Troubled Asset Relief Program, Bank Interest Margin and Default Risk in Equity Return: An Option-Pricing Model

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Abstract: Will banks be willing to sell their toxic loans with the help of the Troubled Asset Relief Program (TARP)? The answer is yes as long as bids are high enough to tempt banks to deal. With the TARP's help, an increase in the toxic loans sold to the government increases the bank's margin and decreases the bank's default probability in equity return when the bank encounters greater risk. This paper concludes that setting up the TARP for the 'bad bank' solution may be a good move for retail banking, resulting in high margin and low default risk when its target banks are willing sellers.

Key-Words: Toxic Loans, Interest Margin, Default Risk, Troubled Asset Relief Program

1. Introduction

The U.S. financial authority created the Troubled Asset Relief Program (TRAP), a $700 billion rescue fund, in October 2008 (Economist, 2009a). The authority is a willing buyer of toxic loans, but are banks willing sellers (Economist, 2009b)? Will setting up the TRAP be a good move for toxic loans? One solution is for the authority to pay more for the target toxic loans than their current market price, assuming that values will eventually rebound. That is a subsidy for bank shareholders and debtholders, and may be one important reason why bank stocks went up 20% on January 28, 2009 (Economist, 2009c). This paper develops an option-pricing model of bank behavior that is used to study bank interest margin with government help.[1] Our results are largely supported the above solution.

The U.S. banking industry is experiencing a renewed focus on retail banking, a trend often attributed to the stability and profitability of retail activities (Hirtle and Stiroh, 2007). Banking firms in retail banking are institutions that engage in two distinct types of activities, deposit-taking and lending.[2] Our primary emphasis on retail banking is the selection of the bank's optimal interest margin which is the difference between the rate of interest the bank charges borrowers and the rate the bank pays to depositors. Mcshane and Sharpe (1985), and Allen (1988) have developed models of bank interest margins based on the bid-ask spread model of Stoll (1978). Zarruk and Madura (1992), and Wong (1997) also provide firm-theoretic models to explain bank margin behavior.

Unlike previous formulations, the model developed here assumes an option-based setting in which the target bank is helped by the TRAP. The authority buys toxic loans held by the bank when bids are high sufficit to tempt the bank to deal. Comparative static results show that the bank's optimal interest margin is an increasing function of the amount of toxic loans sold when the bank encounters greater risk. In addition, the default risk in the bank's equity return is a decreasing function of the amount of toxic loans

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[1] Results to be derived from our model do not extend to the case where the government's subsidy is the swapped high-yield bond (see Lee and Cheng, 2008)

[2] The customer acceptance in the retail banking does not considered in our paper. Our results do not extend to this particular case (Asosheha, Bagherpour, and Yahyapour, 2008).

sold.

The paper is organized as follows. Section 2 develops the basic structure of the model. Section 3 derives the solution of the model and the comparative static analysis. The final section concludes.

2. The Model

We consider a single-period model of a banking firm. At the start of the period, the bank accepts D dollars of deposits. The bank provides depositors with a rate of return equal to the risk-free market rate R_D. Equity capital K held by the bank is tied by regulation to be a fixed proportion q of the bank's deposits, $K \geq qD$. The required capital-to-deposits ratio q is assumed to be an increasing function of the amount of the two types of loans, for example, individual loans (L) and mortgage loans (M), held by the bank at the beginning of the period, $\partial q / \partial L = \partial q / \partial M = q' > 0$. This assumption implies that the system of capital standards is designed to force banks' capital positions to reflect their asset portfolio risks (see Zarruk and Madura, 1992).

The bank makes the two loans $L + M$, which mature and are paid off at the end of the period. Both the loan demands faced by the bank are specified as $L(R_L)$ and $M(R_M)$ where R_L and R_M are individual loan interest rate and mortgage loan interest rate, respectively. We assume that the bank has some market power in lending (see Wong, 1997), which implies that $\partial L / R_L < 0$ and $\partial M / R_M < 0$. Empirical evidence by Slovin and Sushka (1984) supports the presence of rate-setting behavior in loan market. In addition to term loans, the bank can also hold an amount of B liquid assets, for example, Treasury Bills, on its balance sheet during the period. These assets earn the security-market interest rate of R.

When the capital constraint is binding, the bank's balance-sheet constraint is given by

$$L + M + B = D + K = K(\frac{1}{q} + 1) \quad (1)$$

Equation (1) demonstrates the bank's operations management in lending since the total assets in the left-hand side are financed by demandable deposits and equity capital in the right-hand side.

In Merton's (1974) model, the

equity of a firm is viewed as a call option on the firm's assets. The reason is that equity holders are residual claimants on the firm's assets after all other obligations have been met. The strike price of the call option is the book value of the firm's liabilities. When the value of the firm's assets is less than the strike price, the value of equity is zero.

We assume that the bank is willing to sell the amount of toxic loans $\theta(1+R_M)M$, $0<\theta\leq 1$, to the government, what the TARP is meant to target.[3] The market value of the bank's underlying assets follows a geometric Brownian motion of the form:

$$\frac{dV}{V}=\mu_V dt+\sigma_V dW \qquad (2)$$

where

$V=(1+R_L)L+(1-\theta)(1+R_M)M$.

V is the value of the bank's risky loans, with an instantaneous drift μ_V, and an instantaneous volatility σ_V.

W is a standard Wiener process.

With government help when the carrying value of the bank's mortgage loan books is far above market price, it is reasonable to assume that the binding value of toxic loans sold to the government is set to equal its book value. If the TARP falls flat, it is more likely to be because of a lack of sellers.[4] Even with government help, bids may not be high enough to tempt banks to deal since any price below the carrying value will force banks to take a write-down and deplete precious capital.

The market value of equity S will then be given by the Black and Scholes (1973) formula for call options:

$$S=VN(d_1)-Ze^{-\delta}N(d_2) \qquad (3)$$

where

[3] U.S. commercial banks hold 3.8 trillion in residential and commercial mortgages, according to Federal Reserve data, plus $1.1 trillion in mortgage-related securities. That represents more than 50% of banks' total financial assets of $9 trillion, which means that they are highly exposed to the real estate market (Economist, 2009c). Credit-default swap is an insurance policy against a asset default. American International Group (AIG) sold fistfuls on mortgage-related securities that have collapsed in value. Wang (2009) points out that AIG takes the world insurance and the financial service leader, also has paid the huge price.

[4] Banks are still holding assets, particularly whole loans, at values far above their market price because, under accrual accounting, losses can be booked over several years.

$$Z = (1+R_D)\frac{K}{q} - (1+R)[K(\frac{1}{q}+1) - L - M]$$

$$-\theta(1+R_M)M ,$$

$$d_1 = \frac{1}{\sigma_V}(\ln\frac{V}{Z} + \delta + \frac{1}{2}\sigma_V^2) ,$$

$$d_2 = d_1 - \sigma_V ,$$

δ is the spread rate between R and R_D,

$N(\cdot)$ is the cumulative density function of the standard normal distribution.

We note that the amount of toxic loans sold, $\theta(1+R_M)MN(d_1)$, is replaced by its book value of $\theta(1+R_M)Me^{-\delta}N(d_2)$ in the option-pricing valuation.

In addition, the default probability of the bank's equity in equation (3) is the probability when V is less than Z. Using information in equation (3), we apply Vassalou and Xing (2004) and define the default probability as:

$$P_{def} = N(-d_3) \qquad (4)$$

where

$$d_3 = \frac{1}{\sigma_V}(\ln\frac{V}{Z} + \mu_V - \frac{1}{2}\sigma_V^2) .$$

d_3 represents how many standards the log of V/Z needs to deviate from its mean in order for default to occur.

3. Solutions and Results

The first order conditions are given by:

$$\frac{\partial S}{\partial R_L} = \frac{\partial V}{\partial R_L}N(d_1) + V\frac{\partial N}{\partial d_1}\frac{\partial d_1}{\partial R_L}$$

$$-\frac{\partial Z}{\partial R_L}e^{-\delta}N(d_2) - Ze^{-\delta}\frac{\partial N}{\partial d_2}\frac{\partial d_2}{\partial R_L} = 0 \qquad (5)$$

$$\frac{\partial S}{\partial R_M} = \frac{\partial V}{\partial R_M}N(d_1) + V\frac{\partial N}{\partial d_1}\frac{\partial d_1}{\partial R_M}$$

$$-\frac{\partial Z}{\partial R_M}e^{-\delta}N(d_2) - Ze^{-\delta}\frac{\partial N}{\partial d_2}\frac{\partial d_2}{\partial R_M} = 0 \qquad (6)$$

where

$$V\frac{\partial N}{\partial d_1}\frac{\partial d_1}{\partial R_L} = Ze^{-\delta}\frac{\partial N}{\partial d_2}\frac{\partial d_2}{\partial R_L} ,$$

$$V\frac{\partial N}{\partial d_1}\frac{\partial d_1}{\partial R_M} = Ze^{-\delta}\frac{\partial N}{\partial d_2}\frac{\partial d_2}{\partial R_M} ,$$

$$\frac{\partial V}{\partial R_L} = L + (1+R_L)\frac{\partial L}{\partial R_L} < 0 ,$$

$$\frac{\partial Z}{\partial R_L} = [(R - R_D)K\frac{q'}{q^2} + (1+R)]\frac{\partial L}{\partial R_L} < 0$$

,

$$\frac{\partial V}{\partial R_M} = (1-\theta)[M + (1+R_M)\frac{\partial M}{\partial R_M}] < 0,$$

$$\frac{\partial Z}{\partial R_M} = [(R - R_D)K\frac{q'}{q^2} + (1+R)]\frac{\partial M}{\partial R_M}$$

$$-\theta[M + (1+R_M)\frac{\partial M}{\partial R_M}] < 0.$$

We require that the second order conditions be satisfied, there are $\partial^2 S/\partial R_L^2 < 0$ and $\partial^2 S/\partial R_M^2 < 0$. To ensure a unique market equilibrium obtained, we further assume

$$\Delta \equiv (\partial^2 S/\partial R_L^2)(\partial^2 S/\partial R_M^2)$$

$$-(\partial^2 S/\partial R_L \partial R_M)$$

$$\times(\partial^2 S/\partial R_M/\partial R_L) > 0.$$

Both the optimal loan rates, R_L and R_M, are determined when the marginal risk-adjusted value of risky-asset repayments equals to the marginal risk-adjusted value of net-obligation payments. We further substitute these two optimal loan rates to obtain the default probability in equation (4) staying on the equity maximization.

We consider next the impacts on both the bank's optimal loan rates (and thus on the bank's margins) from changes in the amount of toxic loans sold to the government. Implicit differentions of equations (5) and (6) with respect to θ yield:

$$\frac{\partial R_L}{\partial \theta} = -(\frac{\partial^2 S}{\partial R_L \partial \theta}\frac{\partial^2 S}{\partial R_M^2}$$

$$-\frac{\partial^2 S}{\partial R_L \partial R_M}\frac{\partial^2 S}{\partial R_M \partial \theta})/\Delta \qquad (7)$$

$$\frac{\partial R_M}{\partial \theta} = -(\frac{\partial^2 S}{\partial R_L^2}\frac{\partial^2 S}{\partial R_M \partial \theta}$$

$$-\frac{\partial^2 S}{\partial R_L \partial \theta}\frac{\partial^2 S}{\partial R_M \partial R_L})/\Delta \qquad (8)$$

where

$$\frac{\partial^2 S}{\partial R_L \partial \theta} = \frac{\partial V}{\partial R_L}(\frac{\partial N}{\partial d_1} - \frac{N(d_1)}{N(d_2)}\frac{\partial N}{\partial d_2})\frac{\partial d_1}{\partial \theta},$$

$$\frac{\partial^2 S}{\partial R_M \partial \theta} = \frac{\partial^2 V}{\partial R_M \partial \theta}N(d_1)$$

$$-\frac{\partial^2 Z}{\partial R_M \partial \theta}e^{-\delta}N(d_2)$$

$$+\frac{\partial V}{\partial R_M}(\frac{\partial N}{\partial d_1} - \frac{N(d_1)}{N(d_2)}\frac{\partial N}{\partial d_2})\frac{\partial d_1}{\partial \theta},$$

$$\frac{\partial d_1}{\partial \theta} = \frac{1}{\sigma_V}(\frac{1}{V}\frac{\partial V}{\partial \theta} - \frac{1}{Z}\frac{\partial Z}{\partial \theta}) > 0 .$$

Before proceeding with the analyses of the results, we define the term

$$(\partial N / \partial d_1) - (N(d_1) / N(d_2))(\partial N / \partial d_2)$$

as the risk-adjusted factor elasticity effect. This effect reflects the risk state faced by the bank in the option-based optimization. The sign of this effect can be equivalent to the sign of the difference, $(\partial N / \partial d_1)/(N(d_1)/d_1)$ $-(\partial N / \partial d_2)/(N(d_2)/d_2)$. The former represents the reciprocal risk-adjusted factor elasticity of risky-asset repayments, while the latter represents that elasticity of net-obligation payments. When the difference is negative, the effect indicates that the bank is operating under greater risk since the former is more sensitive than the latter. When the difference is positive, the bank has a decreasing risk magnitude for its equity return.

The term $\partial^2 S / \partial R_L \partial \theta$ captures only the variance effect on $\partial S / \partial R_L$ from a change in θ, the term associated with $\partial d_1 / \partial \theta$. This is because term loans L is not the TARP's target. There is $\partial^2 S / \partial R_L \partial \theta > 0$ when the bank is operating under greater risk.

The first two terms of $\partial^2 S / \partial R_M \partial \theta$ represents the mean profit effect on $\partial S / \partial R_M$ from a change in θ, while the third term represents the variance effect. The mean effect is positive in sign. The variance effect is governed by the risk-adjusted factor elasticity effect. There is $\partial^2 S / \partial R_M \partial \theta > 0$ when the bank is operating under greater risk.

The results of equations (7) and (8) are stated as follows. With strategic complements in Bulow, Geankopolos, and Klemperer's (1985) sense, an increase in the toxic loans sold to the government decreases the loan portfolio at an increased R_L in equation (7) and R_M in equation (8) when the bank encounters greater risk. If either loan demand is relatively rate-elastic, a smaller loan portfolio is possible at an increased margin.

Another way to understand the sell of the toxic loans is to see its impact on default risk in equity return since a bank's default probability is related to its volatility, which is a key input in the Black-Scholes (1973) option-pricing formula. Implicit differentiation of equation (4) evaluated at both the optimal loan rates with respect to θ yields:

$$\frac{dP_{def}}{d\theta} = \frac{\partial P_{def}}{\partial \theta} + \frac{\partial P_{def}}{\partial R_L}\frac{\partial R_L}{\partial \theta} + \frac{\partial P_{def}}{\partial R_M}\frac{\partial R_M}{\partial \theta} \tag{9}$$

where

$$\frac{\partial P_{def}}{\partial \theta} = -\frac{\partial N}{\partial d_3}\frac{1}{\sigma_V \theta}\left(\frac{\theta}{V}\frac{\partial V}{\partial \theta} - \frac{\theta}{Z}\frac{\partial Z}{\partial \theta}\right) < 0,$$

$$\frac{\partial P_{def}}{\partial R_L} = -\frac{\partial N}{\partial d_3}\frac{1}{\sigma_V R_L}\left(\frac{R_L}{V}\frac{\partial V}{\partial R_L} - \frac{R_L}{Z}\frac{\partial Z}{\partial R_L}\right) < 0,$$

$$\frac{\partial P_{def}}{\partial R_M} = -\frac{\partial N}{\partial d_3}\frac{1}{\sigma_V R_M}\left(\frac{R_M}{V}\frac{\partial V}{\partial R_M} - \frac{R_M}{Z}\frac{\partial Z}{\partial R_M}\right) < 0.$$

The first term on the right hand side of equation (9) represents the direct effect, while the second term through R_L and the third term through R_M represent the indirect effects. Both the direct and indirect effects are negative since there are $\partial R_L / \partial \theta > 0$ and $\partial R_M / \partial \theta > 0$ known as in equations (7) and (8), respectively. Equation (9) demonstrates that an increase in the toxic loans sold to the government decreases the bank's default probability in equity return.

4. Conclusion

We develop an option-pricing model to determine the market value of bank equity and its default probability in equity return explicitly incorporating the TARP. Within the setting, we show that with strategic complements, an increase in the toxic loans sold to the government by the bank increases the bank's margin and decreases the bank's default probability in equity return when the bank is operating under greater risk. With the TARP's help, banks will be easier to entice as long as bids are high enough to tempt them to deal. Setting up the TARP is a good move for banks. Of course, whether the authority has picked the effective approach is another matter. The alternative chosen by Britain, to leave the assets in place and insure them for a fee, has many proponents (Economist, 2009a). Such concerns are beyond the scope of this paper and so are not addressed here. What this paper does demonstrate, however, is the important role planned by the TARP in affecting the operating decisions of banks.

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