Solving the Problem of the Compressible Fluid Flow around Obstacles by an Indirect Approach with Vortex Distribution and Linear Boundary Elements

LUMINITA GRECU
Department of Applied Sciences, Navigation and Environment Protection
Faculty of Engineering and Management of Technological Systems, Dr Tr Severin,
University of Craiova
1^{st} Calugareni street, Dr Tr Severin, 2220037
ROMANIA
lumigrecu@hotmail.com http://www.imst.ro

Abstract: - In the present paper there is presented a solution with linear boundary elements of lagrangean type for the singular boundary integral equation obtained by an indirect technique with vortex distribution for the bidimensional compressible fluid flow around bodies. The singular boundary integral equation the problem is reduced at is formulated in terms of primary variables-the components of the velocity on the boundary.

Numerical solutions for the components of the velocity and the local pressure coefficient are obtained, for different types of obstacles, with some computer codes made in MATHCAD, based on the method exposed. For some particular cases, when analytical solutions exist a comparison study between the numerical solutions and the exact ones is also done. It can be seen, from the graphics obtained, that the numerical solutions are in good agreement with the exact solutions of the problem.

The paper is also focused on a comparison study between the numerical solutions obtained when the indirect method with sources distribution is used and the numerical solution presented in this paper when boundary elements of same type are used for solving both singular boundary integral equations.

Key-Words: - Compressible fluid flow, boundary element method, vortex distribution, linear boundary elements

1. INTRODUCTION

For solving boundary values problems for systems of partial differential equations different numerical methods can be used. Most of them are able to find the solutions by using the differential equations as they are given, without any further mathematical manipulation. They approximate the differential operators in the equations by simpler ones valid at a series of nodes within the region, like the finite difference method, or they represent the region itself by finite elements which are assembled to provide an approximation of the system involved, like the finite element methods.

The Boundary Element Method (BEM), also known as the Boundary Integral Method, is a modern numerical technique which can be included, together with the Finite Element Method, in the large class of Galerkin methods. These are a class of methods for converting a continuous operator problem to a discrete problem. In principle, this is done by converting the equation to a weak formulation.

There exist two principal techniques of applying BEM method:

- the direct BEM method;
- the indirect BEM method.

Both of these methods offer the principal advantage of the BEM over other numerical methods - the ability to reduce the problem dimension by one. This property is advantageous as it reduces the size of the system the problem is equivalent with, and so improves computational efficiency.

When solving a problem with this method two important steps have to be made: first, we must obtain an equivalent boundary formulation for the problem involved, in fact a boundary integral equation or a system of boundary integral equations, and then, this boundary integral equation which usually is a singular one must be solved.

For solving the boundary integral equation many types of boundary elements can be used: constant, linear quadratic or higher order boundary elements.

The Boundary Element Method reduces the problem to a system of linear equations (see [1], [2], [3]), and further the problem can be solved with a computer.

The aim of the paper is to solve the problem of the compressible fluid flow around an obstacle using a boundary element approach based on the indirect method with a vortex distribution, and to solve the singular boundary equation that results with linear isoparametric boundary elements of Lagrangean type.

2. Advantages brought by applying BEM with vortex distribution

The problem has been studied by many authors, with different kinds of techniques. There have been made different assumptions for simplifying the mathematical model of the problem. Some early techniques deal with the case of the incompressible fluid flow and use linear equations, linear boundary conditions and sometimes the boundary condition was satisfied not on the boundary but on the chord of the profile.

By applying the BEM to solve this problem only the first assumption is still use. So the BEM uses the nonlinear boundary condition which is satisfied on the obstacle's boundary, not on its chord.

The BEM was first applied only for the incompressible case and the boundary integral formulation was obtained in terms of potential function or stream function. The measures of interest for the problem, like the velocity for example, were obtained after evaluating the derivatives of the unknowns of the problem, and so, new errors were introduced at this stage.

The BEM with vortex distribution, presented in this paper, besides the advantages brought by the BEM, offers also the advantage that deals with the compressible case and leads to a boundary formulation of the problem in terms of primary variables-the components of the velocity field – eliminating so the errors that could appear by evaluating the derivatives, and bringing so more accuracy to the numerical solution.

We first present the problem to solve: a uniform, steady, potential motion of an ideal inviscid fluid of subsonic velocity $U_\infty \bar{i}$, pressure p_∞ and density ρ_∞ is perturbed by the presence of a fixed body of a known boundary, noted *C*, assumed to be smooth and closed. We want to find out the perturbed motion, and the fluid action over the body.

Denoting by $\bar{v}$ the perturbation velocity (u, v its components along the axes) and using dimensionless variables we have the following mathematical model:

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \\ \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 \end{cases}, \qquad (1)$$

with the boundary condition:

$$(\beta + u)n_x + \beta^2 v n_y = 0 \text{ on } C, \qquad (2)$$

where $\bar{n}$ is the normal unit vector outward the fluid, β has the usual signification, $\beta = \sqrt{1 - M^2}$, and M the Mach number for the unperturbed motion.

It is also required that the perturbation velocity vanishes at infinity:

$$\lim_{\infty} \bar{v} = 0.$$

3. The boundary integral equation - vortex distribution

The fundamental solution of vortex type is the solution of the following system (see [6]):

$$\begin{cases} \dfrac{\partial u^*}{\partial x} + \dfrac{\partial v^*}{\partial y} = 0 \\ \dfrac{\partial v^*}{\partial x} - \dfrac{\partial u^*}{\partial y} = \delta(x - \xi, y - \eta) \end{cases}$$

Its name comes from the fact that the perturbation produced by the presence of δ appears in the second equation of (1), equation which expresses the fact that the perturbed motion is irrotational, and it has the following expression (see[6]):

$$\begin{cases} u^* = -\dfrac{1}{2\pi}\dfrac{y-\eta}{(x-\xi)^2+(y-\eta)^2} \\ v^* = \dfrac{1}{2\pi}\dfrac{x-\xi}{(x-\xi)^2+(y-\eta)^2} \end{cases}$$

Approximating the boundary with a continuous distribution of such fundamental solutions, having the unknown intensity $g(\bar{x})$, the components of the perturbation velocity for a point situated in the fliud domain are first found. They are given by the formulas:

$$u(\bar{\xi}) = \frac{1}{2\pi}\oint_C g(\bar{x})\frac{y-\eta}{\left|\bar{x}-\bar{\xi}\right|^2}ds$$

$$v(\bar{\xi}) = -\frac{1}{2\pi}\oint_C g(\bar{x})\frac{x-\xi}{\left|\bar{x}-\bar{\xi}\right|^2}ds$$

For obtaining the components of the perturbation velocity we must take the limit of the above integrals for $\bar{\xi} \to \bar{x}_0$, a regular point on the boundary. As it can be observed, the above integrals are singular for such points.

It is necessary to use the concept of the Cauchy Principal Value of an integral for dealing with the singular integrals see for example [10].

This concept is defined in many books and its definition is very simple and natural.

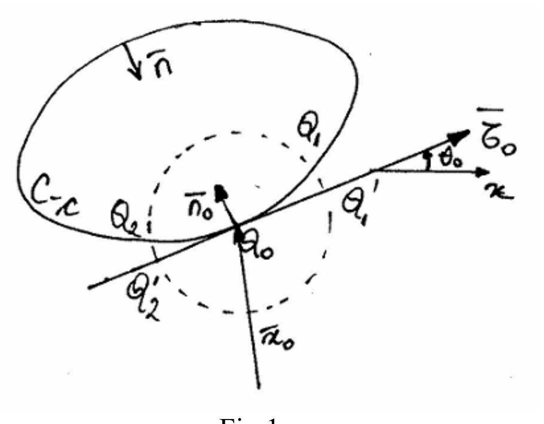

Fig.1.

For evaluate the limit of an integral that has in $\bar{x}_0$ a singularity we have to isolate this point with a circle of a very small radius, noted ε, that intersect the considered boundary along the arc, noted c.

So we have: $\oint_C = \oint_{C-c} + \oint_c$.

If, for $\varepsilon \to 0$, the integral $\oint_{C-c}$ tends to a finite limit, then the limit is called the CPV of the integral. Noting with the prim sign the CPV of an integral, we have the relation:

$$\oint_C{}' = \lim_{\varepsilon \to 0}\oint_{C-c} .$$

Assuming that g is a $H\ddot{o}lder$ function on C, in [6] is obtained an integral formulation for the problem. the components of the perturbation velocity for a regular point on the boundary are found. They are given by the following expressions:

$$\begin{aligned} u(\bar{x}_0) &= \frac{1}{2}g(\bar{x}_0)n_y^0 + \frac{1}{2\pi}\oint_C{}' g(\bar{x})\frac{y-y_0}{\left|\bar{x}-\bar{x}_0\right|^2}ds \\ v(\bar{x}_0) &= -\frac{1}{2}g(\bar{x}_0)n_x^0 - \frac{1}{2\pi}\oint_C{}' g(\bar{x})\frac{x-x_0}{\left|\bar{x}-\bar{x}_0\right|^2}ds \end{aligned} \qquad (3)$$

where n_x^0, n_y^0 are the components of the normal unit vector outward the fluid evaluated at $\bar{x}^0 \in C$.

Using the boundary condition the singular boundary equation is deduced and has the following form:

$$-M^2 g(\bar{x}_0)n_x^0 n_y^0 + \frac{1}{\pi}\int_C{}' g(\bar{x})\frac{\beta^2(x-x_0)n_y^0 - (y-y_0)n_x^0}{\left|\bar{x}-\bar{x}_0\right|^2}ds = $$
$$= 2\beta n_x^0 \qquad (4)$$

with the same notations as before.

The goal of this paper is to solve the singular boundary integral (4) using boundary elements that offer a global continuity for the unknown of the problem, so for the unknown intensity g.

For solving integral equations method of successive approximation, orthogonal polynomials, or Krylov subspaces can be used for example. In case of solving singular boundary integral equations or more general, singular boundary integro-differential equations, approximate solutions can be obtained by using the collocation method as in [4]and [5].

For the singular boundary integral equation (4) in [8] a collocation method is used and good numerical results are obtained.

4. Linear boundary elements for solving the singular boundary inegral equation

In this paper, in order to solve the singular boundary integral equation (4) we use linear isoparametric boundary elements of Lagrangean type (see [1], [2], [3]).

We choose N nodes on the boundary, so on C, and we approximate the boundary with a polygonal line having the segments L_i, $i=1,N$ and the extremes: (x_i^1, y_i^1) and (x_i^2, y_i^2) in a local numbering system.

We have relations:

$$(x_i^2, y_i^2) = (x_{i+1}^1, y_{i+1}^1), \quad 1 \le i \le N-1$$

and

$$(x_N^2, y_N^2) = (x_1^1, y_1^1),$$

contour C being closed.

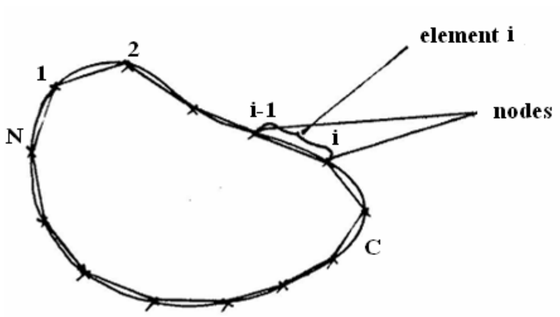

Fig.2.

An isoparametric boundary element uses the same shape functions for local describeing theunknown and the geometry of the element.

For describeing the geometry of a boundary element we use a local system of coordinates which has the origin in the first node of an element , and so we have the relations:

$$\begin{cases} x = x_i^1\varphi^1 + x_i^2\varphi^2 \\ y = y_i^1\varphi^1 + y_i^2\varphi^2 \end{cases}, t \in [0,1], \qquad (5)$$

where φ_1, φ_2 are the shape functions given by:

$$\varphi^1(t) = 1 - t, \varphi^2(t) = t. \qquad (6)$$

Using isoparametric boundary elements we have, for the unknown g, the local representation:

$$g = g_i^1\varphi^1 + g_i^2\varphi^2, \qquad (7)$$

where g_i^1, g_i^2 are the nodal values of the unknown, it means the values of g at the extremes of the boundary element L_i, in the local numbering.

These values satisfy the relations:

$g_i^2 = g_{i+1}^1, \quad 1 \le i \le N-1$, and $g_N^2 = g_1^1$.

For simplifying the writing we shall not use the prim sign to specify that an integral must be understand in its Cauchy sense. For $\bar{x}_0 = \bar{x}_j^1, \forall j = \overline{1,N}$ in equation (4) we obtain an algebraic system of N equations each of them of the following form:

$$-M^2 n_x^j n_y^j g_j^1 + \\ + \frac{1}{\pi} \sum_{i=1}^{N} \int_{L_i} \left(g_i^1\varphi^1 + g_i^2\varphi^2\right) \frac{\beta^2 (x - x_j^1) n_y^j - (y - y_j^1) n_x^j}{\left|\bar{x} - \bar{x}_j^1\right|^2} ds = 2\beta n_x^j$$

(8)

5. Coefficients evaluation

With the notations:

$$a_{ij} = \frac{1}{\pi} \int_{L_i} \varphi^1 \frac{\beta^2 (x - x_j^1) n_y^j - (y - y_j^1) n_x^j}{\left|\bar{x} - \bar{x}_j^1\right|^2} ds,$$

$$b_{ij} = \frac{1}{\pi} \int_{L_i} \varphi^2 \frac{\beta^2 (x - x_j^1) n_y^j - (y - y_j^1) n_x^j}{\left|\bar{x} - \bar{x}_j^1\right|^2} ds \quad (9)$$

we get the following equivalent form for (8):

$$\sum_{i=1}^{N} g_i^1 a'_{ij} + \sum_{i=1}^{N} g_i^2 b_{ij} = 2\beta n_x^j , \qquad (10)$$

where:

$$a'_{ij} = a_{ij} \;\; for\; i \neq j \text{ , and}$$

$$a'_{jj} = \left(-M^2 n_x^j n_y^j\right) + a_{jj} ,\; i, j = \overline{1, N} . \quad (11)$$

After doing some calculous we get the following relations for the above coefficients:

$$a_{ij} = \frac{l_i}{\pi}\int_0^1 (1-t)\frac{\beta^2\left[x_i^1 - x_j^1 + t\left(x_i^2 - x_i^1\right)\right]n_y^j}{at^2 + 2bt + c}dt - $$
$$-\frac{l_i}{\pi}\int_0^1 (1-t)\frac{\left[y_i^1 - y_j^1 + t\left(y_i^2 - y_i^1\right)\right]n_x^j}{at^2 + 2bt + c}dt = $$
$$= \frac{l_i n_y^j \beta^2}{\pi}\left[\left(x_i^1 - x_j^1\right)I_0 + \left(x_i^2 - 2x_i^1 + x_j^1\right)I_1 - \left(x_i^2 - x_i^1\right)I_2\right] - $$
$$-\frac{l_i n_x^j}{\pi}\left[\left(y_i^1 - y_j^1\right)I_0 + \left(y_i^2 - 2y_i^1 + y_j^1\right)I_1\right] - $$
$$-\frac{l_i \beta^2 n_y^j}{\pi}\left(y_i^2 - y_i^1\right)I_2$$

$$b_{ij} = \frac{l_i}{\pi}\int_0^1 t\frac{\beta^2\left[x_i^1 - x_j^1 + t\left(x_i^2 - x_i^1\right)\right]n_y^j}{at^2 + 2bt + c}dt - $$
$$-\frac{l_i}{\pi}\int_0^1 t\frac{\left[y_i^1 - y_j^1 + t\left(y_i^2 - y_i^1\right)\right]n_x^j}{at^2 + 2bt + c}dt = $$
$$= \frac{l_i n_y^j \beta^2}{\pi}\left[\left(x_i^1 - x_j^1\right)I_1 + \left(x_i^2 - x_i^1\right)I_2\right] - $$
$$-\frac{l_i n_x^j}{\pi}\left[\left(y_i^1 - y_j^1\right)I_1 + \left(y_i^2 - y_i^1\right)I_2\right]$$

(12)

With I_k, $k = 0,1,2$ we have noted the following integrals:

$$I_k = \int_0^1 \frac{t^k}{at^2 + 2bt + c}dt,\; k = 0,1,2 ,$$

where $a = l_i^2$,

$$b = \left(x_i^1 - x_j^1\right)\left(x_i^2 - x_i^1\right) + \left(y_i^1 - y_j^1\right)\left(y_i^2 - y_i^1\right),$$

$$c = \left|\overline{x}_i^1 - \overline{x}_j^1\right|^2 .$$

For the components of the the normal unit vector we use the relations:

$$n_x^j = \frac{y_j^2 - y_j^1}{l_j}, n_y^j = \frac{x_j^1 - x_j^2}{l_j},\; \forall j = \overline{1, N} . \quad (13)$$

A computer code can be use to evalute these integrals but, for limitting the errors that appare because of the numerical approach, the nonsingular integrals are computed analitycally and for the singular ones the definition of the Cauchy Principal Value is used.

a)The nonsingular case

For $i \neq j-1$ when $j = \overline{2, N}$ and $i \neq N$ when $j = 1$, we get the following expressions:

$$I_0 = \frac{1}{\sqrt{ac - b^2}} arctg \frac{\sqrt{ac - b^2}}{c + b} ,$$

$$I_1 = \frac{1}{2a}\ln\frac{a + 2b + c}{c} - \frac{b}{a\sqrt{ac - b^2}} arctg \frac{\sqrt{ac - b^2}}{c + b} ,$$

$$I_2 = \frac{1}{a} - \frac{b}{a^2}\ln\frac{a + 2b + c}{c} + \frac{2b^2 - ac}{a^2} I_0 . \quad (14)$$

b) The singular case

For $i = j - 1 \left(j = \overline{2, N}\right)$ and for $i = N$ when $j = 1$, so for the singular integrals that appear we get:

$$I_0 = I_1 = -\frac{1}{l_i^2} \text{ , and } I_2 = 0 . \qquad (15)$$

Achieving this stage we can observe an important aspect: all the coefficients in (10) can be analytically evaluated and they depend only on the coordinates of the nodes chosen for the boundary discretization.

Returning to the global system of notation, so considring that:

$g_i^2 = g_{i+1}^1 = g_{i+1}\; for\; i = \overline{1, N-1}$, $g_N^2 = g_1^1 = g_1$, and noting:

$$A_{ij} = a'_{ij} + b_{i-1j} \ \ for\ i = \overline{2,N},$$

$$A_{1j} = a'_{1j} + b_{Nj} \text{ and } T_j = 2\beta n_x^j, \ \ \forall j = \overline{1,N} \quad (16)$$

we deduce the following equivalent expression for system (10):

$$\sum_{i=1}^{N} A_{ij} g_i = T_j \ , j = \overline{1,N}. \quad (17)$$

For evaluating the singular integrals that appeare when solving second order elliptic equation of Poisson type, for the three dimensional case, with Boundary Element Method, an approximate technique based on the auto solid angle evaluation can be used as in [11].

6. Evaluating the nodal values of the velocity's components and of the local pressure coefficient on the boundary

After solving this system and finding the nodal values for the unknown function, in fact the nodal values of the vortex intensities, noted $g_i, i = \overline{1,N}$, the components of the velocity on the boundary (for the node $\overline{x}_j^1, j = \overline{1,N}$) can be evaluated starting from formulas (3).

With the same notations as before we get the following expressions:

$$u_j^1 = \frac{1}{2} g_j^1 n_y^j + \frac{1}{2\pi} g_j^1 \frac{y_j^1 - y_j^2}{l_j} + \frac{1}{2\pi} g_j^2 \frac{y_j^2 - y_j^1}{l_j} +$$
$$+ \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} g_i^1 \left[\left(y_i^1 - y_j^1\right) I_0 + \left(y_i^2 - 2y_i^1 + y_j^1\right) I_1 - \left(y_i^2 - y_i^1\right) I_2 \right] +$$
$$+ \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} g_i^2 \left[\left(y_i^1 - y_j^1\right) I_1 + \left(y_i^2 - y_i^1\right) I_2 \right], \quad (18)$$

$$v_j^1 = -\frac{1}{2} g_j^1 n_x^j - \frac{1}{2\pi} g_j^1 \frac{x_j^1 - x_j^2}{l_j} - \frac{1}{2\pi} g_j^2 \frac{x_j^2 - x_j^1}{l_j} -$$
$$- \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} g_i^1 \left[\left(x_i^1 - x_j^1\right) I_0 + \left(x_i^2 - 2x_i^1 + x_j^1\right) I_1 - \left(x_i^2 - x_i^1\right) I_2 \right] -$$
$$- \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{l_i}{2\pi} g_i^2 \left[\left(x_i^1 - x_j^1\right) I_1 + \left(x_i^2 - x_i^1\right) I_2 \right] \quad (19)$$

We can compute the fluid velocity for different poi.nts of the fluid domain too using the discretizated expressions of (3) .

Regarding the fluid action over the body, we can evaluate the local pressure coefficient, noted c_p, using the relation:

$$c_p = -u^2 - v^2 - 2u. \quad (20)$$

This coefficient is one of great importance for the problem because it is used to obtain the lift force. It is known that for profiles with smooth boundary the lift force doesn't appear because of the same values of the local pressure coefficient on the intra and extrados of the profile.

It is important to specify that all the coefficients in system (17) have analytical expressions and therefore no errors are introduced for their evaluations. All these coefficients depend only on the coordinates of the nodes used for the boundary discretization, and so, it can be use a computer code to solve the problem.

7. Numerical results and conclusions

For solving system (17) and for evaluating the fluid velocity and the local pressure coefficients there is developed a computer code in MATHCAD that uses relations (18), (19), (20). These numerical solutions are compared with the exact solutions that exist for the particular case of a circular obstacle and an incompressible fluid (M=0).

In [7] the bidimensional problem of the incompressible fluid flow around a circular obstacle is exactly solved. The expressions of the components of the perturbed fluid velocity are obtained and they are given by the following relations:

$$u' = -U_\infty \cos 2\theta \ , v' = -U_\infty \sin 2\theta$$

For the dimensionless components we get:

$$u = -\cos 2\theta \ , v = -\sin 2\theta \ , \quad (21)$$

and further, for the local pressure coefficient, the following expression:

$$cp = -1 + 2\cos 2\theta \ . \quad (22)$$

Another computer code gives us the solution for this case. Both programs can be run for different number of nodes used for the boundary discretization.

For the case when we use 20 nodes for the discretization the solutions obtained are represented in the following graphics.

In Fig. 3. there are represented the values obtained for the velocity component along the Ox axis. In Fig. 4. there are represented the values obtained for the velocity component along the Oy axis.

The pressure coefficient is represented in Fig. 5. The numerical solution is in good agreement with the exact one.

We can verify with this graphic a well known result too: the circular obstacle is a non-lifting profile because of the local pressure coefficient symmetry. For corresponding nodes on the upper and the lower boundary it takes the same value. As we know this is a consequence of the fact that the analyzed profile has a smooth boundary.

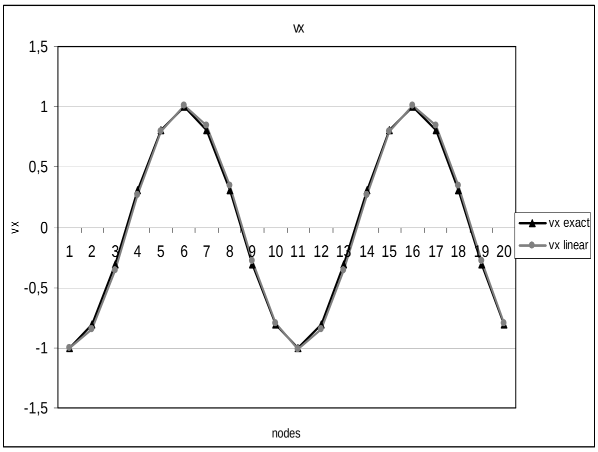

Fig. 3. The velocity along the Ox axis: case of vortex distribution with linear boundary elements and the exact solution.

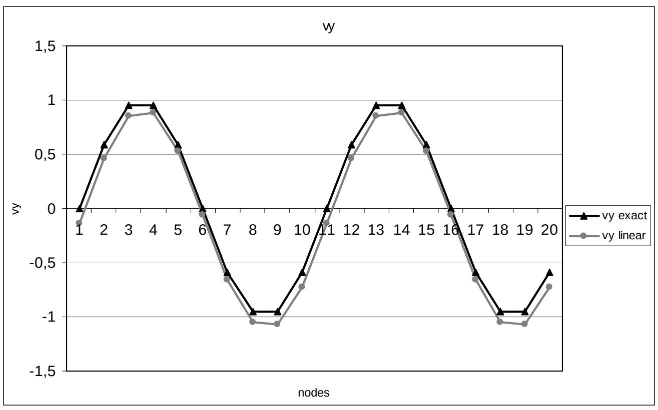

Fig. 4. The velocity along the Oy axis: case of vortex distribution with linear boundary elements and the exact solution.

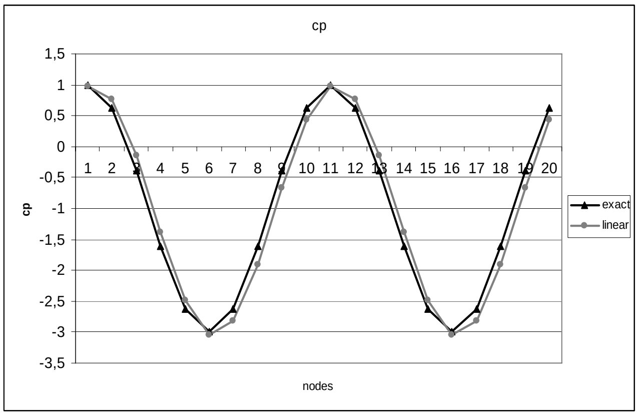

Fig. 5. The local pressure coefficient for the case of vortex distribution with linear boundary elements, and the exact solution.

As we can see from the graphics the numerical solutions are in good agreement with the exact solution, and a small number of elements (20) is sufficient for obtaining satisfactory results.

As it is natural the numerical solution is influenced by the number of nodes chosen for the boundary discretization. We can observe this from the following graphics where the nodal values of the local pressure coefficient are performed for different number of nodes on the boundary. There were considered 10, 15, 25, and 30 nodes for the boundary discretization.

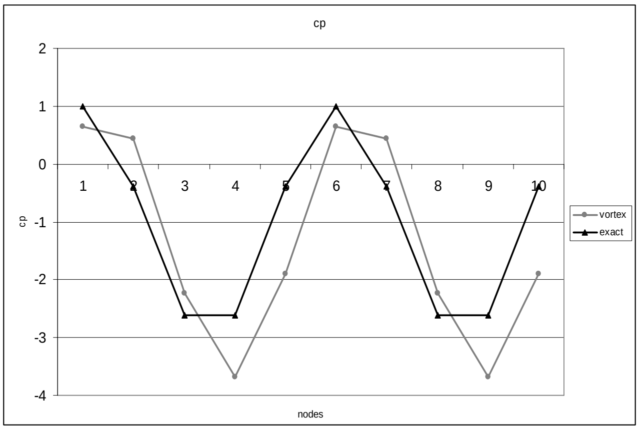

Fig. 6. The local pressure coefficient for the case of 10 nodes: numerical solution and exact solution.

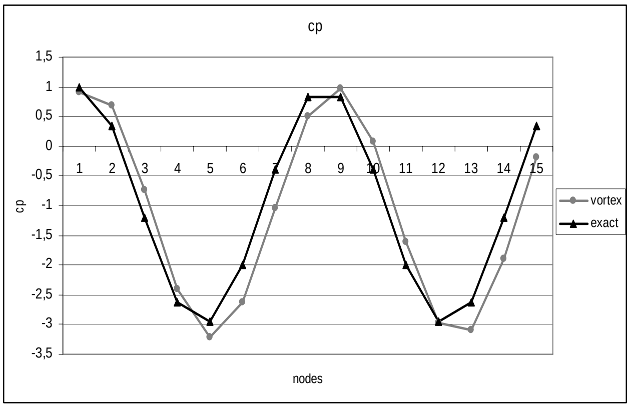

Fig. 7. The local pressure coefficient for the case of 15 nodes: numerical solution and exact solution.

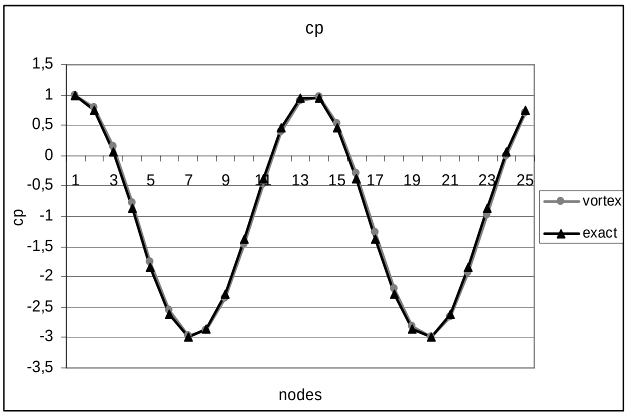

Fig. 8. The local pressure coefficient for the case of 25 nodes: numerical solution and exact solution.

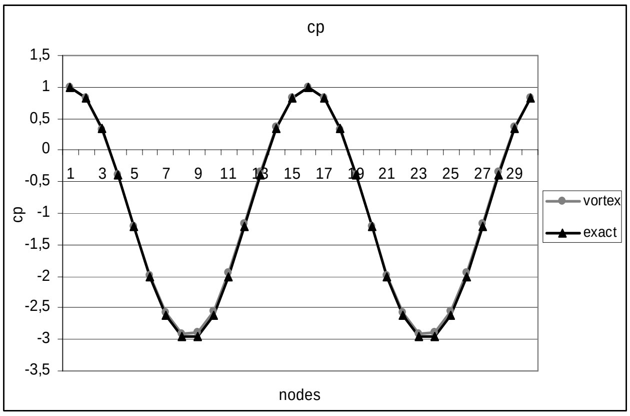

Fig. 7. The local pressure coefficient for the case of 30 nodes: numerical solution and exact solution.

As expected, better results are obtained when using more nodes on the boundary, but the results are very good when using 20, 25 and 30 nodes. For better observing the numerical solution improvement brought by the growth number of nodes we consider in Fig. 8 the maximum values for the errors that appear in each of the above cases. We notice that these values decrease with the growth of the nodes number.

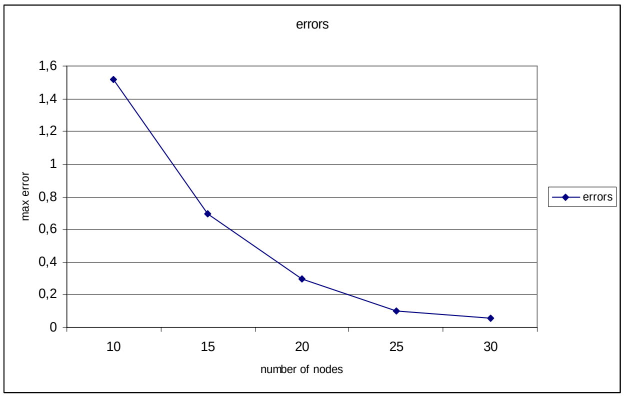

Fig. 8. The maximum errors for 10, 15, 20, 25 and 30 nodes on the boundary.

We can also see from the graphic that a number of nodes bigger than 20 for the boundary discretization does not lead to a substantial improvement so much that to justify the computational effort.

It appears reasonable to expect better results by using higher order boundary elements for solving the singular boundary integral equation because they allow a better approximation of the geometry,

In paper [9] the boundary integral equation, obtained as an equivalent form for the involved problem, by applying the indirect method with sources distribution of unknown intensities, is solved with linear isoparametric boundary elements of Lagrangean type.

In the same paper the numerical solution is compared to the exact one for the same particular case: the circular obstacle and the incompressible fluid, and there were obtained good results. For the boundary discretization there were also used 20 nodes.

In the following paragraphs the numerical solution obtained in this paper is compared to the exact one, and to the one obtained in case of sources distribution, for the mentioned particular case-the circular obstacle. The comparison study is made through the local pressure coefficient, *cp* .

In the following graphics we perform the exact nodal values of *cp* and the numerical ones obtained with sources distribution and vortex distribution and the errors that appear in each case in order to see which of the two numerical solutions offers a better result.

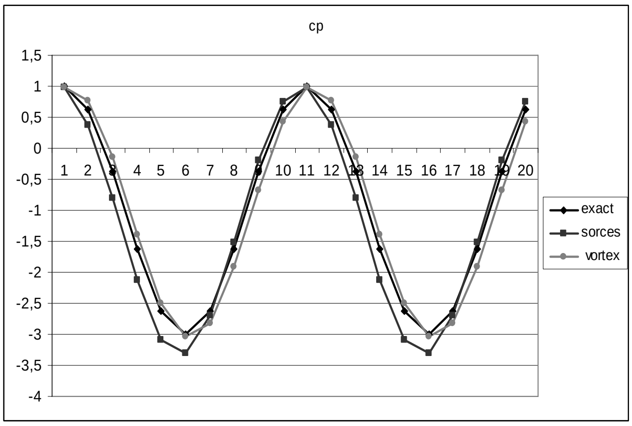

Fig. 9. The local pressure coefficient: exact solution, sources distribution and vortex distribution.

The errors that appear are represented in the following graph. Because of the symmetry of the profile the numerical solution is also symmetrical and so the errors are.

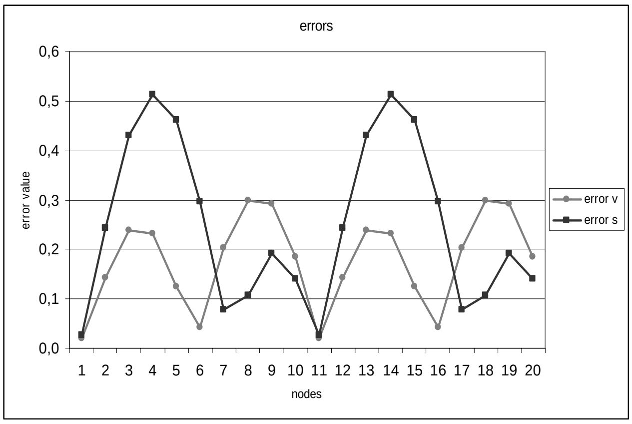

Fig. 10. The errors between the exact nodal values of the local pressure coefficient and the numerical ones obtained: with vortex distribution (error v) and sources distribution (error s).

As we can see the errors obtained when the obstacle's boundary is assimilated with a vortex distribution are smaller than the errors obtained in case of the sources distribution for many nodes (12 from 20 nodes) and also there is a big difference between the two maximum errors values obtained in these cases. This can be better notice from the following figure.

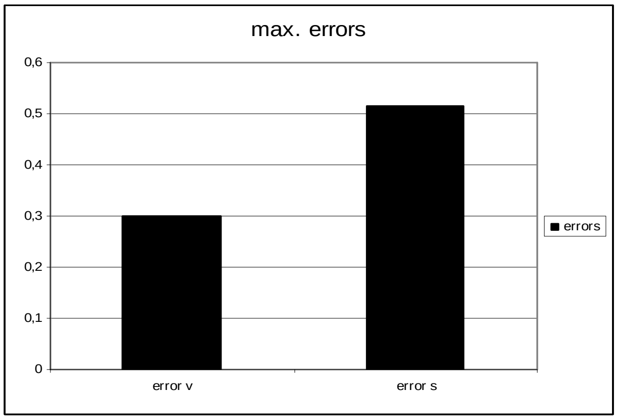

Fig. 11. The maximum errors for the case of vortex and sources distribution.

The numerical results presented in the above paragraphs show that the indirect boundary element method with vortex distribution and linear boundary elements offers for the problem of the compressible fluid flow around an obstacle a better solution than the one that uses a sources distribution and linear boundary elements, and very good results for a quite small number of discretization nodes.

With the same computer code based on the method presented in this paper, numerical solutions can be obtained for any kind of compressible fluid flows, for different values of Mach number, not only for the incompressible case, and for other kinds of obstacles with smooth boundaries too.

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