

# Marangoni Convection in a Variable Viscosity Fluid Layer with Feedback Control

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*Abstract:* - Feedback control was applied to the steady Marangoni convection in a horizontal layer of fluid with variable viscosity and free-slip at the lower boundary heated from below and cooled from above. Prediction for the onset of convection are obtained from the analysis by numerical technique. The effects of feedback control are studied by examining the critical Marangoni numbers and wave numbers. It is shown that the onset of Marangoni convection with variable viscosity can be delayed and the critical Marangoni number can be increased through the use of feedback control.

*Key-Words:* - Linear stability theory, Marangoni convection, Variable viscosity, Feedback control, Free-slip, Deformable free surface

## 1 Introduction

Convection is the transfer of heat by the motion of or within a fluid. It may arise from the temperature differences either within the fluid or between the fluid and its boundary, other sources of density variations (such as variable salinity) or from the application of an external motive force. It is one of the three primary mechanisms of heat transfer, the others being conduction and radiation. Convection has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines and to predict weather patterns. Generally, convection means fluid motion caused by temperature difference with the temperature gradient pointing in any direction (Chapman [1]).

The study of the flow and heat transfer in an electrically conducting fluid has many practical applications in manufacturing process in industry. The thermal fluid flow problem have been extensively studied numerically, theoretically as well as experimentally (see [2-4]). Rayleigh [5] was the first to solve the problem of the onset of thermal convection in a horizontal layer of fluid heated from below. His linear analysis showed that Benard convection occurs when the Rayleigh number exceeds a critical value. This parameter is a Rayleigh number (thermal or solutal) when the convection is induced by buoyancy effects due to variations in density and is a Marangoni number when surface-tension variations induce the

convection. It is well-known fact that the onset of convection in Benard's [6] experiment is produced not simply by buoyancy forces but primarily by variations of the surface tension with the temperature. The latter effect is generally referred to in the literature under the name of thermocapillary or Marangoni instability. Although these flows were studied by Benard in 1900, it was almost sixty years before the critical experiment by Block [7] and the elegant linear stability analysis of Pearson [8] firmly established that Benard cells were a manifestation of the surface tension variations at the free surface (by Ginde et al [9]).

In this paper, we focused on the instability of Marangoni convection which is induced by a surface tension gradient. The Marangoni instability arises whenever the temperature gradient across the layer exceeds a certain critical value. The first theoretical study was done by Pearson [8] where he suggested there exists a surface tension when he observe a polygonal cellular patterns appear in a paint layers even the paint is on the underside of a plane, horizontal surface. He showed that thermocapillary forces can cause convection when the Marangoni number exceeds a critical value in the absence of buoyancy forces in the case of nondeformable free surface and no-slip boundary conditions at the bottom. Pearson [8] obtained the critical Marangoni number,  $M_c = 79.607$  and the critical wave number  $a_c = 1.9929$ . Linear stability analysis of Marangoni convection with free-slip

boundary conditions at the bottom was first investigated by Boeck and Thess [10]. For free-slip case, Boeck and Thess [10] obtained the critical Marangoni number,  $M_c = 57.598$  and the critical wave number  $a_c = 1.7003$ . In particular, Arifin and Rosali [11] extended the work of Boeck and Thess [10] by including magnetic field to suppress the onset of Marangoni convection.

In many fluids with large Prandtl number and in particular, in some oils, the fluids possess a temperature-dependent viscosity as viscosity is more sensitive to temperature variations than heat capacity and thermal conductivity and the effects are important on the onset of convection. Linear and exponential dependence of the viscosity with respect to the temperature for the onset of Marangoni convection has been considered recently. Effect of an exponential viscosity law on the Marangoni convection in a fluid layer with nondeformable free surface was studied by Selak and Lebon [12] and Slatchev and Ouzounov [13]. The effect of a linear viscosity law with deformable free surface on the onset of Marangoni convection was investigated by Clout and Lebon [14] and Kozhoukharova et al. [15]. Kozhoukharova et al. [15] found that the role of a variable viscosity is to promote stability of the fluid layer. Kozhoukharova and Rozé [16] determined the influence of variable viscosity effect and surface deformation on the convective threshold for the primary steady and oscillatory Bénard-Marangoni in a fluid layer and show that the stability threshold for the short wavelength mode depends strongly on the viscosity variation while the long wavelength is determined by the surface deformation.

The importance of understanding the feedback control is to stabilize nonstable states or maintaining a state of no-motion hence we can optimize the process. It may also help in gaining deeper insights into the dynamics of flow. In proportional feedback control of Bau [17], the thermal actuators are placed at the bottom heated surface. Sensors are used to detect the departure of the surface temperature of the fluid from its conductive and they direct the actuators to take action so as to suppress unwanted disturbances. The thermal actuators modify the bottom heated surface temperature using a proportional relationship between upper and lower thermal boundaries. Tang and Bau [18,19] and Howle [20] have shown that the critical Rayleigh number for the onset of Rayleigh-Bénard convection can be delayed. Or et al. [21] studied

theoretically the use of feedback control strategies to stabilize long wavelength instabilities in the Marangoni convection. Bau [17] has shown independently how such a feedback control can delay the onset of Marangoni convection on a linear basis with no-slip boundary conditions at the bottom. Arifin et al. [23] have shown that a feedback control also can delay the onset of Marangoni convection with free-slip boundary conditions at the bottom. Very recently, effect of feedback control on the onset of Marangoni convection in rotating fluid layer with a free-slip bottom has been studied by Hashim and Siri [24]. In this study, we investigate the effects of feedback control on the onset of steady Marangoni convective instability in a horizontal fluid layer with a deformable upper free surface. In so doing, we extend the linear stability analysis results of Arifin et al. [23] to include the variable viscosity. The linear stability theory is applied and the resulting eigenvalue problem is solved numerically to obtain a detail description of the marginal stability curves for the onset of Marangoni convection.

## 2 Problem Formulation

Consider a horizontal fluid layer of depth  $d$  with a free upper surface heated from below subject to a uniform vertical temperature gradient. The fluid layer is bounded below by a horizontal solid boundary at constant temperature  $T_1$  and above by a free surface at constant temperature  $T_2$  which is in contact with a passive gas at constant pressure  $P_0$  and constant temperature  $T_\infty$ . We used Cartesian coordinates with two horizontal  $x$ - and  $y$ - axes located at the lower solid boundary and a positive  $z$ -axis is directed towards the free surface (see Fig. 1). The effect of the surface deformability are measured by the Bond number,  $B_o$  and by the Crispation number,  $C_r$ . Parameter  $B_o$  estimates the effect on the modified static pressure by the gravity forces and  $C_r$  stands for the effect of the rigidity of the deformable surface.

The fluid is supposed to have Newtonian density

$$\rho = \rho_0[1 - \alpha(T - T_0)] \quad (1)$$

where  $T$  is the temperature of the fluid,  $\rho_0$  is its value at a reference temperature  $T_0$  and  $\alpha$  is the positive coefficient of the thermal fluid expansion. We consider the kinematic viscosity to be temperature-dependent, that is, a linear law for the kinematic viscosity is selected

$$\nu = \nu_0 + \zeta(T - T_0) \tag{2}$$

where  $\nu_0$  is viscosity at the reference temperature  $T_0$  and  $\zeta = \partial\nu / \partial T|_{T_0}$  is assumed constant.

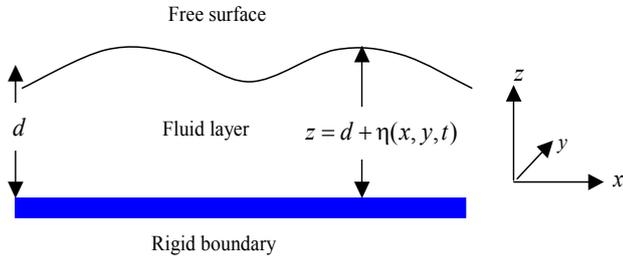


Fig. 1 Problem Set-up

The fluid motion is driven by surface tension,  $\tau$  and it is assumed to be a linear function of the temperature

$$\tau = \tau_0 - \gamma(T - T_0) \tag{3}$$

where  $\tau_0$  is the value of  $\tau$  at temperature  $T_0$  and the constant  $\gamma$  is positive for most fluids. The fluid is assumed to be an incompressible fluid with variations of viscosity satisfying the continuity equation together with the momentum and the heat equation. These equations are, respectively

$$\frac{\partial U_j}{\partial x_j} = 0 \tag{5}$$

$$\rho_0 \left( \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \left[ \mu \frac{\partial}{\partial x_j} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] \tag{6}$$

$$\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = \kappa_f \frac{\partial^2 T}{\partial x_j^2} \tag{7}$$

where  $U_i (i = x, y, z)$  are the velocity components,  $T$  is the temperature of the fluid,  $\rho_0$  is its density value at a reference temperature  $T_0$ ,  $\mu = \rho_0 \nu$  is the dynamic viscosity,  $\kappa$  is the fluid thermal diffusivity and the pressure inside the fluid is denoted by  $p$ .

When motion occurs, the upper free surface of the layer will be deformable with its position at

$$z = d + f(x, y, t) \tag{1}$$

At the free surface, we have the kinematic condition,

$$U_z = \frac{\partial f}{\partial t} \tag{2}$$

together with the conditions of continuity for the normal and tangential stresses,

$$-(p - p_g) + 2\mu \frac{\partial U_z}{\partial z} = 2H\sigma \tag{3}$$

$$\mu \left( \frac{\partial U_f}{\partial z} + \frac{\partial U_z}{\partial x_f} \right) = \frac{\partial \sigma}{\partial x_f} (f = x, y) \tag{4}$$

and the heat transfer balance subjected to Newton's law is of the form

$$k \frac{\partial T}{\partial z} + h(T - T_g) = 0 \tag{5}$$

where  $p_g$  is the gas pressure,  $H$  is the mean curvature of the surface, given by  $H = 1/2 \nabla_d^2 z_d$ ,  $\nabla_d^2$  is the two dimensional Laplacian operator  $\partial^2 / \partial^2 x + \partial^2 / \partial^2 y$ ,  $h$  is the heat transfer coefficient between the fluid and gas phases and  $T_g$  is the temperature of the ambient gas.

We shall investigate the linear stability of a basic in which the fluid is at rest ( $\bar{\mathbf{U}} = (0, 0, 0)$ ), the temperature gradient across the layer,

$$\bar{T} = T_0 - \beta z$$

where  $\beta = h(T_0 - T_g) / k + hd$ , the kinematic viscosity,

$$\bar{\nu} = \nu_0 - \zeta \beta z,$$

and the pressure is hydrostatic

$$\bar{p} = p_0 + \rho_0 g \left[ \frac{1}{2} a \beta (z^2 - d^2) + (z - d) \right],$$

where  $g$  is acceleration due to gravity.

To simplify the analysis, we formulate the stability problem in dimensionless form. We choose  $d$ ,  $\kappa / d$ ,  $d^2 / \kappa$  and  $(T_1 - T_2) / d$  for length, velocity, time, and temperature gradient respectively. As a results the following dimensionless group arises,

$$M = \gamma (T_1 - T_2) d / \rho_0 \nu \kappa \text{ (Marangoni number),}$$

$$Pr = \nu_0 / \kappa \text{ (Prandtl number),}$$

$$C_r = \rho_0 \nu_0 \kappa / \tau_0 d \text{ (Crispation number),}$$

$$B_o = \rho_0 g d^2 / \tau_0 \text{ (Bond number),}$$

$$B_i = h d / k \text{ (Biot number),}$$

$$Rv = \zeta(T_1 - T_2) d / \nu_0 \text{ (Viscosity group).}$$

Our control strategy basically applies a principle similar to that used by Bau [17], which is as follows: Assumed that the sensors and actuators are continuously distributed and that each sensor directs an actuator installed directly beneath it at the same  $\{x,y\}$  location. The sensor detects the deviation of the free surface temperature from its conductive value. The actuator modifies the heated surface temperature according to the following rule (Bau [17]):

$$T(x, y, 0, t) = \frac{1+B_i}{B_i} - K \left( T(x, y, 1, t) - \frac{1}{B_i} \right), \quad (13)$$

where  $K$  is the scalar controller gain. Equation (13) can be rewritten more conveniently as

$$T'(x, y, 0, t) = -K(T'(x, y, 1, t)), \quad (14)$$

where  $T'$  is the deviation of the fluid's temperature from its conductive value. The control strategy in equation (14), in which  $K$  is a scalar will be used to demonstrate that our system can be controlled.

### 3 Linearised Problem

We analyze the linear stability of the basic state by seeking perturbed solutions for any quantity in terms of normal modes in the form

$$\Phi(x, y, z, t) = \Phi_0(x, y, z) + \phi(z) \exp[i(\alpha_x x + \alpha_y y) + st] \quad (15)$$

where  $\Phi_0$  is the value of  $\Phi$  in the basic state,  $a = (\alpha_x^2 + \alpha_y^2)^{1/2}$  is the total horizontal wave number of disturbance and  $s$  is a complex growth rate with a real part representing the growth rate of the instability and an imaginary part representing its frequency.

Substituting equation (15) into equation (5-7), we obtain the corresponding linearized equations involving only the  $z$ -dependent parts of the perturbations to take the temperature and the  $z$ -components of the velocity denoted by  $T$  and  $w$  respectively, namely;

$$(D^2 - a^2)^2 w - s \text{Pr}^{-1} (D^2 - a^2) w - Rv[2(D^2 - a^2)Dw + z(D^2 - a^2)^2 w] = 0, \quad (16)$$

$$(D^2 - \alpha^2 - s)T + w = 0, \quad (17)$$

subject to

$$sf - w = 0, \quad (18)$$

$$C_r \left[ (s \text{Pr}^{-1} + 3a^2(1-Rv) - D^2(1-Rv))Dw \right] + Rv(D^2 + a^2)w + a^2(a^2 + B_o)f = 0, \quad (19)$$

$$(1-Rv)(D^2 - \alpha^2)w + a^2 M(T - f) = 0, \quad (20)$$

$$DT + B_i(T - f) = 0, \quad (21)$$

at  $z = 1$  together with

$$w = 0, \quad D^2 w = 0, \quad (22)$$

at  $z = 0$  and

$$T(0) + KT(1) = 0. \quad (23)$$

The operator  $D = d/dz$  denotes differentiation with respect to  $z$ . As we only consider a steady convection,  $s$  is taken to be zero,  $s = 0$ .

### 4 Solution of the Linearized Problem

Proceeding in the manner of Arifin & Rosali [11], we seek asymptotic solutions for  $w, T$  in the forms

$$w(z) = ACe^{\xi z}, \quad T(z) = Ce^{\xi z} \quad (24)$$

where the exponent  $\xi$  and the complex constants  $A$  and  $C$  are to be determined. Substituting these forms into the Eqs. (16) and (17) and eliminating  $A$  and  $C$  we obtain a sixth-order algebraic equation for  $\xi$ , namely

$$(\xi^2 - a^2)^2(a^2 - \xi^2) - Rv[2(\xi^2 - a^2)\xi(a^2 - \xi^2) + z(\xi^2 - a^2)^2(a^2 - \xi^2)] = 0, \quad (25)$$

with six distinct roots, which we denote by  $\xi_1, \dots, \xi_6$ . Denoting the values of  $A$  and  $C$  corresponding to  $\xi$  for  $i=1, \dots, 6$  by  $A_i$  and  $C_i$ ,

respectively, we can use Eq. (16) to determine  $A_i$ . We can use Eq. (19) to eliminate the free surface deflection

$$f = \frac{C_r}{a^2(a^2 + B_o)} \left\{ -Pr^{-1}sD^2w - 3a^2(1 - Rv)Dw + \right. \\ \left. (1 - Rv)D^3w - Rv(D^2 + a^2)w \right\} \quad (26)$$

evaluated on  $z=1$ , leaving the six boundary conditions, to determine the six unknowns  $C_1, \dots, C_6$ , and the general solution to the stability problem therefore

$$w(z) = \sum_{j=1}^6 A_j C_j e^{\xi_j z}, \quad T(z) = \sum_{j=1}^6 C_j e^{\xi_j z} \quad (27)$$

The dispersion relation between  $M, a, C_r, B_i, B_o$  and  $B_i$  is determined by substituting these solutions into boundary conditions and evaluating the resulting  $6 \times 6$  real determinants of the coefficients of the unknowns, which can be written in the form  $M = -D_1/D_2$ , where the two  $6 \times 6$  real determinants  $D_1$  and  $D_2$  are independent of  $M$ .

The marginal stability curves in the  $(a, M)$  plane on which  $\text{Re}(s) = 0$  separate regions of unstable modes with  $\text{Re}(s) > 0$  from those of stable modes with  $\text{Re}(s) < 0$ . In all the cases investigated in the present work  $M > 0$  and the region above the marginal stability curve corresponds to unstable modes. Hence, the critical Marangoni number for the onset of convection denoted by  $M_c$ , is simply the global minimum of  $M$  on the marginal curves. The marginal stability curves are calculated by setting  $\text{Re}(s) = 0$  and solving the equation  $D_1 + MD_2 = 0$  for the values of  $a$  and  $M$  on the marginal curve. This procedure was implemented numerically using Fortran Powerstation 4.0 IMSL library.

## 5 Results and Discussions

The onset of Marangoni convection comprising an incompressible fluid with a range of viscosity group with a free-slip bottom is investigated numerically. In each case investigated in this paper, we can identify the critical minima of the marginal stability curves in the  $(a, M)$  plane which we denote by  $M_c$  together with corresponding critical wave number  $a_c$ . For a given set of parameters, the critical Marangoni number for the onset of convection defined as the minimum of the global minima of marginal curve. We denote this critical value by

$M_c$  and the corresponding critical wave number,  $a_c$ .

The problem has been solved to obtain a detail description of the marginal stability curves for the onset of Marangoni convection when the free surface is perfectly insulated ( $B_i = 0$ ). The crispation number  $C_r$ , associated with the inverse effect of the surface tension, represents the degree of the free surface deformability and the behaviour of the marginal stability curves depends on whether  $B_o = 0$  or  $B_o \neq 0$ . When  $C_r$  becomes large (corresponding to weak surface tension), the marginal curve has global minimum at zero wavenumber. In contrast, for small values of  $C_r$ , the marginal curve has global minimum at nonzero wavenumber. At some transition value of  $C_r$ , the marginal curve has two local minima that is one at zero wave number and the other at nonzero wave number. The range for parameters  $B_i, B_o$  and  $C_r$  which are respectively given by  $10^{-3} \leq B_i \leq 10^{-1}$ ,  $10^{-3} \leq B_o \leq 10^{-1}$  and  $10^{-6} \leq C_r \leq 10^{-2}$  for most fluids layers of depths ranging from 0.01 cm to 0.1 cm and are in contact with air.

In all cases studied, we use the value of the viscosity group,  $Rv$  between  $-0.5$  and  $0.5$  similar to Kozhoukharova and Rozé [2] and the sign of  $Rv$  depends on the type of the fluid. If the sign of  $Rv$  is positive then the kinematic viscosity is an increasing function of the temperature. Similarly the sign of  $Rv$  is negative then the kinematic viscosity is a decreasing function of temperature. For example, the kinematic viscosity of silicon oil decreases when the temperature increase and its give the viscosity group,  $Rv = -0.5$ . From a physical perspective, if the kinematic viscosity is increase, the onset of convection is governed by a sublayer where it is more unstable than the full layer. If  $Rv = 0$ , the viscosity of the fluid layer becomes uniform.

Before presenting the numerical results, it is useful to explain the physical mechanism for the Marangoni stabilizing effect. When applying heat from below, a hot spot is formed at some point on the free upper surface. With increasing temperature, surface tension decreases and the surface tension at the hot spot will be smaller than at neighboring locations. Thus a surface traction away from the hot spot, giving rise to a convective current. The convected fluid will be replace by a warmer fluid rising from beneath the surface. The proportional feedback control acts to enhance the dissipative

mechanisms in the fluid. The controller senses when a fluid column is about to rise and directs the actuator to drop slightly the heated surface temperature beneath the rising column. In turn, the rising column will carry less excess temperature than in the absence of the controller. As a result, the surface tension-induced currents will be weaker, allowing ample time for heat dissipation to equalize the surface temperature and restore the no-motion state (Bau [17]).

Figure 1-5 shows the marginal stability curves of Marangoni number,  $M$  with wave number,  $a$  for  $B_i=0$ ,  $B_o=0.1$  and  $C_r=0$  in the case  $R_v = -0.5$ ,  $R_v = -0.3$ ,  $R_v = 0$ ,  $R_v = 0.3$  and  $R_v = 0.5$  respectively. As controller gain,  $K$  increases, the marginal stability curves shift upwards and evidently, controller gain,  $K$  stabilizes the no-motion state for all wave numbers. The results of Arifin et al [ ] are also recovered in Figure 3 for the case when  $R_v = 0$ . When  $K = 0$  and  $R_v = 0$ , we will recover the results of Pearson's [4].

Figure 6 shows the contribution of viscosity group,  $R_v$ , where we have revealed that as the viscosity group increases, the marginal stability curves shift downwards, hence showing that the viscosity group destabilizes the no-motion state for all wave numbers when  $C_r = 0$ ,  $B_o = 0$  and  $B_i = 0$ . In the case of non-deformable free surface ( $C_r = 0$ ), the controller can suppress the modes and maintain a no-motion state, but the situation is significantly different if the free surface is deformable.

Figure 7 and 8 illustrate that Crispation number is a destabilizing factor as the marginal stability curves decrease as the Crispation  $C_r$  increases. When  $C_r$  becomes large, the long wavelength instability sets in as a primary one and the critical Marangoni number at  $a = 0$ .

Figure 9 and 10 depict directly the relation between the controller,  $K$ , and the critical Marangoni number,  $M_c$ . With the controller,  $K$ , the critical Marangoni number,  $M_c$  increases depending on the viscosity,  $R_v$ . When  $K \rightarrow \infty$ ,  $M_c$  becomes higher which concludes that the fluid becomes more stable compared to a smaller  $K$ .

## 4 Conclusion

The effect of feedback control on the onset of Marangoni convection in a horizontal layer of electrically-conducting fluid which is free above and rigid below with variation viscosity and surface deformation is studied. The main results for all cases investigated in this paper reveal that the controller gain is a stabilizing factor, where the

parameter  $K$  stabilizes the no-motion state for all wave numbers. It is shown that an increase of parameter  $K$  will also cause an increase of the critical value of the Marangoni number. We also considered the Crispation number,  $C_r$  and viscosity group,  $R_v$  in few cases and we found that fluids with large viscosity variable and surface deformation are clearly a destabilizing factor.

## Acknowledgement

The authors gratefully acknowledged the financial support received from Universiti Putra Malaysia and sciencefund research grant from Kementerian Sains dan Teknologi (MOSTI)

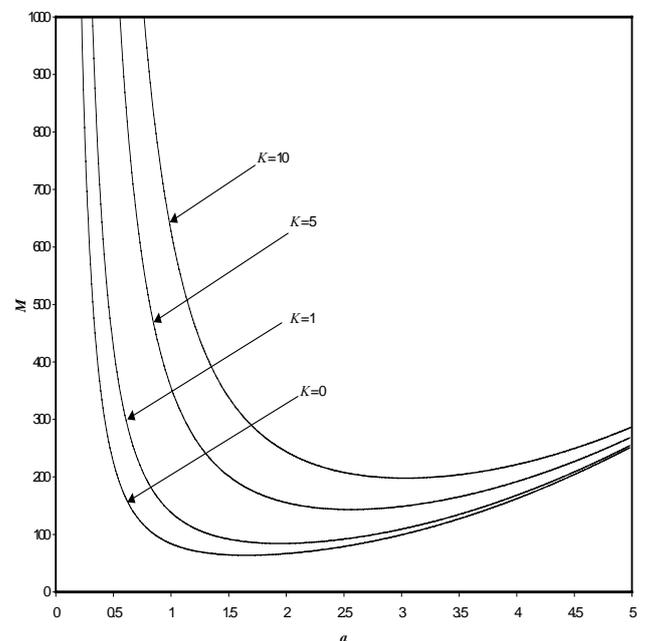


Fig. 1 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of the control gain,  $K$  in the case  $B_o = 0.1$ ,  $B_i = 0$ ,  $C_r = 0$  and  $R_v = -0.5$ .

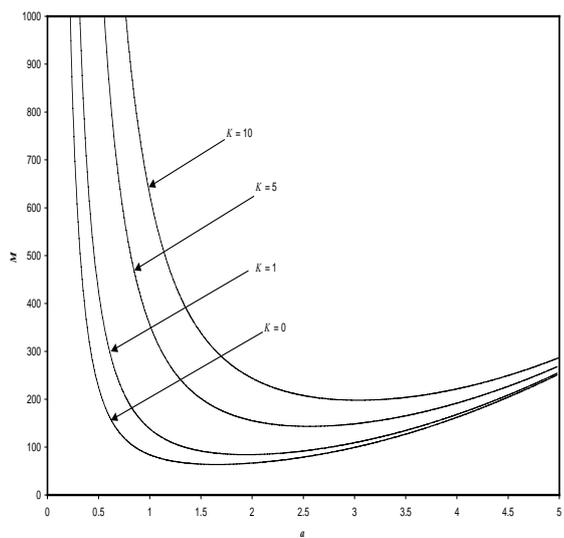


Fig. 2 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of the control gain,  $K$  in the case  $Bo = 0.1, Bi = 0, Cr = 0$  and  $Rv = -0.3$ .

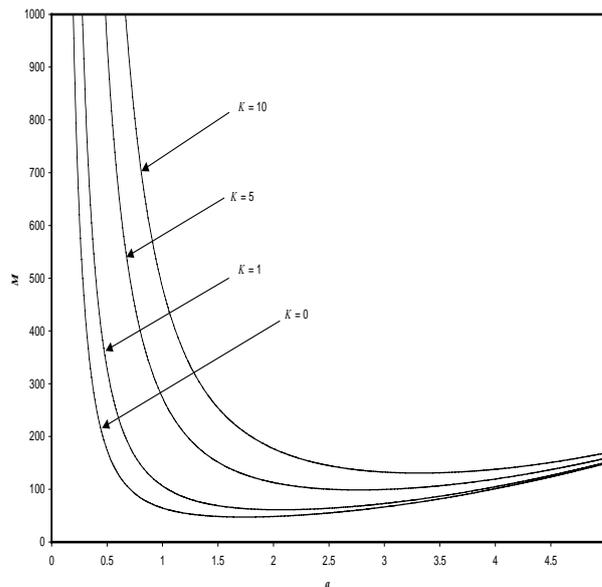


Fig. 4 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of the control gain,  $K$  in the case  $Bo = 0.1, Bi = 0, Cr = 0$  and  $Rv = 0.3$ .

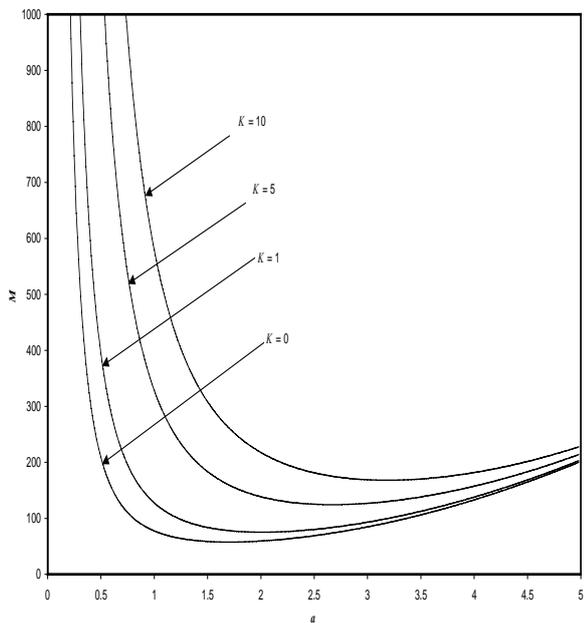


Fig. 3 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of the control gain,  $K$  in the case  $Bo = 0.1, Bi = 0, Cr = 0$  and  $Rv = 0$ .

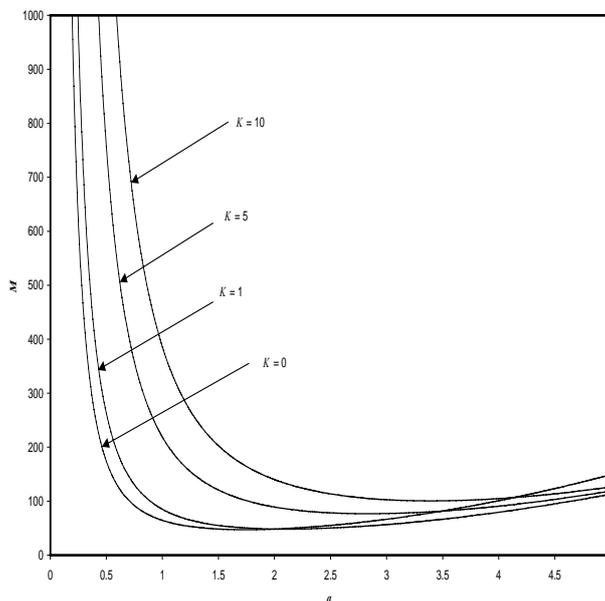


Fig. 5 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of the control gain,  $K$  in the case  $Bo = 0.1, Bi = 0, Cr = 0$  and  $Rv = 0.5$ .

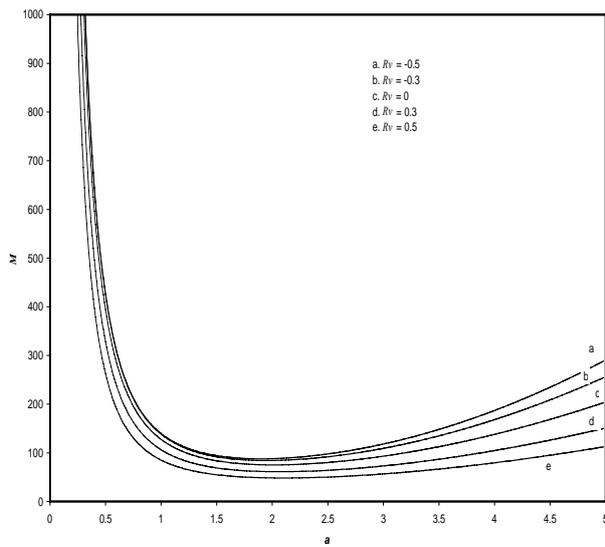


Fig. 6 : Numerically calculated values of  $M$  and the corresponding values of  $a$  different values of the viscosity group,  $Rv$  in the case  $Bo = 0.1$ ,  $Bi = 0$ ,  $Cr = 0$  and  $K = 1$ .

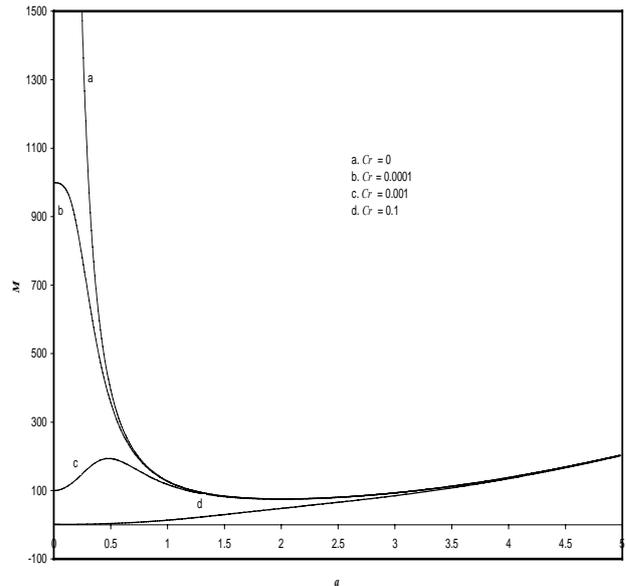


Fig. 8 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of Crispation number,  $Cr$  in the case  $Bo = 0.1$ ,  $Bi = 0$ ,  $K = 1$  and  $Rv = 0$ .

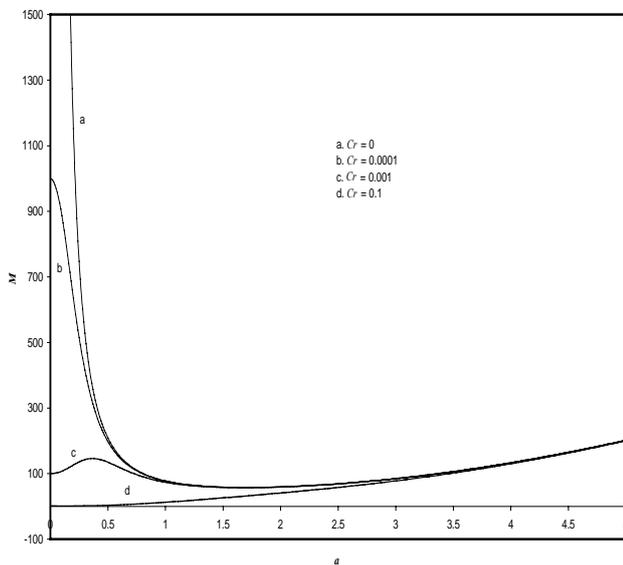


Fig. 7 : Numerically calculated values of  $M$  and the corresponding values of  $a$  for different values of Crispation number,  $Cr$  in the case  $Bo = 0.1$ ,  $Bi = 0$ ,  $K = 0$  and  $Rv = 0$ .

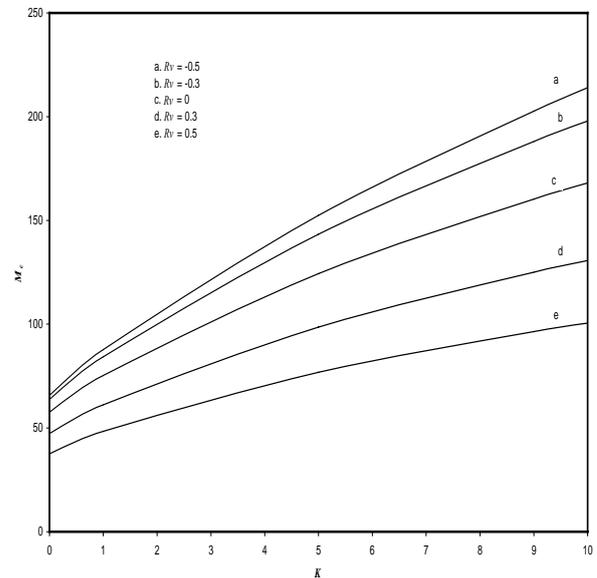


Fig. 9 : Numerically calculated values of critical Marangoni number,  $M_c$  and the corresponding values of controller gain,  $K$  for different values of the viscosity group,  $Rv$  in the case  $Bo = 0.1$ ,  $Bi = 0$  and  $Cr = 0$ .

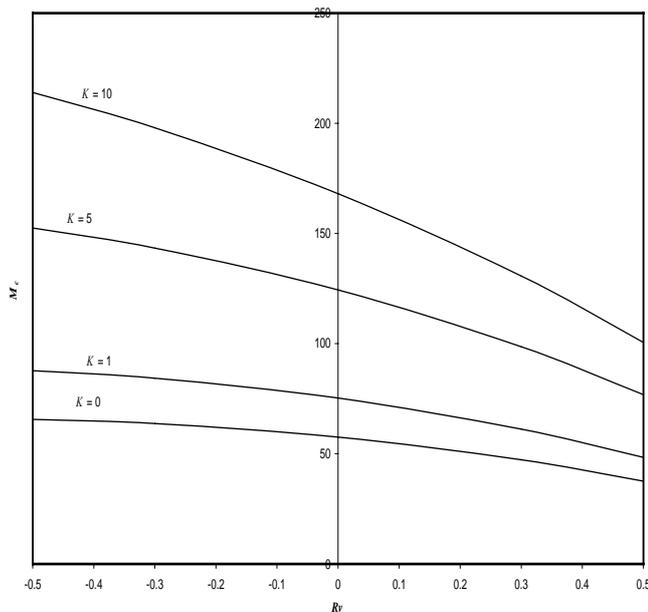


Fig. 10 : Numerically calculated values of critical Marangoni number,  $M_c$  and the corresponding values of  $Rv$ , for different values of controller gain,  $K$ , in the case  $Bo = 0.1$ ,  $Bi = 0$  and  $Cr = 0$ .

#### References:

- [1] Chapman, A. J. *Heat transfer*. 4th edn. New York: MacMillan, 1984.
- [2] Bacharoudis E., Vrachopoulos M.G., Koukou M.K., Filios A.E., Numerical Investigation of the Buoyancy-Induced Flow Field and Heat transfer inside Solar Chimneys. *WSEAS Transactions on Heat and Mass Transfer*, Vol.1, 2006, pp. 193
- [3] Abdell, K., Magpantay, F., Approximate Analytic Solutions for Mixed and Forced Convection Heat Transfer from an Unsteady No-uniform Flow Past a Rotating Cylinder. *WSEAS Transactions on Heat and Mass Transfer*, Vol.2, 2007, pp. 6-16
- [4] Al-Azab, T., Unsteady Mixed Convection Heat and Mass Transfer Past an Infinite Porous Plate with Thermophoresis Effect. *WSEAS Transactions on Heat and Mass Transfer*, Vol.4, 2009, pp. 23-33
- [5] Rayleigh R., On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side, *Phil. Mag.*, Vol.32, No.269-270, 1916, pp. 529-546.
- [6] Benard H., *Rev. Gen. des Sci. pures et appl.*, 1900, **1261**, 1309
- [7] Block M.J., *Nature Lond.*, 1956, **178**, 650
- [8] Pearson J.R.A., On convection cells induce by surface tension. *J. Fluid Mech* 4, 1958, pp. 489-500
- [9] Ginde R.M., Gill W.N. and Verhoeven J.D., An experimental Study of Rayleigh-Benard Convection in Liquid Tin. *Chem. Eng. Comm.* Vol.82, 1989, pp. 223-228.
- [10] Boeck T, Thess A., Inertial Benard-Marangoni convection, *J. Fluid Mech*, Vol.350, 1997, pp. 149-175.
- [11] Arifin, N.M, Rosali, H., Numerical Analysis of Marangoni Convection with Free-Slip Bottom under Magnetic Field. *WSEAS Transactions on Heat and Mass Transfer*, Vol.4, 2007, pp. 89-98
- [12] Selak, R , Lebon, G. Benard-Marangoni thermoconvective instability in presence of a temperature-dependent viscosity, *J. Phys. II France*. Vol. 3, 1993, pp. 1185
- [13] Slatchev, S, Ouzounov, V. Stationary Marangoni instability in a liquid layer with temperature-dependent viscosity in microgravity. *J. Microgravity Q.* Vol. 6, 1994, pp 33
- [14] Clout, A., Lebon, G. Marangoni instability in a fluid layer with variable viscosity and free interface in microgravity, *Physico Chemical Hydrodynamics*, Vol. 6., 1985, pp 453
- [15] Kozoukharova, Zh., Roze, C., Labon, C. Surface tension instability in a fluid layer with variable viscosity and deformable upper surface, *Microgravity Sci. Technol.* Vol. 8, 1995, pp. 249.
- [16] Kozoukharova, Zh. and Roze, C. Influence of the surface deformability and variable viscosity on buoyant-thermocapillary instability in a fluid layer, *Eur. Phys. J. B.* Vol. 8, 1999, pp. 125-135.
- [17] Bau. H. H., Control of Marangoni-Benard convection. *J. Heat Mass Transfer*. Vol. 42, 1999, pp. 1327-1341.
- [18] Tang, J., Bau, H. H., Experiments on the stabilization of the no-motion state of a fluid layer heated from below and cooled from above, *J. Fluid. Mech.* Vol. 363, 1998, pp.153-171

- [19] Tang, J., Bau, H. H. Stabilization of the no-motion state in Rayleigh-Benard problem, *Proc. Royal Soc.* Vol. A447, 1994, pp 587-607.
- [20] Howle, L. E. Linear stability analysis of controlled Rayleigh-Benard convection using shadowgraphic measurement, *Physics of Fluids*, Vol 9, 1997, pp 3111-3113
- [21] Or, A. C., Kelly, R. E., Cortelezzi, L., Speyer, J. L. Control of long wavelength Benard-Marangoni convection. *J. Fluid Mech.*, Vol. 387, 1999, pp 321—341.
- [22] Arifin, N.M, Nazar, R, Sebu, N. Feedback Control of the Marangoni-Benard instability in a fluid layer with free-slip Bottom, *J. Phys. Soc. Japan*. Vol. 76, 2207, pp 014401(1-4).
- [23] Hashim, I., Siri, Z. Feedback Control of Thermocapillary Convection in a rotating fluid layer with Free-Slip Bottom. *Sains Malaysiana*. Vol. 38, 2009, pp 119-124.