A Fuzzy Statistics based Method for Mining Fuzzy Correlation Rules

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Abstract: - Mining fuzzy association rules is the task of finding the fuzzy itemsets which frequently occur together in large fuzzy dataset, but most proposed methods may identify a fuzzy rule with two fuzzy itemsets as interesting when, in fact, the presence of one fuzzy itemsets in a record does not imply the presence of the other one in the same record. To prevent generating this kind of misleading fuzzy rule, in this paper, we construct a new method for finding relationships between fuzzy itemsets based on fuzzy statistics, and the generated rules are called fuzzy correlation rules. In our method, a fuzzy correlation analysis which can show us the strength and the type of the linear relationship between two fuzzy itemsets is used. By using thus fuzzy statistics analysis, the fuzzy correlation rules with the information about that two fuzzy not only frequently occur together in same records but also are related to each other can be generated.

Key-Words: - Fuzzy association rules, Fuzzy itemsets, Fuzzy statistics, Fuzzy correlation analysis, Linear relationship, Fuzzy correlation rules

1 Introduction

Mining association rule is an important data mining task which is often defined as finding the itemsets which frequently occur together in large databases [1, 2, 3, 14, 17, 21, 22]. A popular application of association rule mining is the market basket analysis which identifies the buying behaviours of customers. It is widely used to find the products which are frequently purchased together by same customers in transaction databases. This kind of information is clearly useful for many marketing decisions.

But, in practical databases, many data may be described by fuzzy itemsets but useful, waiting to be explored; hence, methods to discover association rules from fuzzy itemsets are needed, too. To this end, many researchers turn to propose methods for mining fuzzy association rules from various fuzzy dataset recently [5, 6, 9, 12, 15, 16, 19, 20, 24].

Most fuzzy association rules mining methods proposed up to now employ a support-confidence framework, which uses two measures, fuzzy support and fuzzy confidence, to find the fuzzy itemsets which frequently occur together and to identify the fuzzy association rules as interesting.

However, a situation needs to be noticed, if a fuzzy itemset almost occurs in all records, then it may frequently occur with other fuzzy itemsets. Therefore, the support-confidence framework can be misleading in that it may identify a fuzzy rule with two fuzzy itemsets as interesting when, in fact, the presence of one fuzzy itemsets in a record does not imply the presence of the other one in the same record.

To prevent generating this kind of misleading rule, here, we adopt a new method for finding useful relationships between fuzzy itemsets based on fuzzy statistics, and the generated rules are called fuzzy correlation rules. In our method, the analysis of fuzzy correlation [8, 10, 18, 23] is used. The fuzzy correlation analysis which is derived from the conventional statistics and fuzzy set theory can show us the strength and the type of the linear relationship between two fuzzy itemsets. By using the fuzzy correlation analysis, the fuzzy correlation rules with the information about that two fuzzy not only frequently occur together in same records but also are related to each other can be generated.

The rest of this paper is organized as follows: The concepts of mining fuzzy association rules are mentioned in section 2. The concepts of the fuzzy correlation analysis are presented in section 3. The fuzzy correlation rules mining method is introduced in section 4. An experiment is displayed in section 5. The conclusions are given in section 6.

2 Fuzzy Association Rules

Fuzzy association rules mining is a task of finding the fuzzy itemsets which frequently occur together in large databases [1, 2, 3, 14, 17, 21, 22].

Most methods for mining fuzzy association rules employ a support-confidence framework which uses fuzzy support and fuzzy confidence to identify the fuzzy association rules as interesting.

Let $F=\{f_1,f_2,\cdots,f_m\}$ be a set of fuzzy items, $T=\{t_1,t_2,\cdots,t_n)$ be a set of fuzzy records, and each fuzzy record t_i is represented as a vector with m values, $(f_1(t_i),f_2(t_i),\cdots,f_m(t_i))$, where $f_j(t_i)$ is the degree that fuzzy item f_j appears in record t_i, $f_j(t_i)\in[0,1]$. Then, a fuzzy association rule is defined as an implication form, such as $F_X\to F_Y$, where $F_X\subset F, F_Y\subset F$ are two fuzzy itemsets, and $\underset{all\,x}{\forall} f_x\in F_X \neq \underset{all\,y}{\forall} f_y\in F_Y$.

The fuzzy association rule $F_X\to F_Y$ holds in T with the fuzzy support ($fsupp(\{F_X,F_Y\})$ and the fuzzy confidence ($fconf(F_X\to F_Y)$). The fuzzy support and the fuzzy confidence are defined as follows:

$$fsupp(\{F_X,F_Y\}) = \frac{\sum_{i=1}^{n}\min(f_j(t_i)\mid f_j\in\{F_X,F_Y\})}{n} \quad (1)$$

$$fconf(F_X\to F_Y)=\frac{fsupp(\{F_X,F_Y\})}{fsupp(\{F_X\})} \quad (2)$$

If the $fsupp(\{F_X,F_Y\})$ is greater than or equal to a predefined threshold, minimal fuzzy support (s_f), and the $fconf(F_X\to F_Y)$ is also greater than or equal to a predefined threshold, minimal fuzzy confidence(c_f), then $F_X\to F_Y$ is considered as an interesting fuzzy association rule, and it means that the presence of fuzzy itemsets F_X in a record can imply the presence of fuzzy itemsets F_Y in the same record.

Now, let us consider a special situation, if a fuzzy itemset is common, and it almost occurs in all fuzzy records, then according the above framework, we may identify many fuzzy association rules as interesting but in fact, the presence of this observed fuzzy itemsets does not imply the presence of other fuzzy itemsets which are also included in these fuzzy association rules. Therefore, these discovered rules are misleading actually.

Some researchers have also noticed this problem, and thus turned to adopt alternative measure which can show extra information about the relationships between itemsets in mining processes [7, 14, 11, 13, 17, 22]. The famous measure is defined as follows [7, 14, 17]:

Support there are two itemsets, A and B, in a given record set; the probability that A occurs is expressed as $P(A)$; the probability that B occurs is expressed as $P(B)$; the probability that A and B occur both is expressed as $P(A,B)$. Then, the correlation of the association rule $A\to B$ can be expressed as $correl(A\to B)$.

$$correl(A\to B)=\frac{P(A,B)}{P(A)\cdot P(B)} \quad (3)$$

The value computed from (3) lies between in [0, ∞]. If $P(A,B)=P(A)\cdot P(B)$, then $correl(A\to B)$ is equal to 1, meaning A and B are no related, and the presence of one is independent of the presence of the other one. But, if $correl(A\to B)$ is close to 0 or ∞, than it means that A and B are highly related, and the presence of one can imply the presence of the other one.

Although the above probability-based formula can be used to analyze the relationship between crisp itemsets, it is not suitable for analyzing the relationship between fuzzy itemsets.

In order to analyze the relationships between fuzzy itemsets, a useful fuzzy statistics analysis, fuzzy correlation, is adopted. The fuzzy correlation analysis is derived from the conventional statistics and fuzzy set theory, and it can show us the strength and the type of the linear relationship between two fuzzy itemsets. The concepts of fuzzy correlation analysis and how to use the fuzzy correlation analysis in our proposed method will be explained in the next section.

3 Fuzzy Correlation Analysis

The correlation analysis of fuzzy sets is called fuzzy correlation analysis. Many methods have been proposed to calculate the fuzzy correlation coefficient [8, 10, 18, 23]. In our method, we adopt the formula derived by Lin [10], because it can provide the extra information we need.

Suppose there are two fuzzy itemsets $A,B\subset F$, where F is a fuzzy space. A and B are defined on a crisp universal set X with membership functions μ_A and μ_B, and the fuzzy itemsets A and B can be expressed as follows:

$$A = (\ x\ ,\ \mu_A(x)) \mid x \in X\ ), \tag{4}$$

$$B = (\ x\ ,\ \mu_B(x)) \mid x \in X\ ), \tag{5}$$

where $\mu_A, \mu_B \in [\ 0\ ,1\ ]$.

Assume that there is a random sample $(\ x_1,\ x_2,\ \cdots,\ x_n\ ) \in X$, alone with a sequence of paired data, $\{(x_i, \mu_A(x_i), \mu_B(x_i)) \mid i = 1 \cdots n\ \}$, which correspond to the grades of the membership functions of fuzzy itemsets A and B defined on X. Then, the fuzzy correlation coefficient between the fuzzy itemsets A and B, $r_{A,B}$, is:

$$r_{A,B} = {s_{A,B}} \Big/ {s_A \cdot s_B}\ , \tag{6}$$

where

$$s_{A,B} = \frac{\sum_{i=1}^{n}(\mu_A(x_i) - \overline{\mu_A}) \cdot (\mu_B(x_i) - \overline{\mu_B})}{n-1}, \tag{7}$$

$$\overline{\mu_A} = \frac{\sum_{i=1}^{n} \mu_A(x_i)}{n}, \tag{8}$$

$$\overline{\mu_B} = \frac{\sum_{i=1}^{n} \mu_B(x_i)}{n}, \tag{9}$$

$$S_A{}^2 = \frac{\sum_{i=1}^{n}(\ \mu_A(x_i) - \overline{\mu_A}\ )^2}{n-1}, \tag{10}$$

$$S_B{}^2 = \frac{\sum_{i=1}^{n}(\ \mu_B(x_i) - \overline{\mu_B}\ )^2}{n-1}, \tag{11}$$

$$s_A = \sqrt{S_A{}^2}, \tag{12}$$

$$s_B = \sqrt{S_B{}^2}. \tag{13}$$

The value computed from (6) lies between in [-1, 1]. If $r_{A,B} > 0$, then the fuzzy itemsets A and B are positively related. If $r_{A,B} < 0$, then the fuzzy itemsets A and B are negatively related. But, if $r_{A,B} = 0$, then the fuzzy itemsets A and B have no relationship at all. According to these important properties, we can obtain the strength and type of the linear relationship between two fuzzy itemsets, hence, the fuzzy correlation analysis is great useful for mining the interesting fuzzy correlation rules.

4 Mining Fuzzy Correlation Rules

The fuzzy correlation rules mining method will be introduced in this section.

Assume that $F = \{f_1, f_2, \cdots, f_m\}$ be a set of fuzzy items; $T = \{t_1, t_2, \cdots, t_n)$ be a random sample with n fuzzy data records, and each sample record t_i is represented as a vector with m values, $(f_1(t_i),\ f_2(t_i), \cdots, f_m(t_i))$, where $f_j(t_i)$ is the degree that fuzzy item f_j occurs in record t_i, $f_j(t_i) \in\ [0,1]$.

And next, three predefined thresholds are needed to be defined. Here, s_f is the minimal fuzzy support; c_f is the minimal fuzzy confidence; r_f is the minimal fuzzy correlation coefficient. The procedure of mining fuzzy correlation rules is described as the follows:

Step 1: The fuzzy support of each fuzzy item $f_i \in F$, $fsupp(f_i)$ is computed by using formula (1).

Step 2: Let $L_1 = \{f_p \mid f_p \in F, fsupp(f_p) \geq s_f\}$ be the set of frequent fuzzy itemsets whose size is equal to 1.

Step 3: Let $C_2 = \{(F_A, F_B)\}$ be the set of all combinations of two elements belong to L_1, where $F_A, F_B \in L_1$, $F_A \neq F_B$. That is, C_2 is generated by L_1 joint with L_1. Because F_A and F_B are the elements of L_1, the size of each element of C_2 is 2.

Step 4: For each element of C_2, (F_A, F_B), the fuzzy support, $fsupp(\{F_A, F_B\})$, is computed by using formula (2) and then the fuzzy correlation coefficient between F_A and F_B, $r_{A,B}$, is computed by using formula (6), too. Since $r_{A,B}$ is computed from the random sample T, $r_{A,B}$ is needed to be tested to determine if it is really greater than the minimal fuzzy correlation coefficient, r_f. The formula for testing is as follows [4]:

$$t = \frac{r_{A,B} - r_f}{\sqrt{\frac{1 - r_{A,B}^2}{n-2}}} \tag{14}$$

Compare the computed t value to $t_{1-\alpha(n-2)}$, where $t_{1-\alpha(n-2)}$ is the $(1-\alpha)^{th}$ percentile in the t distribution with degree of freedom $n-1$. If we obtain the t value which is greater than $t_{1-\alpha(n-2)}$, then we can conclude that $r_{A,B}$ is greater than the predefined minimal fuzzy correlation coefficient [4].

Step 5: For each element, whose fuzzy support is greater than or equal to s_f and fuzzy correlation coefficient passes the test, of C_2, then it is an element of L_2. Hence, L_2 is the set of the frequent combinations of two fuzzy itemsets, and still, the size of each element of L_2 is 2.

Step 6: Next, each C_k, $k \geq 3$, is generated by L_{k-1} joint with L_{k-1}. Assume that (F_W, F_X) and (F_Y, F_Z) are two elements of L_{k-1}, where $F_X = F_Y$. If the size of the combination $(F_X, \{F_W, F_Z\})$ is k, and (F_W, F_Z) is also a frequent combination of two fuzzy itemsets, then the combination $(F_X, \{F_W, F_Z\})$ is a element with size k of C_k. For each element of C_k, its fuzzy support and fuzzy correlation coefficient are still used to find the elements of L_k.

Step 6: When each L_k, $k \geq 2$, is obtained, for each element of L_k, (F_G, F_H), two candidate fuzzy correlation rules, $F_G \rightarrow F_H$ and $F_H \rightarrow F_G$, can be generated. If the fuzzy confidence of a rule is greater than or equal to c_f, then it is considered as an interesting fuzzy correlation rule.

The algorithm won't stop until no next C_{k+1} can be generated.

5 Example

An experiment will be displayed in this section. Assume that $T = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}\}$ is a random sample with 12 fuzzy records shown in Table 1, and $F = \{f_1, f_2, f_3, f_4, f_5\}$ is the set of observed fuzzy items. Here, s_f is set to 0.30; c_f is set to 0.80; r_f is set to 0.20; α is set to 0.1, and thus $t_{0.9,10}$ is equal to 1.372.

Table 1: A random sample with 12 fuzzy records.

T \ F	f_1	f_2	f_3	f_4	f_5
t_1	0.1	0.9	0.3	0.5	0.2
t_2	0.2	0.8	0.4	0.8	0.1
t_3	0.2	0.7	0.4	0.3	0.3
t_4	0.4	0.5	0.6	0.1	0.5
t_5	0.7	0.3	0.9	0.8	0.2
t_6	0.5	0.6	0.7	0.9	0.8
t_7	0.8	0.1	0.9	0.4	0.9
t_8	0.7	0.1	0.5	0.2	0.7
t_9	0.8	0.4	0.9	0.5	0.9
t_{10}	0.5	0.1	0.6	0.6	0.3
t_{11}	0.6	0.2	0.8	0.5	0.7
t_{12}	0.5	0.3	0.6	0.6	0.3

First, the fuzzy support of each fuzzy item of F is computed and listed in Table 2. Because all $fsupp(f_i), i = 1 \cdots 5$, are greater than s_f, the set of the frequent fuzzy itemsets whose size is equal to 1 is $L_1 = \{f_1, f_2, f_3, f_4, f_5\}$.

Table 2: The fuzzy support of each fuzzy item of F.

F	$fsupp$
f_1	0.50
f_2	0.42
f_3	0.63
f_4	0.52
f_5	0.49

Next, C_2, the set of all combinations of two elements of L_1, is generated by L_1 joint with L_1. $C_2 = \{(f_1, f_2), (f_1, f_3), (f_1, f_4), (f_1, f_5), (f_2, f_3), (f_1, f_5), (f_2, f_3), (f_2, f_4), (f_2, f_5), (f_3, f_4), (f_3, f_5), (f_1, f_4)\}$. For each element of C_2, the fuzzy support, the fuzzy correlation coefficient and t value of testing the fuzzy correlation coefficient are computed and listed in Table 3.

Table 3: The fuzzy support, fuzzy correlation coefficient and t value of testing the fuzzy correlation coefficient of each element of C_2.

C_2	*fsupp*	r	t
$(\{f_1\},\{f_2\})$	0.24	-0.83	-5.79
$(\{f_1\},\{f_3\})$	0.48	0.87	4.38
$(\{f_1\},\{f_4\})$	0.37	-0.03	-0.73
$(\{f_1\},\{f_5\})$	0.42	0.70	2.24
$(\{f_2\},\{f_3\})$	0.31	-0.60	-3.32
$(\{f_2\},\{f_4\})$	0.32	0.19	-0.02
$(\{f_2\},\{f_5\})$	0.26	-0.46	-2.34
$(\{f_3\},\{f_4\})$	0.45	0.18	-0.06
$(\{f_3\},\{f_5\})$	0.47	0.61	1.63
$(\{f_4\},\{f_5\})$	0.33	-0.24	-1.43

In Table 3, an element whose *fsupp* is greater than or equal to s_f (0.30) and t value is greater than or equal to $t_{0.9,10}$ (1.372) is considered an element of L_2. Thus, $L_2=\{(\{f_1\},\{f_3\}),(\{f_1\},\{f_5\}),(\{f_3\},\{f_5\})\}$.

When L_2 is obtained, C_3 is generated by L_2 joint with L_2. $C_3=\{$ $(\{f_1\},\{f_3,f_5\})$, $(\{f_3\},\{f_1,f_5\})$, $(\{f_5\},\{f_1,f_3\})$ $\}$.

Similarly, for each element of C_3, the fuzzy support, the fuzzy correlation coefficient, and the t value of testing the fuzzy correlation coefficient are also computed and displayed in Table 4.

Table 4: The fuzzy support, fuzzy correlation coefficient and t value of testing the fuzzy correlation coefficient of each element of C_3.

C_3	*fsupp*	r	t
$(\{f_1\},\{f_3,f_5\})$	0.40	0.69	2.14
$(\{f_3\},\{f_1,f_5\})$	0.40	0.64	1.81
$(\{f_5\},\{f_1,f_3\})$	0.40	0.67	2.03

In Table 4, because all elements of C_3 satisfy s_f and $t_{0.9,10}$, all elements of C_3 are elements of L_3. Thus, $L_3=C_3$.

No next C_4 can be generated by L_3 joint with L_3, so the mining procedure stops here. By using the elements of L_2 and L_3, 12 candidate fuzzy correlation rules can be generated and listed in Table 5.

Table 5: The fuzzy confidences of the candidate fuzzy correlation rules.

C_2	*fconf*
$\{f_1\}\rightarrow\{f_3\}$	0.97
$\{f_3\}\rightarrow\{f_1\}$	0.76
$\{f_1\}\rightarrow\{f_5\}$	0.83
$\{f_5\}\rightarrow\{f_1\}$	0.85
$\{f_3\}\rightarrow\{f_5\}$	0.78
$\{f_5\}\rightarrow\{f_3\}$	0.95
$\{f_1\}\rightarrow\{f_3,f_5\}$	0.80
$\{f_3,f_5\}\rightarrow\{f_1\}$	0.86
$\{f_3\}\rightarrow\{f_1,f_5\}$	0.63
$\{f_1,f_5\}\rightarrow\{f_3\}$	0.96
$\{f_5\}\rightarrow\{f_1,f_3\}$	0.81
$\{f_1,f_3\}\rightarrow\{f_5\}$	0.83

According to Table 5, we determine 9 interesting fuzzy correlation rules as follows, because their fuzzy confidences are greater than or equal to c_f (0.80).

$$\{f_1\}\rightarrow\{f_3\} \quad (15)$$
$$\{f_1\}\rightarrow\{f_5\} \quad (16)$$
$$\{f_5\}\rightarrow\{f_1\} \quad (17)$$
$$\{f_5\}\rightarrow\{f_3\} \quad (18)$$
$$\{f_1\}\rightarrow\{f_3,f_5\} \quad (19)$$
$$\{f_3,f_5\}\rightarrow\{f_1\} \quad (20)$$
$$\{f_3\}\rightarrow\{f_1,f_5\} \quad (21)$$
$$\{f_1,f_5\}\rightarrow\{f_3\} \quad (22)$$
$$\{f_5\}\rightarrow\{f_1,f_3\} \quad (23)$$
$$\{f_1,f_3\}\rightarrow\{f_5\} \quad (24)$$

From above experiment, we can see that the number of elements of each L_i is effectively reduced. For example, total number of the elements of C_2 is 10, and the number of elements whose fuzzy supports satisfy s_f is 8, but after testing their fuzzy correlation coefficient, the number of elements which belong to L_2 is only 3. Therefore, we can conclude that only really interesting relationships between fuzzy itemsets can be discovered by using our proposed method.

6 Conclusion

In this paper, a fuzzy statistics based method for mining fuzzy correlation rules is proposed. Most methods for mining fuzzy association rules employ a support-confidence framework which adopts fuzzy support and fuzzy confidence to identify the fuzzy association rules as interesting. However, the support-confidence framework may identify many fuzzy association rules as interesting but in fact, the fuzzy itemsets of these rules have no relationship at all.

In our method, a fuzzy correlation analysis which can show us the strength and the type of the linear relationship between two fuzzy itemsets is used. Since the fuzzy correlation coefficient is computed from a random sample, our method is efficient and can be used in large fuzzy dataset. By using the fuzzy correlation analysis, the fuzzy correlation rules with the information about that two fuzzy not only frequently occur together in same records but also are really related to each other are generated.

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