Reliability Mathematics Analysis on Traction Substation Operation

HONGSHENG SU
School of Automation and Electrical Engineering
Lanzhou Jiaotong University
Lanzhou 730070
P.R.CHINA
shsen@163.com

Abstract: - In electrified railway traction power supply systems, the operational qualities and reliabilities of the main traction transformer loop is higher, but ones of bus output units is comparatively low. The traction transformer loop still works when output loops are in failure, and the output loop interrupts working when the main transformer is in failure, and only has residual life when restoring working, the connection formation between them is in series. Traditional reliability analysis methods let their lifetime follow exponential distribution, and reliability is investigated based on the minimal path sets, which lead to a comparatively rough result, consequently. According to Markov theory, in this paper the main loop life is considered as mixed Erlang distribution with order n, and the output unit life follows generic distribution. As compared with conventional series systems, the acquired results are proved to be reliable and sound.

Key-Words: - Transformer; Erlang distribution; Life; Markov theory; Reliability; Traction substation

1 Introduction

In electrified railway traction power supply systems, 110kV High Voltage electric energy transported by power systems is dropped as 27.5kV voltage of overhead contact line to ensure electric locomotive receiving electrical energy well. Traction load is the national first grade loads, hence two circuits are required to supply electric energy for it. In general, two traction transformers with same capacity are located at traction substation together, and they may be in parallel operation or single running according to practical operation demands. The bus outgoing feeder units in Low Voltage side of the traction transformer provide overhead contact lines with electric energy, region 10kV house-service consumption, and traction substation house-service consumption [1-2]. In China many kinds of traction transformers are applied in national electrified railway, for instance, Single-phase connection traction transformer, Single-phase and three-phase V, v connection traction transformer, three-phase y_n, d_{11} traction transformer, Wood-bridge connection traction transformer, Leblanc traction transformer, Kübler and Scott connection traction transformer, and et al. However, the most traction transformer applied by railway departments is y_n, d_{11} connection type, in general, the traction substation installs two transformers, where one is in operation, and another one is spare. The operational characteristic of the transformer is shown in Figure 1. It may be seen from Figure 1 that it has two feeder circuits and one main transformer. Thus, the whole system can be divided as two areas with 27.5kV bus as boundary: one is area 1, which connects electrical sources and implements dropping-voltage, the other is area 2, which feeds out electrical energy and connects loads.

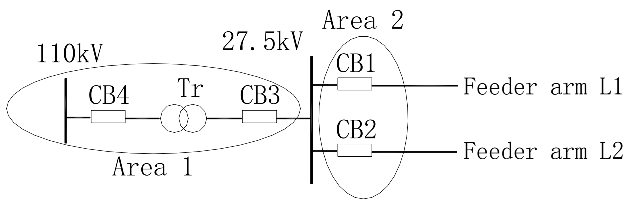

Fig.1 Power supply system model of traction substation

Practically, the failure number in area 1 is far less than one in area 2. The loads in area 1 can reduce some when one outgoing feeder is in failure in area 2, and consequently, the failure possibility in area 1 also for that becomes lower, however it still continues operating, and conversely, if area 1 is in failure, area 2 then interrupt working. It is noticeable that the failure here is referring to behavior of interrupting operation of the parts, for example, the power supply arm will stop working when traction load departs from it, at the moment, although the arm is no in failure, from the angle of reliability it also serves as fault analysis. Moreover, it still includes lines interrupting due to heavy repair and examining repair.

In the conventional analysis methods, failure rate λ is always assumed as a constant, and minimal path sets based analysis method is applied to investigate the reliability of the systems [3-7]. The essential of analysis process is based on approaching of discrete states, and therefore, the error is existent due to lack of continuity. For example, what is described in series system is that the system may then be failure if one part is in failure, and other parts will stop working in system. But in this paper, what is described is that the system may be in failure if one part is in failure, and other parts in system will continue working. For instance, as shown in Figure 1, if line L1 is in failure, the feeder related to L1 is in failure, due to area 1 is normal, line L2 is still works, thus, the traditional analysis methods are incompetent. To approach the true model better, in this paper mixed Erlang distribution is used to approach the life of area 1, more reasonable than traditional exponential distribution. Below taking the example of line L1 in Figure 1, the continuous model is established for the operational characteristic of single line. On the basis of it we perform reliability analysis. According to operational characteristic of traction supply system, two supply arms operate alone, the connections between L1 and L2 be nonexistent.

2 Reliability Mathematics Analysis

Mathematics Model can be established below.

The whole system is composed of two parts, if part 2 is in failure, it then interrupts working and turns into repairing state, at this moment failure rate of part 1 becomes smaller because its loads lighten, however it still continues operating. If part 1 is in failure, it stops working at once and turns into repairing state, at the moment part 2 also interrupts working. Let the whole system have a set of repairman, the repair time follow generic distribution, and the restored failure unit be like the new. If one part is just repairing and the other is in failure, the repair rule is that part 1 is repaired in advance, that is, part 1 possesses a title of preference repair. After the repair of part 1 is completed, the interrupted repair unit then does. Let us assume that bus connections are always reliable, here.

For convenient analysis, definitions are below.

The state 0 expresses the system is fully good; the state 1 expresses part 1 is in failure, here part 2 stops working but not in failure, only has residual life when it restores working; the state 2 expresses part 2 is in failure and part 1 continues operating; the state 3 expresses the two parts are in failure, and part 1 is in repair and part 2 is waiting to be repaired. Assume that the two parts are new from the start, and the life and repair-time of the parts are independently stochastic variables one another. Because repair-time follows arbitrary distribution, the introduced complementary variable $Y_i(t)$, where i=1 or 2, expresses the consumed time when part is repaired. Let the life of part 1 follow Erlang distribution with order n, x_2 be age of part 2 and have the life distribution $F(t)$. Let $S(t)$ express the state of the system at time t, and then $\{S(t),X_2(t),Y_i(t)\}$ is Markov process. According to custom, let failure rate of Part 1 be λ_1, and failure rate of Part 2 be $\lambda_2(x_2)$, failure rate of Part 1 be λ_3 when part 2 is in failure, and $\lambda_3<\lambda_1$. Let repair risk rate function be $u(t)$, and life distribution be $F(t)$ and average value be $\lambda^{-1}<\infty$. Let servicing time distribution be $G(t)$, and average value be $u^{-1}<\infty$. The state transfer diagram is then shown in Figure 2.

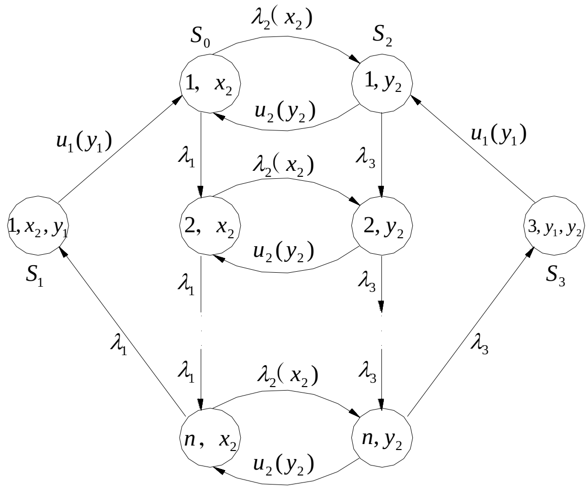

Fig.2 State transfer diagram

According to reliability mathematics theory[8], we have

$$u(t)\Delta t = p\{t < Y \le t + \Delta t \mid Y > t\} = \frac{g(t)\Delta t}{1 - G(t)}$$

After differential we then have

$$\int_0^\infty u(t)dt = \int_0^\infty \frac{1}{1 - G(t)} dG(t)$$

Through simple transformation, then

$$G(y) = 1 - \exp[-\int_0^y u(\tau)d\tau]$$

$$g(y) = u(y)[1 - G(y)] = u(y)\exp[-\int_0^y u(\tau)d\tau]$$

Doing Laplace transform, we then have

$$g(s) = \int_0^\infty g(y)\exp(-sy)dy$$

For $\forall$ $t \geq 0$, $x_2 \geq 0$, $y_1 \geq 0$, we define

$P_{0j}(t,x_2)dx = p\{S(t)=0,\ x_2 < X_2(t) \leq x_2 + dx_2,\ j=1,...,n\}$

$P_1(t, x_2, y_1)dy_1 = p\{S(t)=1,\ dx_2=0,\ y_1 < Y_1(t) \leq y_1 + dy_1\}$

$P_{2j}(t, y_2)dy_2 = p\{S(t)=2,\ y_2 < Y_2(t) \leq y_2 + dy_2,\ j=1,...,n\}$

$P_3(t, y_1, y_2)dy_1 = p\{S(t)=2,\ y_1 < Y_1(t) \leq y_1 + dy_1,\ dy_2=0\}$

According to Figure 2 and reliability mathematics theory [9-11], we can write out Kolmogoro partial calculous equation group below.

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + u_1(y_1)]p_1(t, x_2, y_1) = 0 \qquad (1)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{01}(t, x_2) = \int_0^\infty p_1(t, x_2, y_1)u_1(y_1)dy_1 \qquad (2)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{0j}(t, x_2) = \lambda_1 p_{0j-1}(t, x_2) \quad j = 2 \sim n \qquad (3)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{21}(t, y_2) = \int_0^\infty p_3(t, y_1, y_2)u_1(y_1)dy_1 \qquad (4)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{2j}(t, y_2) = \lambda_3 p_{2j-1}(t, y_2) \quad j = 2 \sim n \qquad (5)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + u_1(y_1)]p_3(t, y_1, y_2) = 0 \qquad (6)$$

The boundary conditions satisfy

$$p_1(t, x_2, 0) = \lambda_1 p_{0n}(t, x_2) \qquad (7)$$

$$p_{0j}(t,0) = \int_0^\infty p_{2j}(t, y_2)u_2(y_2)dy_2 \quad j = 1 \sim n \qquad (8)$$

$$p_3(t,0,y_2) = \lambda_3 p_{2n}(t, y_2) \qquad (9)$$

$$p_{2j}(t,0) = \int_0^\infty p_{0j}(t, x_2)\lambda_2(x_2)dx_2 \quad j = 1 \sim n \qquad (10)$$

The initial conditions satisfy

$p_{01}(0,x) = \delta(x)$, and the rest are zero. (11)

where $\delta(x)$ is Dirac generalized function.
Doing Laplace transform from (1) to (6), and then we have

$$[s + \frac{\partial}{\partial y_1} + u_1(y_1)]p_1(s, x_2, y_1) = 0$$

$$[s + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{01}(s, x_2) - p_{01}(0, x_2) = \\ = \int_0^\infty p_1(s, x_2, y_1)u_1(y_1)dy_1$$

$$[s + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{oj}(s, x_2) = \lambda_1 p_{0j-1}(s, x_2) \quad j = 2 \sim n$$

$$[s + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{21}(s, y_2) = \\ = \int_0^\infty p_3(s, y_1, y_2)u_1(y_1)dy_1$$

$$[s + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{2j}(s, y_2) = \lambda_3 p_{2j-1}(t, y_2) \quad j = 2 \sim n$$

$$[s + \frac{\partial}{\partial y_1} + u_1(y_1)]p_3(s, y_1, y_2) = 0$$

The Laplace transformation of boundary equations (7) to (10) are performed below.

$$p_1(s, x_2, 0) = \lambda_1 p_{0n}(s, x_2)$$

$$p_{0j}(s,0) = \int_0^\infty p_{2j}(s, y_2)u_2(y_2)dy_2, \quad j = 1 \sim n$$

$$p_3(t,0,y_2)=\lambda_3 p_{2n}(s,y_2)$$

$$p_{2j}(s,0)=\int_0^\infty p_{0j}(s,x_2)\lambda_2(x_2)dx_2\ , \quad j=1\sim n$$

Through resolving the above equation group we can get the following formula.

$$p_1(s,x_2,y_1)=\lambda_1 p_{0n}(s,x_2)e^{-sy_1}\bar{G}_1(y_1) \quad (12)$$

$$p_3(s,y_1,y_2)=\lambda_3 p_{2n}(s,y_2)e^{-sy_1}\bar{G}_1(y_1) \quad (13)$$

During calculation we make use of the initial conditions $\delta(x)$, it may be incorporate into $p_{01}(s, 0)$. In fact, the processing is very sound.

$p_{0j}(s, x_2)$ and $p_{2j}(s, y_2)$ satisfy the following equations.

$$[\frac{d}{dx_2}+s+\lambda_1+\lambda_2(x_2)]p_{01}(s,x_2)=\lambda_1 p_{0n}(s,x_2)g_1(s) \quad (14)$$

$$[\frac{d}{dx_2}+s+\lambda_1+\lambda_2(x_2)]p_{0j}(s,x_2)=\lambda_1 p_{0j-1}(s,x_2) \quad j=2\sim n \quad (15)$$

$$[\frac{d}{dy_2}+s+\lambda_3+u_2(y_2)]p_{21}(s,y_2)=\lambda_3 p_{2n}(s,y_2)g_1(s) \quad (16)$$

$$[\frac{d}{dy_2}+s+\lambda_3+u_2(y_2)]p_{2j}(s,y_2)=\lambda_3 p_{2j-1}(s,y_2) \quad j=2\sim n \quad (17)$$

Let

$$p_{0j}(s,x_2)=M_{0j}(s,x_2)\bar{F}_2(x_2) \quad j=1\sim n \quad (18)$$

$$p_{2j}(s,y_2)=M_{2j}(s,y_2)\bar{G}_2(y_2) \quad j=1\sim n \quad (19)$$

Considering

$$\frac{d}{dx_2}p_{0j}(s,x_2)=\frac{d}{dx_2}Q_{0j}(s,x_2)\bar{F}_2(x_2)-Q_{0j}(s,x_2)f_2(x_2)$$

$$\frac{d}{dy_2}p_{2j}(s,y_2)=\frac{d}{dy_2}Q_{2j}(s,y_2)\bar{F}_2(x_2)-Q_{0j}(s,y_2)g_2(y_2)$$

Substituting (18) and (19) into (14), (15), (16), and (17), then we have

$$[\frac{d}{dx_2}+s+\lambda_1]M_{01}(s,x_2)=\lambda_1 M_{0n}(s,x_2)g_1(s) \quad (20)$$

$$[\frac{d}{dx_2}+s+\lambda_1]M_{0j}(s,x_2)=\lambda_1 M_{0j-1}(s,x_2) \quad j=2,\dots,n \quad (21)$$

$$[\frac{d}{dy_2}+s+\lambda_3]M_{21}(s,y_2)=\lambda_3 M_{2n}(s,y_2)g_1(s) \quad (22)$$

$$[\frac{d}{dy_2}+s+\lambda_3]M_{2j}(s,y_2)=\lambda_1 M_{2j-1}(s,y_2) \quad j=2,\dots,n \quad (23)$$

Let

$$A(s,\lambda,f)=\begin{bmatrix}-(s+\lambda) & 0 & \cdots & \lambda f(s)\\ \lambda & -(s+\lambda) & 0 & \cdots\\ \vdots & \ddots & \ddots & \ddots\\ 0 & 0 & \lambda & -(s+\lambda)\end{bmatrix} \quad (24)$$

Characteristic equations of formula (20) and (21) are described by

$$\left|A(s+v,\lambda_1,g_1)\right|=[-(s+\lambda_1)-v]^n+(-1)^{n+1}\lambda_1{}^n g_1(s)=0 \quad (25)$$

Eigenvalue is resolved below.

$$v_i=-(s+\lambda_1)+\lambda_1\omega_i\sqrt[n]{g_1(s)}\ , \quad i=1\sim n \quad (26)$$

where

$$\omega_i=\cos\frac{2i\pi}{n}+j\sin\frac{2i\pi}{n} = e^{\frac{2i\pi}{n}} \quad (27)$$

In the above equation ω_i is a unit circle root.

Let

$$A(s+v_i,\lambda_1,g_1)U_i=0 \quad (28)$$

Then, we have the eigenvector U_i of v_i described by

$$U_i = (1, \beta_i, \cdots, \beta_i^{n-1})^T \qquad (29)$$

where $\beta_i = [\omega_i \sqrt[n]{g_1(s)}]^{-1}$, and then order

$$P = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \beta_1 & \beta_2 & \cdots & \beta_n \\ \vdots & \vdots & \vdots & \vdots \\ \beta_1^{n-1} & \beta_2^{n-1} & \cdots & \beta_n^{n-1} \end{bmatrix}$$

then

$$P^{-1} = \frac{1}{n}\begin{bmatrix} 1 & \beta_1^{-1} & \cdots & \beta_1^{-n+1} \\ 1 & \beta_2^{-1} & \cdots & \beta_2^{-n+1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \beta_n^{n-1} & \cdots & \beta_n^{-n+1} \end{bmatrix}$$

The above matrix P just is noted Vandermond matrix.

According to linear algebra, we then have

$$P^{-1}A(s, \lambda_1, g_1)P = \mathrm{diag}(v_i) \qquad (30)$$

and then

$$\frac{d}{dx_2} M_0(s, x_2) = A(s, \lambda_1, g_1) M_0(s, x_2) \qquad (31)$$

Generic solution of (31) is

$$M_0(s, x_2) = P\mathrm{diag}[\exp(v_i x_2)]c(s) \qquad (32)$$

where $M_0(s,x_2)$ and $c(s)$ are column vector composed of consequent elements, $c_i(s)$ is undetermined constant.

Likewise, we get

$$M_2(s,y_2) = P\mathrm{diag}[\exp(\vartheta_i y_2)]d(s) \qquad (33)$$

where $d_i(s)$ is undetermined constant.

$$\vartheta_i = -(s+\lambda_3) + \lambda_3 \omega_i \sqrt[n]{g_1(s)} \qquad i = 1, \ldots, n \qquad (34)$$

To calculate $c_i(s)$ and $d_i(s)$, in the light of (18) and (32) and boundary conditions, we have

$$p_{oj}(s,0) = M_{0j}(s,0)\bar{F}_2(0) = M_{0j}(s,0)$$

That is

$$\begin{bmatrix} M_{01}(s,0) \\ M_{02}(s,0) \\ \vdots \\ M_{0n}(s,0) \end{bmatrix} = \begin{bmatrix} p_{01}(s,0) \\ p_{02}(s,0) \\ \vdots \\ p_{0n}(s,0) \end{bmatrix} = P\begin{bmatrix} c_1(s) \\ c_2(s) \\ \vdots \\ c_n(s) \end{bmatrix} =$$

$$= \int_0^\infty p_{2j}(s, y_2) u_2(y_2) dy_2$$

$$= \int_0^\infty M_{2j}(s, y_2) \bar{G}_2(y_2) u_2(y_2) dy_2$$

$$= \int_0^\infty M_{2j}(s, y_2) \bar{G}_2(y_2) u_2(y_2) dy_2$$

$$= \int_0^\infty P\mathrm{diag}[\exp(\vartheta_i y_2)] d(s) \bar{G}_2(y_2) u_2(y_2) dy_2$$

$$= P\int_0^\infty \mathrm{diag}[\exp(\vartheta_i y_2)] \bar{G}_2(y_2) u_2(y_2) dy_2 d_j(s)$$

$$= P \times \mathrm{diag}[g_2(-\vartheta_i)] \times \begin{bmatrix} d_1(s) \\ d_2(s) \\ \vdots \\ d_n(s) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (35)$$

Likewise, according to (19) and (33) and boundary conditions, we then have

$$\begin{bmatrix} M_{21}(s,0) \\ M_{22}(s,0) \\ \vdots \\ M_{2n}(s,0) \end{bmatrix} = \begin{bmatrix} p_{21}(s,0) \\ p_{22}(s,0) \\ \vdots \\ p_{2n}(s,0) \end{bmatrix} = P\begin{bmatrix} d_1(s) \\ d_2(s) \\ \vdots \\ d_n(s) \end{bmatrix} =$$

$$= \int_0^\infty p_{0j}(s, x_2) \lambda_2(x_2) dx_2$$

$$= \int_0^\infty M_{0j}(s, x_2) \bar{F}_2(x_2) \lambda_2(x_2) dx_2$$

$$= P\int_0^\infty \mathrm{diag}[\exp(v_i x_2)] \bar{F}_2(x_2) \lambda_2(x_2) dx_2 c_i(s)$$

$$= P\int_0^{\infty} \text{diag}[\exp(v_i x_2)] f_2(x_2) dx_2 c_i(s)$$

$$= P\,\text{diag}[f_2(-v_i)] \times \begin{bmatrix} c_1(s) \\ c_2(s) \\ \vdots \\ c_n(s) \end{bmatrix} \qquad (36)$$

According to (35) and (36), we then have

$$d_i(s) = f_2(-v_i) c_i(s), \quad i=1\sim n \qquad (37)$$

$$c_i(s) = \frac{1}{n}[1 - f_2(-v_i) g_2(-\vartheta_i)]^{-1}, i = 1,...,n \qquad (38)$$

Thus, the state probabilities of the system are determined from (1) to (38)

Theorem1. The availability under steady state is given by

$$A = \lim_{s\to 0} sA(s) = [1 + \frac{\lambda_1}{nu_1} + \frac{\lambda_2}{u_2} + \frac{\lambda_3\lambda_2}{nu_1u_2}]^{-1} \qquad (39)$$

Proof. The instantaneous availability of the system is given by

$$A(t) = \sum_{i=1}^{n} \int_0^{\infty} p_{0j}(t, x_2) dx_2$$

Doing Laplace transform, and according to (18), we then have

$$A(s) = (\int_0^{\infty} M_0(s, x_2) \bar{F}_2(x_2) dx_2)e$$

$$= c^T(s) \text{diag}(\bar{F}_2(-v_i)) P^T e$$

According to $f(s) + s\bar{F}(s) = 1$

Hence, we can write

$$f_2(-v_i) + v_i \bar{F}_2(-v_i) = 1$$

That is,

$$\bar{F}_2(-v_i) = \frac{1 - f_2(-v_i)}{v_i}$$

Substituting above formula in to $A(s)$, and considering c^T (s) and matrix $\boldsymbol{P}$, we have

$$A(s) = \sum_{i=1}^{n} c_i(s) \frac{[1 - f_2(-v_i)]}{v_i} [1 + \beta_i + \cdots + \beta_i^{n-1}]$$

$$= \sum_{i=1}^{n} c_i(s) \frac{[1 - f_2(-v_i)][1 - \beta_i^n]}{v_i[1 - \beta_i]}$$

$$= \sum_{i=1}^{n} c_i(s) \frac{[1 - f_2(-v_i)][1 - \frac{1}{\omega_i^n g_1(s)}]}{v_i[1 - \beta_i]}$$

$$= \sum_{i=1}^{n} c_i(s) \frac{[1 - f_2(-v_i)][1 - \omega_i^n g_1(s)]}{v_i[1 - \beta_i]\omega_i^n g_1(s)}$$

$$= \sum_{i=1}^{n} c_i(s) \frac{[1 - f_2(-v_i)][1 - g_1(s)]}{(1 - \beta_i) v_i g_1(s)}$$

Substituting (38) into $A(s)$, then

$A(s)$=

$$= \sum_i^n \frac{1}{n} \frac{1}{[1 - f_2(-v_i) g_2(-\vartheta_i)]} \frac{[1 - f_2(-v_i)][1 - g_1(s)]}{(1 - \beta_i) v_i g_1(s)}$$

According to the terminative value theorem of Laplace transform, we have

$$A = \lim_{s\to 0} sA(s) =$$

$$= \lim_{s\to 0} \sum_i^n \frac{1}{n} \frac{s}{[1 - f_2(-v_i) g_2(-\vartheta_i)]} \frac{[1 - f_2(-v_i)][1 - g_1(s)]}{(1 - \beta_i) v_i g_1(s)}$$

Considering (26), (27) and (34), then

$$A = \lim_{s\to 0} sA(s) =$$

$$=\lim_{s\to 0}\sum_{i=1}^{n}\frac{1}{n}\frac{s\omega_i}{[1-f_2[\lambda_1(1-\omega_i)]g_2[\lambda_3(1-\omega_i)]]}\times$$

$$\frac{[1-f_2[\lambda_1(1-\omega_i)][1-g_1(s)]]}{(1-\omega_i)^2\lambda_1}$$

$$=\lim_{s\to 0}\sum_{i=1}^{n}\frac{1}{n}\frac{s\omega_i}{[1-f_2[\lambda_1(1-\omega_i)]g_2[\lambda_3(1-\omega_i)]]}\times$$

$$\frac{\bar{F}_2[\lambda_1(1-\omega_i)][1-g_1(s)]]}{(1-\omega_i)}$$

$$=\lim_{s\to 0}\sum_{i=1}^{n}\frac{1}{n}\frac{s^2\omega_i}{[1-f_2[\lambda_1(1-\omega_i)]g_2[\lambda_3(1-\omega_i)]]}\times$$

$$\frac{\bar{F}_2[\lambda_1(1-\omega_i)]\bar{G}_1(s)}{(1-\omega_i)}$$

$$=\frac{u_1^{-1}}{n}\sum_{i=1}^{n}\frac{\omega_i}{1-\omega_i}\frac{\bar{F}_2[\lambda_1(1-\omega_i)]}{1-f_2[\lambda_1(1-\omega_i)]g_2[\lambda_3(1-\omega_i)]}$$

$$=[1+\frac{\lambda_1}{nu_1}+\frac{\lambda_2}{u_2}+\frac{\lambda_3\lambda_2}{nu_1u_2}]^{-1}$$

In the above equation, ***e*** expresses the column vector with all its values being one.

Theorem2. The frequency of system fault under steady state is given by

$$m_f=A(\frac{\lambda_1}{n}+\lambda_2) \qquad (40)$$

Proof. The instantaneous frequency of the system fault is given by

$$m_f(t)=\sum_{i=1}^{n}\int_0^{\infty}p_{0j}(t,x_2)\lambda_2(x_2)dx_2+\int_0^{\infty}p_{0n}(t,x_2)\lambda_1 dx_2$$

Doing Laplace transform, then

$$m_f(s)=\sum_{i=1}^{n}[\int_0^{\infty}p_{0j}(s,x_2)\lambda_2(x_2)dx_2+\int_0^{\infty}p_{0n}(s,x_2\lambda_1 dx_2]$$

$$=\sum_{i=1}^{n}[\int_0^{\infty}M_{0j}(s,x_2)\bar{F}_2(x_2)dx_2+\lambda_1\int_0^{\infty}M_{0n}(s,x_2)\bar{F}_2(x_2)dx_2]$$

$$=\sum_{i=1}^{n}[\int_0^{\infty}P\mathrm{diag}[\exp(v_ix_2)]c(s)dx_2+\lambda_1\int_0^{\infty}P\mathrm{diag}\exp[v_ix_2]c(s)dx_2]$$

Hence, the instant frequency of system fault is

$$m_f(s)=\sum_{i=1}^{n}c_i(s)\frac{f_2(-v_i)[1-g_1(s)]}{(a_i-1)g_1(s)}+\lambda_1\sum_{i=1}^{n}c_i(s)\frac{1-f_2(-v_i)}{-v_i}a_i^{n-1}$$

The frequency of system fault under steady state is

$$m_f=\lim_{s\to 0}sm_f(s)=A(\frac{\lambda_1}{n}+\lambda_2)$$

Theorem3. The frequency of system updating under steady state is

$$m_r=\frac{n}{\lambda_2}+\frac{n}{u_2}+\frac{\lambda_1}{\lambda_2u_1}+\frac{\lambda_3}{u_1u_2} \qquad (41)$$

Proof. The instantaneous frequency of system updating is given by

$$m_r(t)=\int_0^{\infty}p_{21}(t,y_2)u_2(y_2)dy_2$$

$$=\int_0^{\infty}M_{21}(s,y_2)\bar{G}_2(y_2)u_2(y_2)dy_2$$

$$=\int_0^{\infty}P\mathrm{diag}[\exp(\vartheta_iy_2)]d(s)g_2(y_2)dy_2$$

$$=\sum_{i=1}^{n}c_i(s)f_2(-v_i)\int_0^{\infty}\exp(\vartheta_iy_2)g_2(y_2)dy_2$$

$$=\sum_{i=1}^{n}c_i(s)f_2(-v_i)g_2(-\vartheta_i)$$

Then

$$m_r(s)=\sum_{i=1}^{n}c_i(s)f_2(-v_i)g_2(-\vartheta_i)$$

The frequency of system updating under steady state is

$$m_r = \lim_{s\to 0} s m_r(s) = \frac{n}{\lambda_2} + \frac{n}{u_2} + \frac{\lambda_1}{\lambda_2 u_1} + \frac{\lambda_3}{u_1 u_2}$$

The state 1 and 3 must be seen as an absorbing state and construct an absorbing Markov chain to calculate the reliability indexes of part 1, and then, according to the same method as above we easily calculate the reliability and mean time to first failure ($MTTFF_1$) of part 1. Likewise, to get the reliability and $MTTFF_2$ of part 2, the state 1 and 2 must be seen as an absorbing state. Below we take part 1 for an example to illustrate the process, and the absorbing state transfer diagram can be seen in Figure 3.

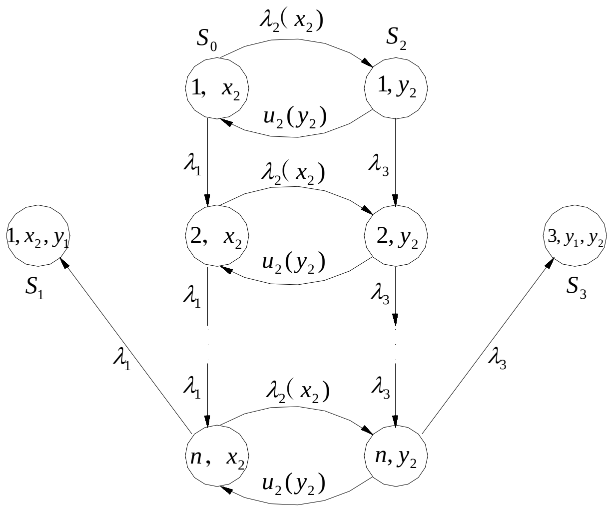

Fig.3 Absorbing state transfer diagram

According to Figure 3, we rewrite the above partial calculous equation group, that is to say, the right sides of (2) and (4) become zero. Then

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + u_1(y_1)]p_1(t, x_2, y_1) = 0 \quad (42)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{01}(t, x_2) = 0 \quad (43)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{0j}(t, x_2) = \lambda_1 p_{0j-1}(t, x_2)$$
$$j = 2 \sim n \quad (44)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{21}(t, y_2) = 0 \quad (45)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{2j}(t, y_2) = \lambda_3 p_{2j-1}(t, y_2)$$
$$j = 2 \sim n \quad (46)$$

$$[\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + u_1(y_1)]p_3(t, y_1, y_2) = 0 \quad (47)$$

In the same way, doing Laplace transform from (42) to (47), and we have

$$[s + \frac{\partial}{\partial y_1} + u_1(y_1)]p_1(s, x_2, y_1) = 0$$

$$[s + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{01}(s, x_2) - p_{01}(0, x_2) = 0$$

$$[s + \frac{\partial}{\partial x_2} + \lambda_1 + \lambda_2(x_2)]p_{oj}(s, x_2) = \lambda_1 p_{0j-1}(s, x_2)$$
$$j = 2 \sim n$$

$$[s + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{21}(s, y_2) = 0$$

$$[s + \frac{\partial}{\partial y_2} + \lambda_3 + u_2(y_2)]p_{2j}(s, y_2) = \lambda_3 p_{2j-1}(t, y_2)$$
$$j = 2 \sim n$$

$$[s + \frac{\partial}{\partial y_1} + u_1(y_1)]p_3(s, y_1, y_2) = 0$$

Noting that here equation (26) and (34) has n multiple equivalent latent roots, that is, $-(s+\lambda_1)$ and $-(s+\lambda_3)$. It is no difficult to prove $P^{-1}A(s, \lambda_1, g_1) = J_n(v_i)$, where $J_n(v_i)$ is the standard Jordan matrix.

Likewise, according to the former method we easily calculate $R_1(s)$.

$$R_1(s) = \sum_{i=1}^{n}[\int_0^{\infty} p_{0j}(s, x_2)dx_2 + \int_0^{\infty} p_{2j}(s, y_2)dy_2] \quad (48)$$

Substituting (18) and (19) into (48), then

$$R_1(s)=\sum_{i=1}^{n}[\int_0^\infty M_{0j}(s,x_2)\bar{F}_2(x_2)dx_2+\int_0^\infty M_{2j}(s,y_2)\bar{G}_2(y_2)dy_2]$$

Substituting (32) and (33) into R_1, and considering (37) and (38), then

$$R_1(s)=\sum_{i=1}^{n}c_i^T(s)[\bar{F}(-v_i)+\bar{G}(-\vartheta_i)f_2(-v_i)]P^T e \qquad (49)$$

According to matrix property [12], we then have

$$R_1(s)=\frac{e_1^T[b(s,\lambda_1,\bar{F}_2)+b(s,\lambda_3,\bar{G}_2)b(s,\lambda_1,f_2)]e_1}{[I-b(s,\lambda_3,g_2)b(s,\lambda_1,f_2)]} \qquad (50)$$

where $e_1=[1,0,\ldots,0]^T$, and

$b(s,\lambda,f)=$

$$\begin{bmatrix} f(s+\lambda) & 0 & \cdots & 0 \\ -\lambda(f^{(1)}(s+\lambda) & f(s+\lambda) & \vdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{(-\lambda)^{(n-1)}}{(n-1)!}f^{(n-1)}(s+\lambda) & \frac{(-\lambda)^{n-2}}{(n-2)!}f^{(n-2)}(s+\lambda) & \cdots & f(s+\lambda) \end{bmatrix}$$

and then $\mathrm{MTTFF}_1=$

$$R_1(0)=\frac{e_1^T[b(0,\lambda_1,\bar{F}_2)+b(0,\lambda_3,\bar{G}_2)(b(0,\lambda_1,f_2)]e_1}{[I-b(0,\lambda_3,g_2)b(0,\lambda_1,f_2)]} \qquad (51)$$

In the same way, if we see the state 1 and 2 as absorbing one, then

$$R_2(s)=\frac{e^T P}{n}[\bar{F}_2(-v_1),\cdots,\bar{F}_2(-v_n)]^T+\frac{\lambda_1\bar{G}(s)}{n}\sum_{i=1}^{n}a_i^{n-1}\bar{F}_2(-v_i) \qquad (52)$$

and then $\mathrm{MTTFF}_2=$

$$R_2(0)=\frac{1}{\lambda_2}+\frac{\lambda_1}{nu_1}\sum_{i=1}^{n}\omega_i^{-1}\bar{F}_2(\lambda_i-\lambda_i\omega_i) \qquad (53)$$

3 Examples

Example 1. As shown in Figure 1, the system only possesses a feeder arm such as L1, the life of part 1 follows Erlong distribution with order 1, and the life of part 2 follows exp(λ_2) distribution, the average values of servicing time are expressed by u_1 and u_2, here $\lambda_3=0$, $n=1$, from (39), (40), and (41), we have

$$A=[1+\frac{\lambda_1}{u_1}+\frac{\lambda_2}{u_2}]^{-1}$$

$$m_f=A(\lambda_1+\lambda_2)$$

$$m_r=\frac{1}{\lambda_2}+\frac{1}{u_2}+\frac{\lambda_1}{\lambda_2 u_1}$$

Let $\lambda_1=\lambda_2=\lambda$, we then have

$$A=\frac{u}{2\lambda+u}$$

$$m_f=\frac{2\lambda u}{2\lambda+u}$$

$$m_r=\frac{2\lambda+u}{\lambda u}$$

This result is fully consistent with series system composed of two parts [10].

Example2. Like example 1, the system possesses two feeder arms such as L_1 and L_2, let $\lambda_1=\lambda_3$, $n=1$, from (39), (40), and (41), we then have

$$A=[1+\frac{\lambda_1}{u_1}+\frac{\lambda_2}{u_2}+\frac{\lambda_1\lambda_2}{u_1u_2}]^{-1}$$

$$m_f=A(\lambda_1+\lambda_2)$$

$$m_r=\frac{1}{\lambda_2}+\frac{1}{u_2}+\frac{\lambda_1}{\lambda_2 u_1}+\frac{\lambda_1}{u_1 u_2}$$

Compared with series system of two parts, the availability of the system in steady state reduces a little because part 1 still works when part 2 is in failure. So long as there is one part in failure the system is in failure, and so fault frequency in steady state is consistent. The reason that system updating frequency increases in steady state is no interruptive of part 1 working when one part 2 is in failure.

4 Conclusion

For series system with its parts being repairable, the reliability mathematics model is proposed based on hierarchical power supply model of traction substation in this paper, and the relevant reliability analysis is also performed. The applied technique is more accurate and ubiquitous than the traditional ones. This indicates that Erlang distribution is more reasonable than exponential one for life approximation of the parts, and calculation is also relatively easy. Under some special conditions, if we apply other exponential distribution types as life approximation of the elements, and then better results can be likely obtained. The model mentioned in the paper can be not only applied to traction substation, but also for other systems it is also available.

5 Acknowledgements

This work is partially supported by the 'Qing Lan' Talent Engineering Funds supported by Lanzhou Jiaotong University (QL-06-19A). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

References:

[1] J. Cao, *Electrified Railway Traction Supply Systems*, China Railway Press, 1983.
[2] Q. Li, and J. He, *Traction Power Supply System Analysis*, Southwest Jiaotong University Press, 2007.
[3] Mahfound Chafai, Larbi Refoufi, and Hamid Bentarzi, Reliability Assessment and Improvement of Large Power Induction Motor Winding Insulation Protection System Using Predictive Analysis, *WSEAS Trans. on Circuits and Systems*, Vol.7, No.4, 2008, pp.193.
[4] H. Li, and G. Li, *Analysis and Design on System Reliability*, Science Press, 2003.
[5] Gabriela Tont, and Dan George Tont, A Multilevel Approach of Reliability Optimization in Complex Systems, *WSEAS Trans. on Systems*, Vol.7, No.7, 2008, pp.833-842.
[6] W. Liu, *Reliability Design on Mechanism*, Qinghua University Press, 1998.
[7] Rita Mahajan, and Renu Vig, Reliability and Performance Analysis of New Fault Tolerant Irregular Network, *WSEAS Trans. on Computer Research*, Vol. 3, No.5, 2008, pp.311-321.
[8] Hongsheng Su, and Youpeng Zhang, Non-Markov Repairable Model Analysis on Two Modular, *Chinese Journal of Applied Probability and Statistics*, Vol. 24, No.2, 2008, pp.166-174.
[9] Hongsheng Su, and Qunzhan Li, Repairable Model Analysis on Triple Modular Redundancy Interlocking System in Railway Signal, in: *Proceedings of 2005 Chinese Control and Decision Conference*, Harbin, China, 2005, pp. 1011-1015.
[10] J. Cao, and G. Chen, *Reliability Mathematics Introduction*, High Education Press, 2006.
[11] H. Dong, J. Li, and J. Xue, Study on Non-Markov Model of Two-computer Redundant Systems, *Journal of China Railway Society*, Vol.36, No.6, 2001, pp. 35-38.
[12] J. Luo, *Matrix Analysis Introduction*, South China University Press, 2005.