NEW ANALYTICAL CAVITATION EROSION MODELS

CONSTANTIN PĂTRĂŞCOIU
Faculty of Engineering and Management of the Technological Systems, Drobeta Turnu Severin,
University of Craiova
Tabla Butii 71, Drobeta Turnu Severin
ROMANIA

patrascoiu@yahoo.com http://www.imst.ro

Abstract: Cavitation erosion prediction for the hydraulic machines is very important in the hydraulics research because that cavitation erosion is a source of failure of pumps water turbine blade, pipelines and other hydraulic devices. In this paper new kinds of theoretical volume loss rate curve of erosion cavitation progress is proposed. The analytical models describing this new kind of erosion curves give a new vision of the volume loss rate curve and produce a good concordance between the experimental and theoretical data if there is a good choice of theoretical model. Instead of using a unique analytical (universal) model for all materials, we give the possibility of a good choice between the proposed models. There may also appear some open problem such as optimally correlating this analytical cavitation erosion models with the properties of the implied materials.

Key-Words: Cavitation, Erosion, Mathematical model, Differential equations, Bessel's equations.

1 Introduction

The *cavities* are formed into a liquid when the static pressure of the liquid is reduced below the vapour pressure of the liquid in current temperature. If the cavities are carried to higher-pressure region they implode violently and very high pressures can occur.
The *cavitation process* is described (Timo Koivula 2000) as follows :
When the local pressure of a liquid is reduced sufficiently, the dissolved air in oil starts to come out of solution.
In this process, air diffuses through cavity wall into the cavity.
When pressure in the liquid is further reduced, evaporation pressure of the liquid is achieved. At this point the liquid starts to evaporate and cavities start to be filled with vapour.
When this kind of a cavity is subjected to a pressure rise cavity growth is stopped and once the pressure gets higher, the cavities start to diminish.
Cavities disappear due to dissolution of air and condensation of vapour.
When a cavity is mostly vapour filled and subjected to a very rapid pressure rise it implodes violently and causes very high pressure peaks. Implosion is less violent if the gas quantity of a cavity is big.
This requires relatively slow nucleation of a cavity.

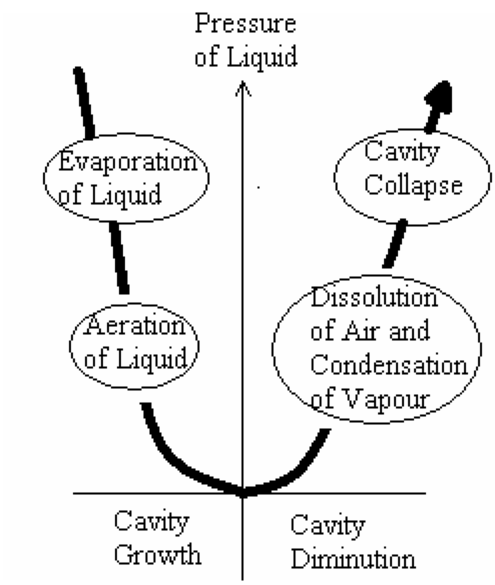

Figure 1. The cavitation process
[Timo Koivula (2000)]

On the cavitation process, the internal phenomena such as the diffusion of vapour, the thermal diffusion, the mist formation due to the homogeneous condensation and the heat and mass transfer through bubble wall, have significant influence. The cavitation phenomenon may cause serious change in microstructure and intrinsic stress level of the material. Macroscopically, the change in hardness is often observed; microscopically, the slip bands and deformation twins appear, and the phase transformations may occur in unstable alloys.

Cavitation erosion is a progressive loss of material from a solid, due to the impact action of the collapsing bubbles or cavities into liquid near the material surface. *Cavitation erosion* can be formed when cavity implosions are violent enough and they take place near enough to the solid material. So, mechanical degradation of a solid material caused by cavitation is called *cavitation erosion*.

The cavitation erosion depends on the material properties (hardness, work hardening capability, and grain size, etc.). The degree of cavitation erosion is affected by stress state and the corrosion resistance of a material.

An important effort of researchers in the field of cavitation is focused on improving the techniques of prediction of cavitation erosion, classified into three main categories: *Empirical* correlations with material properties, *Simulation* techniques using special test devices and *Analytical* methods.

The objective of *Analytical methods* in cavitation erosion study is to predict cavitation erosion without model tests. This method is still in development requiring extensive research efforts before becoming operational.

Many existing models of cavitation erosion give special attention to the volume loss (or weight loss) and the volume loss rate curve (or weight loss rate curve).

The volume loss is often preferred for comparison of materials with great differences between the specific masses.

The volume loss curve ([11], [14]) usually is described by the formula:

$$V(t) = A \cdot U(k_1, k_2, ..., k_m, \Im t) \qquad (1)$$

with: A - the eroded surface area; $\Im$ -measure of cavitations intensity; U- erosion progress function resulting out of applied phenomenological model; $k_1, k_2, ..., k_m$- a set of real parameters (usually 3 parameters are quite sufficient), determined by fitting the erosion curve to the experimental data; t- cumulative exposure duration.

In the study of cavitation erosion Thiruwengadam [16] use the energy E_a absorbed by the volume of the material fractured: $E_a = VS_e$ where E_a is the energy absorbed by eroded material, V is the volume of the eroded material, and S_e stands for an erosion strength which represents the energy – absorbing capability of the material per volume unit under the action of the erosive forces.

The energy flux density J determined by implosion of cavitation micro bubbles may be calculated from the following formula:

$$J = \frac{1}{T}\frac{1}{2\rho s}\sum_{k=1}^{N} n_k p_k \qquad (2)$$

where T – denotes duration of the pressure pulses sampling period; ρ – liquid density; s – sound velocity in liquid; N – number of pressure intervals; n_k – number of pulses in a single interval; p_k – value of pressure amplitude in the k-th interval.

K. Steller [14] proposed a formula based on the cavitations resistance:

$$P_t = R_{cav} V \qquad (3)$$

where P is the power used to erode the material subject to cavitations impingement ,

$$R_{cav} = \frac{\chi + e^{-kt}}{\chi + 1} R_0 \qquad (4)$$

χ is a factor determining the ratio of ultimate and initial cavitations resistance and k is a parameter defining the speed of cavitations resistance.

F.J Heymann [5] give a model according to which the erosion rate of material with unit surface area can be described by the integral equation

$$Y(t) = f(t) + \int_0^1 f(t-T)Y(T)dT \qquad (5)$$

assuming that the eroded material consists of subsequent layers of unit thickness, *f(t)dt* is the probability of removing an element of each layer in the time *dt*.

M. Szkodo [13] uses the probability of cumulative volume loss for elementary volume V_0 the Weibull's function: $P(V_0) = 1 - \exp\left\{ - IV_0 \left(\frac{t}{K_{c_d}} \right)^{\frac{1}{W_{pl}}} \right\}$ (6)

Then, the cumulative probability of volume loss for arbitrary eroded volume $V = nV_0$ is

$$P(V) = [P(V_0)]^n = [P(V_0)]^{\frac{V}{V_0}} =$$
$$= \exp\left\{ \frac{V}{V_0} \ln\left[1 - \exp\left(- IV_0 \left(\frac{t}{K_{c_d}} \right)^{\frac{1}{W_{pl}}} \right) \right] \right\} \quad (7)$$

Taking into account that the volume loss of material is a product of initial volume and cumulative probability of volume loss *P(V)* it follows that

$$V(t) = V \exp\left\{ \frac{H}{h} \ln\left[1 - \exp\left(- IAh \left(\frac{t}{K_{c_d}} \right)^{\frac{1}{W_{pl}}} \right) \right] \right\} \quad (8)$$

where: A – surface area ($V_0 = Ah$); W_{pl} is a relative work of plastic deformation on the eroded surface; K_{cd} is the relative stress intensity factor under cavitation loading ; *h* is the depth of strain hardening or max. length of cracks on the end of incubation period.

J. Noschevich and K. Steller ([11], [14]) , proposed models of erosion kinetics referring directly to the erosion curve pattern. The volume loss curve is obtained by solving the ordinary differential equation with constant coefficients:

$$\frac{d^2 v}{dt^2} + 2\alpha \frac{dv}{dt} + \beta^2 v = I \quad (9)$$

(with $v = \frac{dV}{dt}$ the volume loss rate, α and β - coefficients determining material properties, I – cavitation intensity parameter) or the differential equation:

$$\frac{1}{P}\frac{d^2 v}{dt^2} + \frac{2\alpha}{P}\frac{dv}{dt} + \beta^2 v = I = \gamma P \quad (10)$$

(with *P*- the power of flux energy delivered by the cavitation cloud to the eroded material, γ - the coefficient defined by equation (10)).

In any cavitation (or droplet impact) erosion test, the damage rate is generally time dependent. For V and v , there are proposed [M. Szkodo (2005)] the typical erosion curves for unity eroded surface area as in Fig. 2 and Fig. 3.

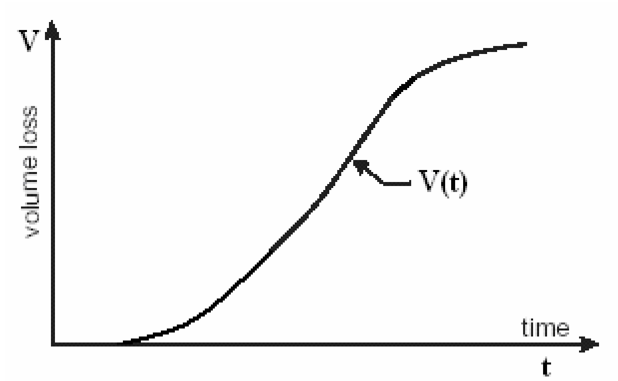

Fig. 2. The volume loss curve

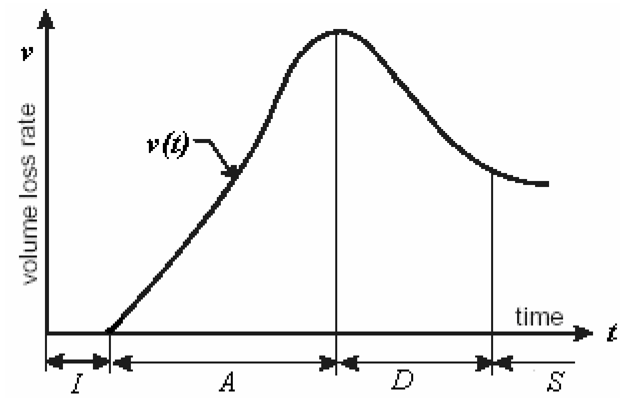

Fig. 3.The volume loss rate curves

The volume loss rate curve (Fig. 3) can be divided into four typical periods: I - Incubation period; A - Acceleration period; D - Deceleration period; S - Steady state erosion period. The signification of this period is clear.

Incubation period I is an initial period of damage in which volume loss of material is nearly zero (non-measurable). During the incubation period, "considerable plastic deformation occurs, without any apparent weight loss" [Leight (1959)]. In this time interval, the material accumulates energy. Leight concluded that the *Incubation* period depended linearly on the corrosion fatigue limit of the material. This does not appear to be generally valid today. Thiruvengadam [16] defined the

Incubation period as “that period during which the first permanent plastic dent is formed.” His definition assumed that cavitation pits (or craters) are due only to fatigue effects. This assumption is also not entirely valid because the incubation period is often characterized by single-blow craters before fatigue effects become significant.

Acceleration period A. In this time interval, the intensification of damage is observed, distinguished by violent increase of volume loss rate of erosion and the volume loss rate reaches maximal value.

Deceleration period D. In this time interval, volume loss rate decreases.

Steady state erosion S, characterized by almost constant volume loss rate of erosion.

In the paper [3] a “damped” model is proposed according to which that the volume loss curve is given by the formula:

$$V(t) = A[v_s t - f(t)] \qquad (11)$$

where v_s is the ultimate value of the volume loss rate and $f(t)$ is the solution of homogenous linear differential equations of the second order with constant coefficients:

$$\frac{d^2 y}{dt^2} + 2\beta \frac{dy}{dt} + \beta^2 y = 0 \qquad (12)$$

describing the “damped” oscillations with “infinite period”. Solving the ODE (11) it follows that the volume loss is given by formula:

$$V(t) = A[v_s t - \lambda t e^{-\beta t}] \qquad (13)$$

and he volume loss rate curve is given by formula:

$$v(t) = A[v_s - \lambda e^{-\beta t} + \lambda \beta t e^{-\beta t}] \qquad (14)$$

The real parameters v_s , λ and β will be determined by fitting the erosion curve to the experimental data (using the least squares method or another numerical method)

Instead of the volume loss curve (13) and the volume loss rate curve (14), may be used the mass loss curve:

$$m(t) = A[at - bte^{-ct}] \qquad (15)$$

and the mass loss rate curve is:

$$q(t) = \frac{dm(t)}{dt} = A[a - be^{-bt} + bcte^{-bt}] \qquad (16)$$

which A- the eroded surface area ;

$$a = \rho\, v_s\,,\ b = \rho\lambda\,,\ c = \rho\beta \qquad (17)$$

and ρ - the density of material. The real parameters a , b and c will be determined by the formulas (16) or fitting the erosion curve to the experimental data. By example using the experimental data given by Laboratories of Politehnic University of Timisoara for OL370, the experimental data for exposure durations T and the loss mass M is :

$$T := \begin{pmatrix} 0 \\ 5 \\ 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \\ 105 \\ 120 \\ 135 \\ 150 \\ 165 \end{pmatrix} \qquad M := \begin{pmatrix} 0 \\ 0.00082 \\ 0.00245 \\ 0.01004 \\ 0.020702 \\ 0.03065 \\ 0.04190 \\ 0.05348 \\ 0.06391 \\ 0.0729 \\ 0.08502 \\ 0.0976 \\ 0.10662 \end{pmatrix}$$

Using the least squares method, the real parameters a , b and c determined by fitting the erosion curve to the experimental data for unity surface area, are:

$a := 0.6571\cdot 10^{-3}$ $\quad b := 0.6709\cdot 10^{-3}$ $\quad c := 0.2551\cdot 10^{-1}$

The mass loss rate curve (continous line given by function $m(t) = 0.6571\cdot 10^{-3}\, t - 0.6709\cdot 10^{-3}\, te^{-0.2551\cdot 10^{-1} t}$) and the experimental data (box) is plotted in Fig. 4

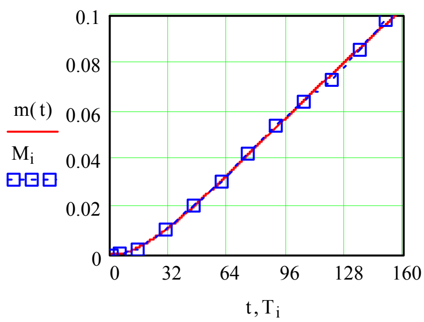

Fig.4. The mass loss rate curve(m) and the experimental data (M_i) for OL370

The mass loss rate is plotted Fig. 5

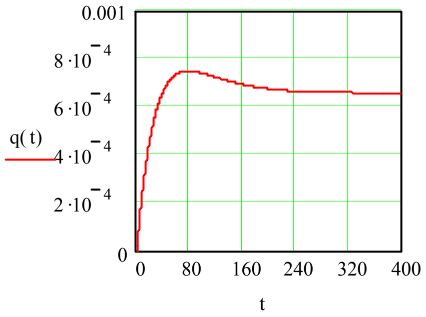

Fig. 5. The mass loss rate (q) for OL370

Taking into account that the density of material is ρ =7.713, The volume loss and the volume loss rate are:

$$V(t) = \frac{m(t)}{7.713} \qquad v(t) = \frac{q(t)}{7.713} \tag{18}$$

Then, the volume loss curve and the volume loss rate curve are plotted in Fig. 6 and Fig.7

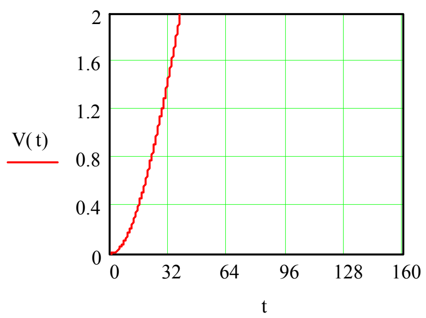

Fig. 6. The volume loss curve for OL370

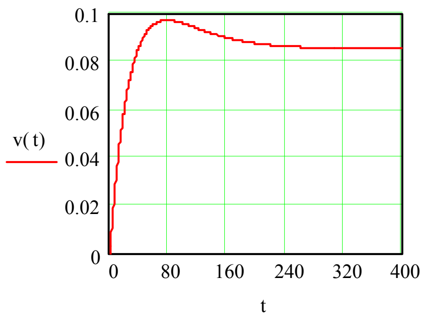

Fig. 7. The volume loss rate curve for OL370

2 A generalization of "damped" model of cavitation erosion.

Like in paper [3] for $v(t)$ we get $v(t) = \frac{d(V(t))}{dt} = A[v_s - \frac{df}{dt}(t)]$,

but now, $\frac{df}{dt}$ must satisfy the ODE:

$$\frac{d^2y}{dt^2} + \alpha\frac{dy}{dt} + \beta y = 0 \tag{19}$$

where α and β are real constants, depending on eroded material; $\alpha \geq 0, \beta > 0$. The above equation (19) has a physical interpretations like as the case of damped vibrations: Because the eroded material tested in Hydraulic Machinery Laboratory using a vibratory device with nickel tube, the eroded material is subject to a frictional force and an a damping force. With Newton's Second Law we have:

$$m\frac{d^2y}{dt^2} = \text{damping force} + \text{restoring force} =$$

$$= -p\frac{dy}{dt} - qy. \tag{20}$$

Putting in (20), $\alpha = \frac{p}{m}$ and $\beta = \frac{q}{m}$, we obtain the equation (19). The equation (19) is a second-order linear differential equation and its auxiliary equation is:

$$r^2 + \alpha r + \beta = 0 \tag{21}$$

The roots of the equations (21) are:

$$r_1 = \frac{-\alpha + \sqrt{\alpha^2 - 4\beta}}{2}, \qquad r_2 = \frac{-\alpha - \sqrt{\alpha^2 - 4\beta}}{2}$$

Case I. If $\alpha = 2\sqrt{\beta}$, the roots of the equations (21) $r_1 = r_2 = -\sqrt{\beta}$, and the equation (19) is like (12) discussed in paper [3]. Taking account that:

$f(0) = 0$, $\lim_{t\to\infty} f(t) = 0$ and $\lim_{t\to\infty} \frac{df}{dt}(t) = 0$

we have the volume loss curve:

$$V(t) = A[v_s t - \lambda t e^{-\sqrt{\beta}\,t}] \tag{21}$$

and the volume loss rate curve is:

$$v(t) = A[v_s - \lambda e^{-\sqrt{\beta}\, t} + \lambda\sqrt{\beta}\; t e^{-\sqrt{\beta}\, t}] \qquad (23)$$

If (-ε,o) is the incubation period, the typical graphs of v as a function of t are shown in Fig. 8.

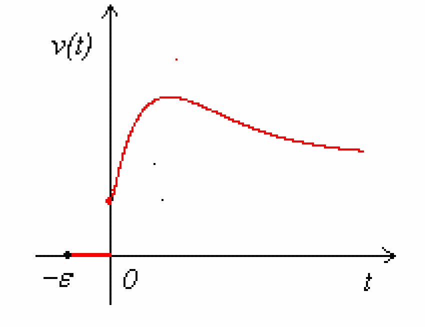

Fig.8. The volume loss rate curve for case I and case III.

Case II. If $\alpha^2 - 4\beta < 0$ then, the roots of auxiliary equation (11) is complex:

$r_1 = \frac{-\alpha}{2} + \gamma i, r_2 = \frac{-\alpha}{2} - \gamma i$ where

$\gamma = \sqrt{4\beta - \alpha^2}$

Because $\frac{df}{dt}$ is the general solution of equation (10) we have:

$\frac{df}{dt}(t) = e^{-\frac{\alpha}{2}t}(C_1 \cos\gamma t + C_2 \sin\gamma t)$, and

$$f(t) = \frac{2e^{-\frac{\alpha}{2}t}}{\alpha^2 + 4\gamma^2}(2C_1\gamma \sin\gamma t - C_2\alpha \sin\gamma t$$
$$- 2C_2\gamma \cos\gamma t - C_1\alpha \cos\gamma t)$$

Evidently, the conditions: $\lim_{t\to\infty} \frac{df}{dt}(t) = 0$ and $\lim_{t\to\infty} f(t) = 0$ are satisfied.

The condition $f(0) = 0$ implies:

$$2C_2\gamma + C_1\alpha = 0 \qquad (24)$$

Then, the volume loss curve is given by:

$$V(t) = A[v_s t + \frac{2e^{-\frac{\alpha}{2}t}}{\alpha^2 + 4\gamma^2} C_2(\frac{4\gamma^2}{\alpha}\sin\gamma t$$
$$+ \alpha \sin\gamma t)] \qquad (25)$$

and the volume loss rate curve is:

$$v(t) = A[v_s - e^{-\frac{\alpha}{2}t} C_2(\sin\gamma t - \frac{2\gamma}{\alpha}\cos\gamma t)] \qquad (26)$$

Typical graphs of v as a function of t are shown in Figure 8.

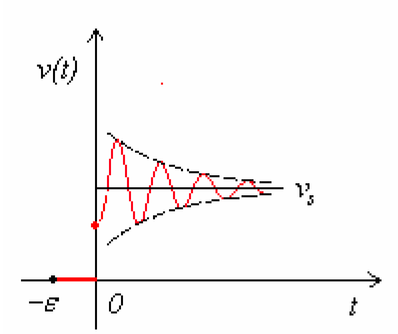

Fig. 9. The volume loss rate curve for case II.

Remark. The graph of volume loss rate curve shown in the Figure 9. differ from the usually(by now) images of the erosion curves, but we explain in next section why it is possible this kind of curve.

Case III. If $\alpha^2 - 4\beta < 0$ then, auxiliary equation (20) have distinct real roots r_1, r_2.

Then we have:

$$\frac{df}{dt}(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} \qquad (27)$$

$$f(t) = \frac{C_1}{r_1} e^{r_1 t} + \frac{C_2}{r_2} e^{r_2 t} \qquad (28)$$

Since α, β, are positive, and $\sqrt{\alpha^2 - 4\beta} < \alpha$, the roots r_1, r_2 must both be negative and the conditions $\lim_{t\to\infty} f(t) = 0$ and $\lim_{t\to\infty} \frac{df}{dt}(t) = 0$ are satisfies. The condition $f(0)=0$ implies

$$\frac{C_2}{r_2} = -\frac{C_1}{r_1}. \qquad (29)$$

Then, the volume loss curve is given by:

$$V(t) = A(v_s t - \frac{C_1}{r_1} e^{r_1 t} + \frac{C_1}{r_1} e^{r_2 t}) \qquad (30)$$

and the volume loss rate curve is:

$$v(t) = A(v_s - C_1 e^{r_1 t} + \frac{C_1 r_2}{r_1} e^{r_2 t}) \qquad (31)$$

Typical graphs of v as a function of t are shown in Fig. 8.

3 A cavitation erosion model based on the Bessel's equations.

We begin with a remark: During the incubation period, The volume loss V (or the weight loss), and the volume loss rate $v(t) = \frac{dV(t)}{dt}$ (or the weight loss rate) are nulls.

At the end of the incubation period, the volume loss rate curve $v(t)$ (or the weight loss rate curve), can have a point of discontinuity, because the volume rate curve, may be not smooth at the end point of the incubation period.

Let $(t_0, 0)$ the end point of the incubation period. If θ is the angle between the time axe and tangent to the volume loss curve at $(t_0, 0)$, and $tan\,\theta \neq 0$ (Fig. 10), then we have an discontinuity point for the volume loss rate curve $v(t)$, like as in Fig. 11.

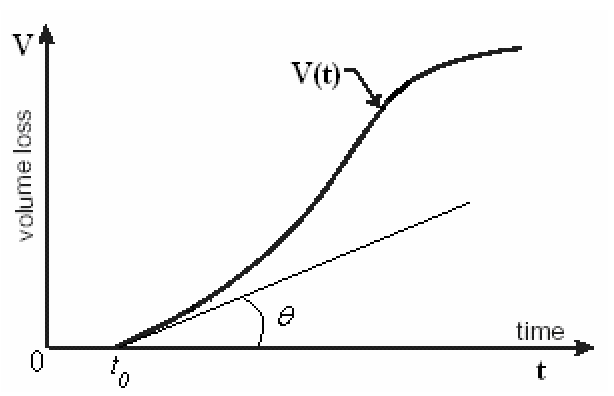

Figure 10. The volume loss curve with nonzero right derivative in the final point of the incubation period.

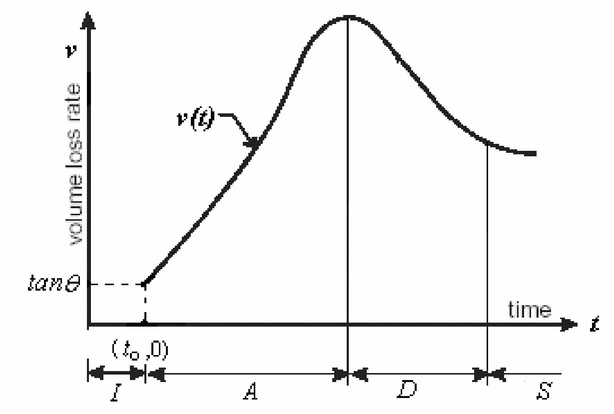

Fig. 11. The volume loss rate curves with a discontinuity in the final point of the incubation period.

For example in the Figure + the slope of weight loss at the end point of the incubation period of first experimentally given curve (▲) is not null.

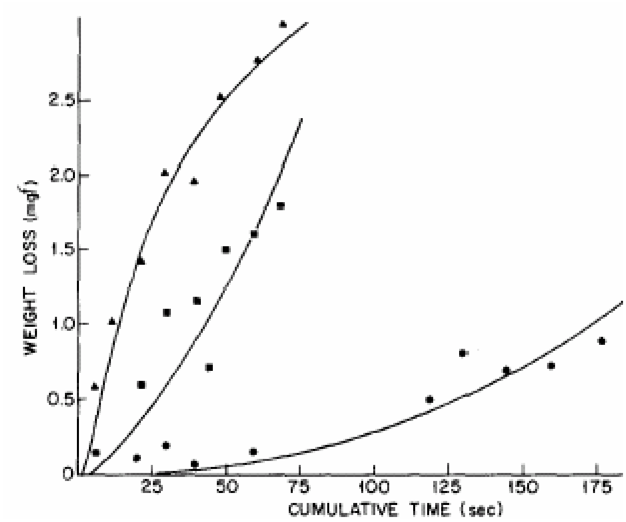

Fig.12 Weight loss at various amplitudes for aluminum alloy 1100-O within the Iincubation period (open-beaker vibratory tests; room temperature): ▲, 1.78×10^{-3} in; ■, 1.38×10^{-3} in; ●, 1×10^{-3} in; [Yu-Kang Zhou and F. G. Hammitt [1983])

During the incubation period, the volume loss, and the volume loss rate curves are nulls and our study is superfluous. To simplify the calculus, we have chosen the time interval $[-\varepsilon, 0]$ as the incubation period, and we shall study the volume loss, and the volume loss rate curves for $t \geq 0$ ($t_0 = 0$ is the end point of the incubation period)

The most accurately experimentally data for very long time, suggested that the volume loss rate curve $v(t)$ can look as in Fig. 13. (The incubation period was chosen the time interval $[-\varepsilon, 0]$)

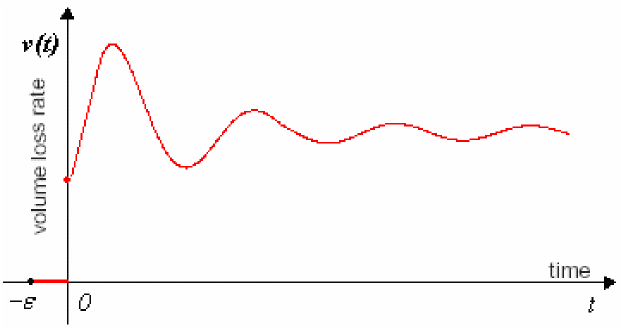

Fig. 13. New model of the volume loss rate curves

For example, the volume loss rate curve of S15C steel [S.Hattori and E.Nakao (2001)] (Figure 1.) have an allure of curve presented in Figure 5.

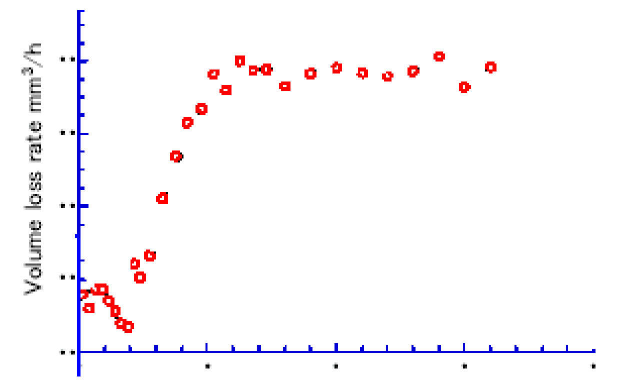

Fig.14. Volume loss rate curve of S15C steel

Based on physical considerations and experimental data, we have chosen that the volume loss curve

$$v(t) = \frac{dV}{dt}(t) \tag{32}$$

is the solution of the ordinary differential equation:

$$t^2 \frac{d^2 v}{dt^2} + t\frac{dv}{dt} + (\beta^2 t^2 - 1)v = \alpha(\beta^2 t^2 - 1) \tag{33}$$

α and β - real constants, depending on the eroded material; $\alpha \geq 0, \beta > 0$.

Setting $v(t) = \alpha + \varphi(\beta t)$, (34)

first we find:

$$t^2 \beta^2 \frac{d^2\varphi}{dt^2}(\beta t) + t\beta \frac{d\varphi}{dt}(\beta t) + (\beta^2 t^2 - 1)\varphi(\beta t) = 0$$

Replacing in the above equation βt by x, we obtain that , φ is the solution of Bessel's differential equation

$$x^2 \frac{d^2 y}{dx^2}(x) + x\frac{dy}{dx}(x) + (x^2 - 1)y = 0 \tag{35}$$

Recall that, the general solution of Bessel's equations (35) is

$$y(x) = C_1 J_1(x) + C_2 Y_1(x) \tag{36}$$

where $J_1(x)$ is the Bessel function of the first kind, and $Y_1(x)$ is the Bessel function of the second kind (also known as the Weber Function).

However, $Y_1(x)$ is divergent at $x = 0$, the associated coefficient C_2 is forced to be zero to obtain a mechanically meaningful result (there is no source or sink at $x = 0$).

So, we get $\varphi(x) = C_1 J_1(x)$, and using (34) we find:

$$v(t) = \alpha + C_1 J_1(\beta t). \tag{37}$$

Then, $V(t) = \int_0^t (\alpha + C_1 J_1(\beta s)ds =$

$$= \alpha s\Big|_0^t + C_1 \int_0^t J_1(\beta s)ds \tag{38}$$

Recall that if $J_n(x)$ is the Bessel function of the first kind, then for any integer n, we have

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x), \tag{39}$$

From formula (39), putting n = 0, we have

$$\frac{d}{dx}[J_0(x)] = J_{-1}(x) \tag{40}$$

But , for any integer n:

$$J_{-n}(x) = (-1)^n J_n(x) \quad (41)$$

and in particular for $n = 1$ we have

$$J_{-1}(x) = -J_1(x) \quad (42)$$

Then, formula (39) become:

$$\frac{d}{dx}[J_0(x)] = -J_1(x) \quad (43)$$

Using this result we get:

$$\int_0^t J_1(\beta s)ds = -\frac{1}{\beta}\int_0^t \frac{d}{ds}[J_0(\beta s)]ds =$$
$$= -\frac{1}{\beta}J_0(\beta s)\Big|_0^t = -\frac{1}{\beta}J_0(\beta t) + \frac{1}{\beta}$$

because $J_0(0) = 1$.

Thus, formula (38) become :

$$V(t) = \alpha t - \frac{C_1}{\beta}J_0(\beta t) + \frac{1}{\beta} \quad (44)$$

But, $0 = V(0) = -\frac{C_1}{\beta}J_0(0) + \frac{1}{\beta}$, $V(0) = 0$, $J_0(0) = 1$,

and follow that $C_1 = 1$.

Thus, finally we obtain:

$$v(t) = \alpha + \frac{1}{\beta}J_1(\beta t) \quad (45)$$

$$V(t) = \alpha t + \frac{1}{\beta}[1 - J_0(\beta t)] \quad (46)$$

for unity eroded surface area.

If A is the eroded surface area, we have the formula:

$$V(t) = A[\alpha t + \frac{1}{\beta}(1 - J_0(\beta t))] \quad (47)$$

Observation.

We know that $\lim_{x\to\infty} J_1(x) = 0$, and $\lim_{\substack{x\to 0\\ x>0}} J_1(x) = 0$.

So, we have:

$$\lim_{t\to\infty} v(t) = \lim_{t\to\infty}[\alpha + \frac{1}{\beta}J_1(\beta t)] = \alpha . \quad (48)$$

$$\lim_{\substack{t\to 0\\ t>0}} v(t) = \alpha \quad (49)$$

From equalities (48) and (49) it follows that the constant α has two physical meanings: α is the ultimate value of the volume loss rate, and the right derivative of $V(t)$ at the point t_0 ($tan\,\theta$). Thus, the ultimate value of the volume loss rate must be equal with $tan\,\theta$. If $tan\,\theta \neq 0$, then the volume loss rate curve is discontinuous at $t_0 = 0$. The constant β is a scale factor. The real parameters α and β will be determined by fitting the erosion curve to the experimental data (using numerical methods).

The new formulas (45) and (46), for the volume loss, and the volume loss rate curves depending only on two real parameters α and β, are based on the Bessel function of the first kind and integer order n, $J_n(x), n = 0,1$. The real parameters α and β (used in above formulas (45) and (46)), can be determined by fitting the erosion curve to the experimental data; using for $J_n(x)$:

Taylor series:

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(m+n)!}\left(\frac{x}{2}\right)^{2m+n} ; \quad (50)$$

Integral formulas:

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(nt - x\sin t)dt ; \quad (51)$$

or their asymptotic forms:

for small x , i.e., fixed n and, $x \to 0$,

$$J_n(x) \approx \frac{1}{2^n n!}x^n \quad (52)$$

and for large x , i.e., fixed n and, $x >> n$,

$$J_n(x) \approx \sqrt{\frac{2}{\pi x}}\cos\left[x - (2n+1)\frac{\pi}{4}\right] \quad (53)$$

4 Another cavitation erosion models based on ODEs.

First, we propose for the volume loss rate curve (for the unity area of eroded material), $v(t)=\frac{d(V(t))}{dt}$, to be the solution of the ordinary differential equation:

$$e^t \frac{d^2v}{dt^2}+e^t\frac{dv}{dt}+k=0\,, \qquad (54)$$

where k is a coefficient determining the material properties.

This ODE have a particular solution

$ke^{-t}+kte^{-t}$, and its general solution is

$$\frac{d(V(t))}{dt}=C_1+C_2e^{-t}+ke^{-t}+kte^{-t}\,. \qquad (55)$$

The initial speed of erosion is

$$V(t)=\alpha t+e^{-t}(te^tC_2-kt-C_1-2k) \qquad (56)$$

The real parameters α , k, C_1 , C_2 can be determined by fitting the erosion curve to the experimental data using numerical method.

The initial speed of cavitation erosion is in this model C_1+C_2 and the final speed of cavitation erosion is C_1

This result is like as the result obtained in the paper [3], but most accurately due to a number of parameters which are used.

Second, we propose for the volume loss curve (for the unity area of eroded material), $V(t)$, (for t>0) to be the solution of the ordinary differential equation:

$$\frac{d^2}{dt^2}(V)=\frac{a}{t}(\frac{d}{dt}V-k)-\frac{be^{bt}}{t^a}(\frac{d}{dt}V-k)^2\,, \qquad (57)$$

This equation has an immediate physical interpretation: If the volume loss rate $\frac{d}{dt}V(t)$ touches the ultimate value of the volume loss rate (the final speed of erosion) k, then the volume loss rate is constant and its derivative must be null.

One of the solution $V(t)$, of equation (57), has the derivative(the volume loss rate) been given by:

$\frac{d}{dt}(V)=k+\frac{t^a}{e^{bt}-1}$, the parameter a, is a scale factor and the parameter b is a dumping factor.

The volume loss $V(t)=kt+\int_0^s\frac{s^a}{e^{bs}-1}ds$ (54)

In this case the characteristic curves of cavitation erosion can be found by using Taylor series.

5 Conclusion

We have presented some new theoretical models of typical cavitation erosion curves based on ODEs. Instead of using a unique theoretical model of characteristic cavitation curves for all studied materials, we think that it is most advantageous to use personalized models for some categories of materials. The advantage using ODEs for theoretical analytical erosion curves is that the parameters appear in a natural way in the solution of ODEs and is is often not necessary to decide aprioristic the number of these parameters.

These parameters, will be determined by fitting the erosion curve to the experimental data (using numerical methods) but in special situations we can reduce their number by using initial conditions.

Depending on the theoretical model used in our application, there is the possibility to make a classification of materials and obtain our special properties. For example, in the case of materials which have an initial speed of erosion equal with the ultimate value of the volume loss rate, the study can be concentrated on one phase (initial or final), and the determination of parameters is simplified.

This work is only a beginning. There are many problems that can be optimized and also there is the open problems such as optimally correlating this analytical cavitation erosion models with the properties of the implied materials

References:

[1] G. B. Arfken and H. J. Weber, Mathematical Methods for Physicists, 6th edition (Harcourt: San Diego, 2005). ISBN 0-12-059876-0

[2] Setia Badlev, Dvs Verma, Physics of Flow Mechanism of Scour Around Submerged Pipelines on an Erodible Bed, 3rd **WSEAS** International Conference on APPLIED and THEORETICAL MECHANICS, Spain, December 14-16, 2007 118

[3] I. Bordeaşu, C. Pătrăşcoiu, V. Bădărău, O. Sucitu, M. Popoviciu, New contribution to cavitation erosion curve modelling, , FME TRANSACTION, University of Belgrade Faculty of Mechanical Enginering, New Series, Volume 34, Number 1, 2006, pp.39-43.

[4] M. Dugonik, D. Goricanec, J. Krope, Pressure Drop in Aqueous Surfactant Solution Flow through Pipes in SIS Structure Degradation Zone, Proceedings of the 2nd IASME / **WSEAS** International Conference on Energy & Environment (EE'07), Portoroz, Slovenia, May 15-17, 2007 50

[5] F.J. Heymann: On the time dependence of the rate of erosion due to impingement or cavitation, A symposium presented at the 69^{th} Annual Meeting, ASTM, pp. 70-82, Atlantic City, 1996.

[6] T. Koivula: On cavitation in fluid power, Proc. of 1st FPNI-PhD Symp. Hamburg 2000, pp. 371-382.

[7] A. Krella, A Zelinski, Influence of cavitation intensity on erosion of an al-mg alloy. 9th International Scientific Conference AMME'2000, Sopot, Poland 2000.

[8] A.A. Lakis, Rotating machinery Fault diagnosis using time-frequency method**,** 7th **WSEAS** International Conference on Electric Power Systems, High Voltages, Electric Machines, Venice, Italy, November 21-23, 2007 139

[9] W. C. Leith, Cavitation damage of metals, Eng. J. (Con.), 42 (1) (1959) 43 - 49.

[10]E.Nakao, S.Hattori, Evaluation of Cavitation Erosion based on Erosion Particles CAV2001: session A4.001

[11]J. Noskievič:Some Aspects With regard to cavitation Resistance of Materials, HIDROTURBO 76, Vol. C7, pp.1-16, Brno, 1976.(In. Polish

[12]Hamed Sadeghinia, M.R.Razfar, Jafar Takabi, 2D finite element modeling of face milling with damage effects, 3rd **WSEAS** International Conference on APPLIED and THEORETICAL MECHANICS, Spain, December 14-16, 2007 145

[13] M. Szkodo Mathematical description and evaluation of cavitation erosion resistance of materials. AMME 13^{th} International Scientific conference, Gliwice, May 2005,Poland, pp. 747-650

[14] J.Steller, P.Kaczmarzyk : Phenomenological Models of Cavitation Erosion Progress, International Cavitation Erosion Test Deminar, Sopot, pp.1-9, $1\text{-}2^{nd}$ June 2000.

[15] A. Thiruvendagam, A unified theory of cavitation damage, J. Basic Eng., 85 (3) (1963) 365 - 376.

[16] Thiruwengadam A. P.: The concept of erosion strength. ASTM no. 408, Atlantic City 1966, pp. 22-35.

[17] G. N. Watson, A Treatise on the Theory of Bessel Functions, Second Edition, (1995) Cambridge University Press ISBN 0-521-48391-3

[18] Yu-Kang Zhou and F. G. Hammitt, Mechanical Engineering and Applied Mechanics Department, University of Michigan, Ann Arbor, MI 48109 (U.S.A.) Wear, 86 (1983) 299 – 313