Marangoni Convection in a Variable Viscosity Fluid Layer with Feedback Control

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Abstract: - Feedback control was applied to the steady Marangoni convection in a horizontal layer of fluid with variable viscosity and free-slip at the lower boundary heated from below and cooled from above. Prediction for the onset of convection are obtained from the analysis by numerical technique.The effects of feedback control are studied by examining the critical Marangoni numbers and wave numbers. It is shown that the onset of Marangoni convection with variable viscosity can be delayed and the critical Marangoni number can be increased through the use of feedback control.

Key-Words: - Linear stability theory, Marangoni convection, Variable viscosity, Feedback control, Free-slip, Deformable free surface

1 Introduction

Convection is the transfer of heat by the motion of or within a fluid. It may arise from the temperature differences either within the fluid or between the fluid and its boundary, other sources of density variations (such as variable salinity) or from the application of an external motive force. It is one of the three primary mechanisms of heat transfer, the others being conduction and radiation. Convection has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines and to predict weather patterns. Generally, convection means fluid motion caused by temperature difference with the temperature gradient pointing in any direction (Chapmen [1]).

The study of the flow and heat transfer in an electrically conducting fluid has many practical applications in manufacturing process in industry. The thermal fluid flow problem have been extensively studied numerically, theorrtically as well as experimentally (see [2-4]). Rayleigh [5] was the first to solve the problem of the onset of thermal convection in a horizontal layer of fluid heated from below. His linear analysis showed that Benard convection occurs when the Rayleigh number exceeds a critical value. This parameter is a Rayleigh number (thermal or solutal) when the convection is induced by buoyancy effects due to variations in density and is a Marangoni number when surface-tension variations induce the convection. It is well-known fact that the onset of convection in Benard's [6] experiment is produced not simply by buoyancy forces but primarily by variations of the surface tension with the temperature. The latter effect is generally referred to in the literature under the name of thermocapillary or Marangoni instability. Although these flows were studied by Benard in 1900, it was almost sixty years before the critical experiment by Block [7] and the elegant linear stability analysis of Pearson [8] firmly established that Benard cells were a manifestation of the surface tension variations at the free surface (by Ginde et al [9]).

In this paper, we focused on the instability of Marangoni convection which is induced by a surface tension gradient. The Marangoni instability arises whenever the temperature gradient across the layer exceeds a certain critical value. The first theoretical study was done by Pearson [8] where he suggested there exists a surface tension when he observe a polygonal cellular patterns appear in a paint layers even the paint is on the underside of a plane, horizontal surface. He showed that thermocapillary forces can cause convection when the Marangoni number exceeds a critical value in the absence of buoyancy forces in the case of nondeformable free surface and no-slip boundary conditions at the bottom. Pearson [8] obtained the critical Marangoni number, M_c= 79.607 and the critical wave number a_c= 1.9929. Linear stability analysis of Marangoni convection with free-slip boundary conditions at the bottom was first investigated by Boeck and Thess [10]. For free-slip case, Boeck and Thess [10] obtained the critical Marangoni number, M_c= 57.598 and the critical wave number a_c= 1.7003. In particular, Arifin and Rosali [11] extended the work of Boeck and Thess [10] by including magnetic field to suppress the onset of Marangoni convection.

In many fluids with large Prandtl number and in particular, in some oils, the fluids posses a temperature-dependent viscosity as viscosity is more sensitive to temperature variations than heat capacity and thermal conductivity and the effects are important on the onset of convection. Linear and exponential dependence of the viscosity with respect to the temperature for the onset of Marangoni convection has been considered recently. Effect of an exponential viscosity law on the Marangoni convection in a fluid layer with nondeformable free surface was studied by Selak and Lebon [12] and Slatchev and Ouzounov[13]. The effect of a linear viscosity law with deformable free surface on the onset of Marangoni convection was investigated by Cloot and Lebon [14] and Kozhoukharova et. al. [15]. Kozhoukharova et. al [15] found that the role of a variable viscosity is to promote stability of the fluid layer. Kozhoukharova and Rozé [16] determined the influence of variable viscosity effect and surface deformation on the convective threshold for the primary steady and oscillatory Bénard-Marangoni in a fluid layer and show that the stability threshold for the short wavelength mode depends strongly on the viscosity variation while the long wavelength is determined by the surface deformation.

The importance of understanding the feedback control is to stabilize nonstable states or maintaining a state of no-motion hence we can optimizes the process. It may also help in gaining deeper insights into the dynamics of flow. In proportional feedback control of Bau [17], the thermal actuators are place at the bottom heated surface. Sensor are used to detect the departure of the surface temperature of the fluid from its conductive and they direct the actuators to take action so as to suppress unwanted disturbances. The thermal actuators modifies the bottom heated surface temperature using a proportional relationship between upper and lower thermal boundaries. Tang and Bau [18,19] and Howle [20] have shown that the critical Rayleigh number for the onset of Rayleigh- Bénard convection can be delayed. Or et al. [21] studied theoretically the use of feedback control strategies to stabilize long wavelength instabilities in the Marangoni convection. Bau [17] has shown independently how such a feedback control can delay the onset of Marangoni convection on a linear basis with no-slip boundary conditions at the bottom. Arifin et. al [23] have shown that a feedback control also can delay the onset of Marangoni convection with free-slip boundary conditions at the bottom. Very recently, effect of feedback control on the onset of Marangoni convection in rotating fluid layer with a free-slip bottom has been studied by Hashim and Siri [24]. In this study, we investigate the effects of feedback control on the onset of steady Marangoni convective instability in a horizontal fluid layer with a deformable upper free surface. In so doing, we extend the linear stability analysis results of Arifin et. al [23] to include the variable viscosity. The linear stability theory is applied and the resulting eigenvalue problem is solved numerically to obtain a detail description of the marginal stability curves for the onset of Marangoni convection.

2 Problem Formulation

Consider a horizontal fluid layer of depth d with a free upper surface heated from below subject ro a uniform vertical temperature gradient. The fluid layer is bounded below by a horizontal solid boundary at constant temperature T_1 and above by a free surface at constant temperature T_2 which is in contact with a passive gas at constant pressure P_0 and constant temperature T_∞. We used Cartesian coordinated with two horizontal x- and y- axes located at the lower solid boundary and a positive z-axis is directed towards the free surface (see Fig. 1). The effect of the surface deformability are measured by the Bond number, B_o and by the Crispation number, C_r. Parameter B_o estimates the effect on the modified static pressure by the gravity forces and C_r stands for the effect of the rigidity of the deformable surface.

The fluid is supposed to have Newtonian density

$$\rho = \rho_0[1-\alpha(T-T_0)] \tag{1}$$

where T is the temperature of the fluid, ρ_0 is its value at a reference temperature T_0 and α is the positive coefficient of the thermal fluid expansion. We consider the kinematic viscosity to be temperature-dependent, that is, a linear law for the kinematic viscosity is selected

$$v = v_0 + \zeta(T - T_0) \qquad (2)$$

where v_0 is viscosity at the reference temperature T_0 and $\zeta = \partial v / \partial T |_{T_0}$ is assumed constant.

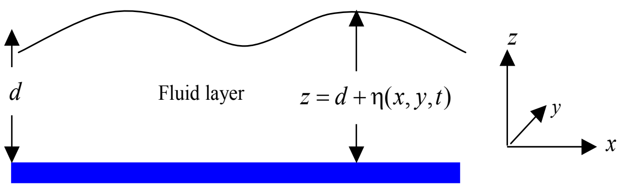

Fig. 1 Problem Set-up

The fluid motion is driven by surface tension, τ and it is assumed to be a linear function of the temperature

$$\tau = \tau_0 - \gamma(T - T_0) \qquad (3)$$

where τ_0 is the value of τ at temperature T_0 and the constant γ is positive for most fluids. The fluid is assumed to be an incompressible fluid with variations of viscosity satisfying the continuity equation together with the momentum and the heat equation. These equations are, respectively

$$\frac{\partial U_j}{\partial x_j} = 0 \qquad (5)$$

$$\rho_0\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \left[\mu \frac{\partial}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\right] \qquad (6)$$

$$\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = \kappa_f \frac{\partial^2 T}{\partial x^2{}_j} \qquad (7)$$

where $U_i (i = x, y, z)$ are the velocity components, T is the temperature of the fluid, ρ_0 is its density value at a reference temperature T_0, $\mu = \rho_0 v$ is the dynamic viscosity, κ is the fluid thermal diffusivity and the pressure inside the fluid is denoted by p.

When motion occurs, the upper free surface of the layer will be deformable with its position at

$$z = d + f(x, y, t) \qquad (1)$$

At the free surface, we have the kinematic condition,

$$U_z = \frac{\partial f}{\partial t} \qquad (2)$$

together with the conditions of continuity for the normal and tangential stresses,

$$-(p - p_g) + 2\mu \frac{\partial U_z}{\partial z} = 2H\sigma \qquad (3)$$

$$\mu\left(\frac{\partial U_f}{\partial z} + \frac{\partial U_z}{\partial x_f}\right) = \frac{\partial \sigma}{\partial x_f} (f = x, y) \qquad (4)$$

and the heat transfer balance subjected to Newton's law is of the form

$$k\frac{\partial T}{\partial z} + h(T - T_g) = 0 \qquad (5)$$

where p_g is the gas pressure, H is the mean curvature of the surface, given by $H = 1/2\nabla_d^2 z_d$, ∇_d^2 is the two dimensional Laplacian operator $\partial^2/\partial^2 x + \partial^2/\partial^2 y$, h is the heat transfer coefficient between the fluid and gas phases and T_g is the temperature of the ambient gas.

We shall investigate the linear stability of a basic in which the fluid is at rest $(\bar{\mathbf{U}} = (0,0,0))$, the temperature gradient across the layer,

$$\bar{T} = T_0 - \beta z$$

where $\beta = h(T_0 - T_g)/k + hd$, the kinematic viscosity,

$$\bar{\nu} = \nu_0 - \zeta\beta z,$$

and the pressure is hydostatic

$$\bar{p} = p_0 + \rho_0 g\left[\tfrac{1}{2}a\beta(z^2 - d^2) + (z - d)\right],$$

where g is acceleration due to gravity.

To simplify the analysis, we formulate the stability problem in dimensionless form. We choose d, κ/d, d^2/κ and $(T_1 - T_2)/d$ for length, velocity, time, and temperature gradient respectively. As a results the following dimensionless group arises,

$M = \gamma(T_1 - T_2)d/\rho_0 v\kappa$ (Marangoni number),

$Pr = v_0/\kappa$ (Prandtl number),

$C_r = \rho_0 v_0 \kappa/\tau_0 d$ (Crispation number),

$B_o = \rho_0 g d^2 / \tau_0$ (Bond number),

$B_i = hd / k$ (Biot number),

$Rv = \zeta(T_1 - T_2)d / \nu_0$ (Viscosity group).

Our control strategy basically applies a principle similar to that used by Bau [17], which is as follows: Assumed that the sensors and actuators are continuously distributed and that each sensor directs an actuator installed directly beneath it at the same {x,y} location. The sensor detects the deviation of the free surface temperature from its conductive value. The actuator modifies the heated surface temperature according to the following rule (Bau [17]):

$$T(x,y,0,t) = \frac{1+B_i}{B_i} - K\left(T(x,y,1,t) - \frac{1}{B_i}\right), \quad (13)$$

where K is the scalar controller gain. Equation (13) can be rewritten more conveniently as

$$T'(x,y,0,t) = -K\left(T'(x,y,1,t)\right), \quad (14)$$

where T' is the deviation of the fluid's temperature from its conductive value. The control strategy in equation (14), in which K is a scalar will be used to demonstrate that our system can be controlled.

3 Linearised Problem

We analyze the linear stability of the basic state by seeking perturbed solutions for any quantity in terms of normal modes in the form

$$\Phi(x,y,z,t) = \Phi_0(x,y,z) + \phi(z)\exp[i(\alpha_x x + \alpha_y y) + st] \quad (15)$$

where Φ_0 is the value of Φ in the basic state, $a = (\alpha_x^2 + \alpha_y^2)^{1/2}$ is the total horizontal wave number of disturbance and s is a complex growth rate with a real part representing the growth rate of the instability and an imaginary part representing its frequency.

Substituting equation (15) into equation (5-7), we obtain the corresponding linearized equations involving only the z-dependent parts of the perturbations to take the temperature and the z-components of the velocity denoted by T and w respectively, namely;

$$(D^2 - a^2)^2 w - s\,\mathrm{Pr}^{-1}(D^2 - a^2)w - Rv[2(D^2 - a^2)Dw + z(D^2 - a^2)^2 w] = 0, \quad (16)$$

$$(D^2 - \alpha^2 - s)T + w = 0, \quad (17)$$

subject to

$$sf - w = 0, \quad (18)$$

$$C_r\left[\left(s\,\mathrm{Pr}^{-1} + 3a^2(1-Rv) - D^2(1-Rv)\right)Dw\right] + Rv\left(D^2 + a^2\right)w + a^2\left(a^2 + B_o\right)f = 0, \quad (19)$$

$$\left(1-Rv\right)(D^2 - \alpha^2)w + a^2 M(T - f) = 0, \quad (20)$$

$$DT + B_i(T - f) = 0, \quad (21)$$

at $z = 1$ together with

$$w = 0, \quad D^2 w = 0, \quad (22)$$

at $z = 0$ and

$$T(0) + KT(1) = 0. \quad (23)$$

The operator $D = d/dz$ denotes differentiation with respect to z. As we only consider a steady convection, s is taken to be zero, $s = 0$.

4 Solution of the Linearized Problem

Proceeding in the manner of Arifin & Rosali [11], we seek asymptotic solutions for w, T in the forms

$$w(z) = ACe^{\xi z}, \quad T(z) = Ce^{\xi z} \quad (24)$$

where the exponent ξ and the complex constants A and C are to be determined. Substituting these forms into the Eqs. (16) and (17) and eliminating A and C we obtain a sixth-order algebraic equation for ξ, namely

$$(\xi^2 - a^2)^2(a^2 - \xi^2) - Rv[2(\xi^2 - a^2)\xi(a^2 - \xi^2) + z(\xi^2 - a^2)^2(a^2 - \xi^2)] = 0, \quad (25)$$

with six distinct roots, which we denote by $\xi_1, \ldots, \xi_6$. Denoting the values of A and C corresponding to ξ for $i = 1, \ldots, 6$ by A_i and C_i, respectively, we can use Eq. (16) to determine A_i. We can use Eq. (19) to eliminate the free surface deflection

$$f = \frac{C_r}{a^2(a^2+B_o)}\begin{Bmatrix} -Pr^{-1}sD^2w - 3a^2(1-Rv)Dw + \\ (1-Rv)D^3w - Rv(D^2+a^2)w \end{Bmatrix} \quad (26)$$

evaluated on $z=1$, leaving the six boundary conditions, to determine the six unknowns $C_1,...C_6$, and the general solution to the stability problem therefore

$$w(z) = \sum_{j=1}^{6} A_j C_j e^{\xi_j z}, \; T(z) = \sum_{j=1}^{6} C_j e^{\xi_j z} \quad (27)$$

The dispersion relation between M, a, C_r, B_i, B_o and B_i is determined by substituting these solutions into boundary conditions and evaluating the resulting 6×6 real determinants of the coefficients of the unknowns, which can be written in the form $M = -D_1/D_2$, where the two 6×6 real determinants D_1 and D_2 are independent of *M*.

The marginal stability curves in the (*a* ,*M*) plane on which Re(s) = 0 separate regions of unstable modes with Re(s) > 0 from those of stable modes with Re(s) < 0. In all the cases investigated in the present work *M* > 0 and the region above the marginal stability curve corresponds to unstable modes. Hence, the critical Marangoni number for the onset of convection denoted by M_c, is simply the global minimum of *M* on the marginal curves. The marginal stability curves are calculated by setting Re(s) = 0 and solving the equation $D_1 + MD_2 = 0$ for the values of *a* and *M* on the marginal curve. This procedure was implemented numerically using Fortran Powerstation 4.0 IMSL library.

5 Results and Discussions

The onset of Marangoni convection comprising an incompressible fluid with a range of viscosity group with a free-slip bottom is investigated numerically. In each case investigated in this paper, we can identify the critical minima of the marginal stability curves in the (*a, M*) plane which we denote by M_c together with corresponding critical wave number a_c. For a given set of parameters, the critical Marangoni number for the onset of convection defined as the minimum of the global minima of marginal curve. We denote this critical value by M_c and the corresponding critical wave number, a_c.

The problem has been solved to obtain a detail description of the marginal stability curves for the onset of Marangoni convection when the free surface is perfectly insulated ($B_i = 0$). The crispation number C_r, associated with the inverse effect of the surface tension, represents the degree of the free surface deformability and the behaviour of the marginal stability curves depends on whether $B_o = 0$ or $B_o \neq 0$. When C_r becomes large (corresponding to weak surface tension), the marginal curve has global minimum at zero wavenumber. In contrast, for small values of C_r, the marginal curve has global minimum at nonzero wavenumber. At some transition value of C_r, the marginal curve has two local minima that is one at zero wave number and the other at nonzero wave number. The range for parameters B_i, B_o and C_r which are respectively given by $10^{-3} \le B_i \le 10^{-1}$, $10^{-3} \le B_o \le 10^{-1}$ and $10^{-6} \le C_r \le 10^{-2}$ for most fluids layers of depths ranging from 0.01 cm to 0.1 cm and are in contact with air.

In all cases studied, we use the value of the viscosity group, *Rv* between –0.5 and 0.5 similar to Kozhoukharova and Rozé [2] and the sign of *Rv* depends on the type of the fluid. If the sign of *Rv* is positive then the kinematic viscosity is an increasing function of the temperature. Similarly the sign of *Rv* is negative then the kinematic viscosity is a decreasing function of temperature. For example, the kinematic viscosity of silicon oil decreases when the temperature increase and its give the viscosity group, Rv =-0.5. From a physical perspective, if the kinematic viscosity is increase, the onset of convection is governed by a sublayer where it is more unstable than the full layer. If Rv = 0, the viscosity of the fluid layer becomes uniform.

Before presenting the numerical results, it is useful to explain the physical mechanism for the Marangoni stabilizing effect. When applying heat from below, a hot spot is formed at some point on the free upper surface. With increasing temperature, surface tension decreases and the surface tension at the hot spot will be smaller than at neighboring locations. Thus a surface traction away from the hot spot, giving rise to a convective current. The convected fluid will be replace by a warmer fluid rising from beneath the surface. The proportional feedback control acts to enhance the dissipative