Application Of The Alternating Group Explicit Method For Convection-Diffusion Equations

Qinghua Feng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
fqhua@sina.com

Bin Zheng
Shandong University of Technology
School of Science
Zhangzhou Road 12, Zibo, 255049
China
zhengbin2601@126.com

Abstract: Based on an unconditionally stable finite difference implicit scheme, we present a concept of deriving a class of effective alternating group explicit iterative method for periodic boundary value problem of convection-diffusion equations, and then give two iterative methods. The methods are verified to be convergent, and have the property of parallelism. Furthermore we construct an alternating group explicit difference method and another iterative method. All of the methods are suitable for parallel computation. Results of numerical experiments show that the methods are of higher accuracy than the known methods in [1,2,6], and will not lead to numerical instability in convection dominant case.

Key–Words: iterative method, iterative method, parallel computing, alternating group, parabolic equations

1 Introduction

In this paper, we will consider the following time-dependent periodic initial boundary value problem of convection-diffusion equations:

$$\begin{cases} \dfrac{\partial u}{\partial t} + k\dfrac{\partial u}{\partial x} = \varepsilon\dfrac{\partial^2 u}{\partial x^2}, \quad 0 \le t \le T \\ u(x,0) = f(x), \\ u(x,t) = u(x+1,t). \end{cases} \tag{1}$$

In scientific and engineering computation, with the development of parallel computer technology, researches on parallel finite difference methods are getting more and more popular [1-3]. As we all know, Most of explicit methods are short in stability and accuracy, while implicit methods usually have good stability, but are complex in computing, and need to solve large equation set in the cost of large memory spaces and CPU cycles. Thus it is necessary to construct methods with the advantages of explicit methods and implicit methods, that is, simple for computation and good stability. Many parallel numerical methods have been presented so far for parabolic partial differential equations, in which a class of alternating group explicit method (AGE) presented in [4-6] is of special meaning for its parallelism and absolute stability. The AGE method is derived by a special composition of two asymmetry schemes, therefore the truncation error can be counteracted much, which leads to high accuracy. Besides the above, In solving large equation set, all the work in the whole domain can be decomposed to many sub-domains for the AGE method. The disadvantage of the original AGE method is that numerical vibration will appear in the case of convection dominant convection-diffusion equations. Based on the original AGE method, many alternating group methods have been presented such as in [7-10]. Rohallah Tavakoli derived a class of domain-split method for diffusion equations in [11-12]. Most of the methods inherit the advantages of the AGE method, that is, parallelism and absolute stability. But we notice researches on alternating group iterative methods are also scarcely presented, and effective methods for convection dominant problems have been scarcely constructed.

We will try to establish a class of parallel unconditionally alternating group explicit method for solving (1). The rest of this paper will be organized as follows:

In section 2, we will get the integral conservative form of (1) by a kind of exponential type transformation [10]. Then a symmetry implicit finite difference scheme based on the form will be presented. Based on the scheme we give four asymmetry iterative schemes, and then construct a class of alternating group explicit iterative method(AGEI). In section 3, we will apply the concept in section 2 to construct another four order alternating group explicit iterative (FOAGEI) method. In section 4, convergence analysis and stability analysis are given. In section 5, we construct an alternating group explicit difference (AGED) method with the accurate of order four in spatial step size. Stability analysis for the AGED method is presented in section 6. In section 7, we apply the concept of constructing the alternating group method to derive another alternating group iterative method. In section 8, results of several numerical examples are presented. Some conclusions are given at the end of the paper.

2 The Parallel AGE Iterative(AGEI) Method

The domain $\Omega : (0,1) \times (0,T)$ will be divided into $(m \times N)$ meshes with spatial step size h=$\frac{1}{m}$ in x direction and the time step size τ=$\frac{T}{N}$. Grid points are denoted by (x_i, t_n) or (i, n), $x_i = ih(i = 0,\ 1, \cdots, m)$, $t_n = n\tau(n = 0,\ 1, \cdots, \frac{T}{\tau})$. The numerical solution of (1) is denoted by u_i^n, while the exact solution $u(x_i,\ t_n)$. In this paper we let $U^n = (u_1^n,\ u_2^n,\ \cdots,\ u_m^n)^T$.

The purpose of this paper is to get the solution of $(n+1)-th$ time level with the solution of $n-th$ time level known. We notice that the equation (1) is equivalent to $e^{-\frac{kx}{\varepsilon}}\frac{\partial u}{\partial t} = \varepsilon\frac{\partial}{\partial x}(e^{-\frac{kx}{\varepsilon}}\frac{\partial u}{\partial x})$. Integral from $x_{i-\frac{1}{2}}$ to $x_{i+\frac{1}{2}}$ we have $(\frac{\partial u}{\partial t})_i^{n+\frac{1}{2}}\int_{x_{i-\frac{1}{2}h}}^{x_{i+\frac{1}{2}h}} e^{-\frac{kx}{\varepsilon}}dx \approx \varepsilon[e^{-\frac{kh}{2\varepsilon}}(\frac{\partial u}{\partial x})_{i+\frac{1}{2}}^{n+\frac{1}{2}} - e^{\frac{kh}{2\varepsilon}}(\frac{\partial u}{\partial x})_{i-\frac{1}{2}}^{n+\frac{1}{2}}]$.

We can derive an implicit scheme for solving (1) as below:

$$(e^{\frac{kh}{2\varepsilon}} - e^{-\frac{kh}{2\varepsilon}})\frac{u_i^{n+1}-u_i^n}{\tau} = \frac{k}{h}[e^{-\frac{kh}{2\varepsilon}}(\frac{u_{i+1}^{n+1}-u_i^{n+1}}{2} + \frac{u_{i+1}^n-u_i^n}{2}) - e^{\frac{kh}{2\varepsilon}}(\frac{u_i^{n+1}-u_{i-1}^{n+1}}{2} + \frac{u_i^n-u_{i-1}^n}{2})]$$

Applying Taylor's formula to the scheme at $(x_i, t_{n+\frac{1}{2}})$, we can easily have that the truncation error of the scheme is $O(\tau^2 + h^2)$.

Let $p = e^{-\frac{kh}{2\varepsilon}}, q = e^{\frac{kh}{2\varepsilon}}, r = \frac{k\tau}{h(q-p)}$, then we have

$$-\frac{rq}{2}u_{i-1}^{n+1} + [1+\frac{r}{2}(p+q)]u_i^{n+1} - \frac{rp}{2}u_{i+1}^{n+1} = \frac{rq}{2}u_{i-1}^n + [1-\frac{r}{2}(p+q)u_i^n + \frac{rp}{2}u_{i+1}^n \quad (2)$$

We denote it as $AU^{n+1} = F^n$. here $F^n = (2I - A)U^n$

In order to solve U^{n+1}, we have to solve an implicit equation set, which is complex in computation. Then we will try to construct an alternating group explicit iterative method instead in the following.

First we will present four asymmetry iterative schemes to solve $u_{i(k+1)}^{n+1}$ with the value at k known. Here k denotes the iterative number.

$$[1+\frac{r}{2}(p+q)]u_{i(k+1)}^{n+1} - \frac{rp}{2}u_{i+1(k+1)}^{n+1} = -rqu_{i-1(k)}^{n+1} + [1+\frac{r}{2}(p+q)]u_{i(k)}^{n+1} - \frac{rp}{2}u_{i+1(k)}^{n+1} \quad (3)$$

$$-\frac{rq}{2}u_{i-1(k+1)}^{n+1} + [1+\frac{r}{2}(p+q)]u_{i(k+1)}^{n+1} - rpu_{i+1(k+1)}^{n+1} = -\frac{rq}{2}u_{i-1(k)}^{n+1} + [1+\frac{r}{2}(p+q)]u_{i(k)}^{n+1} \quad (4)$$

$$-rqu_{i-1(k+1)}^{n+1} + [1+\frac{r}{2}(p+q)]u_{i(k+1)}^{n+1} - \frac{rp}{2}u_{i+1(k+1)}^{n+1} = [1+\frac{r}{2}(p+q)]u_{i(k)}^{n+1} - \frac{rp}{2}u_{i+1(k)}^{n+1} \quad (5)$$

$$-\frac{rq}{2}u_{i-1(k+1)}^{n+1} + [1+\frac{r}{2}(p+q)]u_{i(k+1)}^{n+1} = -\frac{rq}{2}u_{i-1(k)}^{n+1} + [1+\frac{r}{2}(p+q)]u_{i(k)}^{n+1} - rpu_{i+1(k)}^{n+1} \quad (6)$$

If we apply (3)-(6) to four adherent grid points $(i, n+1), (i+1, n+1), (i+2, n+1), (i+3, n+1)$, then we have:

$$B_1\overline{u}_{i(k+1)}^{n+1} = C_1\overline{u}_{i(k)}^{n+1} + D_1$$

here $D_1 = (-rqu_{i-1(k)}^{n+1})^T, 0, 0, -rpu_{i+4(k)}^{n+1})^T$

$$B_1 = \begin{pmatrix} 1+\frac{r}{2}(p+q) & -\frac{rp}{2} & 0 & 0 \\ -\frac{rq}{2} & 1+\frac{r}{2}(p+q) & -rp & 0 \\ 0 & -rq & 1+\frac{r}{2}(p+q) & -\frac{rp}{2} \\ 0 & 0 & -\frac{rq}{2} & 1+\frac{r}{2}(p+q) \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 1+\frac{r}{2}(p+q) & -\frac{rp}{2} & 0 & 0 \\ -\frac{rq}{2} & 1+\frac{r}{2}(p+q) & 0 & 0 \\ 0 & 0 & 1+\frac{r}{2}(p+q) & -\frac{rp}{2} \\ 0 & 0 & -\frac{rq}{2} & 1+\frac{r}{2}(p+q) \end{pmatrix}$$

Then $\overline{u}_{i(k+1)}^{n+1} = B_1^{-1}(C_1\overline{u}_{i(k)}^{n+1} + D_1)$, which shows the values of $(u_{i(k+1)}^{n+1}, u_{i+1(k+1)}^{n+1}, u_{i+2(k+1)}^{n+1}, u_{i+3(k+1)}^{n+1})^T$ can be worked out in one group explicitly.

Let $U_{k+1}^{n+1} = (u_{1(k+1)}^{n+1}, u_{2(k+1)}^{n+1}, \cdots, u_{m(k+1)}^{n+1})^T$, $m = 4s$, s is an integer. we construct the iterative method as below:

First in order to get the solution of U_{k+1}^{n+1} with U_k^{n+1} known, we divide all the grid points into s groups. Four grid points are included in each group, and (3)-(6) are applied. Second in order to get the solution of U_{k+2}^{n+1} with U_{k+1}^{n+1} known, we divide all the grid points into $s+1$ groups. (5) and (6) are used to solve $u_{1(k+2)}^{n+1}$ and $u_{2(k+2)}^{n+1}$. The following $4(s-1)$ grid points are divided into $s-1$ groups, and (3)-(6) are used respectively in each group. (3) and (4) are used to solve $u_{m-1(k+2)}^{n+1}$ and $u_{m-2(k+2)}^{n+1}$.

The alternating use of the asymmetry schemes (3)-(6) can lead to partly counteracting of truncation error, and then can increase the numerical accuracy. On the other hand, grouping explicit computation can be obviously obtained. Thus computing in the whole domain can be splitted into many sub-domains, and can be worked out with several parallel computers independently. So the method has the obvious property of parallelism.

We denote the alternating group explicit iterative method described above as below:

$$\begin{cases} (\rho I+G_1)\tilde{U}_{k+1}^{n+1}=(\rho I-G_2)U_k^{n+1}+\widetilde{F}^n \\ (\rho I+G_2)U_{k+2}^{n+1}=(\rho I-G_1)\tilde{U}_{k+1}^{n+1}+\widetilde{F}^n \end{cases} \quad k=0,1\cdots \tag{7}$$

Here $\widetilde{F}^n=2F^n$, ρ is an iterative parameter.

$$G_1=\begin{pmatrix} B_1 & & & & \\ & \cdots & & & \\ & & \cdots & & \\ & & & \cdots & \\ & & & & B_1 \end{pmatrix}_{m\times m}$$

$$G_2=\begin{pmatrix} B_2 & & & & \widetilde{C} \\ & B_1 & & & \\ & & \cdots & & \\ & & & B_1 & \\ \widehat{C} & & & & B_2 \end{pmatrix}_{m\times m}$$

$$B_2=\begin{pmatrix} 1+\frac{r}{2}(p+q) & -\frac{rp}{2} \\ -\frac{rq}{2} & 1+\frac{r}{2}(p+q) \end{pmatrix}$$

$$\widetilde{C}=\begin{pmatrix} 0 & -rq \\ 0 & 0 \end{pmatrix},\ \widehat{C}=\begin{pmatrix} 0 & 0 \\ -rp & 0 \end{pmatrix}$$

3 The Fourth Order Alternating Group Explicit Iterative(FOAGEI) Method

In section 2, we present a class of alternating group explicit iterative method with intrinsic parallelism. The method is based on an $O(\tau^2+h^2)$ order implicit scheme, which is of absolute stability. Since the construction of the method is universal, of course we can establish another alternating group iterative method based on another high order implicit scheme.

We present another implicit scheme with truncation error $O(\tau^2+h^4)$ for solving (1) as below:

$$(e^{\frac{kh}{2\varepsilon}}-e^{-\frac{kh}{2\varepsilon}})\frac{u_i^{n+1}-u_i^n}{\tau}$$

$$=\frac{k}{2}[e^{-\frac{kh}{2\varepsilon}}(\frac{u_{i+1}^{n+1}-u_i^{n+1}}{h}-\frac{u_{i+2}^{n+1}-3u_{i+1}^{n+1}+3u_i^{n+1}-u_{i-1}^{n+1}}{h}$$

$$+\frac{u_{i+1}^{n}-u_i^{n}}{h}-\frac{u_{i+2}^{n}-3u_{i+1}^{n}+3u_i^{n}-u_{i-1}^{n}}{h})$$

$$-e^{\frac{kh}{2\varepsilon}}(\frac{u_i^{n+1}-u_{i-1}^{n+1}}{h}-\frac{u_{i+1}^{n+1}-3u_i^{n+1}+3u_{i-1}^{n+1}-u_{i-2}^{n+1}}{h}$$

$$+\frac{u_i^{n}-u_{i-1}^{n}}{h}-\frac{u_{i+1}^{n}-3u_i^{n}+3u_{i-1}^{n}-u_{i-2}^{n}}{h})]$$

that is,

$$rqu_{i-2}^{n+1}-(p+27q)ru_{i-1}^{n+1}+[1+27(p+q)r]u_i^{n+1}$$

$$-(q+27p)u_{i+1}^{n+1}+rpu_{i+2}^{n+1}=-rqu_{i-2}^{n+1}+(p+27q)ru_{i-1}^{n+1}$$

$$+[1-27(p+q)r]u_i^{n+1}+(q+27p)u_{i+1}^{n+1}-rpu_{i+2}^{n+1} \tag{8}$$

We denote (8) as $\overline{A}U^{n+1}=\overline{F}^n$. here

$$\overline{F}^n=(2I-\overline{A})U^n$$

Let

$$\overline{A}=\frac{1}{2}(\overline{G}_1+\overline{G}_2)$$

here $\overline{G}_1$=diag$(G_{11},\cdots,G_{11})_{m\times m}$,

$$\overline{G}_2=\begin{pmatrix} G_{21} & & & & \overline{G} \\ & G_{11} & & & \\ & & \cdots & & \\ & & & G_{11} & \\ \overline{\overline{G}} & & & & G_{21} \end{pmatrix}_{m\times m}$$

$$\overline{G}=\begin{pmatrix} 0 & 0 & 2rq & -2(p+27q)r \\ 0 & 0 & 0 & 2rq \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overline{\overline{G}}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2rp & 0 & 0 & 0 \\ -2(q+27p)r & 2rp & 0 & 0 \end{pmatrix}$$

$$G_{21}=\begin{pmatrix} 1+27(p+q)r & -(q+27p)r & rp & 0 \\ -(p+27q)r & 1+27(p+q)r & -(q+27p)r & rp \\ rq & -(p+27q)r & 1+27(p+q)r & -(q+27p)r \\ 0 & rq & -(p+27q)r & 1+27(p+q)r \end{pmatrix}$$

$$G_{11} = \begin{pmatrix} G_{111} & G_{112} \\ G_{113} & G_{114} \end{pmatrix}$$

$$G_{111} =$$
$$\begin{pmatrix} 1+27(p+q)r & -(q+27p)r & rp & \\ -(p+27q)r & 1+27(p+q)r & -(q+27p)r & rp \\ rq & -(p+27q)r & 1+27(p+q)r & -(q+27p)r \\ & rq & -(p+27q)r & 1+27(p+q)r \end{pmatrix}$$

$$G_{112} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2rp & 0 & 0 & \\ -2(q+27p)r & 2rp & 0 & 0 \end{pmatrix}$$

$$G_{113} = \begin{pmatrix} 0 & 0 & 2rq & -2(p+27q)r \\ 0 & 0 & 0 & 2rq \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$G_{114} =$$
$$\begin{pmatrix} 1+27(p+q)r & -(q+27p)r & rp & \\ -(p+27q)r & 1+27(p+q)r & -(q+27p)r & rp \\ rq & -(p+27q)r & 1+27(p+q)r & -(q+27p)r \\ & rq & -(p+27q)r & 1+27(p+q)r \end{pmatrix}$$

Then the fourth order alternating group explicit iterative method can be derived as below:

$$\begin{cases} (\rho I + \overline{G}_1)\tilde{U}_{k+1}^{n+1} = (\rho I - \overline{G}_2)U_k^{n+1} + \widehat{\overline{F}}^n \\ (\rho I + \overline{G}_2)U_{k+2}^{n+1} = (\rho I - \overline{G}_1)\tilde{U}_{k+1}^{n+1} + \widehat{\overline{F}}^n \end{cases} \quad k = 0, 1 \cdots \tag{9}$$

Here $\widehat{\overline{F}}^n = 2\overline{F}^n$, ρ is an iterative parameter.

4 Convergence Analysis and Stability Analysis

Lemma 1[10] Let $\theta > 0$, and $G + G^T$ is nonnegative, then $(\theta I + G)^{-1}$exists, and

$$\begin{cases} \|(\theta I + G)^{-1}\|_2 \leq \theta^{-1} \\ \|(\theta I - G)(\theta I + G)^{-1}\|_2 \leq 1 \end{cases} \tag{10}$$

Theorem 1 The alternating group explicit iterative method given by (7) is convergent.

proof: From the construction of the matrixes we can see $G_1,\ G_2,\ (G_1 + G_1^T),\ (G_2 + G_2^T)$ are all nonnegative matrixes. Then we have $\|(\rho I - G_1)(\rho I + G_1)^{-1}\|_2 \leq 1,\ \|(\rho I - G_2)(\rho I + G_2)^{-1}\|_2 \leq 1$.

From (7), we have $U^{n+1} = GU^n + 2(\rho I + G_2)^{-1}[(\rho I - G_1)(\rho I + G_1)^{-1}F^n + F^n]$, $G = (\rho I + G_2)^{-1}(\rho I - G_1)(\rho I + G_1)^{-1}(\rho I - G_2)$ is the growth matrix.

Let $\widehat{G} = (\rho I + G_2)G(\rho I + G_2)^{-1} = (\rho I - G_1)(\rho I + G_1)^{-1}(\rho I - G_2)(\rho I + G_2)^{-1}$, then $\rho(G) = \rho(\widehat{G}) \leq \|\widehat{G}\|_2 \leq 1$, which shows the alternating group method given by (7) is convergent.

Analogously we have:

Theorem 2 The fourth order alternating group explicit iterative method given by (9) is convergent.

In order to analyze the stability of (2) We will use the Fourier method. Let $u_i^n = \widehat{u}^n e^{i\alpha x_j}$, $x = \frac{r}{2}(p+q) - \frac{r}{2}(p+q)cos(\alpha h),\ y = \frac{r}{2}(p-q)sin(\alpha h)$, then from (2) we have

$$\widehat{u}^{n+1} = \widehat{u}^n \frac{1 - x + iy}{1 + x - iy} \tag{11}$$

Considering $x \geq 0$, it follows that

$$|\frac{1 - x + iy}{1 + x - iy}|^2 = \frac{(1-x)^2 + y^2}{(1+x)^2 + y^2} \leq 1$$

So we have:

Theorem 3 The scheme (2) is unconditionally stable.

In order to analyze the stability of (8) Let $u_i^n = V^n e^{i\alpha x_j}$, $\widehat{x} = 27(p+q)r + r(p+q)cos(2\alpha h) - 28(p+q)rcos(\alpha h),\ \widehat{y} = (p-q)rsin(2\alpha h) - 26(p-q)rsin(\alpha h)$, then from (8) we have

$$V^{n+1} = \frac{1 - \widehat{x} + i\widehat{y}}{1 + \widehat{x} - i\widehat{y}} V^n \tag{12}$$

Considering $\widehat{x} \geq 0$, it follows that

$$|\frac{1 - \widehat{x} + i\widehat{y}}{1 + \widehat{x} - i\widehat{y}}|^2 = \frac{(1-\widehat{x})^2 + \widehat{y}^2}{(1+\widehat{x})^2 + \widehat{y}^2} \leq 1$$

So we have:

Theorem 4 The scheme (8) is unconditionally stable.

5 Alternating Group Explicit Difference (AGED) Method

In this section we will construct another alternating group explicit difference (AGED) method for solving the periodic boundary value problem of convection-diffusion problem denoted by (1). Let

$$\widehat{r} = \frac{\tau}{24h^2} \tag{13}$$

We first present eight asymmetry schemes to approach (1) at $(i, n + \frac{1}{2})$ as follows:

$$[1 + (15\varepsilon - kh)\widehat{r}]u_i^{n+1} + (8kh - 16\varepsilon)\widehat{r}u_{i+1}^{n+1} + (\varepsilon - kh)\widehat{r}u_{i+2}^{n+1}$$
$$= -2(kh + \varepsilon)\widehat{r}u_{i-2}^n + (16kh + 32\varepsilon)\widehat{r}u_{i-1}^n$$
$$+ [1 - (45\varepsilon + kh)\widehat{r}]u_i^n - (8kh - 16\varepsilon)\widehat{r}u_{i+1}^n - (\varepsilon - kh)\widehat{r}u_{i+2}^n \tag{14}$$

$$-(6kh+14\varepsilon)\widehat{r}u_{i-1}^{n+1}+[1+(29\varepsilon-kh)\widehat{r}]u_i^{n+1}$$
$$+(8kh-16\varepsilon)\widehat{r}u_{i+1}^{n+1}+(\varepsilon-kh)\widehat{r}u_{i+2}^{n+1}=-2(\varepsilon+kh)\widehat{r}u_{i-2}^{n}$$
$$+(10kh+18\varepsilon)\widehat{r}u_{i-1}^{n}+[1-(31\varepsilon+kh)\widehat{r}]u_i^{n}$$
$$-(8kh-16\varepsilon)\widehat{r}u_{i+1}^{n}-(\varepsilon-kh)\widehat{r}u_{i+2}^{n} \quad (15)$$

$$(\varepsilon+kh)\widehat{r}u_{i-2}^{n+1}-(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n+1}$$
$$+[1+(31\varepsilon-kh)\widehat{r}]u_i^{n+1}+(10kh-18\varepsilon)\widehat{r}u_{i+1}^{n+1}$$
$$+2(\varepsilon-kh)\widehat{r}u_{i+2}^{n+1}=-(\varepsilon+kh)\widehat{r}u_{i-2}^{n}+(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n}$$
$$+[1-(29\varepsilon+kh)\widehat{r}]u_i^{n}-(6kh-14\varepsilon)\widehat{r}u_{i+1}^{n} \quad (16)$$

$$(\varepsilon+kh)\widehat{r}u_{i-2}^{n+1}-(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n+1}+[1+(45\varepsilon-kh)\widehat{r}]u_i^{n+1}$$
$$+(16kh-32\varepsilon)\widehat{r}u_{i+1}^{n+1}+2(\varepsilon-kh)\widehat{r}u_{i+2}^{n+1}=-(\varepsilon+kh)\widehat{r}u_{i-2}^{n}$$
$$+(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n}+[1-(15\varepsilon+kh)\widehat{r}]u_i^{n} \quad (17)$$

$$2(\varepsilon+kh)\widehat{r}u_{i-2}^{n+1}-(16kh+32\varepsilon)\widehat{r}u_{i-1}^{n+1}+[1+(45\varepsilon+kh)\widehat{r}]u_i^{n+1}$$
$$+(8kh-16\varepsilon)\widehat{r}u_{i+1}^{n+1}+(\varepsilon-kh)\widehat{r}u_{i+2}^{n+1}=[1-(15\varepsilon-kh)\widehat{r}]u_i^{n}$$
$$-(8kh-16\varepsilon)\widehat{r}u_{i+1}^{n}-(\varepsilon-kh)\widehat{r}u_{i+2}^{n} \quad (18)$$

$$2(\varepsilon+kh)\widehat{r}u_{i-2}^{n+1}-(10kh+18\varepsilon)\widehat{r}u_{i-1}^{n+1}+[1+(31\varepsilon+kh)\widehat{r}]u_i^{n+1}$$
$$+(8kh-16\varepsilon)\widehat{r}u_{i+1}^{n+1}+(\varepsilon-kh)\widehat{r}u_{i+2}^{n+1}=(6kh+14\varepsilon)\widehat{r}u_{i-1}^{n}$$
$$+[1-(29\varepsilon-kh)\widehat{r}]u_i^{n}-(8kh-16\varepsilon)\widehat{r}u_{i+1}^{n}-(\varepsilon-kh)\widehat{r}u_{i+2}^{n} \quad (19)$$

$$(\varepsilon+kh)\widehat{r}u_{i-2}^{n+1}-(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n+1}+[1+(29\varepsilon+kh)\widehat{r}]u_i^{n+1}$$
$$+(6kh-14\varepsilon)\widehat{r}u_{i+1}^{n+1}=-(\varepsilon+kh)\widehat{r}u_{i-2}^{n}+(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n}$$
$$+[1-(31\varepsilon-kh)\widehat{r}]u_i^{n}-(10kh-18\varepsilon)\widehat{r}u_{i+1}^{n}-2(\varepsilon-kh)\widehat{r}u_{i+2}^{n} \quad (20)$$

$$(\varepsilon+kh)\widehat{r}u_{i-2}^{n+1}-(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n+1}+[1+(15\varepsilon+kh)\widehat{r}]u_i^{n+1}$$
$$=-(\varepsilon+kh)\widehat{r}u_{i-2}^{n}+(8kh+16\varepsilon)\widehat{r}u_{i-1}^{n}+$$
$$[1-(45\varepsilon-kh)\widehat{r}]u_i^{n}-(16kh-32\varepsilon)\widehat{r}u_{i+1}^{n}-2(\varepsilon-kh)\widehat{r}u_{i+2}^{n} \quad (21)$$

Using the schemes mentioned above, we will have three basic point groups:
"$\omega 1$"group: eight inner points are involved, and $(14)-(21)$ are used at each grid point respectively.
"$\omega 2$"group: four inner points are involved, and $(14)-(17)$ are used respectively.
"$\omega 3$"group: two inner points are involved, and $(18)-(21)$ are used respectively.

Let $m=4s$, here s is an integer. Based on the basic point groups above, the alternating group method will be presented as following:

First at the $(n+1)$-th time level, we will have s point groups. "$\omega 1$" are used in each group. Second at the $(n+2)$-th time level, we will have $(s+1)$ point groups. "$\omega 3$" are used in the left four grid points. "$\omega 1$" are used in the following $s-1$ point groups, while "$\omega 2$" are used in the right four grid points.

As the AGEI method in section 2, We notice the computing in the whole domain can be divided into many sub-domains independently. So the method has also the property of parallelism. We denote the AGED method as follows:

$$\begin{cases} (I+rA)U^{n+1}=(I-rB)U^{n} \\ (I+rB)U^{n+2}=(I-rA)U^{n+1} \end{cases} \quad (22)$$

$$A=\begin{pmatrix} A_1 & & & & \\ & A_1 & & & \\ & & \cdots & & \\ & & & A_1 & \\ & & & & A_1 \end{pmatrix}_{m\times m}$$

$$B=\begin{pmatrix} A_3 & & & & D \\ & A_1 & & & \\ & & \cdots & & \\ & & & A_1 & \\ E & & & & A_2 \end{pmatrix}_{m\times m},$$

$$A_1=\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix}$$

$$A_{11}=\begin{pmatrix} 15\varepsilon-kh & 8kh-16\varepsilon \\ -(6kh+14\varepsilon) & 29\varepsilon-kh \end{pmatrix}$$

$$A_{12}=\begin{pmatrix} \varepsilon-kh & 0 \\ 8kh-16\varepsilon & \varepsilon-kh \end{pmatrix}$$

$$A_{21}=\begin{pmatrix} \varepsilon+kh & -(8kh+16\varepsilon) \\ 0 & \varepsilon+kh \end{pmatrix}$$

$$A_{22}=\begin{pmatrix} 31\varepsilon-kh & 10kh-18\varepsilon \\ -(8kh+16\varepsilon) & 45\varepsilon-kh \end{pmatrix}$$

$$A_{13} = A_{14} = A_{24} = O$$

$$A_{23} = \begin{pmatrix} 2(\varepsilon - kh) & 0 \\ 16kh - 32\varepsilon & 2(\varepsilon - kh) \end{pmatrix}$$

$$A_{31} = A_{41} = A_{42} = O$$

$$A_{32} = \begin{pmatrix} 2(\varepsilon + kh) & -(16kh + 32\varepsilon) \\ & 2(\varepsilon + kh) \end{pmatrix}$$

$$A_{33} = \begin{pmatrix} 45\varepsilon + kh & 8kh - 16\varepsilon \\ -(10kh + 18\varepsilon) & 31\varepsilon + kh \end{pmatrix}$$

$$A_{34} = \begin{pmatrix} \varepsilon - kh & 0 \\ 8kh - 16\varepsilon & \varepsilon - kh \end{pmatrix}$$

$$A_{43} = \begin{pmatrix} \varepsilon + kh & -(8kh + 16\varepsilon) \\ 0 & \varepsilon + kh \end{pmatrix}$$

$$A_{44} = \begin{pmatrix} 29\varepsilon + kh & (6kh - 14\varepsilon) \\ -(8kh + 16\varepsilon) & 15\varepsilon + kh \end{pmatrix}$$

$$A_2 = \begin{pmatrix} A_{21} & A_{22} \\ A_{23} & A_{24} \end{pmatrix}$$

$$A_{21} = \begin{pmatrix} 15\varepsilon - kh & 8kh - 16\varepsilon \\ -(6kh + 14\varepsilon) & 29\varepsilon - kh \end{pmatrix}$$

$$A_{22} = \begin{pmatrix} \varepsilon - kh & 0 \\ 8kh - 16\varepsilon & \varepsilon - kh \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} \varepsilon + kh & -(8kh + 16\varepsilon) \\ 0 & \varepsilon + kh \end{pmatrix}$$

$$A_{24} = \begin{pmatrix} 31\varepsilon - kh & 10kh - 18\varepsilon \\ -(8kh + 16\varepsilon) & 45\varepsilon - kh \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 \\ 2(\varepsilon - kh) & 0 & 0 & 0 \\ 16kh - 32\varepsilon & 2(\varepsilon - kh) & 0 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} A_{31} & A_{32} \\ A_{33} & A_{34} \end{pmatrix}$$

$$A_{31} = \begin{pmatrix} 45\varepsilon + kh & 8kh - 16\varepsilon \\ -(10kh + 18\varepsilon) & 31\varepsilon + kh \end{pmatrix}$$

$$A_{32} = \begin{pmatrix} \varepsilon - kh & 0 \\ 8kh - 16\varepsilon & \varepsilon - kh \end{pmatrix}$$

$$A_{33} = \begin{pmatrix} \varepsilon + kh & -(8kh + 16\varepsilon) \\ 0 & \varepsilon + kh \end{pmatrix}$$

$$A_{34} = \begin{pmatrix} 29\varepsilon + kh & (6kh - 14\varepsilon) \\ -(8kh + 16\varepsilon) & 15\varepsilon + kh \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 2(\varepsilon + kh) & -(16kh + 32\varepsilon) \\ 0 & 0 & 0 & 2(\varepsilon + kh) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Applying Taylor's formula to (14)-(21) at (x_i, t_n), we can easily obtain that the truncation error is $\mathcal{O}(\tau^2 + \tau h + \tau h^2 + \tau h^3 + h^4)$ respectively, and alternating use of (14)-(21) can lead to counteraction of the truncation error for the items containing τh, τh^2 and τh^3. Then we can denote the truncation error of (22) as $\mathcal{O}(\tau^2 + h^4)$.

6 Stability Analysis

Theorem The alternating group method denoted by (22) is unconditionally stable.

Proof: From the construction of the matrices above we can see A and B are both diagonally dominant matrices, which shows A and B are both non-negative definite real matrices. Then we have:

$$\|(I + \widehat{r}A)^{-1}\|_2 \le 1, \|(I - \widehat{r}A)(I + \widehat{r}A)^{-1}\|_2 \le 1$$

$$\|(I + \widehat{r}B)^{-1}\|_2 \le 1, \|(I - \widehat{r}B)(I + \widehat{r}B)^{-1}\|_2 \le 1.$$

Let n be an even integer, from (2.9) we have

$$U^n = \widetilde{A}U^{n-2} = \widetilde{A}^{\frac{n}{2}}U^0$$

here

$$\widetilde{A} = (I + \widehat{r}B)^{-1}(I - \widehat{r}A)(I + \widehat{r}A)^{-1}(I - \widehat{r}B).$$

Let

$$\overline{A} = (I + \widehat{r}B)\widetilde{A}(I + \widehat{r}B)^{-1}$$

$$= (I - \widehat{r}A)(I + \widehat{r}A)^{-1}(I - \widehat{r}B)(I + \widehat{r}B)^{-1}$$

then we have $\rho(\widetilde{A}) = \rho(\overline{A}) \le \|\overline{A}\|_2 \le 1$, which shows the presented method (22) is unconditionally stable.

7 The Further Application Of The Concept Of Alternating Group

We notice that the construction of the methods in section 2,3,5 is an universal process. Based on an implicit symmetry scheme, we present some asymmetry schemes, based on which several independent computation groups are presented. Then with the concept of domain decomposition we construct the alternating group methods. We present an implicit scheme for solving (1) as below:

$$\frac{u_i^{n+1}-u_i^n}{\tau}$$
$$+\frac{k}{2}[(\frac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2h}-\frac{u_{i+2}^{n+1}-2u_{i+1}^{n+1}+2u_{i-1}^{n+1}-u_{i-2}^{n+1}}{12h})$$
$$+(\frac{u_{i+1}^{n}-u_{i-1}^{n}}{2h}-\frac{u_{i+2}^{n}-2u_{i+1}^{n}+2u_{i-1}^{n}-u_{i-2}^{n}}{12h})]$$
$$=\frac{\varepsilon}{2}[(\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{2h}$$
$$-\frac{u_{i+2}^{n+1}-4u_{i+1}^{n+1}+6u_i^{n+1}-4u_{i-1}^{n+1}+u_{i-2}^{n+1}}{12h})$$
$$+(\frac{u_{i+1}^{n}-2u_i^{n}+u_{i-1}^{n}}{2h}-\frac{u_{i+2}^{n}-4u_{i+1}^{n}+6u_i^{n}-4u_{i-1}^{n}+u_{i-2}^{n}}{12h})]$$

Applying Taylor's formula to the scheme at $(x_i, t_{n+\frac{1}{2}})$, we can easily have that the truncation error of the scheme is $O(\tau^2+h^4)$.

Let $U^n=(u_1^n,\ u_2^n,\ \cdots,\ u_m^n)^T$, r=$\frac{\tau}{24h^2}$, then we have:

$$(\varepsilon+kh)ru_{i-2}^{n+1}-(16\varepsilon+8kh)ru_{i-1}^{n+1}+(1+30\varepsilon r)u_i^{n+1}$$
$$+(8kh-16\varepsilon)ru_{i+1}^{n+1}+(\varepsilon-kh)ru_{i+2}^{n+1}=-(\varepsilon+kh)ru_{i-2}^{n}$$
$$+(16\varepsilon+8kh)ru_{i-1}^{n}+(1-30\varepsilon r)u_i^{n}-(8kh-16\varepsilon)ru_{i+1}^{n}$$
$$-(\varepsilon-kh)ru_{i+2}^{n} \tag{23}$$

We denote (23) as $\widehat{A}U^{n+1}=F^n$. here $F^n=(2I-\widehat{A})U^n$.

The alternating group iterative method will be constructed in two conditions as follows:

First let $m=8k$, k is an integer. Let $\widehat{A}=\frac{1}{2}(\widehat{G}_1+\widehat{G}_2)$, here

$$\widehat{G}_1=\begin{pmatrix} G_{2p} & & & & \\ & \cdots & & & \\ & & \cdots & & \\ & & & \cdots & \\ & & & & G_{2p}\end{pmatrix}_{m\times m}$$

$$\widehat{G}_2=\begin{pmatrix} G_{p1} & & & & \widehat{\overline{G}} \\ & G_{2p} & & & \\ & & \cdots & & \\ & & & G_{2p} & \\ \widehat{\overline{\overline{G}}} & & & & G_{p1}\end{pmatrix}_{m\times m}$$

$$\widehat{\overline{\overline{G}}}=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 2(\varepsilon-kh)r & 0 & 0 & 0\\ 2(8kh-16\varepsilon)r & 2(\varepsilon-kh)r & 0 & 0\end{pmatrix}$$

$$\widehat{\overline{G}}=\begin{pmatrix} 0 & 0 & 2(\varepsilon+kh)r & -2(16\varepsilon+8kh)r\\ 0 & 0 & 0 & 2(\varepsilon+kh)r\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$G_{p1}=\begin{pmatrix} G_{p11} & G_{p12}\\ G_{p13} & G_{p14}\end{pmatrix}$$

$$G_{p11}=\begin{pmatrix} 1+30\varepsilon r & (8kh-16\varepsilon)r\\ -(8kh+16\varepsilon)r & 1+30\varepsilon r\end{pmatrix}$$

$$G_{p12}=\begin{pmatrix} (\varepsilon-kh)r & 0\\ (8kh-16\varepsilon)r & (\varepsilon-kh)r\end{pmatrix}$$

$$G_{p13}=\begin{pmatrix} (\varepsilon+kh)r & -(8kh+16\varepsilon)r\\ 0 & (\varepsilon+kh)r\end{pmatrix}$$

$$G_{p14}=\begin{pmatrix} 1+30\varepsilon r & (8kh-16\varepsilon)r\\ -(8kh+16\varepsilon)r & 1+30\varepsilon r\end{pmatrix}$$

$$G_{2p}=\begin{pmatrix} G_{2p1} & G_{2p2}\\ G_{2p3} & G_{2p4}\end{pmatrix}$$

$$G_{2p1}=\begin{pmatrix} 1+30\varepsilon r & (8kh-16\varepsilon)r & (\varepsilon-kh)r & \\ -(8kh+16\varepsilon)r & 1+30\varepsilon r & (8kh-16\varepsilon)r & (\varepsilon-kh)r\\ (\varepsilon+kh)r & -(8kh+16\varepsilon)r & 1+30\varepsilon r & (8kh-16\varepsilon)r\\ & (\varepsilon+kh)r & -(8kh+16\varepsilon)r & 1+30\varepsilon r\end{pmatrix}$$

$$G_{2p2}=\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 2(\varepsilon-kh)r & 0 & 0 & 0\\ 2(8kh-16\varepsilon)r & 2(\varepsilon-kh)r & 0 & 0\end{pmatrix}$$

$$G_{2p3}=\begin{pmatrix} 0 & 0 & 2(\varepsilon+kh)r & -2(8kh+16\varepsilon)r\\ 0 & 0 & 0 & 2(\varepsilon+kh)r\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$G_{2p4}=\begin{pmatrix} 1+30\varepsilon r & (8kh-16\varepsilon)r & (\varepsilon-kh)r & \\ -(8kh+16\varepsilon)r & 1+30\varepsilon r & (8kh-16\varepsilon)r & (\varepsilon-kh)r\\ (\varepsilon+kh)r & -(8kh+16\varepsilon)r & 1+30\varepsilon r & (8kh-16\varepsilon)r\\ & (\varepsilon+kh)r & -(8kh+16\varepsilon)r & 1+30\varepsilon r\end{pmatrix}$$

Then the alternating group iterative method I can be derived as below:

$$\begin{cases} (\rho I+\widehat{G}_1)\tilde{U}^{n+1}=(\rho I-\widehat{G}_2)U^n+2F^n\\ (\rho I+\widehat{G}_2)U^{n+1}=(\rho I-\widehat{G}_1)\tilde{U}^{n+1}+2F^n\end{cases} \tag{24}$$

Let $m = 8k+4,\ k$ is an integer. Let $A = \frac{1}{2}(\widehat{\widehat{G_1}} + \widehat{\widehat{G_2}})$, here

$$\widehat{\widehat{G_1}} = \begin{pmatrix} G_{2p} & & & & \\ & \cdots & & & \\ & & \cdots & & \\ & & & G_{2p} & \\ & & & & G_{p1} \end{pmatrix}_{m\times m}$$

$$\widehat{\widehat{G_2}} = \begin{pmatrix} G_{p1} & & & & \widehat{\widehat{G}} \\ & G_{2p} & & & \\ & & \cdots & & \\ & & & \cdots & \\ \widehat{\widehat{\widehat{G}}} & & & & G_{2p} \end{pmatrix}_{m\times m}$$

$$\widehat{\widehat{\widehat{G}}} = \begin{pmatrix} O & O \\ \widehat{\widehat{G}} & O \end{pmatrix}_{8\times 8}$$

Then the alternating group iterative method II can be derived as below:

$$\begin{cases} (\rho I + \widehat{\widehat{G_1}})\tilde{U}^{n+1} = (\rho I - \widehat{\widehat{G_2}})U^n + 2F^n \\ (\rho I + \widehat{\widehat{G_2}})U^{n+1} = (\rho I - \widehat{\widehat{G_1}})\tilde{U}^{n+1} + 2F^n \end{cases} \tag{25}$$

Similar to the contents in section 2, the method (25) has also the property of intrinsic parallelism.

Also we have:

Theorem 6 The scheme (24)and (25) are converge.

8 Numerical Experiments

Example 1: We consider the following problem:

$$\begin{cases} \frac{\partial u}{\partial t} + k\frac{\partial u}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial x^2}, \quad 0 \le t \le T \\ u(x,0) = sin(2\pi x), \\ u(X,t) = u(x+1,t). \end{cases} \tag{26}$$

The exact solution of the problem above is denoted as below:

$$u(x,t) = e^{-4\epsilon\pi^2 t} sin[2\pi(x-kt)]$$

Let A.E.I. denotes maximum absolute error of the alternating group explicit iterative (AGEI) method (7), while P.E.I. denotes maximum relevant error. Let A.E.II. denotes maximum absolute error of the fourth order alternating group explicit iterative (FOAGEI) method (9), while P.E.II. denotes maximum relevant error. Let $\rho = 1$. Using the iterative error 1×10^{-10} to control the process of iterativeness, We compare the numerical results by the presented method in the paper with the results in [4] in the following tables:

Table 1: Results at $m = 28,\ k = \varepsilon = 1$

	$\tau = 10^{-5}, t = 100\tau$
A.E.I.	1.624×10^{-5}
A.E.II.	8.499×10^{-6}
A.E.[4]	8.879×10^{-2}
P.E.I.	8.211×10^{-3}
P.E.II.	6.221×10^{-3}
P.E.[4]	8.150×10^{-1}

Table 2: Results at $m = 28,\ k = \varepsilon = 1$

	$\tau = 10^{-5}, t = 1000\tau$
A.E.I.	1.132×10^{-4}
A.E.II.	5.953×10^{-5}
A.E.[4]	9.876×10^{-2}
P.E.I.	6.761×10^{-2}
P.E.II.	5.505×10^{-2}
P.E.[4]	27.757

Table 3: Results at $m = 32,\ k = \varepsilon = 1$

	$\tau = 10^{-5}, t = 100\tau$
A.E.I.	1.244×10^{-5}
A.E.II.	6.487×10^{-6}
A.E.[4]	8.837×10^{-2}
P.E.I.	6.290×10^{-3}
P.E.II.	4.762×10^{-3}
P.E.[4]	1.059

Table 4: Results at $m = 32,\ k = \varepsilon = 1$

	$\tau = 10^{-5}, t = 1000\tau$
A.E.I.	8.743×10^{-5}
A.E.II.	4.558×10^{-5}
A.E.[4]	8.424×10^{-2}
P.E.I.	5.177×10^{-2}
P.E.II.	4.214×10^{-2}
P.E.[4]	34.996

Table 5: Results at $m = 28,\ k = 1,\ \varepsilon = 0.01$

	$\tau = 10^{-4}, t = 100\tau$
A.E.I.	3.517×10^{-4}
A.E.II.	3.492×10^{-4}
A.E.[4]	1.138×10^{-2}
P.E.I.	1.180×10^{-1}
P.E.II.	9.757×10^{-2}
P.E.[4]	4.341

Table 6: Results at $m = 28,\ k = 1,\ \varepsilon = 0.01$

	$\tau = 10^{-4}, t = 1000\tau$
A.E.I.	6.305×10^{-3}
A.E.II.	3.315×10^{-3}
A.E.[4]	3.780×10^{-1}
P.E.I.	7.892×10^{-1}
P.E.II.	5.541×10^{-1}
P.E.[4]	234.206

Example 2:

Let $P.E = \frac{|u_i^n - u(x_i,t_n)|}{u(x_i,t_n)}$ denotes maximum relevant error. We compare the numerical results of (22) with the results in [5] and [10] in the follow tables:

Table 7: Results of comparisons at $m = 16$

	$\tau = 10^{-4}, t = 100\tau, \varepsilon = 1$
$P.E.$	6.487 $\times 10^{-2}$
$P.E.^{[5]}$	4.229 $\times 10^{-1}$
$P.E.^{[10]}$	3.673 $\times 10^{-1}$

Table 8: Results of comparisons at $m = 16$

	$\tau = 10^{-5}, t = 100\tau, \varepsilon = 1$
$P.E.$	7.654 $\times 10^{-3}$
$P.E.^{[5]}$	9.456 $\times 10^{-2}$
$P.E.^{[10]}$	8.218 $\times 10^{-2}$

Table 9: Results of comparisons at $m = 24$

	$\tau = 10^{-4}, t = 100\tau, \varepsilon = 0.1$
$P.E.$	1.520 $\times 10^{-2}$
$P.E.^{[5]}$	4.250 $\times 10^{-1}$
$P.E.^{[10]}$	1.241 $\times 10^{-1}$

Table 10: Results of comparisons at $m = 24$

	$\tau = 10^{-4}, t = 1000\tau, \varepsilon = 0.01$
$P.E.$	9.285 $\times 10^{-2}$
$P.E.^{[5]}$	5.364 $\times 10^{-1}$
$P.E.^{[10]}$	2.758 $\times 10^{-1}$

Results in Table 7-10 show that the method introduced in (22) is superior to the methods in [5,10], especially in convection dominant cases.

9 Conclusions

In this paper, based on several unconditionally stable implicit schemes, we present a concept of constructing a class of alternating group method, and derive several alternating group methods. The AGEI method and the FOAGEI method have the property of intrinsic parallelism, and is verified to be convergent. Results of Table 1-4 show that the two methods are of higher accuracy than the original AGE method in [4]. Results of Table 5-6 shows the methods can obtain high accuracy even in the convection dominant case. Considering the construction of the AGEI method mentioned in this paper is a universal process, so we construct another alternating group explicit difference (AGED) method and another alternating group iterative method. All of the methods have the property of intrinsic parallelism. Computation in the whole domain can be divided into many sub-domains and be worked out with several parallel computers independently. Numerical results show the present methods are superior to the known methods in [4,5,10].

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