Incomplete Linguistic Preference Relations to Evaluate Multimedia Authoring System

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Abstract: - MCMD problems with fuzzy preference information on alternatives are essential problems of the importance of weighting and ranking. In this study, the AHP method is reviewed, and then Fuzzy PreRa and incomplete linguistic preference relations methods are elucidated. This study applied above three MCDM methods to a software selection problem proposed by Lai et al. [Software selection: a case study of the application of the analytical hierarchical process to the selection of a multimedia authoring system, *Information and Management*, Vol.36, 1999, pp.221-232.]. The outcome obtained by Fuzzy PreRa and incomplete linguistic preference relations methods almost coincides with that produced by the AHP method, also with the least judgments. The result shows that the approach developed is simple and comprehensible in concept, efficient in computation, and robust in modeling human evaluation processes which make it of general use for solving practical qualitative multi-criteria problems.

Key-Words: - AHP, fuzzy preference relations, incomplete linguistic preference relations, multimedia authoring system (MAS), multi-criteria decision making (MCDM), selection

1 Introduction

Multi-media computing and technology are currently areas of intense interest and future promise for the development of software and hardware. In the past decade, advances in compact disk storage, high quality audio, high resolution video, broadband fiber network, and multimedia database technologies have made possible the creation, processing, storage, management, and communication of multimedia information systems (MMISs) [8, 10, 16, 18, 19]. The tools required for developing these interactive MMISs are called multimedia authoring systems (MASs).

MASs are centralized, stand-alone development tools to create MMISs [2]. MASs support different hardware devices and file formats, thus, providing an effective means of conveying information, such as graphics, text, sound, animation, and video data types. In recent years the MAS tools have increased significantly and prices have declined dramatically. Many of these tools were developed to fit different user needs and were designed to execute on a variety of hardware platforms. Although each tool offers its own particular way of assembling an MMIS, a number of distinct authoring paradigms can be identified: structuring, timeline, flowchart, and script [12]. Owing to the complexity of the product and profusion of alternatives, a systematic process for selection is necessary and important. Clearly, software selection is not a well-defined or structured decision problem. Although it is recognized that MAS selection is a multi-criteria problem and needs a multi-criteria method to solve it.

The typical multi-criteria decision making (MCDM) problem often requires the decision maker to provide qualitative assessments for determining (a) the performance of each alternative with respect to each criterion and (b) the relative importance of the evaluation criteria with respect to the overall objective of the problem. During the past two decades, there has been a steady growth in the number of MCDM methods for assisting decision making with multiple objectives. One of the commonly used methods for multi attribute decision-making is analytic hierarchy process (AHP), which was developed by Saaty [20, 21].

The strength of AHP is that it organizes tangible and intangible factors in a systematic way, and provides a structure, yet relatively simple solution to decision-making problems [24]. Up to present, the AHP method has been applied in many different domains, including risk assessment [25], enterprise resource planning assessment [32], project management [1] and so forth. Although, AHP is a commonly used decision-aiding tool for resolving multi-criteria decision problems. However, decision maker sometimes find it difficult to ensure a consistent pairwise comparison between voluminous decisions. The cause of which is that the consistency ratio (CR) is produced after the evaluation process and its global acceptance criteria is limited. So, there are some methods have been proposed to improve the AHP.

The earliest work in fuzzy AHP appeared in van Laarhoven and Pedrycz [27], which compared fuzzy ratios described by triangular membership functions. Buckley [4] investigated fuzzy weights and fuzzy utility for AHP technique, extending AHP by the geometric mean method to derive the fuzzy weight. Chang [5] introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pairwise comparison scale of fuzzy AHP, and the use of extent analysis method for the synthetic extent values of the pairwise comparisons. Cheng [6] proposed a new algorithm for evaluating naval tactical missile systems by the fuzzy analytical hierarchy process based on grade value of membership function. Cheng et al. [7] proposed a new method for evaluating weapon systems by analytical hierarchy process based on linguistic variable weight. Zhu et al. [36] discussed on extent analysis method and applications of fuzzy AHP. Leung and Cao [14] proposed a fuzzy consistency definition with consideration of a tolerance deviation.

There are other ways to solve AHP problem, such as referenced AHP [23], extended fuzzy AHP [31], modified AHP (MAHP) [26], random-AHP [15], chain-wise paired comparisons (CPC) [17] and DS/AHP [3].

To facilitate the pairwise comparison process and to avoid the complex and unreliable process of comparing fuzzy utilities, Herrera-Viedma et al. [11] developed a new method, fuzzy preference relations (Fuzzy PreRa), which focuses on avoiding the inconsistent solutions in the decision-making processes. Moreover, using AHP method needs to do $n(n-1)/2$ pairwise comparisons, but by using Fuzzy PreRa method only $n-1$ comparisons needed.

However, in real life, because of time pressure and decision maker's limited related knowledge, decision maker may develop an incomplete fuzzy preference relation in which some of the elements cannot be provided. Hence, Xu [33] developed a method that based on the operational laws of the linguistic evaluation scale and acceptable incomplete linguistic preference relation with the least judgments, for constructing a consistent complete linguistic preference relation by using the additive transitivity property. This study applied three MCDM methods (AHP, Fuzzy PreRa method, and incomplete linguistic preference relations methods) to a selection of MAS on the case study in [13] by Lai et al.

The rest of this paper is organized as follows. Section 2 describes the concept of preference relations. In Section 3, we briefly review the AHP, Fuzzy PreRa and incomplete linguistic preference relations methods. An empirical study of software selection is presented in Section 4. Finally, discussion and some concluding remarks are presented in Section 5 and 6, respectively.

2 Preference Relations

In many decision-making approaches, such as AHP, Fuzzy PreRa, the decision information is represented in the format of preference relations. Suppose there is a set of alternatives $X = \{x_1, x_2, ..., x_n\}$, the preference relations are described by decision-maker as follows [28]:

(a) Multiplicative preference relations: A decision maker's preference on a set of alternatives X is denoted by a positive preference relation matrix $A \subset X \times X$, $A = (a_{ij})_{n\times n}, a_{ij} \in [\frac{1}{9}, 9]$, where a_{ij} is the ratio of the preference degree of alternative x_i over x_j. As $a_{ij} = 1$ indicates indifference between x_i and x_j, $a_{ij} = 9$ indicates x_i is extremely preferred to x_j. A is assumed multiplicative reciprocal, given by $a_{ij} \cdot a_{ji} = 1 \ \forall i, j \in \{1, ..., n\}$.

(b) Fuzzy preference relations: A decision maker's preference on a set of alternatives X is denoted by a positive preference relation matrix $P \subset X \times X$, with membership function: $\mu_p : X \times X \to [0,1]$, where $\mu_p(x_i, x_j) = p_{ij}$ indicates the ratio of the preference intensity of alternative x_i to that of x_j. If $p_{ij} = \frac{1}{2}$ implies there is no difference between x_i and x_j $(x_i \sim x_j)$, $p_{ij} > \frac{1}{2}$ implies x_i is preferred to x_j $(x_i > x_j)$, $p_{ij} = 1$ indicates that x_i is absolutely preferred to x_j, $p_{ij} = 0$ indicates that x_j is absolutely preferred to x_i. P is assumed additive reciprocal, given by $p_{ij} + p_{ji} = 1\ \forall i, j \in \{1,...,n\}$.

3 Methodology

In this section, three different MCDM methods are presented. The first one is AHP proposed by Saaty [21]. The second is Fuzzy PreRa method developed by Herrera-Viedma et al. [11]. Finally, an incomplete linguistic preference relations method is introduced.

3.1 Analytic hierarchy process (AHP)

AHP is developed by Saaty [21]. With this method, a complicated system is converted to a hierarchical system of elements. In each hierarchical level, pairwise comparisons of the elements are made by using a nominal scale. Usually, the decision maker uses a 1–9 scale. According to the pair combination method, there should be $n(n-1)/2$ times of evaluation to be completed. These comparisons constitute a comparison matrix. To find the weight of each element, or the score of each alternative, the eigenvector of this matrix is calculated. At the end, the consistency of the pairwise comparisons is calculated by using a consistency ratio. The consistency ratio (CR) is defined as $CR = CI / RI$.

The ‘‘consistency index’’ (CI), is given by Eq.(1).

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{1}$$

The random consistency index (the RI given in Table 1) corresponds to the degree of consistency that automatically arises when completing at random reciprocal matrixes with the values on the 1–9 scale.

Table 1 Random consistency index

r	1	2	3	4	5
RI	0.00	0.00	0.58	0.90	1.12
r	6	7	8	9	10
RI	1.24	1.32	1.41	1.45	1.49
r	11	12	13	14	15
RI	1.51	1.48	1.56	1.56	1.59

Saaty [22] has argued that the inconsistency should not be higher than 10% ($CR \le 0.10$).

The AHP approach involves four essential steps [35] that can be summarized as follows:

1. Reduce the decision problem into a hierarchy of interrelated decision elements (factors/criteria and alternatives).
2. Collect input data by pairwise comparisons of decision elements.
3. Use the eigenvalue method to estimate the relative weights of decision elements.
4. Aggregate the relative weights of decision elements to arrive at a set of ratings for the decision alternatives.

3.2 Fuzzy preference relations (Fuzzy PreRa)

Fuzzy PreRa method was proposed by Herrera-Viedma et al. in 2004 [11], which focus on avoiding inconsistent solutions in the decision-making processes.

The concept of this method is that if there are n attributes $X = \{x_1,...,x_n, n \ge 2\}$, then we can obtain the pairwise preference relation data $\{p_{12}, p_{23},..., p_{n-1n}\}$, from $n-1$ comparing and constructing a consistent reciprocal fuzzy preference relations P'. This method follows the one of traditional AHP method characteristics, which is preference relation satisfied transitivity property.

Herrera-Viedma et al. [11] had proof that for a reciprocal additive fuzzy preference relation $P = (p_{ij})$, the following statements are equivalent:

$$p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \forall i < j < k \tag{2}$$

$$p_{i(i+1)} + p_{(i+1)(i+2)} + \cdots + p_{(j-i)j} + p_{ji} = \frac{j-i+1}{2}\ \ \forall i < j \tag{3}$$

According to Eq.(3), therefore, we can deduce that

$$p_{ji} = \frac{j-i+1}{2} - p_{i(i+1)} - p_{(i+1)(i+2)} - \cdots - p_{(j-1)j} \tag{4}$$

and based on the additive reciprocal

$$p_{ij} + p_{ji} = 1\ \forall i, j \in \{1,\ldots,n\} \tag{5}$$

The steps of using Fuzzy PreRa method are described in the following:

1. Compute the set of preference values B as

$$B = \{p_{ij}, i < j \wedge p_{ij} \notin \{p_{12}, p_{23},..., p_{n-1n}\}\} \tag{6}$$

2. Find P

$$P = \{p_{12}, p_{23},..., p_{n-1n}\} \cup B \cup \{1-p_{12}, 1-p_{23},..., 1-p_{n-1n}\} \cup \neg B \tag{7}$$

3. The consistent fuzzy preference relation P' is obtained as $P' = f(P)$ such that

$f:[-a,1+a] \rightarrow [0,1]$

$$f(x) = \frac{x+a}{1+2a} \quad (8)$$

If there is a set of alternatives $X = \{x_1,...,x_n\}$, which is associated with a reciprocal multiplicative preference relation $A = (a_{ij})$, and $a_{ij} \in [1/9, 9]$, then we can use a transformation function g [9],like Eq.(9) to find the corresponding reciprocal additive fuzzy preference relation $P = (p_{ij})$, and $p_{ij} \in [0,1]$.

$$p_{ij} = g(a_{ij}) = 1/2 \cdot (1 + \log_9 a_{ij}) \quad (9)$$

Fedrizzi [9] proofed that it can be said that by means of function g it is possible to transform, in a certain sense, a "multiplicative" formulation of the problem into an "additive" one. After Fuzzy PreRa method was proposed, Wang and Chen [30] adapted it with the linguistic concept, which uses variables as fuzzy linguistic assessments, and proposed it as the fuzzy linguistic preference relations method.

3.3 Incomplete linguistic preference relations

If decision makers can do pairwise comparison for all properties of preference matrices, it is called "complete linguistic preference relations," otherwise it is called "incomplete linguistic preference relations." The following briefly describes some definitions about incomplete linguistic preference relations presented in [30, 33].

Let $S = \{s_\alpha | \alpha = -t,...,t\}$ be a finite and totally ordered discrete term set, whose cardinality value is odd one, such as 7 and 9, so the medium represents and assessment of indifference, and with the rest of the terms being placed symmetrically around it. Each term, s_i is a linguistic variable and represents a possible value. There are two characteristics about s_i, one is the set ordered $s_\alpha > s_\beta$ if and only if $\alpha > \beta$. The other is the negation operator $neg(s_\alpha) = s_{-\alpha}$ and $neg(s_0) = s_0$. For example, S can be defined as [34] (Table 2).

Table 2 Evaluation of linguistic variables

Item	Linguistic value
Extremely good (EG)	s_4
Very good (VG)	s_3
Good (G)	s_2
Slightly good (SG)	s_1
Fair (F)	s_0
Slightly poor (SP)	s_{-1}
Poor (P)	s_{-2}
Very poor (VP)	s_{-3}
Extremely poor (EP)	s_{-4}

It is clear that the medium represents an assessment of "indifference," and with the rest of the terms being placed symmetrically around it.

Let $A = (a_{ij})_{n \times n}$ be linguistic preference relation, if A is a complete linguistic operation, then the operation is as follows:

$s_\alpha, s_\beta \in S$

$s_\alpha \oplus s_\beta = \max\{s_{-t}, \min\{s_{\alpha+\beta}, s_t\}\}$

$\lambda s_\alpha \oplus s_{\lambda\alpha}, \lambda \in [0,1]$.

Definition 1 Let $A = (a_{ij})_{n \times n}$ be an linguistic preference relation, then A is called a complete linguistic preference relation, if

$$a_{ij} \in S,\ a_{ij} \oplus a_{ji} = s_0,\ a_{ii} = s_0, \text{ for all } i, j \quad (10)$$

Additionally, A is called a consistent complete linguistic preference relation, if

$$a_{ij} = a_{ik} \oplus a_{kj}, \text{ for all } i, j, k \quad (11)$$

Eq.(11) is a type of additive transitivity.

Definition 2 Let $A = (a_{ij})_{n \times n}$ be a linguistic preference relation, if A is an incomplete linguistic preference relation, which means decision makers are unable to provide preference values for all pair of alternatives, and thus some of them are missing. Using the symbol × represents unknown variable while decision makers cannot compare its attribute, which satisfy

$$a_{ij} \in S,\ a_{ij} + a_{ji} = s_0,\ a_{ii} = s_0 \quad (12)$$

Additionally, A is called a consistent incomplete linguistic preference relation, if

$$a_{ij} = a_{ik} \oplus a_{kj} \quad (13)$$

There are some concepts about incomplete linguistic preference define as follows:

(a) Incomplete linguistic preference adjoining relation. Let $A = (a_{ij})_{n \times n}$ be a linguistic preference relation, if A is an incomplete linguistic preference relation, if $(i,j) \cap (k,l) \neq \phi$, the element a_{ij} and a_{kl} are called adjoining.

(b) Incomplete linguistic preference indirect relation. Let $A = (a_{ij})_{n \times n}$ be a linguistic preference relation, if A is an incomplete preference relation, we suppose $a_{i_0 j_0}$ is the unknown value in preference matrix A. The element $a_{i_0 j_0}$ is called "indirect relation" which is obtained from the adjoining elements $a_{i_0 k}$ and $a_{k j_0}$.

(c) Acceptable project of incomplete linguistic preference. Let $A=(a_{ij})_{n\times n}$ be linguistic preference relation, if A is an incomplete linguistic preference relation, it is called "acceptable project" by obtaining all unknown variable × through adjoining known elements.

Therefore, if A is acceptable project of incomplete linguistic preference, it can be the known value in a column or row, and having $n-1$ contrasting values by pairs.

Let $X=\{x_1,x_2,...,x_n\}$ is a set of alternatives; based on s_i linguistic value, every decision maker makes adjoining comparison for the known criterion. It could generate $n-1$ preference value, and obtain acceptable incomplete linguistic relation matrix $A=(a_{ij})_{n\times n}$. Then based on equations preference relation matrix is generated. For different known criterion of decision maker's choice, it can obtain few matrices that are shown as follows:

Type 1: Oblique comparison of each pairs

$$^{r}\mathrm{A}^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & {}^{r}a_{12}^{(e)} & \times & \times & \times & \times \\ \times & 0 & {}^{r}a_{23}^{(e)} & \times & \times & \times \\ \times & \times & 0 & {}^{r}a_{34}^{(e)} & \times & \times \\ \times & \times & \times & 0 & \ddots & \times \\ \times & \times & \times & \times & 0 & {}^{r}a_{m-1\ m}^{(e)} \\ \times & \times & \times & \times & \times & 0 \end{bmatrix}_{m\times m}$$

Type 2: Vertical comparison of each pairs

$$^{r}\mathrm{A}^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & \times & {}^{r}a_{13}^{(e)} & \times & \times & \times \\ \times & 0 & {}^{r}a_{23}^{(e)} & \times & \times & \times \\ \times & \times & 0 & \times & \times & \times \\ \times & \times & {}^{r}a_{43}^{(e)} & 0 & \times & \times \\ \times & \times & \vdots & \times & 0 & \times \\ \times & \times & {}^{r}a_{m3}^{(e)} & \times & \times & 0 \end{bmatrix}_{m\times m}$$

Type 3: Horizontal comparison of each pairs

$$^{r}\mathrm{A}^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & {}^{r}a_{12}^{(e)} & {}^{r}a_{13}^{(e)} & {}^{r}a_{14}^{(e)} & \cdots & {}^{r}a_{1m}^{(e)} \\ \times & 0 & \times & \times & \times & \times \\ \times & \times & 0 & \times & \times & \times \\ \times & \times & \times & 0 & \times & \times \\ \times & \times & \times & \times & 0 & \times \\ \times & \times & \times & \times & \times & 0 \end{bmatrix}_{m\times m}$$

Here, we proposed a transformation function g, it is possible to transform a multiplicative preference relation $A=(a_{ij})_{n\times n}$ with $a_{ij}\in[\frac{1}{9},9]$ into an additive preference relation $P=(p_{ij})_{n\times n}$ with $p_{ij}\in[-t,t]$. It is defined as

$$p_{ij}=g(a_{ij})=k\times\log_9(a_{ij})\text{, where } k=t \tag{14}$$

For example, if $p_{ij}\in[-4,4]$ to $A=(a_{ij})_{n\times n}$ with $a_{ij}\in[\frac{1}{9},9]$, the evaluation of linguistic variables as shown in Table 3.

Table 3 Evaluation of linguistic variables form $[\frac{1}{9},9]$ transform into $[-4,4]$

AHP	New	AHP	New
1/9	-4	9	4
1/8	-3.7856	8	3.7856
1/7	-3.5425	7	3.5425
1/6	-3.2619	6	3.2619
1/5	-2.9299	5	2.9299
1/4	-2.5237	4	2.5237
1/3	-2	3	2
1/2	-1.2619	2	1.2619
1	0		

The steps of using incomplete linguistic preference relations method are described in the following:

1. Determine the decision-makers, alternatives and criteria.
2. Decide the appropriate linguistic variables and its correspondent values $[-t,t]$.
3. Use one of above type to construct decision matrix of preference relations.
4. Utilize the algorithm rules to obtain the entire decision matrix.
5. Integrate all of the alternatives' evaluation values that are apprised by all of the decision makers.
6. Rank the result so as to select the best alternative.

4 Numerical example

This section briefly describes the decision problem and the methods used to derive the priorities to be assigned to the alternatives.

Software selection is not a well-defined or structured decision problem. In the paper, "Software selection: a case study of the application of the analytical hierarchical process to the selection of a multimedia authoring system," Lai et al. [13] applied AHP method to software selection. There are six important factors used to measure the performance of MAS products based on their technical capabilities and their ability to fulfill managerial expectations. It includes: development interfaces (DI), graphic support (GS), multi-media data support (MS), data file support (DS), cost effectiveness (CE) and vendor support (VS).

In development interfaces (DI) criteria, a good MAS should have at least two levels of user support – a novice and an expert mode. The graphic attributes should follow graphic standards, which makes it possible to migrate graphs to different types of hardware systems and to be used in different implementations and applications. In multi-media data support (MS), a good MAS needs to be able to manage dynamic data types and compress audio and video information to appropriate levels before it is stored on disk. A feature that distinguishes the better MAS products is data file support and management. Obviously, good MAS must provide a level of performance that is acceptable for the application, even when large files are associated with audio, images, and video. In cost effectiveness (CE), alternative MAS products must be evaluated cautiously to determine their explicit and implicit costs. Finally, the quality of ongoing vendor support (VS) is of major importance in MAS selection.

4.1 MAS selection with AHP by Lai *et al.* [13]

Lai et al. [13] used the AHP method to solve the selection problem of MASs. Decision makers judge the importance of one alternative over another can be made subjectively and converted to a numerical value using a scale of 1-9 where 1 denotes equal importance and 9 denotes the highest degree of favoritism. The preference relation matrix for pairwise comparison of criteria is shown in Table 4 (which represents the evaluation of member 1 of the group) [13].

Table 4 Preference relation matrix for pairwise comparison of criteria (AHP)

	DI	GS	MS	DS	CE	VS
DI	1	1/3	1/4	1/3	5	7
GS	3	1	1/3	1/4	5	6
MS	4	3	1	2	7	8
DS	3	4	1/2	1	6	8
CE	1/5	1/5	1/7	1/6	1	5
VS	1/7	1/6	1/8	1/8	1/5	1

Notice the reciprocals across the diagonal. That is, (development interface, graphics support) is 3 while (graphics support, development interface) is 1/3.

After obtaining the pairwise comparisons, a normalized matrix was developed in Table 5 by dividing each element by the sum of its respective columns. The row entries in the last two columns of the normalized matrix table comprised the sum of six elements in the row and the average of those row elements (principal vector), respectively.

Table 5 Normalized matrix of Table 4

	DI	GS	MS	DS	CE	VS
DI	0.0881	0.0383	0.1063	0.0853	0.2066	0.2000
GS	0.2645	0.1149	0.1417	0.0645	0.2066	0.1714
MS	0.3527	0.3448	0.4255	0.5168	0.2893	0.2286
DS	0.2645	0.4598	0.2127	0.2583	0.2479	0.2286
CE	0.0176	0.0230	0.0607	0.0432	0.0413	0.1429
VS	0.0126	0.0192	0.0531	0.0322	0.0083	0.0285

Thus, the relative weights for each attribute, after calculation, are found to be: DI=0.1208, GS=0.1606, MS=0.3596, DS=0.2786, CE=0.0548 and VS=0.0256. Given by the mathematical expression shown below:

MS>DS>GS>DI>CE>VS.

4.2 Fuzzy PreRa for MAS selection [29]

In this section, we use Fuzzy PreRa method re-computation with the six evaluation criteria in [13].

First, this study applies Eq.(9) to transform the original data $a_{12}, a_{23}, a_{34}, a_{45}, a_{56}$, which transforms the reciprocal multiplicative preference relation with an interval scale $[1/9,9]$ into a reciprocal additive fuzzy preference relation with an interval scale $[0,1]$.

In this case, there are only five comparisons required for six evaluation criteria, which are as follows:

$p_{12} = (1 + \log_9 1/3)/2 = 0.25$
$p_{23} = (1 + \log_9 1/3)/2 = 0.25$
$p_{34} = (1 + \log_9 2)/2 = 0.6577$
$p_{45} = (1 + \log_9 6)/2 = 0.9077$
$p_{56} = (1 + \log_9 5)/2 = 0.8662$

Then, using Eqs.(2)-(5), we obtain the entire preference relation matrix, which is shown in Table 6. For clarity, examples for p_{21}, p_{31}, p_{42} and p_{52} are shown below:

$p_{21} = 1 - p_{12} = 1 - 0.25 = 0.75$
$p_{31} = 1.5 - p_{12} - p_{23} = 1.5 - 0.25 - 0.25 = 1.00$
$p_{42} = 1.5 - p_{23} - p_{34} = 1.5 - 0.25 - 0.6577 = 0.5923$
$p_{52} = 2 - p_{23} - p_{34} - p_{45} = 0.1845$

Table 6 Preference relation matrix for pairwise comparison of criteria (Fuzzy PreRa)

	DI	GS	MS	DS	CE	VS
DI	0.5000	0.2500	0.0000	0.1577	0.5655	0.9317
GS	0.7500	0.5000	0.2500	0.4077	0.8155	1.1817
MS	1.0000	0.7500	0.5000	0.6577	1.0655	1.4317
DS	0.8423	0.5923	0.3423	0.5000	0.9077	1.2740
CE	0.4345	0.1845	-0.0655	0.0923	0.5000	0.8662
VS	0.0683	-0.1817	-0.4317	-0.2740	0.1338	0.5000

Notably, the primary values in Table 6 are not in the interval $[0,1]$, but in an interval $[-a, 1+a]$, being $a > 0$, we need to transform the obtained values using Eq.(8) a transformation function which preserves reciprocity and additive consistency, that is a function $f : [-a, 1+a] \rightarrow [0,1]$.

According to Eq.(8), $a = 0.4317$, therefore, the entire preference relation matrix can be transformed as shown in Table 7 and the normalized matrix is shown in Table 8.

Table 7 After transformation matrix

	DI	GS	MS	DS	CE	VS
DI	0.5000	0.3658	0.2317	0.3163	0.5351	0.7317
GS	0.6342	0.5000	0.3658	0.4505	0.6693	0.8658
MS	0.7683	0.6342	0.5000	0.5846	0.8035	1.0000
DS	0.6837	0.5495	0.4154	0.5000	0.7188	0.9154
CE	0.4649	0.3307	0.1965	0.2812	0.5000	0.6965
VS	0.2683	0.1342	0.0000	0.0846	0.3035	0.5000

Table 8 Normalized matrix of Table 7

	DI	GS	MS	DS	CE	VS
DI	0.1506	0.1455	0.1355	0.1427	0.1516	0.1554
GS	0.1910	0.1989	0.2140	0.2032	0.1896	0.1839
MS	0.2315	0.2522	0.2925	0.2637	0.2276	0.2123
DS	0.2060	0.2185	0.2430	0.2255	0.2036	0.1944
CE	0.1400	0.1315	0.1150	0.1268	0.1416	0.1479
VS	0.0808	0.0534	0.0000	0.0382	0.0860	0.1062

Finally, the relative weights for each attribute, after calculation, are found to be: DI=0.1469, GS=0.1968, MS=0.2466, DS=0.2152, CE=0.1338 and VS=0.0608. Given by the mathematical expression shown below:

MS>DS>GS>DI>CE>VS.

4.3 Incomplete linguistic preference relations for MAS selection

In this section, we re-examine the numerical example investigated by Lai et al. [13].

We take Type 2: Vertical comparison of each pairs, to illustrate. First, this study applies $p_{ij} = g(a_{ij}) = 4 \times \log_9 a_{ij}$ to transform the original data $a_{41}, a_{42}, a_{43}, a_{45}, a_{46}$, which transforms the reciprocal multiplicative preference relation with an interval sacle $[\frac{1}{9}, 9]$ into a reciprocal additive fuzzy preference relation with interval sacle $[-4, 4]$. In this case, there are only five comparisons required for six evaluation criteria (Table 9).

Second, by Definition 2 and the known elements, one can obtain the upper diagonal triangle of an 6×6 matrix, then, using Eqs. (12) and (13) to obtain other missing elements.

Table 9 Preference relation matrix for pairwise comparison of criteria (incomplete linguistic preference relations)

	DI	GS	MS	DS	CE	VS
DI	0	0.5237	-3.2619	-2	1.2619	1.7856
GS	-0.5237	0	-3.7856	-2.5237	0.7381	1.2619
MS	3.2619	3.7856	0	1.2619	4.5237	5.0474
DS	2	2.5237	-1.2619	0	3.2619	3.7856
CE	-1.2619	-0.7381	-4.5237	-3.2619	0	0.5237
VS	-1.7856	-1.2619	-5.0474	-3.7856	-0.5237	0

Notably, there are some elements are not in the interval $[-4, 4]$, according to operational law, therefore, the entire preference relation matrix can be transformed Table 10.

Table 10 After transformation matrix

	DI	GS	MS	DS	CE	VS
DI	0	0.5237	-3.2619	-2	1.2619	1.7856
GS	-0.5237	0	-3.7856	-2.5237	0.7381	1.2619
MS	3.2619	3.7856	0	1.2619	4	4
DS	2	2.5237	-1.2619	0	3.2619	3.7856
CE	-1.2619	-0.7381	-4	-3.2619	0	0.5237
VS	-1.7856	-1.2619	-4	-3.7856	-0.5237	0

Table 11 Incomplete linguistic preference relations compute result

	Oblique	Horizontal	Vertical
DI	0.153 (4)	0.134 (3)	0.134 (3)
GS	0.187 (3)	0.060 (4)	0.060 (4)
MS	0.220 (1)	0.567 (1)	0.567 (1)
DS	0.199 (2)	0.415 (2)	0.415 (2)
CE	0.145 (5)	-0.049(5)	-0.049(5)
VS	0.096 (6)	-0.127(6)	-0.127(6)

From Table 11, we can find whatever the decision-maker gives any types pairwise result, the six criteria show the same ranking. Given by the mathematical expression shown below:

MS>DS>GS>DI>CE>VS.

5 Discussion

This study we re-examine the numerical example investigated by Lai et al. [13]. The purpose of re-examination is to provide a comparison of our results with those obtained by Lai et al. and also to show the advantage of incomplete linguistic preference relations method.

The analyzed outcome obtained by Fuzzy PreRa and incomplete linguistic preference relations methods almost coincides with that produced by the AHP method, shown in Table 12.

Table 12 Comparing compute result

	AHP	Fuzzy PreRa	incomplete linguistic preference relations
DI	0.1208	0.1469	0.1340
GS	0.1606	0.1968	0.0605
MS	0.3596	0.2466	0.5667
DS	0.2786	0.2152	0.4149
CE	0.0548	0.1338	-0.0486
VS	0.0256	0.0608	-0.1274

Notably, the ratio of the pairwise comparison times of the priority weight for the six influential factors between Fuzzy PreRa, incomplete linguistic preference relations and AHP is 5:5:15.

6 Conclusion

The main purpose of employing MCMD with fuzzy preference information on alternatives are weighting and ranking. Although the AHP method has been widely applied in many research domains, it still has some problems that need to be solved. Because taking into account that the AHP method must perform very complicated pairwise comparisons amongst elements (attributes or alternatives). Additionally, it is difficult to obtain a convincing consistency index with an increasing number of attributes or alternatives.

Then, Herrera-Viedma et al. [11] developed Fuzzy PreRa method to improve above problem. Fuzzy PreRa method differs from traditional AHP in the number of comparing times. If we have n attributes, using the AHP method we need to count the pairwise comparison times of $n(n-1)/2$ to be able to obtain a result. However, using Fuzzy PreRa method we only need to count the pairwise comparison times of $n-1$.

In this study, there are six criteria, using AHP we need to compare the criteria 15 times, but using Fuzzy PreRa we only need to compare them 5 times. Regarding the pairwise comparing times, Fuzzy PreRa requires less comparisons than AHP by 10 times. Fuzzy PreRa is clearly faster to execute and more efficient than AHP.

Applying Fuzzy PreRa method, it is possible to assure better consistency of the fuzzy preference relations provided by the decision makers, and in such a way also to avoid inconsistent solutions in the decision making processes.

However, in real life, because of time pressure and decision maker's limited related knowledge, decision maker may develop an incomplete fuzzy preference relation in which some of the elements cannot be provided. Hence, Xu [33] proposed a method with incomplete linguistic preference relations method that decision makers obtain a matrix by choosing a finite and fixed set of alternatives and performing a pairwise comparison based on their different preference and knowledge.

In this paper we reviewed the Fuzzy PreRa and incomplete linguistic preference relations methods in the AHP, which was proposed by Lai et al. [13]. The result shows that the approach introduced is simple and comprehensible in concept, efficient in computation, and robust in modeling human evaluation processes which make it of general use for solving practical qualitative multi-criteria problems.

Comparing with AHP and Fuzzy PreRa approaches, the incomplete linguistic preference relations method introduced here provide a greater flexibility for solving MCDM problems with preference information on alternatives and/or attributes. The concept of incomplete linguistic preference relations method is: when doing pairwise comparison decision makers can choose a clear preference item for standard, and then compare with the adjacent items. Therefore, the evaluating process can be more flexible by using incomplete linguistic preference relations method.

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