Nonlinear Growth Models in the Age Measurement Of The European Rabbits In Australia in the Context of Alternative Models in Multivariate Statistics

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Abstract: Examining the special model

$$\text{Lens} = \alpha \exp\{-\beta / (\text{Age} + \gamma)\}$$

which belongs to Dudzinski and Mykytowycz (1961), and was used in a study made by them to find the age of Oryctolagus Caniculus, which lives in Europe and is known as the European rabbit, with live eye lens weight, we showed that similar results can be obtained with the Gompertz and Logistic nonlinear regression models. We determined that the results given by the models are showing the goodness of fit, as a result of a statistical analysis. Furthermore we showed that adding various parameters to Dudzinski and Mykytowycz model will not change the results.

Key-Words*:* nonlinear regression, logistic and gompertz model, gauss-newton iteration method, SPSS

Table 1 Dry weight of eye lens and age

Data No	X(Age/Day)	Y(Eye lens weight(mg))	Data No	X(Age/Day)	Y(Eye lens weight(mg))	Data No	X(Age/Day)	Y(Eye lens weight(mg))
1	15	21.66	25	98	104.30	49	285	189.66
2	15	22.75	26	125	134.90	50	300	186.09
3	15	22.30	27	142	130.68	51	301	186.70
4	18	31.25	28	142	140.58	52	305	186.80
5	28	44.79	29	147	155.30	53	312	195.10
6	29	40.55	30	147	152.20	54	317	216.41
7	37	50.25	31	150	144.50	55	338	203.23
8	37	46.88	32	159	142.15	56	347	188.38
9	44	52.03	33	165	139.81	57	354	189.70
10	50	63.47	34	183	153.22	58	357	195.31
11	50	61.13	35	192	145.72	59	375	202.63
12	60	81.00	36	195	161.10	60	394	224.82
13	61	73.09	37	218	174.18	61	513	203.30
14	64	79.09	38	218	173.03	62	535	209.70
15	65	79.51	39	219	173.54	63	554	233.90
16	65	65.31	40	224	178.86	64	591	234.70
17	72	71.90	41	225	177.68	65	648	244.30
18	75	86.10	42	227	173.73	66	660	231.00
19	75	94.60	43	232	159.98	67	705	242.40
20	82	92.50	44	232	161.29	68	723	230.77
21	85	105.00	45	237	187.07	69	756	242.57
22	91	101.70	46	246	176.13	70	768	232.12
23	91	102.90	47	258	183.40	71	860	246.70
24	97	110.00	48	276	186.26			

1 Introduction

In their study, Dudzinski and Mykytowycz [1], determining that eye lens weight tends much less to alter than the total body weight due to the enviromental conditions, thought that life period of free living and wild European rabbit Oryctolagus Cuniculus, which is an important animal for Australia, can be an indicator of the age. 71 of the rabbits mentioned are chosen randomly and dry weight of their eye lenses are measured. And because they are under examination, beginning their ages from the youngest rabbit (Age / Day) the data table below is prepared. Similarly it will be someway determined if echological enviromental factors have an affect on the lifes of all the other animals which live in wild nature. To obtain similar data, we worked hard for a long time in Turkey. When we reached the responsible for both Government and Private wild animal living areas, it was revealed that they don't care such information. Thus, applying some data to various "nonlinear regression models", we thought that the results may be helpful to the echological studies [1].

The graph of Table 1 above is given below:

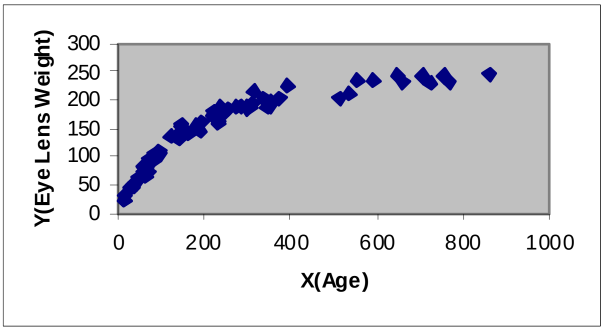

Fig.1 Graph of the Table 1

For each of the rabbits simple Logistic and Gompertz growth curve models which were denoted by equations [1] and [2] have thought without a constant term, to use the measured live eye lens weight values. The mathematical equations belonging to the models which will be used are given below:

$$y = f(x) = \frac{\alpha}{1+\beta e^{-\kappa x}} = \alpha(1+\beta e^{-\kappa x})^{-1} \qquad (1)$$

$$y = f(x) = \alpha e^{\{-e^{-\kappa(x-\gamma)}\}} \qquad (2)$$

Here, f(x): age in time {as day} x, α: asymptotic eye lens weight, β: a contant which defines the growth curve, κ: growth velocity, γ: a parameter about the distortion (twist) point, e:represents natural logarithm.

Least squares method normal equations for these models are given below:

$$\sum_{i=1}^{71}\left[y_i - \left\{\alpha/1+\beta e^{-\kappa x_i}\right\}\right]\left(1/\ \ 1+\beta e^{-\kappa x_i}\right) = 0$$

$$\sum_{i=1}^{71}\left[y_i - \left\{\alpha/1+\beta e^{-\kappa x_i}\right\}\right]\left(-\alpha e^{-\kappa x_i}/\{1+\beta e^{-\kappa x_i}\}^2\right) = 0$$

$$\sum_{i=1}^{71}\left[y_i - \left\{\alpha/1+\beta e^{-\kappa x_i}\right\}\right]\left(\alpha\beta x_i e^{-\kappa x_i}/\{1+\beta e^{-\kappa x_i}\}^2\right) =$$

and

$$\sum_{i=1}^{71}\left[y_i - \alpha e^{-e^{-\kappa(x_i-\gamma)}}\right]\left(e^{-e^{-K(x_i-\gamma)}}\right) = 0$$

$$\sum_{i=1}^{71}\left[y_i - \alpha e^{-e^{-\kappa(x_i-\gamma)}}\right]\left(\alpha(x_i-\gamma)e^{-\kappa(x_i-\gamma)}e^{-e^{-K(x_i-\gamma)}}\right) = 0$$

$$\sum_{i=1}^{71}\left[y_i - \alpha e^{-e^{-\kappa(x_i-\gamma)}}\right]\left(-\alpha\gamma e^{-\kappa(x_i-\gamma)}e^{-e^{-K(x_i-\gamma)}}\right) = 0$$

First of these normal equation systems is nonlinear to β and κ, and the latter is nonlinear to κ and γ. Thus there is no simple and closed-form solution. Using iterative methods is needed to reach the solution.

For Gauss-Newton method [2] if the initial vectors are taken as [200.8,12.9,-0.03] for logistic model and as [217.7,-2.8,-0.14] for Gompertz model (see Table II and III), Logistic model converges in iteration 11 and Gompertz model converges in iteration 9. The solution vectors are found as [α,β,κ] = [223.45,4.39,-0.01] for Logistic model and as [α,κ,γ] = [227.9,-1.98,-0.009] for Gompertz model.

Table 2 The initial values of Gompertz model

```
                                x(1,i)
0.150000E+02 0.150000E+02 0.150000E+02 0.180000E+02 0.280000E+02
0.290000E+02 0.370000E+02 0.370000E+02 0.440000E+02 0.500000E+02
0.500000E+02 0.600000E+02 0.610000E+02 0.640000E+02 0.650000E+02
0.650000E+02 0.720000E+02 0.750000E+02 0.750000E+02 0.820000E+02
0.850000E+02 0.910000E+02 0.910000E+02 0.970000E+02 0.980000E+02
0.125000E+03 0.142000E+03 0.142000E+03 0.147000E+03 0.147000E+03
0.150000E+03 0.159000E+03 0.165000E+03 0.183000E+03 0.192000E+03
0.195000E+03 0.218000E+03 0.218000E+03 0.219000E+03 0.224000E+03
0.225000E+03 0.227000E+03 0.232000E+03 0.232000E+03 0.237000E+03
0.246000E+03 0.258000E+03 0.276000E+03 0.285000E+03 0.300000E+03
0.301000E+03 0.305000E+03 0.312000E+03 0.317000E+03 0.338000E+03
0.347000E+03 0.354000E+03 0.357000E+03 0.375000E+03 0.394000E+03
0.513000E+03 0.535000E+03 0.554000E+03 0.591000E+03 0.648000E+03
0.660000E+03 0.705000E+03 0.723000E+03 0.756000E+03 0.768000E+03
0.860000E+03

                                sigmax(1,i)
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00

                                y(i)
0.216600E+02 0.227500E+02 0.223000E+02 0.312500E+02 0.447900E+02
0.405500E+02 0.502500E+02 0.468800E+02 0.520300E+02 0.634700E+02
0.611300E+02 0.810000E+02 0.730900E+02 0.790900E+02 0.795100E+02
0.653100E+02 0.719000E+02 0.861000E+02 0.946000E+02 0.925000E+02
0.105000E+03 0.101700E+03 0.102900E+03 0.110000E+03 0.104300E+03
0.134900E+03 0.130680E+03 0.140580E+03 0.155300E+03 0.152200E+03
0.144500E+03 0.142150E+03 0.139810E+03 0.153220E+03 0.145720E+03
0.161100E+03 0.174180E+03 0.173030E+03 0.173540E+03 0.178860E+03
0.177680E+03 0.173730E+03 0.159980E+03 0.161290E+03 0.187070E+03
0.176130E+03 0.183400E+03 0.186260E+03 0.189660E+03 0.186090E+03
0.186700E+03 0.186800E+03 0.195100E+03 0.216410E+03 0.203230E+03
0.188380E+03 0.189700E+03 0.195310E+03 0.202630E+03 0.224820E+03
0.203300E+03 0.209700E+03 0.233900E+03 0.234700E+03 0.244300E+03
0.231000E+03 0.242400E+03 0.230770E+03 0.242570E+03 0.232120E+03
0.246700E+03

                                sigmay(i)
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
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0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01

              INITIAL VALUES OF THE PARAMETERS
0.217700E+03 -.280600E+01 -.137100E-01
```

```
                                x(1,i)
0.150000E+02 0.150000E+02 0.150000E+02 0.180000E+02 0.280000E+02
0.290000E+02 0.370000E+02 0.370000E+02 0.440000E+02 0.500000E+02
0.500000E+02 0.600000E+02 0.610000E+02 0.640000E+02 0.650000E+02
0.650000E+02 0.720000E+02 0.750000E+02 0.750000E+02 0.820000E+02
0.850000E+02 0.910000E+02 0.910000E+02 0.970000E+02 0.980000E+02
0.125000E+03 0.142000E+03 0.142000E+03 0.147000E+03 0.147000E+03
0.150000E+03 0.159000E+03 0.165000E+03 0.183000E+03 0.192000E+03
0.195000E+03 0.218000E+03 0.218000E+03 0.219000E+03 0.224000E+03
0.225000E+03 0.227000E+03 0.232000E+03 0.232000E+03 0.237000E+03
0.246000E+03 0.258000E+03 0.276000E+03 0.285000E+03 0.300000E+03
0.301000E+03 0.305000E+03 0.312000E+03 0.317000E+03 0.338000E+03
0.347000E+03 0.354000E+03 0.357000E+03 0.375000E+03 0.394000E+03
0.513000E+03 0.535000E+03 0.554000E+03 0.591000E+03 0.648000E+03
0.660000E+03 0.705000E+03 0.723000E+03 0.756000E+03 0.768000E+03
0.860000E+03

                                sigmax(1,i)
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
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0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
0.000000E+00
```

Table 3 The initial values for Logistic model

```
                              y(i)
0.216600E+02 0.227500E+02 0.223000E+02 0.312500E+02 0.447900E+02
0.405500E+02 0.502500E+02 0.468800E+02 0.520300E+02 0.634700E+02
0.611300E+02 0.810000E+02 0.730900E+02 0.790900E+02 0.795100E+02
0.653100E+02 0.719000E+02 0.861000E+02 0.946000E+02 0.925000E+02
0.105000E+03 0.101700E+03 0.102900E+03 0.110000E+03 0.104300E+03
0.134900E+03 0.130680E+03 0.140580E+03 0.155300E+03 0.152200E+03
0.144500E+03 0.142150E+03 0.139810E+03 0.153220E+03 0.145720E+03
0.161100E+03 0.174180E+03 0.173030E+03 0.173540E+03 0.178860E+03
0.177680E+03 0.173730E+03 0.159980E+03 0.161290E+03 0.187070E+03
0.176130E+03 0.183400E+03 0.186260E+03 0.189660E+03 0.186090E+03
0.186700E+03 0.186800E+03 0.195100E+03 0.216410E+03 0.203230E+03
0.188380E+03 0.189700E+03 0.195310E+03 0.202630E+03 0.224820E+03
0.203300E+03 0.209700E+03 0.233900E+03 0.234700E+03 0.244300E+03
0.231000E+03 0.242400E+03 0.230770E+03 0.242570E+03 0.232120E+03
0.246700E+03

                            sigmay(i)
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
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0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01 0.100000E+01
0.100000E+01

           INITIAL VALUES OF THE PARAMETERS
0.200800E+03 0.129000E+02 -.297100E-01
```

```
        THE CONVERGENCE HAPPENED IN THE ITERATION       9
proje.txt

 N.  80:   Y = A*EXP(B*EXP(C*X)) <--- Gompertz

    PARAMETERS: Mean              UNCERTAINTIES: SD              t          P(t)

  A =  0.22791665313E+03      SIGMAA =  0.33397553359E+01    0.682435E+02    0.000
  B = -0.19809727482E+01      SIGMAB =  0.80239241358E-01    -.246883E+02    0.000
  C = -0.88862799166E-02      SIGMAC =  0.44268518325E-03    -.200736E+02    0.000

                 Attention:  SIGMA(p) = f * SIGMA
p=68.3%: f= 1.00   p=90%: f= 1.67   p=95.4%: f= 2.00   p=99%: f= 2.75

                            Chi-Square:
Deg. Freed.= 68  ChiSq.=0.680000E+02  Red. ChiSq.=0.100000E+01 => P(Red. ChiSq.)=0.477
```

```
        THE CONVERGENCE HAPPENED IN THE ITERATION      11
proje.txt

 N.  81:   Y = A/(1+B*EXP(C*X)) <--- Logistic

    PARAMETERS: Mean              UNCERTAINTIES: SD              t          P(t)

  A =  0.22344679745E+03      SIGMAA =  0.36701079241E+01    0.608829E+02    0.000
  B =  0.43883422345E+01      SIGMAB =  0.35060263395E+00    0.125166E+02    0.000
  C = -0.12296460845E-01      SIGMAC =  0.69956602499E-03    -.175773E+02    0.000

                 Attention:  SIGMA(p) = f * SIGMA
p=68.3%: f= 1.00   p=90%: f= 1.67   p=95.4%: f= 2.00   p=99%: f= 2.75

                            Chi-Square:
Deg. Freed.= 68  ChiSq.=0.680000E+02  Red. ChiSq.=0.100000E+01 => P(Red. ChiSq.)=0.477
```

Thus the fitted models are

$$\hat{y} = \frac{223.45}{1 + 4.39e^{-0.01x}} \text{ (Logistic Model)}$$

and (Gompertz Model)

For each model the growth models constituted with the obtained parameter estimation results are given in Fig.2 and Fig.3.

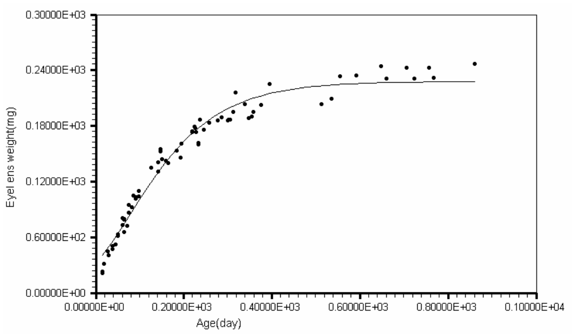


Fig.2 Logistic growth model for live eye lens weight of European rabbit

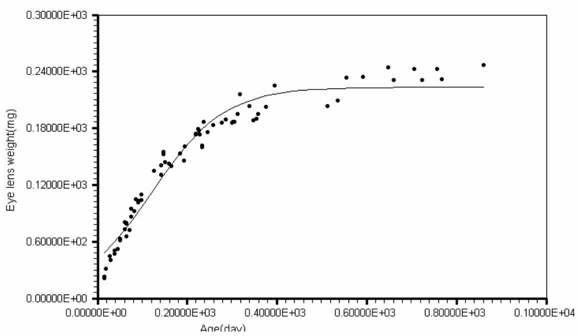


Fig.3 Gompertz growth model for live eye lens weight of European rabbit

The resulting residual sum of squares is 12394 for logistic model and 8474.7 for Gompertz model.

```
                    Analysis of Variance:
F = (Sum Sq./Deg. Freed.)_reg / (Sum Sq./Deg. Freed.)_error  =>
F = (0.28287E+06/2) / (0.84747E+04/ 68) = 0.1135E+04  =>  P(F) = 0.00

Standard Deviation of the Fitting: 0.111637E+02

                   Correlation Coeficient:
  R²yy(x) = 0.9718060E+00            adjR²yy(x) = 0.9709768E+00

        Ryy(x) =  0.985802E+00  =>  P(NP,|R|) = 0.300E-07
```

```
                    Analysis of Variance:
F = (Sum Sq./Deg. Freed.)_reg / (Sum Sq./Deg. Freed.)_error   =>
F = (0.27279E+06/2) / (0.12394E+05/ 68) = 0.7484E+03   =>   P(F) = 0.00

Standard Deviation of the Fitting: 0.135003E+02

                    Correlation Coeficient:
  R²yy(x) = 0.9591523E+00              adjR²yy(x) = 0.9579509E+00
        Ryy(x) =  0.979363E+00   =>   P(NP,|R|) = 0.300E-07
```

An estimation of error variance

$$\hat{\sigma}^2 = RSS_E = \frac{\sum_{i=1}^{n}(y_i-\hat{y}_i)^2}{n-p} = \frac{\sum_{i=1}^{n}[y_i - f(x_i,b)]^2}{n-p} = \frac{S(b)}{n-p}$$

can be found as S(b) = 12394 / n - p = 71 – 3 = 182.27 for logistic model and as

S(b) = 8474.7 / n – p = 71 – 3 = 124.63 for Gompertz model. Residuals can be obtained by

$e_i = y_i - \hat{y}_i$ (i = 1,2,…,71).

The plots of the estimated values against the residuals for each model are given in Fig.4 and Fig.5.

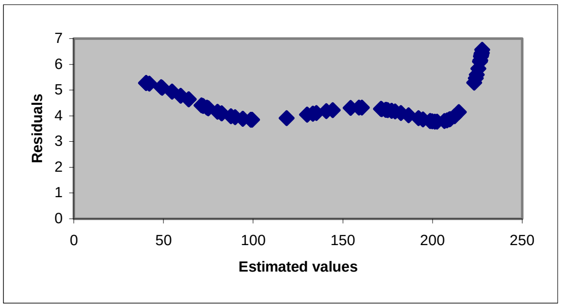


Fig.4. Estimated values versus residuals for Gompertz model

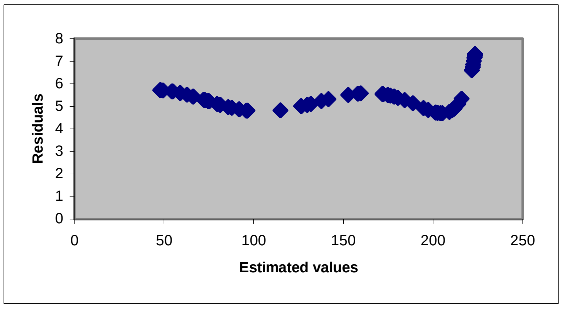


Fig.5 Estimated values versus residuals for logistic model

The asymptotic (large-sample) [3] covariance matrix of the regression can be obtained by $\mathrm{var}(b) = \hat{\sigma}_e^{\,2}(D'D)^{-1}$. Here b is the parameter estimates vector, $\hat{\sigma}_e^{\,2}$ is the error variance and D is the partial derivatives matrix.

If the numeric values put on their places, for Logistic model

$$D'D = \begin{bmatrix} 36.43 & -293.83 & -222855.04 \\ -293.83 & 5613.72 & 2995779.35 \\ -222855.04 & 2995779.35 & 2179116573 \end{bmatrix}$$

$$(D'D)^{-1} = \begin{bmatrix} 0.0739 & -0.00062 & 8.41x10^{-6} \\ -0.00062 & 0,000674 & -9.9x10^{-8} \\ 8.41x10^{-6} & -9.9x10^{-8} & 2.68x10^{-9} \end{bmatrix}$$

$$\mathrm{var}(b) = \hat{\sigma}_e^{\,2}(D'D)^{-1} = \begin{bmatrix} 13.47 & -0.1138 & 0.0015 \\ -0.1138 & 0.123 & -0.00018 \\ 0.0015 & -0.00018 & 4.9x10^{-7} \end{bmatrix}$$

and for Gompertz model

$$D'D = \begin{bmatrix} 27.63 & 618.77 & -216251.33 \\ 618.77 & 52096.72 & -9646228.8 \\ -216251.33 & -9646228.8 & 2920767264 \end{bmatrix}$$

$$(D'D)^{-1} = \begin{bmatrix} 0.09 & 0.0004 & 8.02x10^{-6} \\ 0.0004 & 5.14x10^{-5} & 2.01x10^{-7} \\ 8.02x10^{-6} & 2.01x10^{-7} & 1.6x10^{-9} \end{bmatrix}$$

$$\text{var}(b) = \hat{\sigma}_e{}^2 (D'D)^{-1} = \begin{bmatrix} 11.154 & 0.0526 & 0.001 \\ 0.0526 & 0.0064 & 0.000025 \\ 0.001 & 0.000025 & 2x10^7 \end{bmatrix}.$$

By taking squareroots of the diagonal elements of these covarince matrices standart errors of the model parameters can be found

assh(α) = 3.67

sh(β) = 0.35

sh(κ) = 0.0007

for Logistic model and

sh(α) = 3.34

sh(κ) = 0.08

sh(γ) = 0.0004

for Gompertz model. t-values of the parameters are

t(α) = 223.45/3.67 = 60.88

t(β) = 4.39/0.35 = 12.52

t(κ) = -0.01/0.0007 = -17.58

for Logistic model and

t(α) = 227.9/3.34 = 68.24

t(κ) = -1.98/0.08 = -24.69

t(γ) = -0.009/0.0004 = -20.07

for Gompertz model The F-test ratio which we will use to test the significance of the regression can be found by dividing the residual sum of squares of the model to mean of error squares and it is

S(h) = 136395/182.27 = 748.4

for Logistic model and

S(h) = 141435/124.63 = 1135

for Gompertz model. The test statistics are too big so H_0(null hypothesis) is rejected and we can conclude that at least one of the parameters of the both models is nonzero.

%95 confidence regions for the model parameters can be found with the inequalities below:

$\alpha - z_{0.025}\, sh(\alpha) \leq \alpha \leq \alpha + z_{0.025}\, sh(\alpha)$

$\beta - z_{0.025}\, sh(\beta) \leq \beta \leq \beta + z_{0.025}\, sh(\beta)$

$\kappa - z_{0.025}\, sh(\kappa) \leq \kappa \leq \kappa + z_{0.025}\, sh(\kappa)$

and

$\alpha - z_{0.025}\, sh(\alpha) \leq \alpha \leq \alpha + z_{0.025}\, sh(\alpha)$

$\kappa - z_{0.025}\, sh(\kappa) \leq \kappa \leq \kappa + z_{0.025}\, sh(\kappa)$

$\gamma - z_{0.025}\, sh(\gamma) \leq \gamma \leq \gamma + z_{0.025}\, sh(\gamma)$.

The value of $z_{0.025}$ is 1.96 so if the numerical values put on their places we find

223.45 – 1.96(3.67) ≤ α ≤ 223.45 + 1.96(3.67)

4.39 – 1.96(0.35) ≤ β ≤ 4.39 + 1.96(0.35)

-0.01 – 1.96(0.0007) ≤ κ ≤ -0.01 + 1.96(0.0007)

227.9 – 1.96(3.34) ≤ α ≤ 227.9 + 1.96(3.34)

-1.98 –1.96(0.08) ≤ κ ≤ -1.98 + 1.96(0.08)

$-0.009 - 1.96(0.0004) \leq \gamma \leq -0.009 + 1.96(0.0004)$

$216.2568 \leq \alpha \leq 230.6432$

$3.704 \leq \beta \leq 5.076$

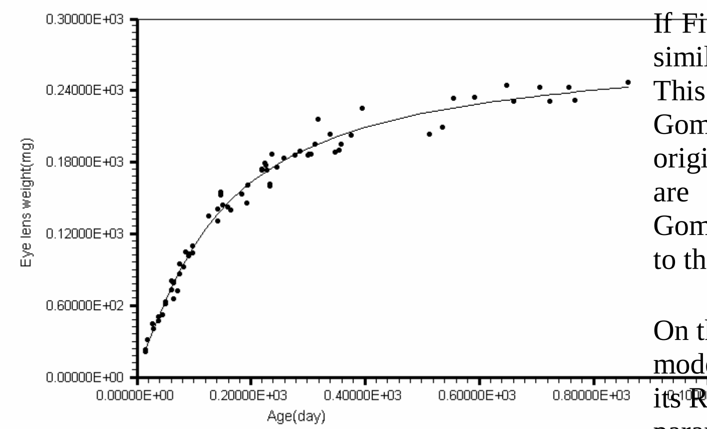
$-0.011372 \leq \kappa \leq -0.008628$

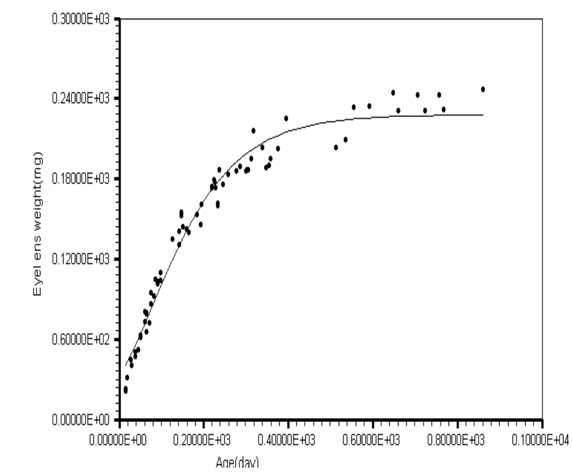
$221.3536 \leq \alpha \leq 234.4464$

$-2.1368 \leq \kappa \leq -1.8232$

$-0.009784 \leq \gamma \leq -0.008216.$

The growth curve of the model of Dudzinski and Mykytowycz is given in Fig.6





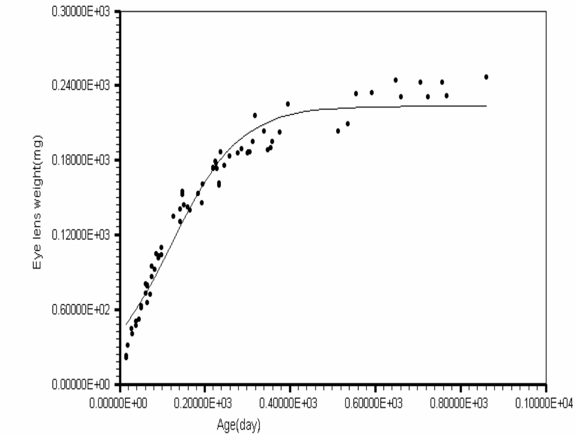


Fig.6 Growth curve for the model of Dudzinski and Mykytowycz

4 Conclusion

If Fig.2 and Fig.3 are compared with Fig.6 the similarity between them can be seen clearly. This shows that the fitting level of Logistic and Gompertz models is very high and close to the original model. Furthermore the R^2 values [2] are 0.96 for logistic model and 0.97 for Gompertz model and this values are very close to the R^2 value of original model 0.99.

On the other hand if the Gompertz and logistic models are compared with each other, for both its R^2 value is bigger and standart values of its parameters are smaller, we can say that Gompertz model fits the data better than logistic model.

4.1 Applying the Logistic Model to the Same Data With SPSS (Statistics Package for Social Sciences) and The Interpretation of the Results

GET

FILE='C:\Program Files\SPSS\adnan-reha.sav'.

DATASET NAME DataSet1 WINDOW=FRONT.

- NonLinear Regression.
- The Initial vector for Logistic Model

MODEL PROGRAM a=200.8 b=12.9 c=-0.03 .

COMPUTE PRED_ = a / (1 + b*exp(-c*Age)).

NLR Eyelens

/OUTFILE='C:\DOCUME~1\dente\LOCALS~1\Temp\spss3972\SPSSFNLR.TMP'

/PRED PRED_

/CRITERIA SSCONVERGENCE 1E-8 PCON 1E-8 .

Nonlinear Regression Analysis

[DataSet1] C:\Program Files\SPSS\adnan-reha.sav

Iteration History[b]

Iteration Number[a]	Residual Sum of Squares	Parameter		
		a	b	c
1.0	1791260,4	200,800	12,900	-,030
1.1	3,3E+010	21699,075	1700,536	1,023
1.2	26009472	-456,570	54,269	,548
1.3	594946,14	319,456	4,771	,017
2.0	594946,14	319,456	4,771	,017
2.1	441099,41	310,569	5,134	,016
3.0	441099,41	310,569	5,134	,016
3.1	210324,39	291,738	5,732	,015
4.0	210324,39	291,738	5,732	,015
4.1	22161,922	243,014	5,560	,013
5.0	22161,922	243,014	5,560	,013
5.1	12559,780	224,060	4,136	,012
6.0	12559,780	224,060	4,136	,012
6.1	12393,828	223,565	4,391	,012
7.0	12393,828	223,565	4,391	,012
7.1	12393,604	223,460	4,387	,012
8.0	12393,604	223,460	4,387	,012
8.1	12393,602	223,454	4,388	,012
9.0	12393,602	223,454	4,388	,012
9.1	12393,602	223,451	4,388	,012
10.0	12393,602	223,451	4,388	,012
10.1	12393,602	223,450	4,388	,012

Derivatives are calculated numerically.

a. Major iteration number is displayed to the left of the decimal, and minor iteration number is to the right of the decimal.

b. Run stopped after 22 model evaluations and 10 derivative evaluations because the relative reduction between successive residual sums of squares is at most SSCON = 1,00E-008.

Parameter Estimates

Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
a	223,450	3,670	216,126	230,774
b	4,388	,351	3,689	5,088
c	,012	,001	,011	,014

Correlations of Parameter Estimates

	a	b	c
a	1,000	-,088	-,597
b	-,088	1,000	,734
c	-,597	,734	1,000

ANOVA[a]

Source	Sum of Squares	df	Mean Squares
Regression	1787899	3	595966,3
Residual	12393,602	68	182,259
Uncorrected Total	1800293	71	
Corrected Total	298613,0	70	

Dependent variable: Eyelens

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = ,958.

We applied the same data with the SPSS computer program package and we obtained the lists below. It is obvius that there is a one-on-one perfect similarity with Fig.3, because that the correlation is R = 0.97.

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[2] Gauss-Newton iteration method is program package which used under computer. The computer output is given.

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