The Returns Policy for Perishable Commodities under Fuzzy Demand

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Abstract: This paper studies the effect of the returns policy on channel coordination and Pareto efficiency in a two-echelon supply chain with fuzzy demand. As in the traditional probabilistic analysis, we prove that the profits for the whole supply chain, the manufacturer and the retailer in the coordination situation are larger than the corresponding one in the non- coordination situation. Not like the probabilistic analysis, the optimal quantity is not unique in fuzzy demand. The goal of channel coordination and Pareto efficiency can be achieved by the returns policy if the optimal quantity is smaller than the most possible value of fuzzy demand; otherwise, it may not be achieved.

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1 Introduction

If total profits summing up profits of members of supply chain do not reach the attainable maximum profit of the whole supply chain, then the supply chain lacks of efficiency. Literature suggests that contract terms such as returns policy may be used to coordinate profit distribution among supply chain to improve the efficiency of supply chain (Emmons and Gilbert, 1980; Pasternack, 1985; Jeuland and Shugan, 1983; Weng, 1995). It is well known that members of supply chain enter into a contract with some certain terms leading to maximization of total expected profits of supply chain, then the supply chain is called as coordinative one. But only if the coordinative contract is also Pareto efficient[1] which means that if after members of supply chain enters into a contract, profits for all members will not decrease and profit of at least one member will increase, the contract will be accepted by all parties and rejected by one party at least otherwise.[2]

These researches mentioned above mostly discussed the models by using probability distribution with known parameters. The method of probability distribution is based on the assumption that the demand distribution can be inferred from sufficient data. However, past data are not always available or reliable because of new products and market turbulence. The challenges for using traditional models are increasing with the change of environment and the fuzzy set theory provides an alternative method for dealing with demand uncertainty.

The fuzzy set theory is widely applied in many academic areas, [3] for example, inventory problem (Chang et al, 2004; Wang, 2011; Vijayan and Kumaran, 2008), EOQ model (Roy and Maiti, 1997; Yao and Lee, 1999), etc. However, there have been a few researches on channel coordination and Pareto efficiency under fuzzy demand even how to achieve channel coordination and Pareto efficiency simultaneously has been the focus of intensive research in traditional method by using probability distribution.

In fact, only a few articles have been published on the fuzzy inventory problem to discuss channel coordination and Pareto efficiency. Xu and Zhai (2008) consider a two-stage supply chain coordination problem with revenue-sharing policy and focuses on the fuzziness aspect of demand uncertainty. They prove that the maximum expected supply chain profit in a coordination situation is greater than the total profit in a non-coordination situation and also prove that Pareto efficiency can simultaneously occur with coordination.

Returns policy often occur in practice, but to our knowledge, the discussion on channel coordination and Pareto efficiency under fuzzy demand is absent in the literature. In this paper, we study channel coordination and Pareto efficiency of a two-echelon

[1] Varian (1992).

[2] Bose and Anand (2007).

[3] See Sirbiladze et al (2009), Cheng et al (2010), and Lai et al (2010).

supply chain with fuzzy demand by using the returns policy. Just like Xu and Zhai (2008), we also adopt triangular fuzzy number to model market demand. First, we find the optimal quantity for the profit of the whole supply chain in the integrated system, then analyze the behavior of both the manufacturer and the retailer in a non-coordinated situation. Finally, we establish a model in which the supply chain is decentralized, and the manufacturer adopts the returns policy that allows the retailer to return all unsold items at a buyback price per unit, but the retailer has to order the quantity which maximizes the channel-wide profit. By comparing the two situations with or without the returns policy in the decentralized supply chain, we conclude that the returns policy is channel coordinating and Pareto efficient if the optimal quantity is smaller than the most possible value of fuzzy demand; but the goal of coordination and efficiency could not be achieved if the optimal quantity is larger than the most possible value of fuzzy demand.

2 Preliminaries

Consider a single-period supply chain problem in which the manufacturer produces a product at unit cost c, sells the product to the retailer and charges the retailer a unit wholesale price w (>c). The retailer faces demand from consumers and sells the product at the price $p(> w)$. We assume that the wholesale price w and the retail price p both are exogenously given. The demand is subjectively believed to be a normal fuzzy number $\tilde{A}$ described by a general membership function $\mu_{\tilde{A}}$

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(m-\sigma)}{\sigma}, & \text{if} \quad m-\sigma < x \le m \\ \dfrac{(m+\sigma)-x}{\sigma}, & \text{if} \quad m < x \le m+\sigma \\ 0 & \text{otherwise} \end{cases}$$

where m is the most possible value of fuzzy number $\tilde{A}$, and σ is the spread of $\tilde{A}$. We denote it as $\tilde{A} = (m,\sigma)$. Figure 1 is a general membership function for the demand $\tilde{A}$.

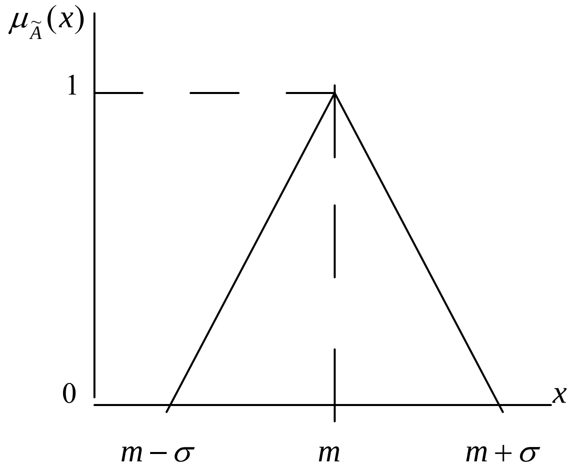

Figure 1. The membership function of $\tilde{A}$

It can easily be shown that the λ-level set of $\tilde{A}$, defined as $A_\lambda = \{x \mid \mu_{\tilde{A}}(x) \ge \lambda\}$ is a closed bounded interval for $0 \le \lambda \le 1$, and can be denoted as

$$A_\lambda = [m + (\lambda - 1)\alpha, m - (\lambda - 1)\beta].$$

For linear operations of closed interval, by classical extension principle (Luo, 1984) , we have the following conclusions:

(1) $[a,b] + [c,d] = [a + c, b + d]$;

(2) $[a,b] - [c,d] = [a - d, b - c]$;

(3) $\alpha[a,b] = [\alpha a, \alpha b]$ if $\alpha \ge 0$, $\alpha \in R$ and

(4) $\alpha[a,b] = [\alpha b, \alpha a]$ if $\alpha < 0$, $\alpha \in R$.

In order to measure the mean value of a fuzzy number $\tilde{A}$, we use the fuzzy mean introduced by

Dubois and Prade (1987)

$$E(\widetilde{A}) = \int_0^1 \frac{a_1(\lambda) + a_2(\lambda)}{2} d\lambda, \qquad (1)$$

where $[a_1(\lambda), a_2(\lambda)]$ is the λ-level set of $\widetilde{A}$.

3 The Basic Model

We first consider a single-period model for a supply chain in which the manufacturer and the retailer are independent to each other. That is, each member in the supply chain aims to maximize his own profit. In this scenario, the manufacturer's profit is

$$\widetilde{\pi}_M^D(Q^D) = (w-c)Q^D,$$

and the retailer's profit is

$$\widetilde{\pi}_R^D(Q^D) = p\min\{Q^D, \widetilde{A}\} - s\max\{0, \widetilde{A} - Q^D\} - wQ^D. \qquad (2)$$

where the superscript D represents the decentralized supply chain. Without doubt, the retailer's profit is a fuzzy set. Since $\widetilde{\pi}_R^D(Q^D)$ is a fuzzy number, we cannot directly maximize it. Therefore we first have to transform $\widetilde{\pi}_R^D(Q^D)$ into $E(\widetilde{\pi}_R^D(Q^D))$ by using the fuzzy number's probabilistic mean as defined by Eq. (1), and then maximize $E(\widetilde{\pi}_R^D(Q^D))$ .We discuss this optimal problem by the following two cases.

(1). Case 1: $m-\sigma \le Q < m$

According to the operation properties of triangular fuzzy numbers, we have the following expressions about the k-level sets of fuzzy sales volume and goodwill loss quantity:

$$(\min\{Q^D, \widetilde{A}\})_\lambda = \begin{cases} [m + (\lambda-1)\sigma, Q^D], & \text{if } 0 < \lambda \le y_0; \\ [Q^D, Q^D], & \text{if } y_0 < \lambda \le 1. \end{cases}$$

$$(\max\{0, \widetilde{A} - Q^D\})_\lambda = \begin{cases} [0, m - Q^D - (\lambda-1)\sigma], & \text{if } 0 < \lambda \le y_0; \\ [m - Q^D + (\lambda-1)\sigma, \; m - Q^D - (\lambda-1)\sigma], & \text{if } y_0 < \lambda \le 1 \end{cases}$$

where $y_0 = 1 + (Q^D - m)/\sigma$. It follows that the λ-level sets of $\widetilde{\pi}_R^D(Q^D)$ are

$$(\widetilde{\pi}_R^D(Q^D))_\lambda = [(p-s)m + (s-w)Q^D + (\lambda-1)(p+s)\sigma, (p-w)Q^D]$$

, if $0 < \lambda \le y_0$; and

$$(\widetilde{\pi}_R^D(Q^D))_\lambda = [(p+s-w)Q^D - sm + (\lambda-1)s\sigma, \; (p+s-w)Q^D - sm - (\lambda-1)s\sigma], \text{if } y_0 < \lambda \le 1.$$

According to Eq. (1), the probabilistic mean of a retailer's profit is given by

$$E(\widetilde{\pi}_R^D(Q^D))_\lambda = \frac{1}{2\sigma}[(p-s)m + (p+s-2w)Q^D](\sigma + Q^D - m) + \frac{1}{4\sigma}(p+s)[(Q^D - m)^2 - \sigma^2] + \frac{1}{\sigma}[(p+s-w)Q^D - sm](m - Q^D).$$

We can determine the profit-maximizing quantity by setting $dE(\widetilde{\pi}_R^D(Q^D))_\lambda / dQ^D = 0$ since the second-order condition $d^2E(\widetilde{\pi}_R^D(Q^D))_\lambda / dQ^{D2} = -(p+s)/2\sigma < 0$ and solving the profit-maximizing quantity Q_l^{D*} as

$$Q_l^{D*} = m + \frac{p+s-2w}{p+s}\sigma, \quad (3)$$

where $p+s<2w$, and the corresponding maximum probabilistic mean profit for the retailer and the manufacturer are

$$E(\tilde{\pi}_R^D(Q_l^{D*})) = (p-w)m - \frac{w(p+s-w)}{p+s}\sigma. \quad (4)$$

$$E(\tilde{\pi}_M^D(Q_l^{D*})) = (w-c)[m + \frac{(p+s-2w)}{p+s}\sigma].$$

The profit for the whole supply chain is equal to the sum of $E(\tilde{\pi}_M^D(Q))$ and $E(\tilde{\pi}_R^D(Q))$. That's,

$$E(\tilde{\Pi}^D(Q_l^{D*})) = E(\tilde{\pi}_M^D(Q_l^{D*})) + E(\tilde{\pi}_R^D(Q_l^{D*})),$$

where $E(\tilde{\Pi}^D(Q_l^{D*}))$ denotes the profit for the whole supply chain in the decentralized chain without returns policy (i.e., $u=0$) and $Q_l^{D*}<m$, and

$$E(\tilde{\Pi}^D(Q_l^{D*})) = (p-c)m - \frac{w^2+c(p+s-2w)}{p+s}\sigma. \quad (5)$$

where $p+s<2w$.

(2). Case 2. $m \le Q \le m+\sigma$

Similar to the discussion in case 1, the λ-level sets of fuzzy sales volume and goodwill loss quantity can be respectively expressed as

$$(\min\{Q,\tilde{A}\})_\lambda = \begin{cases} [m+(\lambda-1)\sigma, Q], \text{ if } 0<\lambda\le y_1; \\ [m+(\lambda-1)\sigma, m-(\lambda-1)\sigma], \text{ if } y_1<\lambda\le 1. \end{cases}$$

$$(\max\{0,\tilde{A}-Q^D\})_\lambda = \begin{cases} [0, m-Q^D-(\lambda-1)\sigma], \text{ if } 0<\lambda\le y_1; \\ [0,0], \text{ if } y_1<\lambda\le 1. \end{cases}$$

where $y_1 = 1-(Q^D-m)/\sigma$. Similarly, the λ-level sets of $\tilde{\pi}_R^D(Q^D)$ are

$$(\tilde{\pi}_R^D(Q^D))_\lambda = [(p-s)m+(s-c)Q^D+(\lambda-1)\sigma(p+s), (p-c)Q^D], \text{ if } 0<\lambda\le y_1; \text{ and}$$

$$(\tilde{\pi}_R^D(Q^D))_\lambda = [pm-cQ^D+\sigma(\lambda-1)p, pm-cQ^D-\sigma(\lambda-1)p], \text{ if } 0<\lambda\le y_1.$$

According to Eq. (1), the probabilistic mean of a retailer's profit is given by

$$E(\tilde{\pi}_R^D(Q^D)) = \frac{1}{2\sigma}[(p-s)m + (p+s-2c)Q^D](\sigma+m-Q^D) + \frac{1}{4\sigma}(p+s)[(Q^D-m)^2-\sigma^2] + \frac{1}{\sigma}[pm-cQ^D](Q^D-m).$$

Differentiating $E(\tilde{\pi}_R^D(Q^D))$ with respect to Q^D, the first-order condition is

$$\frac{dE(\tilde{\pi}_R^D(Q^D))}{dQ^D} = \frac{1}{2\sigma}[(p+s-2c)\sigma + (p+s)(m-Q)] = 0.$$

We can determine the optimal order quantity is

$$Q_r^{D*} = m + \frac{p+s-2w}{p+s}\sigma, \quad (6)$$

where $p+s \ge 2w$. The second-order condition is satisfied. and the corresponding maximum probabilistic mean profit for the retailer and the manufacturer are

$$E(\tilde{\pi}_R^D(Q_r^{D*})) = (p-w)m - \frac{w(p+s-w)}{p+s}\sigma. \quad (7)$$

$$E(\tilde{\pi}_M^D(Q_l^{D*})) = (w-c)[m + \frac{(p+s-2w)}{p+s}\sigma]$$

The profit for the whole supply chain in the case is
$E(\widetilde{\Pi}^D(Q_r^{D*})) =$

$$(p-c)m - \frac{w^2 + c(p+s-2w)}{p+s}\sigma. \quad (8)$$

where $p+s \geq 2w$.

There is one thing worth noting. The two optimal order quantity in Eqs. (3) and (6) look alike, but in fact they are different since $p+s<2w$ for Q_l^{D*} and $p+s \geq 2w$ for Q_r^{D*}. In other words, $Q_l^{D*} < Q_r^{D*}$.

Subsequently, we consider the situation in which the manufacturer acts as his own retailer. Hence, the profit ($\widetilde{\Pi}^I$) is

$$\widetilde{\Pi}^I(Q^I) = p\min\{Q^I, \widetilde{A}\} - s\max\{0, \widetilde{A} - Q^I\} - cQ^I. \quad (9)$$

where the superscript I represents the integrated supply chain.

$\widetilde{\Pi}^I(Q^I)$ is also a fuzzy number, and we can not directly maximize it. Similarly, we have to transform $\widetilde{\Pi}^I(Q^I)$ into $E(\widetilde{\Pi}^I(Q^I))$ by Eq. (1), and then maximize $E(\widetilde{\Pi}^I(Q^I))$. In this situation, the optimal quantity is produced or ordered based on maximizing the chain-wide profit, not like Q_i^{D*} ($i=l,r$) maximizing the retailer's profit. Since compared to Eq. (2), the only difference of Eq. (9) is the unit cost, hence, it is easy to derive the optimal quantity for the whole supply chain by replacing w in Eqs. (3) and (6) with c. That is,

$$Q_i^{I*} = m + \frac{p+s-2c}{p+s}\sigma, \quad i=l,r, \quad (10)$$

where $p+s \geq 2c$ if $m \leq Q_l^{I*} \leq m+\sigma$;

and $p+s<2c$ if $m-\sigma \leq Q_r^{I*} < m$.

The corresponding optimal profit for the whole channel in this model is

$$E(\widetilde{\Pi}^I(Q_i^{I*})) = (p-c)m - \frac{c(p+s-c)}{p+s}\sigma, i=l,r. \quad (11)$$

From Eqs.(3) (or (6)) and (10), we obtain

$$Q_i^{I*} - Q_i^{D*} = \frac{2(w-c)\sigma}{p+s} > 0, i=l,r.$$

The optimal quantity in the decentralized system is smaller than that in the integrated system. This is the so-called "double-marginalization effect" which arises from the conflict of the profits between the manufacturer and the retailer (Spengler, 1950;. Ding and Chen, 2008).

Next, from Eqs.(5) (or (8)) and (11), we derive

$$E(\widetilde{\Pi}^I(Q_i^{I*})) - E(\widetilde{\Pi}^D(Q_i^{D*})) = \frac{(w-c)^2}{p+s}\sigma > 0,$$

$i=l,r$.

The above result is obvious since the profit of a vertically integrated firm is the maximum attainable in the supply chain. If the total profits summing up profits of members of supply chain do not reach the maximum attainable profit of the whole supply chain, then such supply chain lacks of efficiency. That is to say, the manufacturer and the retailer do not reach their respective maximum at the same time in the decentralized supply chain.

4 Returns Policy

In order to improve the efficiency of the whole supply chain, we now turn to consider the situation in which both players are willing to cooperate to get their optimal joint profit. In this situation, the retailer orders the quantity Q_i^{I*} ($i=l,r$), and the manufacturer promise to buy back all unsold items by paying the retailer a buyback price u at the end of the selling season. If $u = w$, the returns policy is with a full refund; if $u < w$, then, the returns policy is with a partial refund. In the scenario, the profits for the manufacturer and the retailer are as follows, respectively,

$$\widetilde{\pi}_M^P(u, Q_i^{I*}) = (w-c)Q_i^{I*} - u\max\{0, Q_i^{I*} - \widetilde{A}\},$$

$$\widetilde{\pi}_R^P(u, Q_i^{I*}) = p\min\{Q_i^{I*}, \widetilde{A}\} + u\max\{0, Q_i^{I*} - \widetilde{A}\} - s\max\{0, \widetilde{A} - Q_i^{I*}\} - wQ_i^{I*}.$$

where the superscript P represents the supply chain with the returns policy. $\widetilde{\pi}_R^P(w,u,Q_i^{I*})$ is a fuzzy number like $\widetilde{\pi}_R^D(Q^D)$. Similar to the precedent, we discuss the expressions of $\min\{Q_i^{I*}, \widetilde{A}\}$, $\max\{0, Q_i^{I*} - \widetilde{A}\}$ and $\max\{0, \widetilde{A} - Q_i^{I*}\}$ by the following two cases.

(1). Case 1: $m - \sigma \le Q < m$

$$(\min\{Q_l^{I*}, \widetilde{A}\})_\lambda = \begin{cases} [m + (\lambda-1)\sigma, Q_i^{I*}], \text{ if } 0 < \lambda \le y_0^*; \\ [Q_l^{I*}, Q_l^{I*}], \text{ if } y_0^* < \lambda \le 1. \end{cases}$$

$$(\max\{0, Q_l^{I*} - \widetilde{A}\})_\lambda = \begin{cases} [0, Q_l^{I*} - m - (\lambda-1)\sigma], \\ \quad \text{if } 0 < \lambda \le y_0^*; \\ [0,0], \text{ if } y_0^* < \lambda \le 1. \end{cases}$$

$$(\max\{0, \widetilde{A} - Q_l^{I*}\})_\lambda = \begin{cases} [0, m - Q_l^{I*} - (\lambda-1)\sigma], \\ \quad \text{if } 0 < \lambda \le y_0^*; \\ [m - Q_l^{I*} + (\lambda-1)\sigma, \\ \quad m - Q_l^{I*} - (\lambda-1)\sigma], \\ \quad \text{if } y_0^* < \lambda \le 1. \end{cases}$$

where $y_0^* = 1 + (Q_l^{I*} - m)/\sigma$. We can determine that the λ-level sets of $\widetilde{\pi}_R^P(u, Q_l^{I*})$ are

$$\widetilde{\pi}_R^P(u, Q_l^{I*}) = [(p-s)m + (p+s)(\lambda-1)\sigma + (s-w)Q_l^{I*}, (p+u-w)Q_l^{I*} - um - u(\lambda-1)\sigma] \text{ if } 0 < \lambda \le y_0^*;$$

$$\widetilde{\pi}_R^P(u, Q_l^{I*}) = [(p+s-w)Q_l^{I*} - sm + s(\lambda-1)\sigma, (p+s-w)Q_l^{I*} - sm - s(\lambda-1)\sigma] \text{ if } y_0^* < \lambda \le 1.$$

According to Eq. (1) and Q_l^{I*} defined in Eq.(10), the probabilistic mean of a retailer's profit in this situation is given by

$$E(\widetilde{\pi}_R^P(u, Q_l^{I*})) = (p-w)m + (p+s-w)\left(\frac{p+s-2c}{p+s}\right)\sigma - (p-u+s)\left(\frac{p+s-c}{p+s}\right)^2\sigma. \quad (12)$$

In order to entice the retailer to increase his stocking level from Q_i^{D*} to Q_i^{I*} ($i = l, r$), the manufacturer has to ensure that the retailer's profit is at least the profit without returns policy. That is,

$$E(\widetilde{\pi}_R^P(u, Q_i^{I*})) - E(\widetilde{\pi}_R^D(Q_i^{D*})) \ge 0. \quad (13)$$

This is the retailer participation constraint (Lau and

Lau, 1999; Tsay, 2001). Let Eq.(13) equal zero and set $i = l$, so the retailer's profit with returns policy is equal to its counterpart without returns policy:

$$E(\tilde{\pi}_R^P(u, Q_l^{I*})) - E(\tilde{\pi}_R^D(Q_l^{D*})) = 0 .$$

From the above equation, we obtain the following condition:

$$u_l = \frac{(p+s)(w-c)^2}{(p+s-c)^2} > 0 . \qquad (14)$$

If the buyback price is set according to Eq.(14), then we can find that the manufacturer is strictly better off with the returns policy than without it, and neither of the parties is worse off, the returns policy is Pareto-efficient with respect to the contract without the returns policy between the manufacturer and the retailer in the decentralized supply chain. And it is found that the joint profit $E(\tilde{\Pi}^P(u, Q_l^{I*}))$ $(= E(\tilde{\pi}_R^P(u, Q_l^{I*})) + E(\tilde{\pi}_M^P(u, Q_l^{I*})))$ is maximized in the supply chain. Therefore, channel coordination will occur alongside Pareto-efficiency if Eq.(14) is satisfied.

Proposition 1. *If the optimal quantity is smaller than the most possible value of fuzzy demand (m), and if the returns policy is implemented according to Eq.(14), then the supply chain achieves the goal of channel coordination and Pareto efficiency.*

Now we assume $u_l = w$. If $u_l = w$, from (14), we derive

$$w(p+s) - c^2 = 0 .$$

This is a contradiction since $(p+s) > c$ and $w > c$.

Proposition 2. *A policy which allows for unlimited return at full refund can not achieves the goal of channel coordination and Pareto efficiency.*

The result in **Proposition 2** is similar to the Theorem 1 of Pasternack (1985), which neglects the discussion on Pareto efficiency. It is obvious that, if the manufacturer does not allow the retailer to make any returns at all, it is impossible to achieve channel coordination and Pareto efficiency. If $u_l = 0$, from (14), we derive

$$\frac{(p+s)(w-c)^2}{(p+s-c)^2} = 0 .$$

This is also a contradiction since $w > c$ and $(p+s) > c$.

Differentiating Eq.(14) with respect to *w*, *p*, *s*, and *c*, respectively, then we determine the following conditions:

$$\frac{du}{dw} = \frac{2(p+s)(w-c)}{(p+s-c)^2} > 0 ;$$

$$\frac{du}{dp} = \frac{du}{ds} = \frac{-(p+s+c)(w-c)^2}{(p+s-c)^3} < 0 ;$$

$$\frac{du}{dc} = \frac{-2(p+s-w)(p+s)(w-c)}{(p+s-c)^3} < 0 .$$

From the results mentioned above, we conclude that *u* is an increasing function of *w* and a decreasing function of *p*, *s*, and *c*.

Proposition 3. *To maintain Pareto efficiency and channel coordination, the larger the wholesale price, the larger the buyback price will be; but the larger the retail price, the goodwill loss, and the unit cost, the smaller the buyback price will be.*

Finally, we discuss the effect of the most possible value of fuzzy demand (m) and the spread of fuzzy demand (σ) on the buyback price (u).

It is easy to find that u has nothing to do with m and σ from Eq.(14) since $du/dm = du/d\sigma = 0$. In other words, the buyback price is not affected by the change of the most possible value and the spread of fuzzy demand in order to keep the existence of channel coordination and Pareto efficiency.

Proposition 4. *The buyback price is independent to the most possible value and the spread of fuzzy demand in order to achieve the goal of channel coordination and Pareto efficiency.*

(2). Case 2. $m \le Q \le m + \sigma$

$$(\min\{Q_r^{I*}, \widetilde{A}\})_\lambda = \begin{cases} [m + (\lambda - 1)\sigma, Q_r^{I*}], \text{ if } 0 < \lambda \le y_1^*; \\ [m + (\lambda - 1)\sigma, m - (\lambda - 1)\sigma], \text{ if } y_1^* < \lambda \le 1. \end{cases}$$

$$(\max\{0, Q_r^{I*} - \widetilde{A}\})_\lambda = \begin{cases} [0, Q_r^{I*} - m - (\lambda - 1)\sigma], \text{ if } 0 < \lambda \le y_1^*; \\ [Q_r^{I*} - m + (\lambda - 1)\sigma, Q_r^{I*} - m - (\lambda - 1)\sigma], \text{ if } y_1^* < \lambda \le 1. \end{cases}$$

$$(\max\{0, \widetilde{A} - Q_r^{I*}\})_\lambda = \begin{cases} [0, m - Q_r^{I*} - (\lambda - 1)\sigma], \text{ if } 0 < \lambda \le y_1^*; \\ [0, 0], \text{ if } y_1^* < \lambda \le 1. \end{cases}$$

where $y_1^* = 1 - (Q_r^{I*} - m)/\sigma$. We can determine that the λ-level sets of $\widetilde{\pi}_R^P(u, Q_r^{I*})$ are

$\widetilde{\pi}_R^P(u, Q_r^{I*}) =$

$[(p - s)m + (p + s)(\lambda - 1)\sigma + (s - w)Q_r^{I*},$

$(p + u - w)Q_r^{I*} - um - u(\lambda - 1)\sigma]$, if $0 < \lambda \le y_1^*$;

$\widetilde{\pi}_R^P(u, Q_r^{I*}) =$

$[(p - u)m + (p + u)(\lambda - 1)\sigma + (u - w)Q_r^{I*},$

$(p - u)m - (p + u)(\lambda - 1)\sigma + (u - w)Q_r^{I*}]$, if $y_1^* < \lambda \le 1$.

According to Eq. (1) and Q_r^{I*} defined in Eq.(10), the probabilistic mean of a retailer's profit in this situation is given by

$$E(\widetilde{\pi}_R^P(u, Q_r^{I*})) = (p - w)m - (w - u)\sigma \left(\frac{p + s - c}{p + s}\right)^2.$$

By means of the similar procedure to case 1, we set

$$E(\widetilde{\pi}_R^P(u, Q_r^{I*})) - E(\widetilde{\pi}_R^D(Q_l^{D*})) = 0.$$

From the above equation, we obtain the following condition:

$$u_r = \frac{w[(p + s)(w - 2c) + c^2)]}{(p + s - c)^2} > (\le) 0, \text{ if}$$

$$w(p + s) > (\le) c[2(p + s) - c]. \qquad (15)$$

Therefore, it is impossible to induce the retailer to increase his stocking level from Q_r^{D*} to Q_r^{I*} by adopting the returns policy only if $w(p + s) \le c[2(p + s) - c]$.

If $w > 2c$, then $u_r > 0$. It occurs in the case that the optimal order quantity is larger than the most possible value of fuzzy demand. Since $w > 2c$

and $p+s>2c$,[4] the difference between $p+s$ and w could be not large enough which means that the benefit from selling an extra unit of product could be equivalent or the difference is not big. Therefore, the manufacturer is willing to adopt the returns policy; otherwise, the manufacturer may have no intension to implement the returns policy if $w<2c$ and $p+s>2c$. Hence, the sign of u_r in Eq.(15) is ambiguous if $w<2c$.

Proposition 5. *If the optimal quantity is larger than the most possible value of fuzzy demand (m) and if $w(p+s)>c[2(p+s)-c]$, the supply chain achieves the goal of channel coordination and Pareto efficiency if the returns policy is implemented according to Eq.(15); otherwise, the goal can not be achieved by using the returns policy if $w(p+s)\le c[2(p+s)-c]$.*

If $w(p+s)>c[2(p+s)-c]$, then it is easy to examine that **Proposition 2**, **3** and **4** can be applied in case 2, omitting them.

5 Numerical Example

Assume that the product discussed in the paper has a limited shelf life because of either product obsolescence or physical decay and its salvage value is zero.

The retail price would be \$40, and the wholesale price is \$30. The goodwill cost per unit due to stockout would be \$5 and the unit cost for the product is \$25. Assume that the retailer feels that the demand in the selling season will most likely be 5000 units; not less than 3380 units not more than 6620 units. Therefore, the fuzzy demand $\widetilde{A}(Q)$ = (5000,1620) and the parameters are (p,w,s,c) = (40,30,5,25).

In the centralized supply chain, the optimal quantity is

$$Q_l^{I*}=m+\frac{p+s-2c}{p+s}\sigma=4{,}820<m=5{,}000\,;$$

and the profit for the whole supply chain is

$$E(\widetilde{\Pi}^I(Q_l^{I*}))=$$
$$(p-c)m-\frac{c(p+s-c)}{p+s}\sigma=\$57{,}000\,.$$

But when the manufacturer and the retailer take action independently, who aim to maximize their individual profit, in other words, in the non-coordinated situation, the optimal quantity, the profits for the manufacturer, the retailer and the whole supply chain are, respectively,

$$Q_l^{D*}=m+\frac{p+s-2w}{p+s}\sigma=4{,}460<m=5{,}000\,;$$

$$E(\widetilde{\pi}_R^D(Q_l^{D*}))=(p-w)m-\frac{w(p+s-w)}{p+s}\sigma$$
$$=\$33{,}800$$

$$E(\widetilde{\pi}_M^D(Q_l^{D*}))=(w-c)[m+\frac{(p+s-2w)}{p+s}\sigma]$$
$$=\$22{,}300\,.$$

$$E(\widetilde{\Pi}^D(Q_r^{D*}))=\$56{,}100\,.$$

From the above calculation, we obtain that the whole supply chain gets 900 more profit in the coordinated situation (i.e., the centralized supply chain) than that in the non-coordinated situation.

[4] If p+s=2c, w>2c is impossible.

In order to improve the efficiency of the whole supply chain, both parties are willing to cooperate by the returns policy with the buyback price $u_l = 45/16$ (from Eq. (14)). In this situation,

$$E(\tilde{\pi}_R^P(u=\frac{45}{16}, Q_l^{I*})) = \$33{,}800;$$

$$E(\tilde{\pi}_M^P(u=\frac{45}{16}, Q_l^{I*})) = \$23{,}200;$$

$$E(\tilde{\Pi}^P(u=\frac{45}{16}, Q_l^{I*})) = \$57{,}000.$$

Since $E(\tilde{\pi}_R^P(u=\frac{45}{16}, Q_l^{I*})) = E(\tilde{\pi}_R^D(Q_l^{D*}))$, $E(\tilde{\pi}_M^P(u=\frac{45}{16}, Q_l^{I*})) > E(\tilde{\pi}_M^{D*}(Q_l^{D*}))$ and $E(\tilde{\Pi}^P(u=\frac{45}{16}, Q_l^{I*})) = E(\tilde{\Pi}^D(Q_l^{D*}))$, thus, the goal of channel coordination and Pareto efficiency can be achieved by the returns policy if the optimal quantity which maximizes the joint profit is smaller than the most possible value of fuzzy demand.

Keep the others fixed, but the retail price is increased to 76. Notice that $p+s=81>2c=50$, and $p+s=81>2w=60$, and $w=30<2c=50$ now. In this situation, the corresponding values in the centralized and the non-coordinated supply chain are as follows.

$$Q_r^{I*} = 5{,}620 > m = 5{,}000;$$

$$E(\tilde{\Pi}^I(Q_l^{I*})) = \$227{,}000.$$

$$5{,}000 = m < Q_r^{D*} = 5{,}420 < Q_r^{I*} = 5{,}620;$$

$$E(\tilde{\pi}_R^D(Q_l^{D*})) = \$199{,}400;$$

$$E(\tilde{\pi}_M^D(Q_l^{D*})) = \$27{,}100;$$

$$E(\tilde{\Pi}^D(Q_r^{D*})) = \$226{,}500.$$

In order to keep $E(\tilde{\pi}_R^P(u, Q_r^{I*})) = E(\tilde{\pi}_R^D(Q_r^{D*}))$, from Eq.(15) the buyback price is

$$u_r = \frac{w[(p+s)(w-2c)+c^2)]}{(p+s-c)^2} = -\$9.25.$$

It is impossible for the retailer to accept the contract in which he has to pay the manufacturer \$9.25 per unit when he returns unsold items to the manufacturer.

Therefore, when the optimal quantity is larger than the most possible value of fuzzy demand, the goal of Pareto efficiency and channel coordination can not be achieved through the returns policy if $w(p+s) \le c[2(p+s)-c]$.

6 Conclusion

The returns policy is widely studied in the literature to discuss the effect of the policy on channel coordination. In this paper, we study channel coordination and Pareto efficiency of a two-echelon supply chain with demand uncertainty which is expressed by fuzzy demand rather than probabilistic demand inferred by the past record, using the returns policy. But realistically, it is not always possible for the retailer to gather enough data recorded in the past. In fact, external demand is often estimated by the retailer's experience. So it should be necessary to use fuzzy set theory to solve the uncertainty (Xu and Zhai, 2008).

When the demand is a trapezoidal fuzzy number, the optimal quantity is not necessarily unique like the probabilistic demand. The returns policy is a risk-sharing mechanism. [5] The larger the order

[5] Padmanabhan and Png (1997) do not agree with the opinion.

quantity, the larger the risk faced by firms due to unsold items at the end of the selling season will be. When the optimal order quantity is smaller than the most possible value of fuzzy demand, the returns policy is channel coordinating and Pareto-efficient for appropriately chosen value of the problem parameters. If all others equal but the retail price increases to make that the optimal order quantity is larger than the most possible value of fuzzy demand, then, the manufacturer may have no intension to implement the returns policy unless the retailer is willing to accept the negative buyback price.

Although the increase in the retail price makes both parties in the supply chain, it is not certain for the manufacturer to encourage the retailer to increase his stocking level to the quantity maximizing the profit of the whole supply chain if the optimal quantity is larger than the most possible value of fuzzy demand.

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