Finite Difference Method And Iterative Method With Parallelism For Dispersive Equations

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Abstract: In this paper, based on the concept of domain decomposition and alternating group, we construct a class of Finite Difference method for fifth order dispersive equations, Stability Analysis for he method is given. Then we construct a new alternating group explicit iterative method. Both the two methods are suitable for parallel computation. Results of numerical experiments show the methods are effective in computing.

Key–Words: parallel computing, dispersive equations, finite difference, iterative method, asymmetry schemes, alternating group

1 Introduction

Finite difference method is one of the most frequently used numerical methods in solving differential equations [1-5]. Many numerical methods have been established for third order dispersive equations [6-9], But researches on high order dispersive equations have been scarcely presented. Recently with the development of parallel computer many scientists pay much attention to the finite difference methods with the property of parallelism. D. J. Evans presented an AGE method in [10] originally. The AGE method is used in computing by applying the special combination of several asymmetry schemes to a group of grid points, and then the numerical solutions at the group of points can be denoted explicitly. Furthermore, by alternating use of asymmetry schemes at adherent grid points and different time levels, the AGE method can lead to the property of unconditional stability. But the original AGE method has only two order accurate for spatial step. The AGE method is soon applied to convection-diffusion equations and hyperbolic equations in [11,12]. In [13-16], AGE method is applied to solve semi-linear and non-linear equations. Several AGE methods are given for two-point linear and non-linear boundary value problems in [17-18]. To our knowledge AGE methods for fifth order dispersive equations have scarcely been presented.

In this paper we will consider the fifth order dispersive equations:

$$u_t + au_{xxxxx} = 0,\ 0 \le t \le T \qquad (1)$$

with initial and periodic boundary value:

$$\begin{cases} u(x,0) = f(x), \\ u(x,t) = u(x+L,t). \end{cases} \qquad (2)$$

The paper is organized as follows: In section 2, we present a group of asymmetric schemes. Based on the schemes a class of unconditionally stable alternating group explicit finite difference method will be derived. Stability analysis for the alternating group method is given in section 3. In section 4, We construct an iterative method based on the concept of decomposition and alternating group. Convergence analysis for the iterative method is given in section 5. Results of numerical experiments are presented in section 6. Some conclusions are presented at the end of the paper.

2 The Alternating Group Explicit Finite Difference (AGEFD) Method

The domain $\Omega : [0,L] \times [0,T]$ will be divided into $(m \times N)$ meshes with spatial step size h=$\frac{1}{m}$ in x direction and the time step size τ=$\frac{T}{N}$. Grid points are denoted by (x_i, t_n) or (i,n), $x_i = ih(i = 0,\ 1, \cdots, m)$, $t_n = n\tau(n = 0,\ 1, \cdots, \frac{T}{\tau})$. The numerical solution of (1) is denoted by u_i^n, while the exact solution $u(x_i,\ t_n)$. Let $r = \frac{a\tau}{4h^5}$.

We first present twelve saul'yev asymmetry schemes to approach (1) at $(i, n+\frac{1}{2})$ as follows:

$$(1-2r)u_i^{n+1} + 5ru_{i+1}^{n+1} - 4ru_{i+2}^{n+1} + ru_{i+3}^{n+1} = 2ru_{i-3}^n$$

$$-8ru_{i-2}^{n}+10ru_{i-1}^{n}+(1-2r)u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \tag{3}$$

$$-2ru_{i-1}^{n+1}+u_i^{n+1}+5ru_{i+1}^{n+1}-4ru_{i+2}^{n+1}+ru_{i+3}^{n+1}=2ru_{i-3}^{n}$$
$$-8ru_{i-2}^{n}+8ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \tag{4}$$

$$3ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+5ru_{i+1}^{n+1}-4ru_{i+2}^{n+1}+ru_{i+3}^{n+1}$$
$$=2ru_{i-3}^{n}-5ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \tag{5}$$

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+5ru_{i+1}^{n+1}-5ru_{i+2}^{n+1}$$
$$+2ru_{i+3}^{n+1}=ru_{i-3}^{n}-4ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+3ru_{i+2}^{n} \tag{6}$$

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+8ru_{i+1}^{n+1}-8ru_{i+2}^{n+1}$$
$$+2ru_{i+3}^{n+1}=ru_{i-3}^{n}-4ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-2ru_{i+1}^{n} \tag{7}$$

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+(1-2r)u_i^{n+1}+10ru_{i+1}^{n+1}$$
$$-8ru_{i+2}^{n+1}+2ru_{i+3}^{n+1}=ru_{i-3}^{n}-4ru_{i-2}^{n}+5ru_{i-1}^{n}+(1-2r)u_i^{n} \tag{8}$$

$$-2ru_{i-3}^{n+1}+8ru_{i-2}^{n+1}-10ru_{i-1}^{n+1}+(1+2r)u_i^{n+1}+5ru_{i+1}^{n+1}$$
$$-4ru_{i+2}^{n+1}+ru_{i+3}^{n+1}=(1+2r)u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \tag{9}$$

$$-2ru_{i-3}^{n+1}+8ru_{i-2}^{n+1}-8ru_{i-1}^{n+1}+u_i^{n}+5ru_{i+1}^{n+1}-4ru_{i+2}^{n+1}$$
$$+ru_{i+3}^{n+1}=2ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \tag{10}$$

$$-2ru_{i-3}^{n+1}+5ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+5ru_{i+1}^{n+1}-4ru_{i+2}^{n+1}$$
$$+ru_{i+3}^{n+1}=-3ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \tag{11}$$

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+5ru_{i+1}^{n+1}-3ru_{i+2}^{n+1}$$
$$=ru_{i-3}^{n}-4ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+5ru_{i+2}^{n}-2ru_{i+3}^{n} \tag{12}$$

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+2ru_{i+1}^{n+1}=ru_{i-3}^{n}$$
$$-4ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-8ru_{i+1}^{n}+8ru_{i+2}^{n}-2ru_{i+3}^{n} \tag{13}$$

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+(1+2r)u_i^{n+1}=ru_{i-3}^{n}$$
$$-4ru_{i-2}^{n}+5ru_{i-1}^{n}+(1+2r)u_i^{n}-10ru_{i+1}^{n}+8ru_{i+2}^{n}-2ru_{i+3}^{n} \tag{14}$$

Using the schemes mentioned above, we will have three basic independent computation groups:
"ω_1"group: twelve grid points are involved, and $(3)-(14)$ are used at each grid point respectively.
"ω_2"group: six inner points are involved, and $(3)-(8)$ are used respectively.
"ω_3"group: six inner points are involved, and $(9)-(14)$ are used respectively.

Based on the basic point groups above, we construct the alternating group explicit (AGE) finite difference method in two cases as follows:

Case 1: Let $m=12s$, here s is an integer. First at the (n+1)-th time level, we divide all the m grid points into s "ω_1" groups. Twelve grid points are included in each group, named $(i+k, n+1), k=0,1,\cdots,11$, and (2.1)-(2.12) are applied respectively. Second at the (n+2)-th time level, we divide all the m grid points into $(s+1)$ groups. "ω_3" group is used to get the solution of the left six grid points $u_1^{n+2}, u_2^{n+2}, u_3^{n+2}, u_4^{n+2}, u_5^{n+2}, u_6^{n+2}$. "$\omega_1$" group is used in each of the following $s-1$ point groups, while "ω_2" group is used in the right six grid points $u_{m-5}^{n+2}, u_{m-4}^{n+2}, u_{m-3}^{n+2}, u_{m-2}^{n+2}, u_{m-1}^{n+2}, u_m^{n+2}$.

It is obvious that computation in the whole domain can be fulfilled in many sub domains independently, and the basic computation groups above are properly used in each sub domain. So the alternating group method has the property of parallelism.

Let $U^n=(u_1^n,\ u_2^n,\ \cdots,\ u_m^n)^T$, then we can denote the AGE finite difference method method (AGEFD1) as follows:

$$\begin{cases} (I+rH_1)U^{n+1}=(I-rH_2)U^n \\ (I+rH_2)U^{n+2}=(I-rH_1)U^{n+1} \end{cases} \tag{15}$$

$$H_1=\begin{pmatrix} H_{11} & & & & \\ & H_{11} & & & \\ & & \cdots & & \\ & & & H_{11} & \\ & & & & H_{11} \end{pmatrix}_{m\times m}$$

$$H_2=\begin{pmatrix} H_{21} & & & & \widetilde{Q} \\ & H_{11} & & & \\ & & \cdots & & \\ & & & H_{11} & \\ \widehat{Q} & & & & H_{22} \end{pmatrix}_{m\times m},$$

$$H_{11}=\begin{pmatrix} H_{111} & H_{112} \\ H_{113} & H_{114} \end{pmatrix}$$

$$H_{111}=\begin{pmatrix} -2 & 5 & -4 & 1 & & \\ -2 & 0 & 5 & -4 & 1 & \\ 3 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -5 \\ & -1 & 4 & -5 & 0 & 8 \\ & & -1 & 4 & -5 & -2 \end{pmatrix},$$

$$H_{112}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -8 & 2 & 0 & 0 & 0 & 0 \\ 10 & -8 & 2 & 0 & 0 & 0 \end{pmatrix},$$

$$H_{113}=\begin{pmatrix} 0 & 0 & 0 & -2 & 8 & -10 \\ 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H_{114}=\begin{pmatrix} 2 & 5 & -4 & 1 & & \\ -8 & 0 & 5 & -4 & 1 & \\ 5 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -3 \\ & -1 & 4 & -5 & 0 & 2 \\ & & -1 & 4 & -5 & 2 \end{pmatrix},$$

$$H_{22}=\begin{pmatrix} -2 & 5 & -4 & 1 & & \\ -2 & 0 & 5 & -4 & 1 & \\ 3 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 5 & 5 & -5 \\ & -1 & 4 & -5 & 0 & 8 \\ & & -1 & 4 & -5 & -2 \end{pmatrix}$$

$$\widehat{Q}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -8 & 2 & 0 & 0 & 0 & 0 \\ 10 & -8 & 2 & 0 & 0 & 0 \end{pmatrix},$$

$$H_{21}=\begin{pmatrix} 2 & 5 & -4 & 1 & & \\ -8 & 0 & 5 & -4 & 1 & \\ 5 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -3 \\ & -1 & 4 & -5 & 0 & 2 \\ & & -1 & 4 & -5 & 2 \end{pmatrix}$$

$$\widetilde{Q}=\begin{pmatrix} 0 & 0 & 0 & -2 & 8 & -10 \\ 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Case 2: Let $m = 12s + 6$, here s is an integer. First at the (n+1)-th time level, we divide all the m grid points into $(s + 1)$. "ω_1" group is used in each of the left s groups, while "ω_2" group is used in the right six grid points $u_{m-5}^{n+2}, u_{m-4}^{n+2}, u_{m-3}^{n+2}, u_{m-2}^{n+2}, u_{m-1}^{n+2}, u_m^{n+2}$. Second at the (n+2)-th time level, we still divide all the m grid points into $(s + 1)$ groups. "ω_3" group is used to get the solution of the left six grid points $u_1^{n+2}, u_2^{n+2}, u_3^{n+2}, u_4^{n+2}, u_5^{n+2}, u_6^{n+2}$, while "$\omega_1$" group is used in each of the following s point groups.

We denote the AGE finite difference method II (AGEFD2) as follows:

$$\begin{cases} (I + r\widetilde{H}_1)U^{n+1} = (I - r\widetilde{H}_2)U^n \\ (I + r\widetilde{H}_2)U^{n+2} = (I - r\widetilde{H}_1)U^{n+1} \end{cases} \quad (16)$$

$$\widetilde{H}_1=\begin{pmatrix} H_{11} & & & & \\ & H_{11} & & & \\ & & ... & & \\ & & & H_{11} & \\ & & & & H_{22} \end{pmatrix}_{m\times m}$$

$$\widetilde{H}_2=\begin{pmatrix} H_{21} & & & & \widetilde{P} \\ & H_{11} & & & \\ & & ... & & \\ & & & H_{11} & \\ \widehat{P} & & & & H_{11} \end{pmatrix}_{m\times m},$$

$$\widetilde{P}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\widehat{P}=\begin{pmatrix} 0&0&0&0&0&0&0&0&0&2&-8&10\\ 0&0&0&0&0&0&0&0&0&0&2&-8\\ 0&0&0&0&0&0&0&0&0&0&0&2\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}^T.$$

The alternating use of the asymmetry schemes (3)-(14) can lead to partly counteracting of truncation error, and then can increase the numerical accuracy.

3 Stability Analysis

Kellogg Lemma[19] Let $r>0$, and $G+G^T$ is nonnegative definite real matrix, then:

$$\begin{cases} \|(I+rG)^{-1}\|_2\le 1\\ \|(I-rG)(I+rG)^{-1}\|_2\le 1 \end{cases} \quad (17)$$

Theorem 1 The AGEFD1 method denoted by (15) is unconditionally stable.

Proof: Obviously $H_1+H_1^T$ and $H_2+H_2^T$ are both nonnegative definite real matrices. Then we have:

$$\|(I+rH_1)^{-1}\|_2\le 1, \|(I-rH_1)(I+rH_1)^{-1}\|_2\le 1$$

$$\|(I+rH_2)^{-1}\|_2\le 1, \|(I-rH_2)(I+rH_2)^{-1}\|_2\le 1.$$

Let n be an even number. From (15) we have

$$U^n=HU^{n-2}$$

here

$$H=(I+rH_2)^{-1}(I-rH_1)(I+rH_1)^{-1}(I-rH_2).$$

Let $\overline{H}=(I+rH_2)H(I+rH_2)^{-1}=(I-rH_1)(I+rH_1)^{-1}(I-rH_2)(I+rH_2)^{-1}$, then we have $\rho(H)=\rho(\overline{H})\le\|\overline{H}\|_2\le 1$, which shows the alternating group method given by (15) is unconditionally stable. So theorem 1 is proved.

Analogously we have:

Theorem 2 The AGEFD2 method denoted by (16) is also unconditionally stable.

4 The AGE Iterative Method

We first present an implicit scheme for (1) as follows:

$$-ru_{i-3}^{n+1}+4ru_{i-2}^{n+1}-5ru_{i-1}^{n+1}+u_i^{n+1}+5ru_{i+1}^{n+1}-4ru_{i+2}^{n+1}+ru_{i+3}^{n+1}$$
$$=ru_{i-3}^{n}-4ru_{i-2}^{n}+5ru_{i-1}^{n}+u_i^{n}-5ru_{i+1}^{n}+4ru_{i+2}^{n}-ru_{i+3}^{n} \quad (18)$$

Applying Taylor's formula to (18), we can easily have that the truncation error is $O(\tau^2+h^2)$.

We use Fourier method to analyze the stability of (18). Let $u_i^n=V^ne^{j\alpha x_i}$, here j is the unit of imaginary number. Then from (18) we have

$$V^{n+1}=\frac{1+(-10rsin\alpha h+8rsin2\alpha h-2sin3\alpha h)j}{1+(10rsin\alpha h-8rsin2\alpha h+2sin3\alpha h)j}V^n \quad (19)$$

Let $E=\dfrac{1+(-10rsin\alpha h+8rsin2\alpha h-2sin3\alpha h)j}{1+(10rsin\alpha h-8rsin2\alpha h+2sin3\alpha h)j}$. Obviously $|E|=1$, which shows (18) is unconditionally stable.

Let $F_i^n=ru_{i-3}^n-4ru_{i-2}^n+5ru_{i-1}^n+u_i^n-5ru_{i+1}^n+4ru_{i+2}^n-ru_{i+3}^n$. In order to get the solution of U^{n+1} with U^n known, we first present a group of asymmetry iterative schemes based on (18). Here k is the iterative number.

$$u_{i(k+1)}^{n+1}+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}+ru_{i+3(k+1)}^{n+1}$$
$$=2ru_{i-3(k)}^{n+1}-8ru_{i-2(k)}^{n+1}+10ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}$$
$$-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-ru_{i+3(k)}^{n+1}+2F_i^n \quad (20)$$

$$-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}$$
$$+ru_{i+3(k+1)}^{n+1}=2ru_{i-3(k)}^{n+1}-8ru_{i-2(k)}^{n+1}+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}$$
$$-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-ru_{i+3(k)}^{n+1}+2F_i^n \quad (21)$$

$$4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}+5ru_{i+1(k+1)}^{n+1}$$
$$-4ru_{i+2(k+1)}^{n+1}+ru_{i+3(k+1)}^{n+1}=2ru_{i-3(k)}^{n+1}-4ru_{i-2(k)}^{n+1}$$
$$+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-ru_{i+3(k)}^{n+1}+2F_i^n \quad (22)$$

$$-ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}+2ru_{i+3(k+1)}^{n+1}$$
$$=ru_{i-3(k)}^{n+1}-4ru_{i-2(k)}^{n+1}+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}$$
$$-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}+2F_i^n \quad (23)$$

$$-ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+5ru_{i+1(k+1)}^{n+1}-8ru_{i+2(k+1)}^{n+1}+2ru_{i+3(k+1)}^{n+1}=ru_{i-3(k)}^{n+1}$$
$$-4ru_{i-2(k)}^{n+1}+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}-5ru_{i+1(k)}^{n+1}+2F_i^n \quad (24)$$

$$-ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+10ru_{i+1(k+1)}^{n+1}-8ru_{i+2(k+1)}^{n+1}+2ru_{i+3(k+1)}^{n+1}=ru_{i-3(k)}^{n+1}$$
$$-4ru_{i-2(k)}^{n+1}+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}+2F_i^n \quad (25)$$

$$-2ru_{i-3(k+1)}^{n+1}+8ru_{i-2(k+1)}^{n+1}-10ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}+ru_{i+3(k+1)}^{n+1}=-u_{i(k)}^{n+1}$$
$$-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-ru_{i+3(k)}^{n+1}+2F_i^n \quad (26)$$

$$-2ru_{i-3(k+1)}^{n+1}+8ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n}$$
$$+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}+ru_{i+3(k+1)}^{n+1}=5ru_{i-1(k)}^{n+1}$$
$$-u_{i(k)}^{n+1}-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-ru_{i+3(k)}^{n+1}+2F_i^n \quad (27)$$

$$-2ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}+ru_{i+3(k+1)}^{n+1}=-4ru_{i-2(k)}^{n+1}$$
$$+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-ru_{i+3(k)}^{n+1}+2F_i^n \quad (28)$$

$$-ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+5ru_{i+1(k+1)}^{n+1}-4ru_{i+2(k+1)}^{n+1}=ru_{i-3(k)}^{n+1}-4ru_{i-2(k)}^{n+1}$$
$$+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}-5ru_{i+1(k)}^{n+1}+4ru_{i+2(k)}^{n+1}-2ru_{i+3(k)}^{n+1}+2F_i^n \quad (29)$$

$$-ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$+5ru_{i+1(k+1)}^{n+1}=ru_{i-3(k)}^{n+1}-4ru_{i-2(k)}^{n+1}+5ru_{i-1(k)}^{n+1}$$
$$-u_{i(k)}^{n+1}-5ru_{i+1(k)}^{n+1}+8ru_{i+2(k)}^{n+1}-2ru_{i+3(k)}^{n+1}+2F_i^n \quad (30)$$

$$-ru_{i-3(k+1)}^{n+1}+4ru_{i-2(k+1)}^{n+1}-5ru_{i-1(k+1)}^{n+1}+u_{i(k+1)}^{n+1}$$
$$=ru_{i-3(k)}^{n+1}-4ru_{i-2(k)}^{n+1}+5ru_{i-1(k)}^{n+1}-u_{i(k)}^{n+1}$$
$$-10ru_{i+1(k)}^{n+1}+8ru_{i+2(k)}^{n+1}-2ru_{i+3(k)}^{n+1}+2F_i^n \quad (31)$$

Using the schemes mentioned above, we will have three basic independent computation groups:
"κ_1"group: twelve grid points are involved, and $(20)-(31)$ are used at each grid point respectively.
"κ_2"group: six grid points are involved, and $(20)-(25)$ are used respectively.
"κ_3"group: six grid points are involved, and $(26)-(31)$ are used respectively.

Let $U^{n+1(k)}=(u_{1(k)}^{n+1},u_{2(k)}^{n+1},\cdots,u_{m(k)}^{n+1})^T$. Based on the basic point groups above, we construct the alternating group explicit iterative method in two cases as follows:

Case 1: Let $m=12s$, here s is an integer. First in order to get the solution of $U^{n+1(k+\frac{1}{2})}$ with $U^{n+1(k)}$ known, we divide all the m grid points into s "κ_1" groups. Twelve grid points are included in each group, named $(i+p,n+1),p=0,1,\cdots,11$, and (20)-(31) are applied to get the solution of $u_{i+p(k+\frac{1}{2})}^{n+1}$, $p=0,1,\cdots,11$ respectively. Second in order to get the solution of $U^{n+1(k+1)}$ with $U^{n+1(k+\frac{1}{2})}$ known, we divide all the grid points into $s+1$ groups. "κ_3" group is used to get the solution of the left six grid points $u_{1(k+1)}^{n+1},u_{2(k+1)}^{n+1},u_{3(k+1)}^{n+1},u_{4(k+1)}^{n+1},u_{5(k+1)}^{n+1},u_{6(k+1)}^{n+1}$. "$\kappa_1$" group is used in each of the following $s-1$ point groups, while "κ_2" group is used in the right six grid points $u_{m-5(k+1)}^{n+1},u_{m-4(k+1)}^{n+1},u_{m-3(k+1)}^{n+1}$, $u_{m-2(k+1)}^{n+1},u_{m-1(k+1)}^{n+1},u_{m(k+1)}^{n+1}$.

It is obvious that the computation in the whole domain can be fulfilled in many sub domains independently, and the basic computation groups above are properly used in each sub domain. So the alternating group explicit iterative method has the property of parallelism.

We denote the AGE iterative method I (AGEI1) as follows:

$$\begin{cases}(I+rA_1)U^{n+1(k+\frac{1}{2})}=(I-rA_2)U^{n+1(k)}+\widehat{F}^n\\(I+rA_2)U^{n+1(k+1)}=(I-rA_1)U^{n+1(k+\frac{1}{2})}+\widehat{F}^n\end{cases} \quad (32)$$

$$A_1=\begin{pmatrix}A_{11}&&&&\\&A_{11}&&&\\&&\cdots&&\\&&&A_{11}&\\&&&&A_{11}\end{pmatrix}_{m\times m}$$

$$A_2=\begin{pmatrix}A_{21}&&&&\widetilde{B}\\&A_{11}&&&\\&&\cdots&&\\&&&A_{11}&\\\widehat{B}&&&&A_{22}\end{pmatrix}_{m\times m},$$

$$A_{11} = \begin{pmatrix} A_{111} & A_{112} \\ A_{113} & A_{114} \end{pmatrix}$$

$$A_{111} = \begin{pmatrix} 0 & 5 & -4 & 1 & 0 & 0 \\ -5 & 0 & 5 & -4 & 1 & 0 \\ 4 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -4 \\ 0 & -1 & 4 & -5 & 0 & 5 \\ 0 & 0 & -1 & 4 & -5 & 0 \end{pmatrix},$$

$$A_{112} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -8 & 2 & 0 & 0 & 0 & 0 \\ 10 & -8 & 2 & 0 & 0 & 0 \end{pmatrix},$$

$$A_{113} = \begin{pmatrix} 0 & 0 & 0 & -2 & 8 & -10 \\ 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A_{114} = \begin{pmatrix} 0 & 5 & -4 & 1 & 0 & 0 \\ -5 & 0 & 5 & -4 & 1 & 0 \\ 4 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -4 \\ 0 & -1 & 4 & -5 & 0 & 5 \\ 0 & 0 & -1 & 4 & -5 & 0 \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} 0 & 5 & -4 & 1 & & \\ -5 & 0 & 5 & -4 & 1 & \\ 4 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -4 \\ & -1 & 4 & -5 & 0 & 5 \\ & & -1 & 4 & -5 & 0 \end{pmatrix}$$

$$\widehat{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ -8 & 2 & 0 & 0 & 0 & 0 \\ 10 & -8 & 2 & 0 & 0 & 0 \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} 0 & 5 & -4 & 1 & & \\ -5 & 0 & 5 & -4 & 1 & \\ 4 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -4 \\ & -1 & 4 & -5 & 0 & 5 \\ & & -1 & 4 & -5 & 0 \end{pmatrix}$$

$$\widetilde{B} = \begin{pmatrix} 0 & 0 & 0 & -2 & 8 & -10 \\ 0 & 0 & 0 & 0 & -2 & 8 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\widehat{F}^n = 2(I - rF^n)U^n$$

$$F^n = \begin{pmatrix} F_1^n & F_2^n \\ F_3^n & F_4^n \end{pmatrix}$$

$$F_1^n = \begin{pmatrix} 0 & 5 & -4 & 1 & 0 & 0 \\ -5 & 0 & 5 & -4 & 1 & 0 \\ 4 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -4 \\ 0 & -1 & 4 & -5 & 0 & 5 \\ 0 & 0 & -1 & 4 & -5 & 0 \end{pmatrix}$$

$$F_2^n = \begin{pmatrix} 0 & 0 & 0 & -1 & 4 & -5 \\ 0 & 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 & 0 & 0 \\ 5 & -4 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$F_3^n = \begin{pmatrix} 0 & 0 & 0 & -1 & 4 & -5 \\ 0 & 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 & 0 & 0 \\ 5 & -4 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$F_4^n = \begin{pmatrix} 0 & 5 & -4 & 1 & 0 & 0 \\ -5 & 0 & 5 & -4 & 1 & 0 \\ 4 & -5 & 0 & 5 & -4 & 1 \\ -1 & 4 & -5 & 0 & 5 & -4 \\ 0 & -1 & 4 & -5 & 0 & 5 \\ 0 & 0 & -1 & 4 & -5 & 0 \end{pmatrix}$$

Case 2: Let $m = 12s + 6$, here s is an integer. First in order to get the solution of $U^{n+1(k+\frac{1}{2})}$ with $U^{n+1(k)}$ known, we divide all the m grid points into $(s+1)$groups. "κ_1" group is used in each of the left s groups, while "κ_2" group is used in the right six grid points $u_{m-5(k+1)}^{n+1}, u_{m-4(k+1)}^{n+1}, u_{m-3(k+1)}^{n+1}, u_{m-2(k+1)}^{n+1}, u_{m-1(k+1)}^{n+1}$, $u_{m(k+1)}^{n+1}$. Second in order to get the solution of $U^{n+1(k+1)}$ with $U^{n+1(k+\frac{1}{2})}$ known, we still divide all the m grid points into $(s+1)$ groups. "κ_3" group is used to get the solution of the left six grid points

$u^{n+1}_{1(k+1)}, u^{n+1}_{2(k+1)}, u^{n+1}_{3(k+1)}, u^{n+1}_{4(k+1)}$, $u^{n+1}_{5(k+1)}, u^{n+1}_{6(k+1)}$, while "$\kappa_1$" group is used in each of the following s point groups.

We denote the AGE iterative method II (AGEI2) as follows:

$$\begin{cases} (I + r\widetilde{A}_1)U^{n+1(k+\frac{1}{2})} = (I - r\widetilde{A}_2)U^{n+1(k)} + \widehat{F}^n \\ (I + r\widetilde{A}_2)U^{n+1(k+1)} = (I - r\widetilde{A}_1)U^{n+1(k+\frac{1}{2})} + \widehat{F}^n \end{cases} \quad (33)$$

$$\widetilde{A}_1 = \begin{pmatrix} A_{11} & & & & \\ & A_{11} & & & \\ & & \ldots & & \\ & & & A_{11} & \\ & & & & A_{22} \end{pmatrix}_{m\times m}$$

$$\widetilde{A}_2 = \begin{pmatrix} A_{21} & & & & \widetilde{C} \\ & A_{11} & & & \\ & & \ldots & & \\ & & & A_{11} & \\ \widehat{C} & & & & A_{11} \end{pmatrix}_{m\times m},$$

$$\widetilde{C} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&-2&8&-10 \\ 0&0&0&0&0&0&0&0&0&0&-2&8 \\ 0&0&0&0&0&0&0&0&0&0&0&-2 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$\widehat{C} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&2&-8&10 \\ 0&0&0&0&0&0&0&0&0&0&2&-8 \\ 0&0&0&0&0&0&0&0&0&0&0&2 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}^T.$$

5 Convergence Analysis For The AGE Iterative Method

Theorem 3 The AGEI1 method denoted by (32) is convergent.

Proof: Obviously $A_1 + A_1^T$ and $A_2 + A_2^T$ are both nonnegative definite real matrices. Then from Kellogg lemma we have:

$$\|(I+rA_1)^{-1}\|_2 \le 1, \|(I-rA_1)(I+rA_1)^{-1}\|_2 \le 1$$

$$\|(I+rA_2)^{-1}\|_2 \le 1, \|(I-rA_2)(I+rA_2)^{-1}\|_2 \le 1.$$

Let n be an even number. From (32) we have

$$U^n = AU^{n-2}$$

$$+[(I+rA_2)^{-1}(I-rA_1)(I+rA_1)^{-1}+(I+rA_2)^{-1}]\widehat{F}^n,$$

here

$$A = (I + rA_2)^{-1}(I - rA_1)(I + rA_1)^{-1}(I - rA_2)$$

is the growth matrix.

Let $\overline{A} = (I + rA_2)H(I + rA_2)^{-1} = (I - rA_1)(I+rA_1)^{-1}(I-rA_2)(I+rA_2)^{-1}$, then we have $\rho(A) = \rho(\overline{A}) \le \|\overline{A}\|_2 \le 1$, which shows the method given by (32) is convergent. So theorem 3 is proved.

Similarly we have:

Theorem 4 The AGEI2 method denoted by (33) is also convergent.

6 Numerical Experiments

Example 1: Let $a = 1,\ L = 2.0,\ u(x, 0) = cos(\pi x)$, then the exact solution of (1.1) is denoted as below:

$$u(x, t) = cos(\pi x - \pi^5 t)$$

Let $A.E. = |u_i^n - u(x_i, t_n)|$, $P.E = \dfrac{|u_i^n - u(x_i, t_n)|}{u(x_i, t_n)}$ denote maximum absolute error and relevant error respectively. we compare the numerical results of the presented AGEFD method in this paper with the results of the full implicit Crank-Nicolson scheme(IC-N) as follows:

Table 1: results of comparison $m = 96, r = 0.5$

	$t = 1000\tau$	$t = 3000\tau$	$t = 7000\tau$
$A.E.$	3.618×10^{-6}	1.112×10^{-5}	2.623×10^{-5}
$A.E.^{(IC-N)}$	3.462×10^{-6}	1.030×10^{-5}	2.405×10^{-5}
$P.E.$	5.453×10^{-3}	1.813×10^{-2}	5.004×10^{-2}
$P.E.^{(IC-N)}$	5.319×10^{-3}	1.749×10^{-2}	4.923×10^{-2}

Table 2: results of comparison $m = 96, r = 2$

	$t = 1000\tau$	$t = 3000\tau$	$t = 7000\tau$
$A.E.$	1.426×10^{-5}	4.125×10^{-5}	9.631×10^{-5}
$A.E.^{(IC-N)}$	1.375×10^{-5}	4.114×10^{-5}	9.597×10^{-5}
$P.E.$	2.506×10^{-2}	1.147×10^{-1}	3.589×10^{-1}
$P.E.^{(IC-N)}$	2.440×10^{-2}	1.121×10^{-1}	3.505×10^{-1}

Table 3: results of Comparison $m = 120, r = 2$

	$t = 1000\tau$	$t = 3000\tau$	$t = 7000\tau$
$A.E.$	2.942×10^{-6}	8.713×10^{-6}	2.085×10^{-5}
$A.E.^{(IC-N)}$	2.918×10^{-6}	8.651×10^{-6}	2.017×10^{-5}
$P.E.$	5.717×10^{-3}	2.014×10^{-2}	6.647×10^{-2}
$P.E.^{(IC-N)}$	5.706×10^{-3}	1.988×10^{-2}	6.616×10^{-2}

Table 4: results of Comparison $m = 120, r = 10$

	$t = 1000\tau$	$t = 3000\tau$	$t = 7000\tau$
$A.E.$	1.454×10^{-5}	4.362×10^{-5}	1.024×10^{-4}
$A.E.^{(IC-N)}$	1.443×10^{-5}	4.317×10^{-5}	1.006×10^{-4}
$P.E.$	3.933×10^{-2}	8.412×10^{-1}	1.854
$P.E.^{(IC-N)}$	3.915×10^{-2}	8.393×10^{-1}	1.838

Results of Table 1-4 show the presented AGEFD method is stable even in large r, and is of nearly the same accuracy as the full implicit Crank-Nicolson scheme. With the increase of grid points, we can obtain higher accuracy.

Example 2: Let $a = 1$, $L = 2.0$, $u(x,0) = cos(\pi x)$, ω denotes the parameter of the SOR iterative method. We use the iterative error 1×10^{-6} to control the process of iterativeness, and the results of comparison between AGEI method and the SOR iterative method are listed in the following tables:

Table 5: Results at $m = 96, r = 0.5, \rho = 10^6$

	$t = 1000\tau$
average iterative times	1.999
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 6: Results at $m = 96, r = 0.5, \rho = 10^6$

	$t = 3000\tau$
average iterative times	1.956
average iiterative times(SOR):	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 7: Results at $m = 96, r = 0.5, \rho = 10^6$

	$t = 7000\tau$
average iterative times	1.637
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 8: Results at $m = 96, r = 0.5, \rho = 10^6$

	$t = 10000\tau$
average iterative times	1.637
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 9: Results at $m = 96, r = 0.01, \rho = 1$

	$t = 1000\tau$
average iterative times(AGE)	1.499
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	2.646
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 10: Results at $m = 96, r = 0.01, \rho = 1$

	$t = 3000\tau$
average iterative times(AGE)	1.500
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	2.759
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 11: Results at $m = 96, r = 0.01, \rho = 1$

	$t = 7000\tau$
average iterative times(AGE)	1.500
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	3.394
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 12: Results at $m = 96, r = 0.01, \rho = 1$

	$t = 10000\tau$
average iterative times(AGE)	1.500
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	3.563
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 13: Results at $m = 96, r = 2, \rho = 10^{-6}$

	$t = 1000\tau$
average iterative times(AGE)	2
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 14: Results at $m = 96, r = 2, \rho = 10^{-6}$

	$t = 3000\tau$
average iterative times(AGE)	2
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 15: Results at $m = 96, r = 2, \rho = 10^{-6}$

	$t = 7000\tau$
average iterative times(AGE)	2
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Table 16: Results at $m = 96, r = 2, \rho = 10^{-6}$

	$t = 10000\tau$
average iterative times(AGE)	2
average iiterative times(SOR):$\omega = 10^{-6}$	can't converge
average iiterative times(SOR):$\omega = 10^{-3}$	can't converge
average iiterative times(SOR):$\omega = 1$	can't converge
average iiterative times(SOR):$\omega = 10$	can't converge
average iiterative times(SOR):$\omega = 10^3$	can't converge

Results of Table 5-16 show the presented AGEI method is superior to the known SOR iterative method obviously.

7 Conclusions

In this paper, we present a class of alternating group explicit finite difference method (AGEFD) and an alternating group explicit iterative method (AGEI) for fifth order dispersive equations, which are both of intrinsic parallelism. Numerical results show that both of the two methods are effective. Considering the absolute stability of the AGEFD method, it doesn't lead to numerical vibration in computation. Numerical results of comparison between the presented AGEI method and the SOR iterative method show that the AGEI method is superior to the SOR method.

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