Manufacturing Lot Sizing with Backordering, Scrap, and Random Breakdown Occurring in Inventory-Stacking Period

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Abstract: - This paper is concerned with determination of optimal lot size for an economic manufacturing quantity model with backordering, scrap and breakdown occurring in inventory-stacking period. Generation of defective items and random breakdown of production equipment are inevitable in most real-life manufacturing systems. To cope with the stochastic machine failures, production planners practically calculate the mean time between failures (MTBF) and establish the robust plan accordingly, in terms of optimal lot size that minimizes total production-inventory costs for such an unreliable system. Random scrap rate is considered in this study, and breakdown is assumed to occur in inventory stacking period. Mathematical modeling and analysis is used and the renewal reward theorem is employed to cope with the variable cycle length. An optimal manufacturing lot size that minimizes the long-run average costs for such an imperfect system is derived. Numerical example is provided to demonstrate its practical usages.

Key-Words: - Optimization, Manufacturing systems, Production lot size, Machine breakdown, Backordering, Scrap, Inventory

1 Introduction

The economic manufacturing quantity (EMQ) model (also known as economic production quantity (EPQ) model) implicitly assumes that items produced are of perfect quality. But in real-life production systems, generation of defective items is inevitable. Hence, studies have been carried out to enhance the classic EMQ model by addressing the issue of imperfection quality items produced [1-6]. Ben-Daya [2] studied an imperfect process having a general deterioration distribution with increasing hazard rate. A model with joint determination of economic production quantity and preventive maintenance level was investigated. Cheng [3] formulated inventory as a geometric program and obtained the closed-form optimal solutions for an EOQ model with demand-dependent unit production cost and imperfect production processes. Zhang and Gerchak [6] considered joint lot sizing and inspection policy in an EOQ model with random yield. Despite the simplicity of EMQ model, it is still extensively applied world-wide and are still the basis for the analyses of more complex systems [7-11].

Stock-out situations may occur due to the excess demand. Sometimes, these shortages can be backordered and satisfied at a future time, hence the overall production-inventory costs can be reduced significantly [11-17]. Hayek & Salameh [11] derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items. Chiu and Chiu [14] examined an EPQ model with reworking of defective items and failure-in-repair. Optimal lot size and backordering level that minimize overall production costs was derived. Chiu [16] studied the effect of service level constraint on EPQ model with random defective rate. Relationship between “imputed backorder cost” and maximal shortage level is derived for decision-making on whether the required service level is achievable. Then an equation is proposed for calculating the intangible backorder cost for the situation when the required service level is not attainable. By including this intangible backorder cost in the mathematical analysis, one can derive a new optimal lot-size policy that minimizes expected total costs as well as satisfies the service level constraint.

Random breakdown of production equipment is another common and inevitable reliability factors that trouble the production planners and practitioners most. To effectively manage and control the disruption and minimize overall production costs, become the primary task of most manufacturing firms. It is no wonder that determining optimal lot-size (or production uptime) for systems with machine failures has received attention from researchers in recent decades [18-32].

Groenevelt et al. [22] studied two inventory control policies that deal with machine failures. The first one assumes that the production of the interrupted lot is not resumed (called no resumption - NR policy) after a breakdown. The second policy considers that the production of the interrupted lot will be immediately resumed (called abort/ resume (AR) policy) after the breakdown is fixed and if the current on-hand inventory falls below a certain threshold level. The repair time is assumed to be negligible and the effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions are investigated. Chiu et al. [20] investigated optimal run time for EPQ model with scrap, rework and random breakdown. They proposed and proved theorems on conditional convexity of the integrated cost function and on bounds of the production run time. Then, an optimal run time was located by the use of the bisection method based on the intermediate value theorem. Chung [21] gave approximations to production lot sizing with machine breakdowns. He showed that the long-run average cost function per unit time for the case of exponential failure is uni-modal and it is neither convex nor concave. In his paper, he derived the better lower and upper bounds of the optimal lot sizes for EPQ model with random machine breakdowns that improve some existing results.

Kim and Hong [23] investigated the optimal production run length in deteriorating production processes. They assumed that production process is subject to a random deterioration from an in-control state to an out-of-control state with an arbitrary probability distribution. As a result, the optimal production run lengths were derived under three different assumptions of deteriorating processes, namely constant, linearly increasing, and exponential increasing. Kuhn [24] considered a dynamic lot sizing model with exponential machine breakdowns. Two different situations were examined. Case one, after a machine breakdown the setup is totally lost and new setup cost is incurred. Case two, the cost of resuming the production run after a failure might be substantially lower than the production setup cost. Kuhn showed that under the first case the cost penalty for ignoring machine failures will be noticeably higher than that of the classical EPQ model. For the second case, a conditional resumption was recommended which is based on the sizes of future demands versus the incomplete lot sizes. He also suggested a stochastic dynamic programming model for finding optimum lot sizing decisions for both cases. Giri and Dohi [26] presented the exact formulation of stochastic EMQ model for an unreliable production system. Their EMQ model is formulated based on the net present value (NPV) approach and by taking limitation on the discount rate the traditional long-run average cost model is obtained. They also provided the criteria for the existence and uniqueness of the optimal production time and computational results showing that the optimal decision based on the NPV approach is superior to that based on the long-run average cost approach. Boone et al. [27] investigated the impact of imperfect processes on the production run time. They built a model in an attempt to provide managers with guidelines to choose the appropriate production run times to cope with both the defective items and stoppages occurring due to machine breakdowns.

Lin and Kroll [28] examined an EMQ model for an imperfect production process that is subject to random machine breakdowns. In their study, the time-to-shift and time-to-breakdown are two random variables that follow different exponential distributions. Abboud [29] considered an EMQ model with Poisson machine failures and random machine repair time. An approximation model was developed to describe the behavior of such systems, and specific formulations were derived for the cases where the repair times are exponential and constant. Chiu [30] investigated a production run time problem with assumptions of rework, shortage not allowed, and random machine breakdown under AR inventory control policy. An optimal run time that minimizes overall production- inventory cost is derived.

This paper investigates the optimal manufacturing lot size for EMQ model with scrap, backlogging, and random breakdown occurring in inventory-stacking period. Since little attention was paid to the aforementioned area, this paper intends to fill the gap.

2 Mathematical Modeling

Consider that a manufacturing process may randomly produce *x* portion of defective items at a rate *d*. All imperfect quality items are assumed not repairable, are treated as scrap. The production rate *P* is much larger than the demand rate λ and the production rate of scrap items *d* can be expressed as $d=Px$. Further, shortages are backordered. They will be satisfied first when the next replenishment production cycle begins.

Suppose that according to mean time between failures (MTBF) data, a random machine breakdown may take place randomly in the inventory-stacking period (refer to Figure 1). The abort/resume (AR) inventory control policy is adopted in this study and under such policy, when a breakdown takes place, the machine is under corrective maintenance immediately. The repair time is assumed to be constant. The interrupted lot will be resumed right after the machine is restored.

Cost parameters include unit production cost *C*, unit holding cost *h*, setup cost *K*, disposal cost per scrap item C_S, unit shortage/backordered cost *b*, and cost for repairing and restoring machine *M*.

Additional notation is listed as follows.

T_1 = *the optimal production time (i.e. production uptime) to be determined for the proposed EPQ model,*

t = *production time before a random breakdown occurs,*

t_r = *time required for repairing and restoring the machine,*

t_2 = *time required for depleting all available perfect quality on-hand items,*

t_3 = *shortage permitted time,*

t_4 = *time required for filling backorder quantity,*

H_1 = *level of on-hand inventory when machine breakdown occurs,*

H_2 = *level of on-hand inventory when machine is repaired and restored,*

H_3 = *the maximum level of on-hand inventory for each production cycle,*

Q = *production lot size for each cycle,*

B = *maximum backorder level per cycle,*

T = *the production cycle length,*

$TC(T_1,B)$ = *total production-inventory costs per cycle,*

$TCU(T_1,B)$ = *total production-inventory costs per unit time (e.g. annual),*

$E[TCU(T_1,B)]$ = *the expected total production-inventory costs per unit time.*

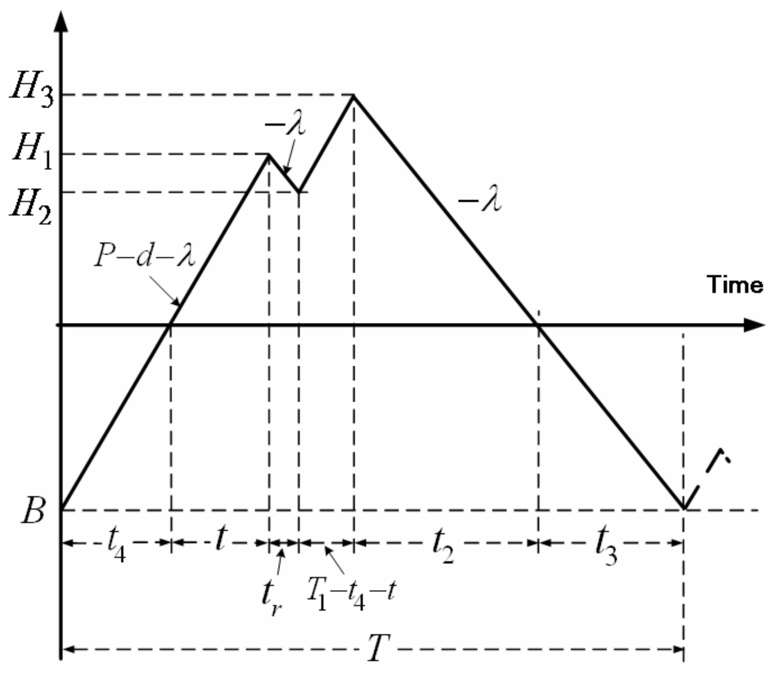

Fig.1: On-hand inventory of perfect quality items in EPQ model with scrap and breakdown occurring in inventory-stacking period

The production rate *P* of perfect quality items must always be greater than or equal to the sum of the demand rate λ and the production rate of defective items d. Hence, the following condition must hold: $(P\text{-}d\text{-}\lambda)>=0$ or $(1\text{-}x\text{-}\lambda/P)>=0$.

Let t denote production time before a breakdown taking place in the inventory-stack period and let the constant machine repair time $t_r = g$. The following derivation procedure is similar to what was used by prior studies [13-14]. From Figure 1, one can obtain the following:

$$H_1 = (P - d - \lambda)t \tag{1}$$

$$H_2 = H_1 - t_r\lambda = H_1 - g\lambda \tag{2}$$

$$T_1 = \frac{Q}{P} \tag{3}$$

$$H_3 = H_2 + (P - d - \lambda)\cdot(T_1 - t_4 - t) \tag{4}$$

$$T = T_1 + t_2 + t_3 + t_r \tag{5}$$

$$t_2 = \frac{H_3}{\lambda} \tag{6}$$

$$t_3 = \frac{B}{\lambda} \tag{7}$$

$$t_4 = \frac{B}{P - d - \lambda} \tag{8}$$

where $d=Px$.

Total scrap items produced during production uptime T_1 are (see Figure 2):

$$d \cdot T_1 = x \cdot Q \tag{9}$$

Total production-inventory cost per cycle $TC(T_1,B)$ is:

$$\begin{aligned} TC(T_1,B) &= K + C\cdot(P\cdot T_1) + Cs\cdot(T_1\cdot P\cdot x) + M \\ &+ h\cdot\left[\frac{H_1}{2}(t) + \frac{H_1+H_2}{2}(t_r) + \frac{H_2+H_3}{2}(T_1-t_4-t) + \frac{H_3}{2}(t_2)\right] \\ &+ h\cdot\left[\frac{d(t_4+t)}{2}(t_4+t) + (t_4+t)t_r + \frac{(t_4+t)+dT_1}{2}(T_1-t_4-t)\right] \\ &+ b\cdot\left[\frac{B}{2}(t_4) + \frac{B}{2}(t_3)\right] \end{aligned} \tag{10}$$

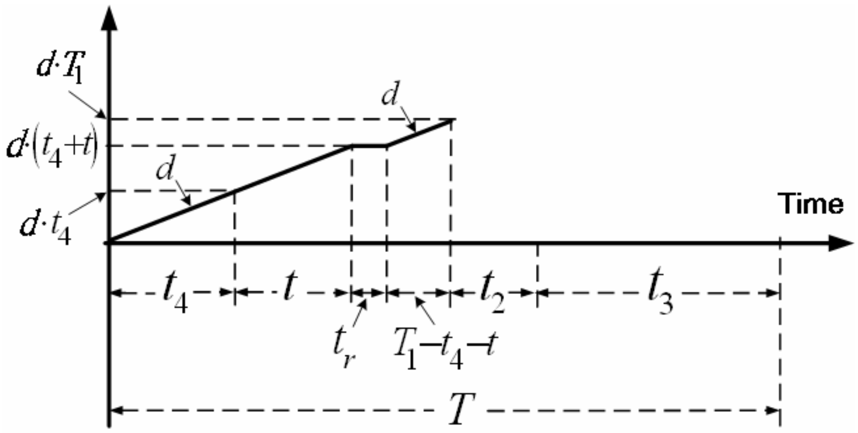

Fig.2: On-hand inventory of scrap items in EPQ model with scrap and breakdown occurring in inventory-stacking period

Substituting all related parameters from Eqs. (1) to (9) in Eq. (10), one obtains $TC_1(T_1,B)$ as follows.

$$\begin{aligned} TC(T_1,B) &= C\cdot P\cdot T_1 + K + M + C_S\cdot T_1\cdot P\cdot x - h\frac{P}{\lambda}T_1B(1-x) \\ &+ \frac{h}{2}\left\{\frac{P^2}{\lambda}(1-2x+x^2)T_1^2 + \frac{(1-x)}{\lambda\left(1-x-\frac{\lambda}{P}\right)}B^2 - PT_1^2 + 2PxT_1^2\right\} \\ &+ \frac{b(1-x)}{2\lambda(1-x-\frac{\lambda}{P})}B^2 + \frac{hg}{(1-x-\frac{\lambda}{P})}(B+g\lambda) - hPgT_1(1-x) + hPgt \end{aligned} \tag{11}$$

The production cycle length is not constant due to the assumption of random scrap rate and a uniformly distributed breakdown is assumed to occur in the inventory stacking time.

Thus, to take randomness of scrap and breakdown into account, one can use the renewal reward theorem in inventory cost analysis to cope with variable cycle length and use integration of $TC(T_1,B)$ to deal with the random breakdown happening in inventory stacking time.

The expected total production-inventory costs per unit time can be calculated as follows.

$$E\left[TCU\left(T_1,B\right)\right]=\frac{E\left[\int_0^{T_1-t_4} TC\left(T_1,B\right)\cdot f\left(t\right)dt\right]}{E\left[T\right]} = \frac{E\left[\int_0^{T_1-t_4} TC\left(T_1,B\right)\cdot\left(1/t_4\right)dt\right]}{T_1 P\left(1-E\left[x\right]\right)/\lambda} \quad (12)$$

Substituting equations (1) through (11) in (12), we have:

$$\begin{aligned} E[TCU(T_1,B)] &= \lambda\left[C\frac{1}{1-E[x]}+C_S\frac{E[x]}{1-E[x]}\right] \\ &+\frac{\lambda(K+M)}{PT_1}\frac{1}{1-E[x]}+\frac{h}{2}\left[\left(1-\frac{\lambda}{P}\right)PT_1-2B\right]\frac{1}{1-E[x]}+\frac{hPT_1}{2}\frac{E\left[x^2\right]}{1-E[x]} \\ &+\frac{B^2}{2PT_1}(b+h)E\left[\frac{1-x}{1-x-\frac{\lambda}{P}}\right]\frac{1}{1-E[x]}+h\left[B-\left(1-\frac{\lambda}{P}\right)PT_1\right]\frac{E[x]}{1-E[x]} \\ &+\frac{h\lambda g}{2PT_1}(B+\lambda g)E\left[\frac{1}{1-x-\frac{\lambda}{P}}\right]\frac{1}{1-E[x]}+h\lambda g\frac{E[x]}{1-E[x]}-\frac{h\lambda g}{2}\frac{1}{1-E[x]} \end{aligned} \quad (13)$$

Let $E_0=\frac{1}{1-E[x]}$; $E_1=\frac{E[x]}{1-E[x]}$; $E_2=\frac{E\left[x^2\right]}{1-E[x]}$;

$$E_3=E\left[\frac{1-x}{1-x-\frac{\lambda}{P}}\right]\frac{1}{1-E[x]};\quad E_4=E\left[\frac{1}{1-x-\frac{\lambda}{P}}\right]\frac{1}{1-E[x]}$$

Then Eq. (13) becomes:

$$\begin{aligned} E\left[TCU(T_1,B)\right] &= \lambda\left[CE_0+C_S E_1\right]+\frac{\lambda(K+M)}{PT_1}E_0 \\ &+\frac{h}{2}\left[\left(1-\frac{\lambda}{P}\right)PT_1-2B\right]E_0+\frac{hPT_1}{2}E_2 \\ &+\frac{B^2}{2PT_1}(b+h)E_3+h\left[B-\left(1-\frac{\lambda}{P}\right)PT_1\right]E_1 \\ &+\frac{h\lambda g}{2PT_1}(B+\lambda g)E_4+h\lambda gE_1-\frac{h\lambda g}{2}E_0 \end{aligned} \quad (14)$$

3 Convexity of E[TCU(T_1,B)]

The optimal inventory operating policy can be obtained by minimizing the expected cost function. For the proof of convexity of *E[TCU(T_1,B)]*, one can utilize the Hessian matrix equation [33] and verify the existence of the following:

$$\begin{bmatrix}T_1 & B\end{bmatrix}\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial T_1^2} & \frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} \\ \frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} & \frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix} > 0 \quad (15)$$

E[TCU(T_1,B)] is strictly convex only if Eq. (15) is satisfied, for all T_1 and B different from zero. By computing all the elements of the Hessian matrix equation, one obtains:

$$\begin{bmatrix}T_1 & B\end{bmatrix}\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial T_1^2} & \frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} \\ \frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} & \frac{\partial^2 E\left[TCU\left(T_1,B\right)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix} = \frac{2\lambda(K+M)}{PT_1}E_0+\frac{hg^2\lambda^2}{PT_1}E_4 > 0 \quad (16)$$

Eq. (16) is resulting positive, because all parameters are positive. Hence, *E[TCU(T_1,B)]* is a strictly convex function. It follows that for optimal production uptime T_1 and maximal backorder level B, one can differentiate *E[TCU(T_1,B)]* with respect to T_1 and with respect to B, and solve linear systems of Eqs.(17) and (18) by setting these partial derivatives equal to zero.

$$\begin{aligned} \frac{\partial E\left[TCU(T_1,B)\right]}{\partial T_1} &= -\frac{\lambda(K+M)}{PT_1^2}E_0+\frac{h}{2}P\left(1-\frac{\lambda}{P}\right)E_0 \\ &-\frac{B^2}{2PT_1^2}(b+h)E_3-hP\left(1-\frac{\lambda}{P}\right)E_1 \\ &+\frac{hP}{2}E_2-\frac{h\lambda gB}{2PT_1^2}E_4-\frac{hg^2\lambda^2}{2PT_1^2}E_4 \end{aligned} \quad (17)$$

$$\frac{\partial E\left[TCU(T_1,B)\right]}{\partial B}=-hE_0+\frac{B}{PT_1}(b+h)E_3+hE_1+\frac{h\lambda g}{2PT_1}E_4 \quad (18)$$

$$\therefore T_1^*=\frac{1}{P}\sqrt{\frac{2\lambda(K+M)E_0+h\lambda^2g^2E_4\left(1-\frac{hE_4}{4(b+h)E_3}\right)}{h\left(1-\frac{\lambda}{P}\right)E_0-\frac{h^2}{(b+h)E_3}-2h\left(1-\frac{\lambda}{P}\right)E_1+hE_2}} \quad (19)$$

$$\therefore B^*=\frac{h}{(b+h)E_3}\left(PT_1^*-\frac{\lambda g}{2}E_4\right) \quad (20)$$

Plugging E_0, E_1, E_2, E_3, and E_4 in equations (19) and (20), the optimal production run time and optimal backordering quantity become:

$$T_1^* = \frac{1}{P}\left[\frac{2\lambda(K+M)+\frac{h\lambda^2g^2}{1-E[x]}\cdot E[1/(1-x-\lambda/P)]\cdot\left(1-\frac{h\cdot E[1/(1-x-\lambda/P)]}{4(b+h)\cdot E[(1-x)/(1-x-\lambda/P)]}\right)}{h\left(1-\frac{\lambda}{P}\right)-2h\left(1-\frac{\lambda}{P}\right)E[x]+h\cdot E[x^2]-\frac{h^2\cdot[1-E[x]]^2}{(b+h)\cdot E[(1-x)/(1-x-\lambda/P)]}}\right]^{\frac{1}{2}} \tag{21}$$

$$B^* = \frac{h\cdot P[1-E[x]]\cdot T_1^* - \frac{\lambda gh}{2}E\left[\frac{1}{1-x-\frac{\lambda}{P}}\right]}{(b+h)\cdot E\left[\frac{1-x}{1-x-\frac{\lambda}{P}}\right]} \tag{22}$$

From Eq. (4) and Eqs. (19-20) one can obtain the optimal lot-size Q^* and optimal backorder level B^* as follows:

$$\therefore Q^* = \sqrt{\frac{2\lambda(K+M)E_0 + h\lambda^2g^2E_4\left(1-\frac{hE_4}{4(b+h)E_3}\right)}{h\left(1-\frac{\lambda}{P}\right)E_0 - \frac{h^2}{(b+h)E_3} - 2h\left(1-\frac{\lambda}{P}\right)E_1 + hE_2}} \tag{23}$$

$$\therefore B^* = \frac{h}{(b+h)E_3}\left(Q^* - \frac{\lambda g}{2}E_4\right) \tag{24}$$

Plugging E_0, E_1, E_2, E_3, and E_4 in equations (23) and (24), the optimal production lot-size and optimal backordering quantity become:

$$Q^* = \left[\frac{2\lambda(K+M)+\frac{h\lambda^2g^2}{1-E[x]}\cdot E[1/(1-x-\lambda/P)]\cdot\left(1-\frac{h\cdot E[1/(1-x-\lambda/P)]}{4(b+h)\cdot E[(1-x)/(1-x-\lambda/P)]}\right)}{h\left(1-\frac{\lambda}{P}\right)-2h\left(1-\frac{\lambda}{P}\right)E[x]+h\cdot E[x^2]-\frac{h^2\cdot[1-E[x]]^2}{(b+h)\cdot E[(1-x)/(1-x-\lambda/P)]}}\right]^{\frac{1}{2}} \tag{25}$$

$$B^* = \frac{h\cdot[1-E[x]]\cdot Q^* - \frac{\lambda gh}{2}E\left[\frac{1}{1-x-\frac{\lambda}{P}}\right]}{(b+h)\cdot E\left[\frac{1-x}{1-x-\frac{\lambda}{P}}\right]} \tag{26}$$

3.1 Results and Verification

Suppose that machine breakdown factor is not an issue to be considered, then the cost and time for repairing failure machine M=0 and g=0, equations (25) and (26) become the same equations as were given by Chiu and Chiu [17]:

$$Q^* = \sqrt{\frac{2\lambda K}{\left\{h\left(1-\frac{\lambda}{P}\right)-2h\left(1-\frac{\lambda}{P}\right)E[x]+h\cdot E[x^2] - \frac{h^2\cdot[1-E[x]]^2}{(b+h)\cdot E[(1-x)/(1-x-\lambda/P)]}\right\}}} \tag{27}$$

$$B^* = \left(\frac{h}{b+h}\right)\cdot\frac{1-E[x]}{E\left[\frac{1-x}{1-x-\lambda/P}\right]}\cdot Q^* \tag{28}$$

Further, suppose that regular production process produces no defective items, i.e. x=0, then equations (27) and (28) become the same equations as were presented by the classic EPQ model with backordering permitted [34-35]:

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}}\cdot\sqrt{\frac{b+h}{b}} \tag{29}$$

$$B^* = \left[\frac{h}{(b+h)}\left(1-\frac{\lambda}{P}\right)\right]\cdot Q^* \tag{30}$$

4 Numerical Example

4.1 Example 1

Assume annual production rate of a manufactured item is 18,000 units and demand of this item is 3,000 units per year. The percentage of random scrap items produced x, follows a uniform distribution over the range [0,0.15]. Other parameters used are as follows:

K = \$240 for each production run,
C_S = \$1.00 disposal cost for each scrap item,
C = \$2.00 per item,
M = \$500 repair cost for each breakdown,
h = \$0.6 per item per unit time,
b = \$0.8 per item backordered per unit time,

g = 0.018 years, time needed to repair and restore the machine.

Applying equations (25), (26) and (13), one can obtain the optimal production time T_1* =0.2345 years, the optimal lot size *Q** =4,221, the backorder *B**=1,368 and *E[TCU(Q*,B*)]*=$7,851.30. Variation of defective rate *x* effects on optimal production lot size is shown in Figure 3. One notes that as *x* increases, the optimal production lot-size increases accordingly.

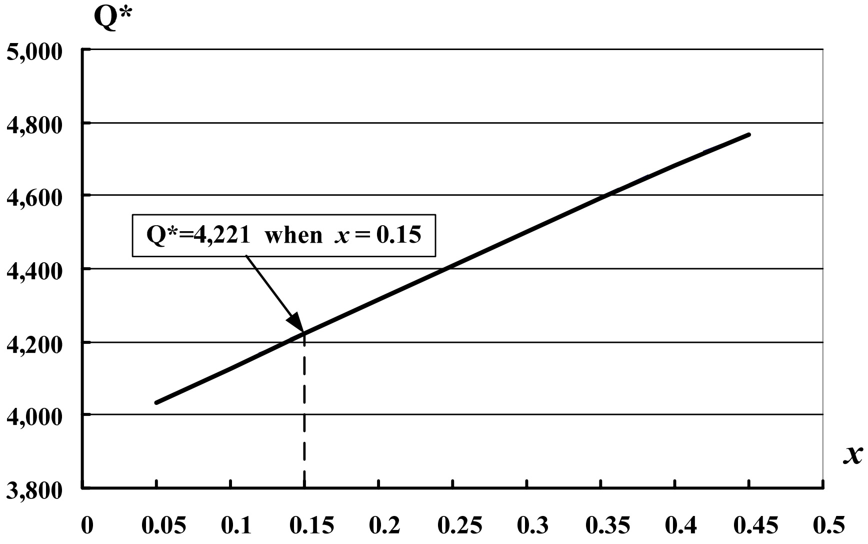

Fig.3: Variation of defective rate effects on optimal production lot size *Q**

Effect of defective rate *x* on optimal backorder quantity is depicted in Figure 4. One notes that as *x* increases, the optimal backorder quantity decreases accordingly.

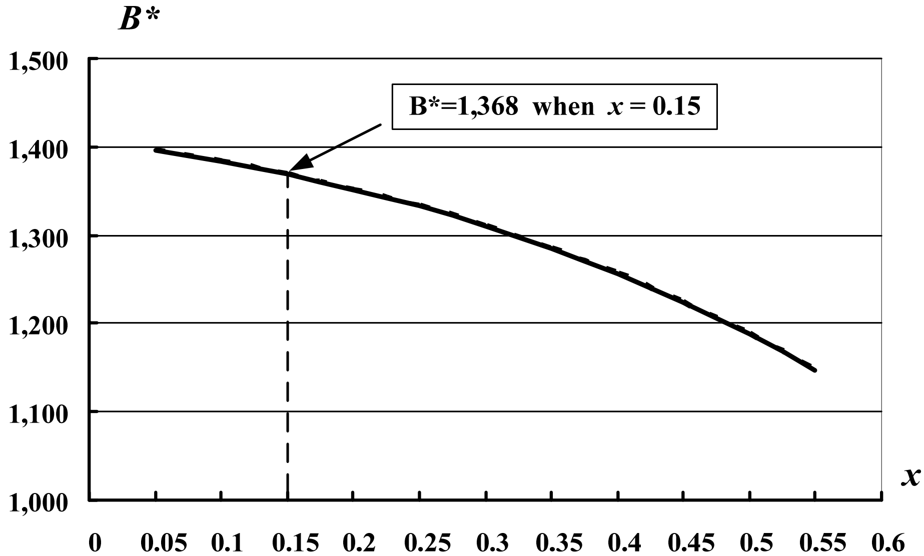

Fig.4: Effect of defective rate *x* on optimal backorder quantity

A demonstration of the convexity of the long-run average costs *E[TCU(Q,B)]* is depicted in Figure 5. Effect of defective rate *x* on the expected cost *E[TCU(Q*,B*)]* is illustrated in Figure 6. One notes that as defective rate *x* increases, the long-run average production-inventory cost increases too.

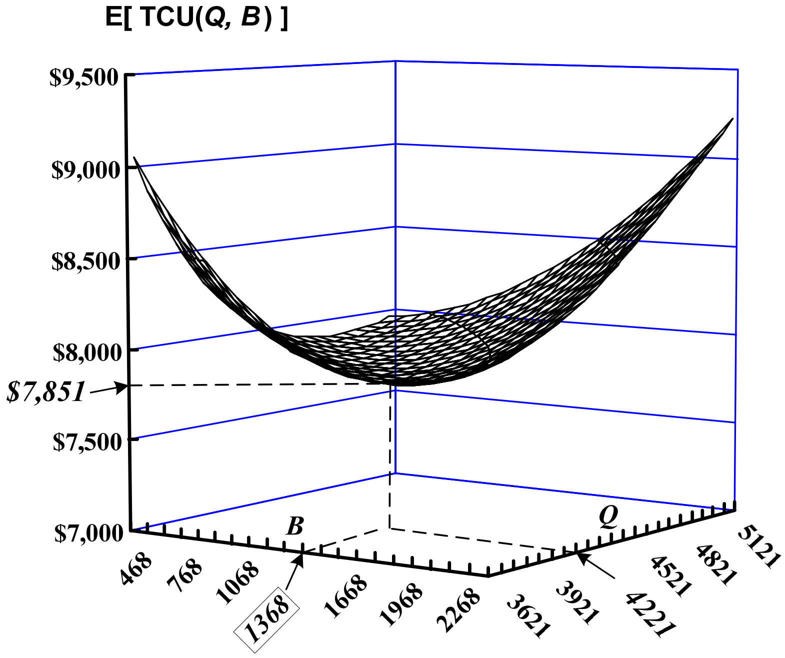

Fig.5: Convexity of the expected cost function E[TCU(Q,B)]

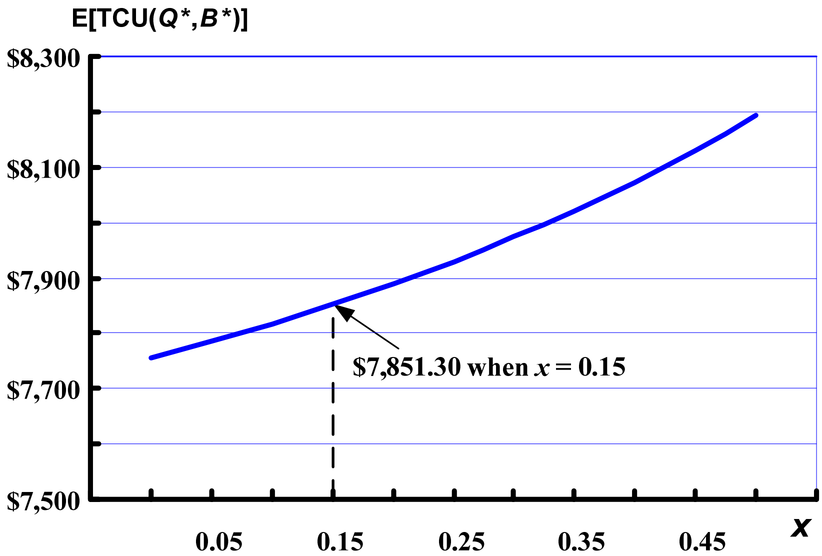

Fig.6: Effect of defective rate x on *E[TCU(Q*,B*)]*

Suppose the result of this paper is not available, one can only use a closely related lot-size solution given by [13] for solving such an unreliable system and obtaining Q=2,404 (or T_1=0.1335) and B=781. Plugging this lot-size solution into Eq. (13), one has $E[TCU(Q,B)]$ =$8,036.39. As a result, it pays extra 9.99% on total setup and holding costs than the optimal production-inventory costs obtained by using lot-size decisions of the present study.

4.2 Example 2

Assume another manufactured item can be produced at a rate of 44,000 units and its annual demand is 11,000 units. A random percentage of scrap items produced x follows a uniform distribution over the interval [0, 0.20]. Other parameters used are:

K = $350 for each production run,
M = $600 repair cost for each breakdown,
g = 0.036 years, time needed to repair and restore the machine,

C = \$2.40 per item,
C_S = \$0.80 disposal cost for each scrap item,
h = \$1.00 per item per unit time,
b = \$1.40 per item backordered per unit time.

Applying equations (21)-(22) one obtains the optimal T_1*=0.1711 years or 8.90 weeks, and backorder level B*=2,008. From equation (14), the long-run average costs $E[TCU(T_1*,B*)]$= \$28,324. The optimal production run time T_1* can be used to determine a multi-item production schedule.

From equation (23), one also obtains optimal lot-size Q*=7,526, it can then be used for consequent materials requirement planning based on product structure diagram [36-37].

Figure 7 shows variation of defective rate x effects on optimal production run time T_1*. It shows that as x increases, the value of T_1* increases significantly.

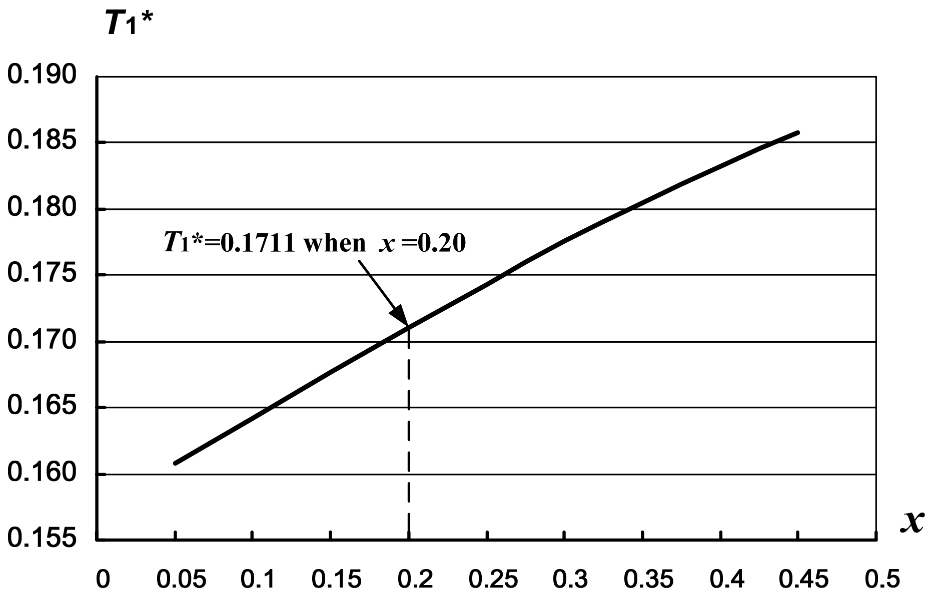

Fig.7: Variation of defective rate effects on optimal production run time T_1*

Variation of defective rate effects on optimal expected inventory cost function E[TCU(T_1*,B*)] is depicted in Figure 8.

5 Conclusion

Generation of random scrap items and stochastic breakdown of production equipment are inevitable in most real-life manufacturing systems. Groenevelt *et al*. [18] and others [19-28] studied manufacturing systems with machine breakdowns as stated earlier in Section 1. In real life production environments, in addition to stochastic breakdowns the generation of random scrap items during the regular production process is also inevitable. Such a realistic EMQ model must be studied specifically in order to minimize the total costs.

This paper investigates joint effects of stochastic machine breakdowns, backlogging, and scrap on the optimal lot-size decision of EMQ model. With the purpose of minimizing overall production-inventory costs. Random scrap rate is considered in this study, and breakdown is assumed to occur in inventory stacking period. Mathematical modeling and analysis is used and the renewal reward theorem is employed to cope with the variable cycle length. An optimal manufacturing lot size that minimizes the long-run average costs for such an imperfect system is derived. Numerical example is provided to demonstrate its practical usages. Since little attention was paid to the aforementioned area, this paper intends to fill the gap.

For future research, to investigate the effect of random breakdown with abort/resume inventory control policy and occurring in the backorder filling time on same model will be one of the interesting topics.

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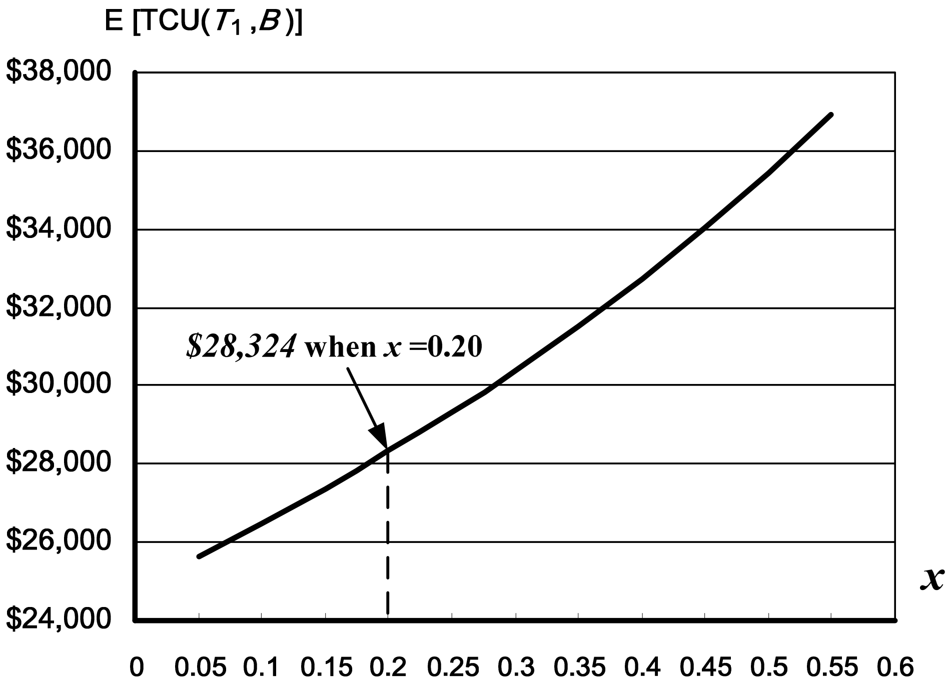

Fig.8: Variation of defective rate effects on optimal cost function E[TCU(T_1*,B*)]

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