Transient Temperature Analysis of a Cylindrical Heat Equation

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Abstract: - The method of superposition and separation variables is applied to gain analytical solutions to the transient heat conduction for a two dimensional cylindrical fin. The temperature distributions are generalized for a linear combination of the product of Bessel function, Fourier series and exponential type for nine different cases. The solutions presented in this study can be used to verify the two- or three-dimensional numerical conduction codes. Relevant connections with some other closely-related recent works are also indicated.

Key-Words: -Fourier series, Heat conduction, Separation variables, Transcendent equation, Superposition method, Temperature distribution.

1 Introduction

A systematic procedure for determining the separation of variables for a given partial differential equation can be found in [1] and [2]. Analytical solutions are particularly important and useful. However, a heat problem involving two dimensions and more general boundary conditions of the type considered in our present study is presumably not solved in the existing literature on this subject. The separation variables method is applied in this study. The partial differential equations are transferred into ordinary differential equations by separating the independent variables involved in the problem.

The temperature distributions of fins under transient condition are important for proper prediction and control of the fin performance. Closed-form analytical solution for the transient temperature distribution would provide continuous physical insight which is much better than discrete numbers from a numerical computation. The main purpose of this study is to investigate the analytical transient solutions by using the method of separation of variables. A group of theory, being a systematic procedure of determining the separation variables can be found in [3-8]. However, a two-dimensional rectangular problem in present study is presumed to be absent. The superposition and the separation method are used in this study to get the analytical solutions for nine different cases.

The superposition method is widely used in other research area, such as [9-11] V. M. Guibout, D. J. Scheeres [9] focuses on applications of a general methodology that we developed for solving two-point boundary value problems. By the Hamilton-Jacobi theory in conjunction with canonical transformation induced by the phase flow, they provided that the generating functions for the transformation solve any two-point boundary value problem in phase space. This paper solved optimal control problems without an initial guess, studied the phase structure and plan spacecraft formation flights. Sandberg [10] focused on continuous-space shift-invariant systems with continuous system maps and inputs and outputs. It presented that infinite superposition can fail in this important setting and a continuous shift-invariant linear mappings need not commute with the operation of integration. JUAN ZAPATA and RAM´ON RUIZ [11] presented a modified snake model assisted by a hybrid force. The internal energy of the proposed snake model is given in terms of two geometric characteristics, the first derivative (tension) and the second derivative (rigidity) functions. When the proposed external energy is given in terms of a hybrid energy which combine short for a head in movement, and the target will be located in a close position. Then, the active contour will be extracted of a frame and it used like seed contour for the next frame. LIGIA and VETURIA [12] studied the motion of two pendulums coupled by an elastic spring. By extending the linear equivalence method (LEM), the solutions of its simplified set of nonlinear equations are written as a linear superposition of Coulomb vibrations. The motion of pendulum is describable as a linear superposition of cnoidal vibrations and additional terms, which include nonlinear interactions among the vibrations. The LEM represents of the solutions of the superposition of Coulomb vibrations. The cnoidal solutions are described as a superposition of cnoidal vibrations and nonlinear interactions among vibrations. The cnoidal method generalize the Fourier series with the cnoidal wave as the fundamental basis function, but is a completely different than an ordinary Fourier series expressed as a linear superposition of sine waves.

2 Problem Formulation

One can apply Fourier's law and energy conservation law to form a set of dimensionless governing equation, the initial condition and the boundary conditions as following.

$$\frac{\partial u(r,z,t)}{\partial t} = \frac{\partial^2 u(r,z,t)}{\partial z^2} + \frac{\partial^2 u(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,z,t)}{\partial r}, \quad (1)$$

$$t = 0, \quad u(r,z,0) = 0 \quad (2)$$

$t > 0$,

$$r = 0, \; u(0,z,t) = \text{finite} \quad (3)$$

$$r = 1, \; u(1,z,t) = 0 \quad (4)$$

$$z = 0, \; -u_z(r,0,t) + Biu(r,0,t) = Bi + q \quad (5)$$

$$z = L, \; u_z(r,L,t) + Bi_L u(r,L,t) = 0 \quad (6)$$

Where $u(r,z,t)$ denotes the temperature, $\mathrm{Bi_L}$ Biot number, L fin length. Note that a Bessel's equation satisfied a cylindrical coordinate in r-direction can be made orthogonal, one can separate the temperature distribution as follows:

$$u(r,z,t) = \sum_{n=1}^{\infty} u_n(z,t) J_0(\beta_n r). \quad (7)$$

Where α_n are the positive characteristic values of the transcendent equation (8),

$$J_0(\beta_n) = 0, \quad n = 1,2,3,... \quad (8)$$

And the boundary conditions in equations (3) and (4) are automatically satisfied. Then, the partial differential equation and initial and boundary conditions can be simplified to be:

$$\frac{\partial u_n(z,t)}{\partial t} = \frac{\partial^2 u_n(z,t)}{\partial z^2} - \beta_n u_n(z,t), \quad (9)$$

B. Cs.,

$$z = 0, \; -u_{nz}(0,t) + Biu_n(0,t) = g_n, \quad (10)$$

$$z = L, \; u_{nz}(L,t) + B_{iL} u_n(L,t) = 0. \quad (11)$$

Where

$$g_n = \frac{2(Bi+q)J_1(\beta_n)}{[J_0^2(\beta_n) + J_1^2(\beta_n)]\beta_n} \quad (12)$$

Revised the boundary conditions to a homogeneous one which lead to the required coefficients are

$$A = \frac{g_n}{(Bi + Bi_L + LBiBi_L)L}$$
$$B = \frac{(LBi_L + 1)g_n}{(Bi + Bi_L + LBiBi_L)L}. \quad (13)$$

The differential equation (9) and initial condition and boundary condition could be written as

$$\frac{\partial \bar{u}_n(z,t)}{\partial t} = \frac{\partial^2 \bar{u}_n(z,t)}{\partial z^2} - \alpha_n[zA + (L-z)B + \bar{u}_n(z,t)] \quad (14)$$

I.C. $\bar{u}_n(z,0) = -[zA + (L-z)B]$ (15)

B.Cs. $t > 0$, $z = 0$, $-\bar{u}_{nz}(0,t) + Bi\bar{u}_n(0,t) = 0$ (16)

$z = L$, $\bar{u}_{nz}(L,t) + Bi_L \bar{u}_n(L,t) = 0$ (17)

$$\bar{u}_n(z,t) = \sum_{m=1}^{\infty} u_{nm}(t)(\cos\alpha_m z + \frac{Bi}{\alpha_m}\sin\alpha_m z) \quad (18)$$

In the last equation, α_m are the characteristic values of the transcendent equation (19),

$$\tan\alpha_m L = \frac{\alpha_m(Bi + Bi_L)}{\alpha_m - BiBi_L}. \quad (19)$$

Thus only time variable is left and the ordinary differential equation could be written as

$$\frac{\partial u_{nm}(t)}{\partial t} = -(\beta_n^2 + \alpha_m^2)u_{nm}(t) + \beta_n^2 C_{nm} \quad (20)$$

$$u_{nm}(0) = C_{nm} \quad (21)$$

In the last equation, C_{nm} are the values of the equation (22),

$$C_{nm} = \frac{\int [Az + (L-A)B](\cos\alpha_m z + \frac{Bi}{\alpha_m}\sin\alpha_m z)dz}{\int (\cos\alpha_m z + \frac{Bi}{\alpha_m}\sin\alpha_m z)^2 dz} \quad (22)$$

The analytic solution and the initial condition is shown as

$$u_{nm}(t) = \frac{C_{nm}}{\beta_n^2 + \alpha_m^2}[\beta_n^2 + \alpha_m^2 e^{-(\alpha_m^2 + \beta_n^2)t}]. \quad (23)$$

The analytical temperature profile for above governing equation and initial condition and boundary conditions obtained by the method of separation variables and the method of superposition is shown below. The temperature of lateral surface is the temperature of surrounding, heat dissipates rapidly through lateral surface, and heat flux can dissipate through tip and lateral surface. The temperature profile involved nine different boundary conditions in z direction are presented.

3 Problem Solution

Case 1: $Bi =$ constant, $Bi_L =$ constant

Bi, Bi_L, Bi_r are the base Biot number; tip Biot number; lateral Boit number respectively. While Bi_L constant, heat convection condition on the tip of the fin, the larger $\mathrm{Bi_L}$ will result in the faster heat dissipation though the fin. The analytical solution is

$$u(r,z,t)=\sum_{n=1}^{\infty}\{zA+(L-z)B$$
$$+\sum_{m=1}^{\infty}u_{nm}(t)(\cos\alpha_m z+\frac{B_i}{\lambda_m}\sin\alpha_m z)\}J_0(\beta_n r) \tag{24}$$

Where

$$u_{nm}(t)=\frac{C_{nm}}{\beta_n^2+\alpha_m^2}[\beta_n^2+\alpha_m^2 e^{-(\alpha_m^2+\beta_n^2)t}] \tag{25}$$

And the relating coefficients are as following:

$$g_n=\frac{2(Bi+q)J_1(\beta_n)}{[J_0^2(\beta_n)+J_1^2(\beta_n)]\beta_n} \tag{26}$$

$$C_{nm}=\frac{E}{F}$$
$$E=-2[(A-B-ABiL)\alpha_m\cos\alpha_m L$$
$$+(A\alpha_m^2 L+ABi-BBi)\sin\alpha_m L-(A-B-BBiL)\alpha_m]$$
$$F=(\alpha_m^2-Bi^2)[\cos\alpha_m L\sin\alpha_m L$$
$$-2Bi\alpha_m\cos^2\alpha_m L+(\alpha_m^2 L+Bi^2 L+2Bi)\alpha_m] \tag{27}$$

$$A=\frac{g_n}{(Bi+Bi_L+LBiBi_L)L},$$

$$B=\frac{(LBi_L+1)g_n}{(Bi+Bi_L+LBiBi_L)L} \tag{28}$$

α_m are the positive roots of equations (29), and β_n are the positive roots of equations (30) which fits all following cases.

$$J_0(\beta_n)=0, \tag{29}$$

$$\tan\alpha_m L=\frac{\alpha_m(Bi+Bi_L)}{\alpha_m-BiBi_L}\ . \tag{30}$$

Case 2: $Bi=0$, $Bi_L=0$

As $B_i=0$, the fin roots is constraint to constant heat flux and a constant heat flux conduct into fin through fin roots. While $B_{iL}=0$, the tip of the fin is adiabatic, heat cannot dissipate though the fin and the lateral surface also adiabatic. All energy will be stored in the fin.

The equations (5) and (6) are revised to be

$$-u_z(r,0,t)=q \tag{31}$$

$$u_z(r,L,t)=0 \tag{32}$$

Note that a Bessel's equation satisfied a cylindrical coordinate in r-direction can be made orthogonal, one can separate the temperature distribution as follows:

$$u(r,z,t)=\sum_{n=1}^{\infty}u_n(z,t)J_0(\sqrt{\alpha_n}r)\ . \tag{33}$$

Where α_n are the positive characteristic values of the transcendent equation (34),

$$J_0(\sqrt{\alpha_n})=0,\quad n=1,2,3,... \tag{34}$$

And the boundary conditions in equations (3) and (4) are automatically satisfied. Then, the partial differential equation and boundary conditions can be simplified to be:

$$\frac{\partial u_n(z,t)}{\partial t}=\frac{\partial^2 u_n(z,t)}{\partial z^2}-\alpha_n u_n(z,t), \tag{35}$$

B. Cs. $t > 0$, $z = 0$, $-u_{nz}(0,t) = g_n$,

(36)

$z = L$, $u_{nz}(L,t) = 0$.

(37)

Where

$$g_n = \frac{2qJ_1(\beta_n)}{[J_0^2(\beta_n) + J_1^2(\beta_n)]\beta_n}.$$

(38)

Revised the boundary conditions to a homogeneous one which lead to the required coefficients are

, $B = -\frac{g_n}{L}$.

(39)

The differential equation (9) and initial condition and boundary condition could be written as

B.Cs. $t > 0$, $z = 0$, $-\bar{u}_{nz}(0,t) = 0$

(40)

$z = L$, $\bar{u}_{nz}(L,t) = 0$

(41)

$$\bar{u}_n(z,t) = \sum_{m=1}^{\infty} u_{nm}(t)(\cos \alpha_m z).$$

(42)

In the last equation, α_m are the characteristic values of the transcendent equation (43),

$$\alpha_m = \frac{m\pi}{L}; m, n = 1, 2, 3, \dots.$$

(43)

Thus only time variable is left and the ordinary differential equation could be written as

$$\frac{\partial u_{nm}(t)}{\partial t} = -(\beta_n^2 + \alpha_m^2)u_{nm}(t) + \beta_n^2 C_{nm}$$

(44)

In the last equation, C_{nm} are the values of the equation (45),

$$\begin{cases} C_{n0} = \frac{-BL^2}{3}, m = 0 \\ C_{nm} = \frac{2BL^2}{(m\pi)^2}, m > 0 \end{cases}$$

(45)

The analytic solution and the initial condition is shown as

$$u_{nm}(t) = \frac{C_{nm}}{\beta_n^2 + \alpha_m^2}[\beta_n^2 + \alpha_m^2 e^{-(\alpha_m^2 + \beta_n^2)t}].$$

(46)

Then the analytic temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{[Lz - \frac{z^2}{2}]B + C_{n0}$$
$$+\sum_{m=1}^{\infty} u_{nm}(t)\cos \alpha_m z\} J_0(\beta_n r)$$

(47)

Case 3: $Bi = 0$, $Bi_L \to \infty$

While $B_{iL} \to \infty$, the tip of the fin is isothermal to environment, the lateral surface also isothermal and heat dissipates fast though the fin. The temperature of the fin, left sitting on the base temperature, will fall until it reaches the surrounding temperature. The equations (5) and (6) are revised to be

$$-u_z(r,0,t) = q\,, \tag{48}$$

$$u(r,L,t) = 0\,. \tag{49}$$

Let

$$u(r,z,t) = \sum_{n=1}^{\infty} u_n(z,t) J_0(\sqrt{\alpha_n} r)\,.$$

(50)

Where α_n are the positive characteristic values of the transcendent equation (51),

$$J_0(\sqrt{\alpha_n}) = 0\,,\quad n = 1,2,3,...$$

(51)

And the boundary conditions in equations (3) and (4) are automatically satisfied. Then, the partial differential equation and initial and boundary conditions can be simplified to be:

$$\frac{\partial u_n(z,t)}{\partial t} = \frac{\partial^2 u_n(z,t)}{\partial z^2} - \alpha_n u_n(z,t)\,,$$

(52)

B. Cs. $t > 0$, $z = 0$, $-u_{nz}(0,t) = g_n$,

(53)

$z = L$, $u_n(L,t) = 0$.

(54)

Where

$$g_n = \frac{2q}{\beta_n J_1(\beta_n)}$$

(55)

Revised the boundary conditions to a homogeneous one which lead to the required coefficients are

$$B = g_n\,.$$

(56)

The differential equation (9) and initial condition and boundary condition could be written as

B.Cs. $t > 0$, $z = 0$, $-\bar{u}_{nz}(0,t) = 0$

(57)

$z = L$, $\bar{u}_{nz}(L,t) = 0$

(58)

$$\bar{u}_n(z,t) = \sum_{m=1}^{\infty} u_{nm}(t)(\cos\alpha_m z)\,.$$

(59)

In the last equation, α_m are the characteristic values of the transcendent equation (60),

$$\alpha_m = \frac{(2m-1)\pi}{2L},\quad m = 1,2,3,\ldots.$$

(60)

Thus only time variable is left and the ordinary differential equation could be written as

$$\frac{\partial u_{nm}(t)}{\partial t} = -(\beta_n^2 + \alpha_m^2)u_{nm}(t) + \beta_n^2 C_{nm} \quad (61)$$

In the last equation, C_{nm} are the values of the equation (62),

$$u_{nm}(0) = C_{nm} = \frac{-8BL}{[(2m-1)\pi]^2} \quad (62)$$

The analytic solution and the initial condition is shown as

$$u_{nm}(t) = \frac{C_{nm}}{\alpha_n + \beta_m^2}[\alpha_n + \beta_m^2 e^{-(\beta_m^2+\alpha_n)t}]. \quad (63)$$

The analytic temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{(L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\cos\alpha_m z\} J_0(\beta_n r) \quad (64)$$

Case 4: $Bi = 0$, $Bi_L =$ constant

While B_{iL} = constant, heat convection condition on the tip of the fin, the larger B_{iL} will result in the faster heat dissipation though the fin tip surface. Note that a Bessel's equation satisfied a cylindrical coordinate in r-direction can be made orthogonal, one can separate the temperature distribution as follows:

$$u(r,z,t) = \sum_{n=1}^{\infty} u_n(z,t) J_0(\beta_n r). \quad (65)$$

Where α_n are the positive characteristic values of the transcendent equation (66),

$$J_0(\beta_n) = 0, \quad n = 1,2,3,... \quad (66)$$

And the boundary conditions r-direction are automatically satisfied. Then, the partial differential equation and initial and boundary conditions can be simplified to be:

$$\frac{\partial u_n(z,t)}{\partial t} = \frac{\partial^2 u_n(z,t)}{\partial z^2} - \beta_n u_n(z,t), \quad (67)$$

B. Cs. $t > 0$, $z = 0$, $-u_{nz}(0,t) = g_n$, (68)

$z = L$, $u_{nz}(L,t) + B_{iL} u_n(L,t) = 0$. (69)

Where

$$g_n = \frac{2q}{\beta_n J_1(\beta_n)} \quad (70)$$

Revised the boundary conditions to a homogeneous one which lead to the required coefficients are

$$A = \frac{g_n}{B_{iL} L}, \quad B = \frac{(LB_{iL}+1)g_n}{B_{iL} L}. \quad (71)$$

The differential equation (9) and initial condition and boundary condition could be written as

$$\frac{\partial \bar{u}_n(z,t)}{\partial t} = \frac{\partial^2 \bar{u}_n(z,t)}{\partial z^2} - \alpha_n[zA + (L-z)B + \bar{u}_n(z,t)] \quad (72)$$

I.C. $\bar{u}_n(z,0) = -[zA + (L-z)B]$

(73)

B.Cs. $t > 0$, $z = 0$, $\bar{u}_{nz}(0,t) = 0$

(74)

$z = L$, $\bar{u}_{nz}(L,t) + Bi_L \bar{u}_n(L,t) = 0$

(75)

$\bar{u}_n(z,t) = \sum_{m=1}^{\infty} u_{nm}(t)(\cos\alpha_m z)$.

(76)

In the last equation, β_m are the characteristic values of the transcendent equation (77),

$\alpha_m \tan\alpha_m L = Bi_L$.

(78)

Thus only time variable is left and the ordinary differential equation could be written as

$$\frac{\partial u_{nm}(t)}{\partial t} = -(\beta_n^2 + \alpha_m^2)u_{nm}(t) + \beta_n^2 C_{nm}$$

(77)

$u_{nm}(0) = C_{nm}$

(78)

In the last equation, C_{nm} are the values of the equation (79),

$$C_{nm} = \frac{2\{(-A+B)\cos\beta_m L - [(\beta_m L)A\sin\beta_m L + (A-B)\}}{(\beta_m)\cos\beta_m L\sin\beta_m L + \beta_m^2 L}$$

(79)

The analytic solution and the initial condition is shown as

$$u(r,z,t) = \sum_{n=1}^{\infty}\{zA + (L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\cos\alpha_m z\} J_0(\sqrt{\beta_n}r) \quad (80)$$

Case 5: $Bi \to \infty$, $Bi_L = 0$

As $B_i \to \infty$, the fin root is constraint to isothermal conductivity, the root interface temperature is kept constant. While $B_{iL} = 0$, the tip of the fin is adiabatic, heat cannot dissipate though the fin and the lateral surface also isothermal. All energy will be dissipated by lateral surface. The equations (5) and (6) are revised to be

$$u(r,0,t) = 1 \quad (81)$$

$$u_z(r,L,t) = 0 \quad (82)$$

The analytic temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{zA + (L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\sin\alpha_m z\} J_0(\beta_n r)$$

(83)

And the relating coefficients are as following:

$$g_n = \frac{2}{\beta_n J_1(\beta_n)}$$

(84)

$$C_{nm} = \frac{4L[2(A-B)(-1)^m - (2m-1)B\pi]}{[(2m-1)\pi]^2}$$

(85)

$$A = \frac{g_n}{L},\ B = \frac{g_n}{L},$$

(86)

$$\alpha_m = \frac{(2m-1)\pi}{2L},\ m = 1,2,3,\ldots$$

(87)

Case 6: $Bi \to \infty$, $Bi_L \to \infty$

While $B_{iL} \to \infty$, the tip and root of the fin are isothermal, heat dissipates fast though the tip of the fin.

The equations (5) and (6) are revised to be

$$u(r,0,t) = 1 \tag{88}$$

$$u(r,L,t) = 0 \tag{89}$$

The analytical temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{(L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\sin\alpha_m z\} J_0(\beta_n r) \tag{90}$$

And the relating coefficients are as following:

$$g_n = \frac{2}{\beta_n J_1(\beta_n)} \tag{91}$$

$$C_{nm} = \frac{-2BL}{m\pi} \tag{92}$$

$$B = \frac{g_n}{L}, \quad \alpha_m = \frac{m\pi}{L}, m = 1,2,3,... \tag{93}$$

Case 7: $Bi \to \infty$, $Bi_L =$ constant

The equations (5) and (6) are revised to be

$$u(r,0,t) = 1 \tag{94}$$

$$u_z(r,L,t) + Bi_L u(r,L,t) = 0 \tag{95}$$

The analytical temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{Az + (L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\sin\alpha_m z\} J_0(\beta_n r) \tag{96}$$

And the relating coefficients are as following:

$$g_n = \frac{2}{\beta_n J_1(\beta_n)} \tag{97}$$

$$C_{nm} = \frac{2[(A-B)\sin\alpha_m L - A\alpha_m L\cos\alpha_m L + B\alpha_m L]}{\alpha_m(\cos\alpha_m L\sin\alpha_m L - \alpha_m L)} \tag{98}$$

$$A = \frac{g_n}{L(LBi_L+1)}; \; B = \frac{g_n}{L}, \tag{99}$$

α_m are the positive roots of the following equation

$$\alpha_m \cot\alpha_m L = -Bi_L. \tag{100}$$

Case 8: $Bi =$ constant, $Bi_L = 0$

The equations (5) and (6) are revised to be

$$-u_z(r,0,t) + Biu(r,0,t) = Bi + q \tag{101}$$

$$u_z(r,L,t) = 0 \tag{102}$$

The analytical temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{Az + (L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\frac{\cos\alpha_m(L-z)}{\cos\alpha_m L}\} J_0(\beta_n r). \tag{103}$$

And the relating coefficients are as following:

$$g_n = \frac{2(Bi+q)}{\beta_n J_1(\beta_n)} \quad (104)$$

$$C_{nm} = \frac{2[(A-B)(\cos\alpha_m L - 1) - B\alpha_m L \sin\alpha_m L]\cos\alpha_m L}{\alpha_m(\cos\alpha_m L \sin\alpha_m L + \alpha_m L)} \quad (105)$$

$$A = B = \frac{g_n}{LBi_L}, \quad (106)$$

α_m are the positive roots of the following equation

$$\alpha_m \tan\alpha_m L = Bi\,. \quad (107)$$

Case 9: $Bi =$ constant, $Bi_L \to \infty$

The equations (5) and (6) are revised to be

$$-u_z(r,0,t) + Biu(r,0,t) = Bi + q \quad (108)$$

$$u(r,L,t) = 0 \quad (109)$$

The analytical temperature profile is

$$u(r,z,t) = \sum_{n=1}^{\infty}\{Az + (L-z)B + \sum_{m=1}^{\infty} u_{nm}(t)\frac{\sin\alpha_m(L-z)}{\sin\alpha_m L}\}J_0(\beta_n r) \quad (110)$$

And the relating coefficients are as following:

$$g_n = \frac{2(Bi+q)}{\beta_n J_1(\beta_n)} \quad (111)$$

$$C_{nm} = \frac{E}{F}$$
$$E = -2[(A-B-ABiL)\alpha_m\cos\alpha_m L + (A\alpha_m^2 L + ABi - BBi)\sin\alpha_m L - (A-B-BBiL)\alpha_m] \quad (112)$$
$$F = (\alpha_m^2 - Bi^2)[\cos\alpha_m L\sin\alpha_m L - 2Bi\alpha_m\cos^2\alpha_m L + (\alpha_m^2 L + Bi^2 L + 2Bi)\alpha_m]$$

$$A = \frac{g_n}{(Bi + Bi_L + LBiBi_L)L};$$

$$B = \frac{(LBi_L + 1)g_n}{(Bi + Bi_L + LBiBi_L)L}, \quad (113)$$

α_m are the positive roots of the following equation

$$\tan\alpha_m L = \frac{\alpha_m(Bi + Bi_L)}{\alpha_m - BiBi_L}. \quad (114)$$

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4 Conclusion

The principle of superposition and separable variables are applied to the transient heat conduction in a cylindrical fin subjected to convective lateral surface to provide a simplified formulation that can be used to identify the temperature distribution. The temperature distributions are formed in a Fourier Bessel series and exponential type and are given by nine different cases.

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