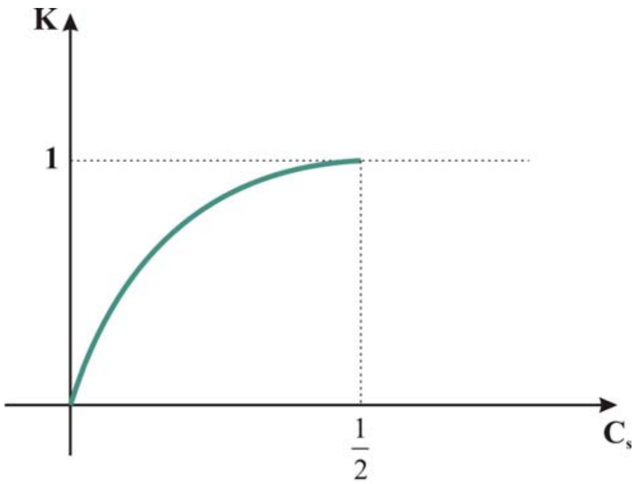

Fig.4 Function $K(C_s)$

An actual problem concerning the dependence between the energy consumption reduction and the increase of the safe functioning of the analyzed system is represented by the determination of the consumption reduction when the increasing value of the system's safe functioning is by 1%.
Thus:

$$K(C_s + 0,01) - K(C_s) = 0,04(1-2C_s) \qquad (17)$$

From which results:

$$K(0,44+0,01) - K(0,44) = 0,004 < K(C_s + 0,01) - K(C_s) < 0,04 = K(0+0,01) - K(0).$$

An explanatory conclusion to the 1% increase of the system's safe functioning ensures a linear variation of the energy consumption between 0,4% and 4%.

This variation is obviously influenced by the increasing level of reliability with C_S value and it is clearly noticed that the higher the reduction is, the 1% variation of the safe functioning starts from the lowest value of C_S.

Synthesizing, the results are as follows:

- the analytical expression of the increase in the safe functioning of the system (assuming that it is constituted from n elements) is:

$$C_s = e^{-\frac{t_e}{m}} \sum_{i=0}^{n} \frac{\left(\frac{t_e}{m}\right)^i}{i!} \qquad (18)$$

- the energy consumption reduction:

$$K = 4\cdot C_s \cdot (1-C_s) \qquad (19)$$

- intervention cost in t_e moment:

$$C(t_e) = \frac{m_1 + m_2}{m_1} \cdot \frac{\sum_{i=1}^{n} e^{-\frac{r_i}{a_i}}}{n} \qquad (20)$$

In which m_1 represents the average time of the system's proper functioning, m_2 the average time of the system's recovery, a_{ir} the failure rate, and r_i the recovery rate of the component S_i.

b) Interventions on the system's components are performed until the stability moment of the system (the stability points of the components being spread around the stability moment of the system)

In this case, the interventions (and their possible arrangements) are analyzed according to the costs.

The increase in safe functioning is:

$C_s = \frac{n_0^2}{3\cdot n\cdot (n_0 - 1)}$ (n_0 represents the components number on which interventions are made with their stability point lower than the system's stability point).

The energy consumption reduction has the following form:

$$K(C_s) \approx \frac{4 \cdot n_0 \cdot (3 \cdot n - n_0)}{9 \cdot n^2} \qquad (21)$$

c) Interventions on the n-n_0 components are performed according to the system's stability point. In this case it is recommended that a global intervention to occur in a moment that is close to the falling moment t_2 of the system. The purpose of studying this is that of analyzing only the interventions which are performed during moments higher than the system's stability point. Anyway, the intervention moments can practically be chosen, the only condition is not to overstep the system's t_2 faulting moment.

d) The energy consumption reduction evaluation if the intervention is performed during an arbitrary moment .

Solving this problem is practically done by evaluating the relation G(t) between curvilinear triangles' areas AB'C' and ABC (figure 5).

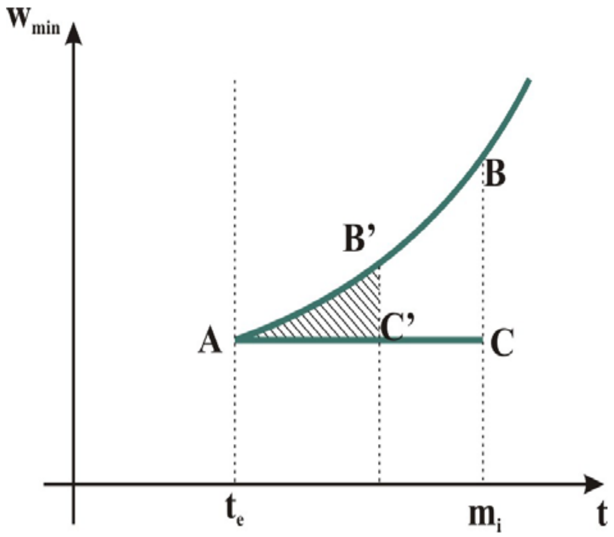

Fig.5

If C_S is the safe profit gained from the interventions which were performed during the stability moments (of the components) lower than the system's stability point, then:

$$G(t) \approx \frac{(t-m)\left[2m - t - t_1 + m\left(1 + \frac{C_s}{a-1}\right)\right]}{(m_i - m)(2m + t_1 - m_i)} \qquad (22)$$

Because the G(t) relation has the constant denominator (considering the fact that the system would fail until the m_i moment), the energy consumption reduction analysis following the intervention in t moment can be performed only according to G(t) numerator.

To make notations easier, m represents the average time of the system's proper functioning and m_i represents the average time of the component S_i proper functioning.

Keeping in mind that $a \approx 0{,}55$ and C_S varies between 0 and 0,45 there can be established the dependence between the energy consumption reduction and the increase of the safe functioning of the system, as well as some important conclusions in the case of individualizing moment t.

If we note: $C = (m_i - m) \cdot (2m + t_1 - m_i)$, we have:

$$G(t) = \frac{1}{C} \cdot (t-m) \cdot \left(3 \cdot m + \frac{m \cdot C_s}{a-1} - t_1 - t\right) \quad t \approx m \qquad (23)$$

Function G graph is presented in figure 6.

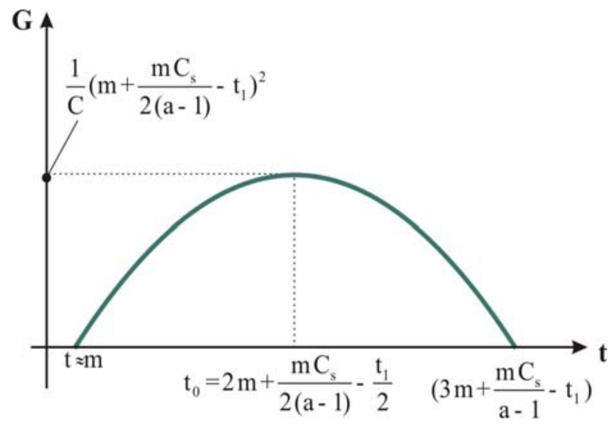

Fig.6 The function G

We can observe that the maximum reduction G_{max} is performed in the $t_0 = 2m + \frac{mC_s}{2(a-1)} - \frac{t_1}{2}$ moment and has the following value:

$$G_{max} = \frac{1}{C}\left(m + \frac{mC_s}{2(a-1)} - t_1\right)^2 \qquad (24)$$

Considering that in t_0 moment it is performed a maximum increase in the safe functioning ($C_s = 0{,}45$), than results $G_{max} = \frac{1}{4} \cdot \frac{n - n_0}{n}$, so we accomplish a maximum 25%. The maximum increase in the safe functioning is obviously accomplished when an intervention in moment t_0

takes place on every component which hasn't been intervened until moment t_e (n_0 represents the components number upon which hasn't been intervened until the stability moment t_e).

The case C_s=0,45 refers to a particular situation, when the interventions performed until moment t_e were very few and during moment t_e no interventions were performed.

If we take into consideration that the interventions performed until the stability moment can lead to an increase in safe functioning with at least 33%, then the intervention in moment t_0 leads to a consumption reduction $G_{max} = \frac{0,052 \cdot m^2}{0,4 \cdot m^2}\left(\frac{n-n_0}{n}\right)$, therefore a maximum reduction of 13% is accomplished.

The explanation of this result sets off the fact that the previous successive interventions have already led to an increase in the system's safe functioning and implicitly to a reduction within the energy consumption.

For a=0,55 and $C_s \in [0,...0,33]$ results $t_0 \in [1,2 \cdot m, \quad 1,3 \cdot m]$, so very closet to t_2. Therefore it is recommended that the interventions performed on the system's components to be done in a moment that is very close to the one of its failure.

According to the components number which suffered interventions and to the previous profit in safe functioning increase, the intervention performed on the components in moment t_0 (close to t_2) varies within the interval $\left[13\frac{n-n_0}{n}\%, \quad 25\frac{n-n_0}{n}\%\right]$.

4 Cost accounts of the intervention performed on the system

As mentioned above it is recommended that the intervention takes place in t_0 on every component that hasn't been intervened on until the stability moment of the system, the cost of this intervention can be evaluated as an average of all the costs corresponding to the n-n_0 components.

The intervention cost in moment t_0 can be evaluated as a BRP (Black Replacement Policy) renewal cost.

The cost $C_i(t_0)$ of intervening on the component S_i is given by:

$$C_i(t_0) \approx \frac{m_1 + m_2}{m_1} \cdot e^{-\frac{r_i}{a_i}}, \quad i = \overline{1, n-n_0} \qquad (25)$$

Therefore, the total cost $C(t_0)$ is given by:

$$C(t) = \frac{m_1 + m_2}{m_1(n-n_0)} \sum_{i=1}^{n-n_0} e^{-\frac{r_i}{a_i}} \qquad (26)$$

It is noticed that this cost (which actually represents the intervention's cost share within the system's acquisition cost) depends explicitly on the reliability components belonging to the system as well as on the reliability elements belonging to the components on which the intervention is performed.

5 The dynamic aspect

The analysis of the reduction of electrical energy consumption through the increase of the operation safety may be done detailed if we calculate effectively the coefficients C_s^1, C_s^2, …, C_s^k, …, which represent the increase coefficients of system reliability from the interventions done in the moments t_1, t_2, …, t_k, … .

Taking into consideration the serial type system structure, we have:

$$C_s^1 = R_2(t_1)R_3(t_1).....R_n(t_1)(1-R_1(t_1))$$
$$C_s^2 = R_3(t_2)R_4(t_2).....R_n(t_2)(1-R_2(t_2))R_1(t_2-t_1)$$
$$C_s^3 = R_4(t_3)R_5(t_3).....R_n(t_3)(1-R_3(t_3))$$
$$R_2(t_3-t_{22})R_1(t_3-t_1)$$
$$\ldots$$
$$C_s^k = R_{k+1}(t_k)R_{k+2}(t_k)...R_n(t_k)(1-R_k(t_k))$$
$$R_{k-1}(t_k-t_{k-1})R_{k-2}(t_k-t_{k-2})....R_1(t_k-t_1)$$

We can concentrate these equations into:

$$C_s^k = \prod_{i=k}^{n} R_{i+1}(t_i)(1-R_i(t_i)) \prod_{j=1}^{k-1} R_j(t_k - t_j) \qquad (27)$$
$$k = \overline{1,n}$$

Since $R_i(t) = e^{-\frac{t}{m_i}}$, $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} + ... + \frac{1}{m_n}$, after a short calculation we obtain:

$$C_s^k = e^{-\frac{t_k}{m} + \frac{t_1}{m_1} + \frac{t_2}{m_2} + ... + \frac{t_k}{m_k}}\left(1 - e^{-\frac{t_k}{m_k}}\right), k = \overline{1,n} \qquad (28)$$

If the intervention moments on the system t_1, t_2, …, t_n are also equilibrium type, after calculations it results:

$$C_s \approx e^{-k-\frac{m_k}{m}-\frac{1}{2}} \tag{29}$$

Based on the relation (8) it is obvious that $0 \le C_s^k \le 0,55$, $k=\overline{1,n}$. From here it results immediately the possibilities:

a) $C_s^k = 0{,}55$, so $C_s^k \approx e^{-\frac{1}{2}}$ resulting $k=\frac{m_k}{m}$. Since $k \ge 1$, meaning that $m_k \ge m$.

b) $C_s^k = 0$, so $k=\frac{2m_k+m}{2m}$. Since $k \ge 1$, it results that $m_k \ge \frac{m}{2}$.

Knowing that the equilibrium point of the system t_e is around the value m, the analysis of the extreme values of C_s^k, leads to the following results:

1. In the interval $(0,t_1]$ the system operates with high safety (basically the reliability is close to 1) and the electrical energy consumption is low (these results were presented by in the paragraph 2, t_1 is given by (7)).

2. In the interval $(t_1, \frac{1}{2}t_e)$, there may appear failures in the system, but its renewal may be done with high safety, which leads to low energy consumption.

3. In the interval $[\frac{1}{2}t_e, t_e)$ any system failure because of one component leads to a low possibility to increase the reliability after the intervention on the component. It is basically the same as case b) analyzed previously. In this case, because the reliability of the system maintains relatively low, it is obvious that we will have relatively high consumption of energy in the analyzed interval.

4. If the intervention on the system is done around the equilibrium point, then the system safety may be maximized, which leads to a minimum consumption of electrical energy. It is the case analyzed in the paragraph 3.

All these situations are represented in figure 7.

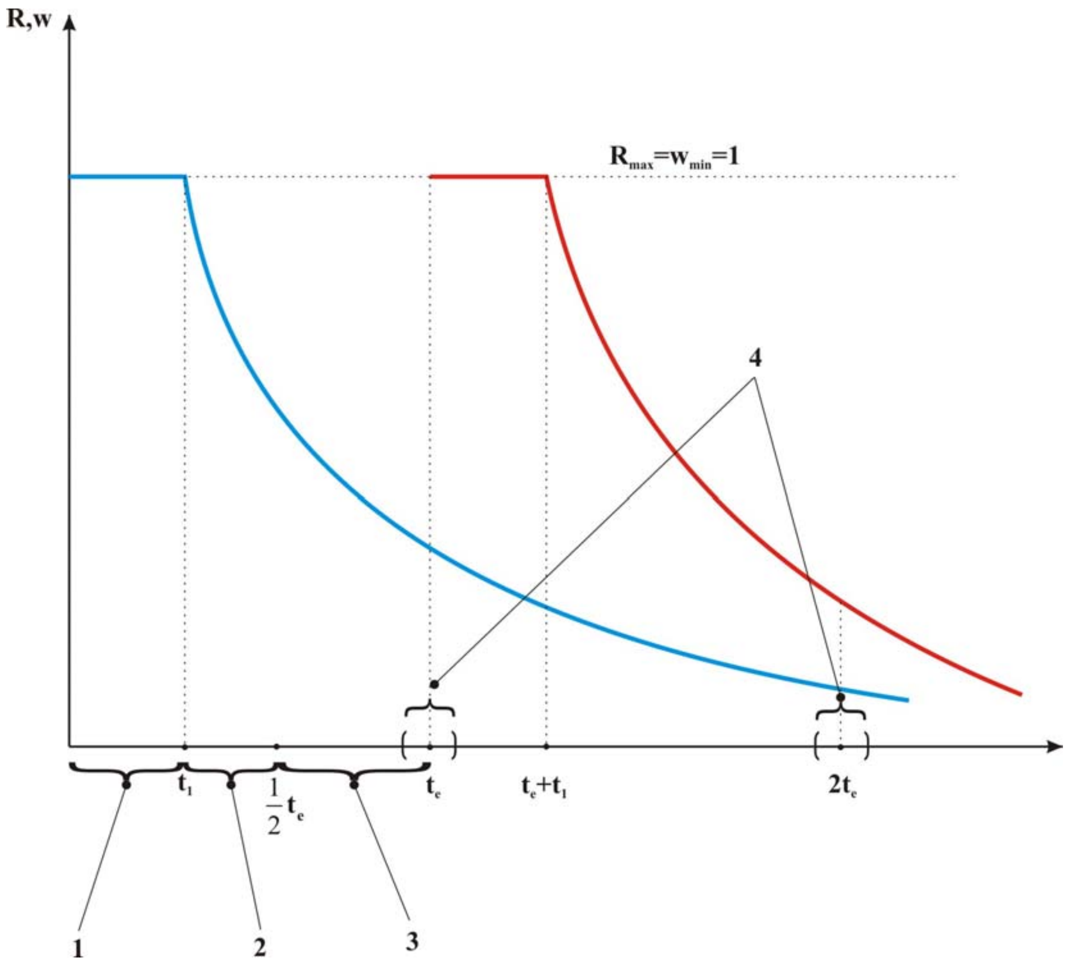

Fig.7 The dynamic aspect

1 Correspond to:
- Increased reliability;
- Low consumption;

2 Correspond to:
- accidental breakdowns;
- Relatively low consumption;

3 Relatively numerous breakdowns may lead to:
- Decreased reliability and increased consumption during individual interventions;
- Increased reliability and low consumption for group interventions;

4 Optimal interventions which lead to:
- Maximum safety;
- Minimum consumption.

Considering that the successive interventions on the system lead to a continuous movement of the equilibrium points towards right, the recurrence relation between two consecutive equilibrium type moments is the following:

$$t_e^n = t_e^{n-1}\left(1 + \frac{m}{t_e^{n-1} + mC_s^n - t_1}\right) \tag{30}$$

In the particular case where the interventions are done only around equilibrium type moments, this recurrence relation becomes:

$$t_e^n = t_e^{n-1} + m \tag{31}$$

so $t_e^n \approx nm$, result according to the situations observed practically.

6 Conclusions

Having into consideration the results presented in this study, there can be drawn the following conclusions:

1. If the stability moments of the components are found around the system's stability moment t_e, then the intervention performed on the system's elements is accomplished in moment t_e.

2. If the stability moments of the components are found spread around the stability point then:

2.1 Interventions are performed first of all in the stability moments of the components lower than t_e, being possible a grouping of the interventions, and they perform concerning the interventions cost.

2.2 Interventions can be performed on the components if these components have stability moments higher than t_e either in moment t_e, or in moments higher than t_e. In this last case it is recommended that the intervention to be performed altogether in a moment close to the system's failure moment t_2.

Basically, through measurements we highlight both the electrical energy consumption and the reliability, then it is done the dependency between them (usually has the form of a non-linear regression which depends on a considerable number of parameters).

The approached working technique is based mainly on the method of least squares, on calculations of several important statistical indicators (average, mean square deviation, different order moments, correlation coefficient etc.), and also on using statistical tests of concordance (especially the test Kolmogorov – Smirnov).

The results may be synthesized in:

1. Solving the optimal problem type max min in case of efficiency function (1);

The optimal solutions searched are (5) and (6);

This equation was solved approximately and was mentioned the calculation error.

2. There was determined the equilibrium point t_e (and the integration constant k, so it does not depend on definite data) and was economically interpreted.

3. The effective calculation of the electrical energy consumption until the equilibrium moment t_e and until the system failure moment:

$$\int_0^{t_e} w_{min}(t) = \frac{\left(1 - z - e^{-z}\right)^2}{2z} -$$
$$-\frac{k}{2}\gamma\left[\left(1 - z - e^{-z}\right)\left(e^{z+e^{-z}} + e^z\right)\right] -$$
$$-\frac{k}{2}\gamma\left[2\left(mz + me^{-z}\right)e^{e^{-z}}\right] + k\cdot m\left(e - e^{e^{-z}}\right)$$

where:

$$\gamma = \frac{1 - z - e^{-z}}{z}$$

$$\int_0^{t_2} W_{min}(t) = \frac{mz}{2} - \frac{mk}{2}\left[z\left(e^{z+e^{-z}} + e^{e^{-z}}\right) - 2\left(e^{z+e^{-z}} - e^{e^{-z}}\right)\right]$$

where:

$$z = \sqrt{1 - \frac{D}{m^2}}\,.$$

4. The calculation of the equilibrium point t_e^i

of each component of the analyzed system

$$t_e^i = m^i \ln\left(\frac{z}{1-e^{-z}}\right),\ i = \overline{1,n}$$

5. The analysis of the interventions on system components in different circumstances:

- when the interventions were done taking into consideration only the equilibrium points, it was analyzed the reduction of electrical energy consumption, the fundamental result being $R = 4\ C_s\ (1 - C_s)$; C_s being the increase of operation safety;
- when the interventions were done taking into consideration the failure points or the equilibrium and failure points, the calculations were focused on costs (and not on the reduction of energy consumption).

The mathematical device used was focused on a special optimization technique (max min optimization), on differential calculation, and also on basic results of the probability and statistical calculation (especially on the calculation of statistical indicators).

Some advanced calculations were approximated but the calculation error was insignificant.

The results presented in this chapter depend mainly on the precision of calculating the statistical indicators m and D. Since the experimental data are obtained through measurements, it is important to mention that, usually, the experimental data collection presents errors (systematic or casual).

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