Existence, Uniqueness and Finite Difference Solution for the Dirichlet problem of the Schrodinger-Maxwell equations

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Abstract: - In this paper, the existence, the Uniqueness and the Finite Difference Scheme for the Dirichlet problem of the Schrodinger-Maxwell equations is going to be presented.

Keywords: - Schrodinger-Maxwell equations, Finite Difference, Finite Difference Schemes.

1 Introduction

Recently many authors have examined the following system of the non-linear Partial Differential Equations (PDEs) in $\mathbb{R}^3$

$$-\nabla^2 u + q\phi u = g(u) \qquad (1)$$
$$-\nabla^2 \phi = qu^2 \qquad (2)$$

with $g(.)$ being a known function. The system of (1) and (2) is called: Schrodinger-Maxwell equations. This system of Equations arises in many mathematical physics contexts, such as in quantum electrodynamics, in nonlinear optics, in nano-mechanics and in plasma physics.
The greatest part of the literature focuses on the study of the previous system for the very special nonlinearity $g(u) = -u + |u|^{p-1} u$ and existence, nonexistence and multiplicity results are provided in many papers for this particular problem (see [18]÷[28]).
In [29], Azzollini, D'Avenia and Pomponio that a solution of a boundary problem of (1) and (2) yields the minimization of some functional.
In this paper we will solve the boundary value problem (i.e. Dirichlet problem) of

$$-\nabla^2 u + q\phi u = g(u) \qquad (1)$$
$$-\nabla^2 \phi = qu^2 \qquad (2)$$

where $g(.)$ is a known using Finite Difference Schemes. Here also we consider $g(.)$ as a known differentiable function.

2 The Existence and the Uniqueness of the solution of the discretization of (1) and (2)

For clarity only, in this paper, we will restrict attention to boundary value problems defined on **rectangles.** The ideas and methods extend in a natural way to more complex regions.
Consider a cube Γ with vertices $(a_1,a_2,a_3),(a_1,b_2,a_3),(a_1,a_2,b_3),(a_1,b_2,b_3)$, $(b_1,a_2,a_3),(b_1,b_2,a_3),(b_1,a_2,b_3),(b_1,b_2,b_3)$ and let Ω be the interior of Γ

Discretization concerns the process of transferring continuous models and equations into discrete counterparts. One proceeds as follows. Subdivide $[a_1,b_1]$ into n_1 equal parts, each of length $h_1=\dfrac{b_1-a_1}{n_1}$, subdivide $[a_2,b_2]$ into n_2 equal parts, each of length $h_2=\dfrac{b_2-a_2}{n_2}$, subdivide $[a_3,b_3]$ into n_3 equal parts, each of length $h_3=\dfrac{b_3-a_3}{n_3}$,

This discretization in (1) leads from $u(x,y,z)$ to $u_{j,k,l}$ where *j,k,l* the spatial coordinates, i.e.
$u_{i,k,l}=u(a_1+ih_1,a_2+kh_2,a_3+lh_3)$
and $\phi_{i,k,l}=\phi(a_1+ih_1,a_2+kh_2,a_3+lh_3)$
Therefore the steps of the discretization are h_1,h_2,h_3 with respect to *x,y,z*
So, we have introduced a grid of points in the $\mathbb{R}^3$ space of (*x,y,z*)
So from (1) one takes

$$\frac{u_{j+1,k,l}-2u_{j,k,l}+u_{j-1,k,l}}{h_1^2}+\frac{u_{j,k+1,l}-2u_{j,k,l}+u_{j,k-1,l}}{h_2^2}+$$
$$+\frac{u_{j,k,l+1}-2u_{j,k,l}+u_{j,k,l-1}}{h_3^2}-q\phi_{j,k,l}u_{j,k,l}=-g(u_{j,k,l})$$

(3)

and from (2)

$$\frac{\phi_{j+1,k,l}-2\phi_{j,k,l}+\phi_{j-1,k,l}}{h_1^2}+\frac{\phi_{j,k+1,l}-2\phi_{j,k,l}+\phi_{j,k-1,l}}{h_2^2}+$$
$$+\frac{\phi_{j,k,l+1}-2\phi_{j,k,l}+\phi_{j,k,l-1}}{h_3^2}=-qu_{i,j,k}^2$$

(4)

Eq.(3) can be written:

$$h_2^2h_3^2(u_{j+1,k,l}+u_{j-1,k,l})+h_1^2h_3^2(u_{j,k+1,l}+u_{j,k-1,l})+$$
$$+h_1^2h_2^2(u_{j,k,l+1}+u_{j,k,l-1})-2u_{j,k,l}(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)$$
$$-qh_1^2h_2^2h_3^2\phi_{j,k,l}u_{j,k,l}=-h_1^2h_2^2h_3^2g(u_{j,k,l})$$

(5)

Eq.(4) can be written:

$$h_2^2h_3^2(\phi_{j+1,k,l}+\phi_{j-1,k,l})+h_1^2h_3^2(\phi_{j,k+1,l}+\phi_{j,k-1,l})+$$
$$+h_1^2h_2^2(\phi_{j,k,l+1}+\phi_{j,k,l-1})-2\phi_{j,k,l}(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)$$
$$=-qh_1^2h_2^2h_3^2u_{j,k,l}^2$$

(6)

Considering these equations on each node, one can write a **mildly non-linear system.**
What is **mildly non-linear system.**
We recall [31] that a **mildly non-linear system is** a system of *n* equations in *n*

$$a_{11}x_1+a_{12}x_2+...+a_{1n}x_n+f_1(x_1)=0 \qquad (7.1)$$
$$a_{21}x_1+a_{22}x_2+...+a_{2n}x_n+f_2(x_2)=0 \qquad (7.2)$$
$$\vdots$$
$$a_{n1}x_1+a_{n2}x_2+...+a_{nn}x_n+f_n(x_n)=0 \qquad (7.n)$$

where $f_1(x_1)$, $f_2(x_2)$,..., $f_n(x_n)$ are non-linear functions in only one variable. It can be easily proved ([31]) that if the linear part our **mildly non-linear system is diagonally dominant (that means**

$$|a_{ii}|\geq\sum_{\substack{j=1\\j\neq i}}^{n}a_{ij} \quad i=1,2,...,n$$

with strict inequality for at least one value of i)

and satisfy $a_{ii} < 0$ and $\underset{j \neq i}{a_{ij}} > 0$ **and in addition**

$f_i(x_i)$, $i = 1,2,...,n$ are differentiable with $\frac{df_i(x)}{dx} < 0$ then the system of (7.1),(7.2),…,(7,n) has one and only one solution.

So, we can see the system of (5) and (6) can be written as a system of $2(n_1 - 1)(n_2 - 1)(n_3 - 1)$ Equations in $2(n_1 - 1)(n_2 - 1)(n_3 - 1)$ unknowns. Our equations are (5) and (6) at the points of Ω (Ω is the interior of Γ).

From (6) we obtain

$$\phi_{j,k,l} = \frac{h_2^2 h_3^2(\phi_{j+1,k,l} + \phi_{j-1,k,l}) + h_1^2 h_3^2(\phi_{j,k+1,l} + \phi_{j,k-1,l}) + +h_1^2 h_2^2(\phi_{j,k,l+1} + \phi_{j,k,l-1}) + q h_1^2 h_2^2 h_3^2 u_{j,k,l}^2}{2(h_2^2 h_3^2 + h_1^2 h_3^2 + h_1^2 h_2^2)} \tag{8}$$

Introducing it in (5) we obtain Eq.(9)

$$\begin{aligned}
&h_2^2 h_3^2(u_{j+1,k,l} + u_{j-1,k,l}) + h_1^2 h_3^2(u_{j,k+1,l} + u_{j,k-1,l}) + \\
&+h_1^2 h_2^2(u_{j,k,l+1} + u_{j,k,l-1}) - 2u_{j,k,l}(h_2^2 h_3^2 + h_1^2 h_3^2 + h_1^2 h_2^2) \\
&-q h_1^2 h_2^2 h_3^2 \left(\frac{h_2^2 h_3^2(\phi_{j+1,k,l} + \phi_{j-1,k,l}) + h_1^2 h_3^2(\phi_{j,k+1,l} + \phi_{j,k-1,l}) + h_1^2 h_2^2(\phi_{j,k,l+1} + \phi_{j,k,l-1}) + q h_1^2 h_2^2 h_3^2 u_{j,k,l}^2}{2(h_2^2 h_3^2 + h_1^2 h_3^2 + h_1^2 h_2^2)}\right) u_{j,k,l} \\
&= -h_1^2 h_2^2 h_3^2 g(u_{j,k,l})
\end{aligned} \tag{9}$$

or

$$\begin{aligned}
&\left(-q h_1^2 h_2^2 h_3^2 \left(\frac{h_2^2 h_3^2(\phi_{j+1,k,l} + \phi_{j-1,k,l}) + h_1^2 h_3^2(\phi_{j,k+1,l} + \phi_{j,k-1,l}) + h_1^2 h_2^2(\phi_{j,k,l+1} + \phi_{j,k,l-1})}{2(h_2^2 h_3^2 + h_1^2 h_3^2 + h_1^2 h_2^2)}\right) - 2(h_2^2 h_3^2 + h_1^2 h_3^2 + h_1^2 h_2^2)\right) u_{j,k,l} \\
&+h_2^2 h_3^2(u_{j+1,k,l} + u_{j-1,k,l}) + h_1^2 h_3^2(u_{j,k+1,l} + u_{j,k-1,l}) + h_1^2 h_2^2(u_{j,k,l+1} + u_{j,k,l-1}) \\
&+h_1^2 h_2^2 h_3^2 g(u_{j,k,l}) - q h_1^2 h_2^2 h_3^2 \left(\frac{q h_1^2 h_2^2 h_3^2}{2(h_2^2 h_3^2 + h_1^2 h_3^2 + h_1^2 h_2^2)}\right) u_{j,k,l}^3 \\
&= 0
\end{aligned} \tag{10}$$

One can easily see that the Equations (10) with (8) is a system of $2(n_1 - 1)(n_2 - 1)(n_3 - 1)$ equations in $2(n_1 - 1)(n_2 - 1)(n_3 - 1)$ unknowns

After these mathematical preparations we can prove the following Theorem 1.

Theorem 1:
A solution in the mildly non-linear system of $2(n_1-1)(n_2-1)(n_3-1)$ equations in $2(n_1-1)(n_2-1)(n_3-1)$ unknowns of (5) and (6) (discretization of the Dirichlet problem of (1), (2) in Ω) exists and is unique if for the known function g:

$$g'(u) < \frac{3}{2} q \frac{h_1^2 h_2^2 h_3^2}{h_1^2 h_2^2 + h_1^2 h_3^2 + h_2^2 h_3^2} u^2 \qquad (11)$$

For Ω **where** $g'(u)$ is the derivative of g

Proof:
Suppose that ϕ is positive, then at any iterative numerical scheme this positivity is maintained because of (8).
Now, for every $\phi_{j,k,l}$ from (8), the (10) forms a $(n_1-1)(n_2-1)(n_3-1)$ equations in $(n_1-1)(n_2-1)(n_3-1)$ unknowns

So, the system of (10) the coefficient of $u_{j,k,l}$ is

$$-qh_1^2h_2^2h_3^2\left(\frac{\begin{array}{l}h_2^2h_3^2(\phi_{j+1,k,l}+\phi_{j-1,k,l})+\\ h_1^2h_3^2(\phi_{j,k+1,l}+\phi_{j,k-1,l})+\\ h_1^2h_2^2(\phi_{j,k,l+1}+\phi_{j,k,l-1})\end{array}}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)$$

$$-2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)$$

which in absolute value is

$$qh_1^2h_2^2h_3^2\left(\frac{\begin{array}{l}h_2^2h_3^2(\phi_{j+1,k,l}+\phi_{j-1,k,l})+\\ h_1^2h_3^2(\phi_{j,k+1,l}+\phi_{j,k-1,l})+\\ h_1^2h_2^2(\phi_{j,k,l+1}+\phi_{j,k,l-1})\end{array}}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)$$

$$+2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)$$

Denoting now for sake of abbreviation :

i.e. greater than

$$2h_2^2h_3^2+2h_1^2h_3^2+2h_1^2h_2^2$$

(sum of absolute values of the coefficients of $u_{j+1,k,l}, u_{j-1,k,l}, u_{j,k+1,l}, u_{j,k-1,l}, u_{j,k,l+1}, u_{j,k,l-1}$
Hence, we have that the linear part of the non-linear system of (10) is **diagonally dominant.**

On the other hand, let us consider the fulfillment of

$$\frac{df_i(x)}{dx} < 0$$

In (10), we must have

$$d\left(h_1^2h_2^2h_3^2 g(u) - qh_1^2h_2^2h_3^2\left(\frac{qh_1^2h_2^2h_3^2}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)u^3\right)/du \le 0$$

Q.E.D. (quod erat demonstrandum)

3 Numerical Scheme for the solution of (1) and (2)

From (8), using ([31]), we have

$$\phi^{(n+1)}{}_{j,k,l} = \frac{\begin{array}{l}h_2^2h_3^2(\phi^{(n)}{}_{j+1,k,l}+\phi^{(n+1)}{}_{j-1,k,l})+h_1^2h_3^2(\phi^{(n)}{}_{j,k+1,l}+\phi^{(n+1)}{}_{j,k-1,l})+\\ +h_1^2h_2^2(\phi^{(n)}{}_{j,k,l+1}+\phi^{(n+1)}{}_{j,k,l-1})+qh_1^2h_2^2h_3^2u^{(n)2}{}_{j,k,l}\end{array}}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}$$

Where(n) and (n+1) denote the n, n+1 iterations

$$C=-qh_1^2h_2^2h_3^2\left(\frac{h_2^2h_3^2(\phi^{(n)}{}_{j+1,k,l}+\phi^{(n+1)}{}_{j-1,k,l})+h_1^2h_3^2(\phi^{(n)}{}_{j,k+1,l}+\phi^{(n+1)}{}_{j,k-1,l})+h_1^2h_2^2(\phi^{(n)}{}_{j,k,l+1}+\phi^{(n+1)}{}_{j,k,l-1})}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)-2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)$$

then (10) can be written

$$Cu^{(n+1)}{}_{j,k,l}+h_2^2h_3^2(u_{j+1,k,l}+u_{j-1,k,l})+h_1^2h_3^2(u_{j,k+1,l}+u_{j,k-1,l})+h_1^2h_2^2(u_{j,k,l+1}+u_{j,k,l-1})$$
$$+h_1^2h_2^2h_3^2g(u_{j,k,l})-qh_1^2h_2^2h_3^2\left(\frac{qh_1^2h_2^2h_3^2}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)u_{j,k,l}^3$$
$$=0$$

and so

$$u^{(n+1)}{}_{j,k,l}=u^{(n)}{}_{j,k,l}-\omega\frac{\begin{array}{l}(Cu^{(n)}{}_{j,k,l}+h_2^2h_3^2(u^{(n)}{}_{j+1,k,l}+u^{(n+1)}{}_{j-1,k,l})+h_1^2h_3^2(u^{(n)}{}_{j,k+1,l}+u^{(n+1)}{}_{j,k-1,l})+\\+h_1^2h_2^2(u^{(n)}{}_{j,k,l+1}+u^{(n+1)}{}_{j,k,l-1})+\\+h_1^2h_2^2h_3^2g(u^{(n)}{}_{j,k,l})-qh_1^2h_2^2h_3^2\left(\frac{qh_1^2h_2^2h_3^2}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)u_{j,k,l}^{(n)3})\end{array}}{C+h_1^2h_2^2h_3^2g'(u^{(n)}{}_{j,k,l})-3qh_1^2h_2^2h_3^2\left(\frac{qh_1^2h_2^2h_3^2}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)u_{j,k,l}^{(n)2}}$$

That means that finally we can have the following numerical scheme

$$\phi^{(n+1)}{}_{j,k,l}=\frac{\begin{array}{l}h_2^2h_3^2(\phi^{(n)}{}_{j+1,k,l}+\phi^{(n+1)}{}_{j-1,k,l})+h_1^2h_3^2(\phi^{(n)}{}_{j,k+1,l}+\phi^{(n+1)}{}_{j,k-1,l})+\\+h_1^2h_2^2(\phi^{(n)}{}_{j,k,l+1}+\phi^{(n+1)}{}_{j,k,l-1})+qh_1^2h_2^2h_3^2u^{(n)2}{}_{j,k,l}\end{array}}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}$$

$$u^{(n+1)}{}_{j,k,l}=u^{(n)}{}_{j,k,l}-\omega\frac{\begin{array}{l}(Cu^{(n)}{}_{j,k,l}+h_2^2h_3^2(u^{(n)}{}_{j+1,k,l}+u^{(n+1)}{}_{j-1,k,l})+h_1^2h_3^2(u^{(n)}{}_{j,k+1,l}+u^{(n+1)}{}_{j,k-1,l})+\\+h_1^2h_2^2(u^{(n)}{}_{j,k,l+1}+u^{(n+1)}{}_{j,k,l-1})+\\+h_1^2h_2^2h_3^2g(u^{(n)}{}_{j,k,l})-qh_1^2h_2^2h_3^2\left(\frac{qh_1^2h_2^2h_3^2}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)u_{j,k,l}^{(n)3})\end{array}}{C+h_1^2h_2^2h_3^2g'(u^{(n)}{}_{j,k,l})-3qh_1^2h_2^2h_3^2\left(\frac{qh_1^2h_2^2h_3^2}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)u_{j,k,l}^{(n)2}}$$

where C is given by

$$C=-qh_1^2h_2^2h_3^2\left(\frac{h_2^2h_3^2(\phi^{(n)}{}_{j+1,k,l}+\phi^{(n+1)}{}_{j-1,k,l})+h_1^2h_3^2(\phi^{(n)}{}_{j,k+1,l}+\phi^{(n+1)}{}_{j,k-1,l})+h_1^2h_2^2(\phi^{(n)}{}_{j,k,l+1}+\phi^{(n+1)}{}_{j,k,l-1})}{2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)}\right)-2(h_2^2h_3^2+h_1^2h_3^2+h_1^2h_2^2)$$

3 Conclusion

In this paper, the Numerical Solution of the system of PDEs of Schrodinger-Maxwell equations (with a general nonlinear term) via an appropriate Finite Difference Scheme is introduced. The Existence and Uniqueness of the discretization of the system of the PDEs of Schrodinger-Maxwell equation is also provided. The problem has been also solved by the method of Finite Elements and Genetic Algorithms with Nelder-Mead in [32].

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