Integration of Item Hierarchy and Concept Tree based on Clustering Approach with Application in Statistics Learning

YUAN-HORNG LIN
Department of Mathematics Education, National Taichung University
140 Min-Sheng Rd., Taichung City 403, Taiwan
Taiwan
lyh@mail.ntcu.edu.tw

Abstract: Item hierarchy and concept tree provide references for cognition diagnosis and remedial instruction. Therefore, integration of data analysis on item hierarchy and concept tree should be important. The purpose of this study is to provide an integrated methodology of item hierarchy and concept tree analysis. Besides, fuzzy clustering is adopted to classify sample so that homogeneity appear in the same cluster and adaptive instruction will be more feasible. Polytomous item relational structure (PIRS) is the foundation of item hierarchy analysis. Interpretive structural modeling (ISM) combined with calculation of ordering coefficient is to construct concept tree. Source data sets of PIRS and ISM are based on response data matrix and item-attribute matrix respectively. In this study, the empirical test data is the statistics assessment of university students. The results show that the integration of PIRS and ISM based on fuzzy clustering are useful for cognition diagnosis and adaptive instruction. Finally, further suggestions and recommendations based on findings are discussed.

Key-Words: cognition diagnosis, concept tree, fuzzy clustering, ISM, item hierarchy, PIRS

1 Introduction

Item hierarchy and concept tree provide effective information for cognition science and psychometrics [2][4]. Quite a few researchers, such as information science, psychometrics, and e-learning, focus on methodological development of item hierarchy and concept tree [1][31]. As to item hierarchy, item relational structure (IRS) is a useful method [14]. In accordance with item hierarchy, knowledge structures of examinee are clearly realized. However, there exit some limitations on IRS. One is its limitation on dichotomous items and IRS is not suitable for polytomous items [15]. Owing to the limitation of dichotomous IRS, Lin, Bart and Huang had proposed a generalized IRS ordering coefficient formula and this formula is suitable for polytomous or mixed scoring items, which is called polytomous item relational structure (PIRS) [28]. Furthermore, dichotomous IRS is a special case of PIRS and PIRS extends the application in real assessment environment.

According to the viewpoint cognition science, concept is the basic unit to construct knowledge structures [17][25]. Concept is also the node to activate formation of new knowledge [26][27]. As to the assessment tool, the relationship of item and concept make up the binary Q-matrix, which is also called item-attribute matrix [12]. Each item measures at least one concept. Concept tree of examinee could be organized based on response matrix and item-attribute matrix [11][16][19]. Interpretive structural modeling (ISM) flourishes the mathematical algorithm to construct concept tree. Original data of ISM is the adjacent matrix of concepts and the analytic results will display concept tree. In this study, formula of ordering coefficient from ordering theory is used to calculate the adjacent relation among concepts. Information of concept tree should be more beneficial for cognition diagnosis and pedagogy [9][18][33].

Clustering technique has been widely used in many fields. One of these fields is cognition diagnosis and it helps represent concept structure [30][32]. For the purpose of pedagogy, fuzzy clustering based on response matrix is suitable for remedial instruction. Fuzzy clustering improves homogeneity within group and heterogeneity between groups [22]. If item hierarchy and concept tree are discussed based on the results of fuzzy clustering, this integration will help adaptive and remedial instruction more feasible.

Statistics is an important one branch of applied mathematics and it helps explanation of research data greatly. Therefore, investigation on item hierarchy and concept tree of statistics learning is rarely discussed. With the integration item hierarchy and concept tree based on fuzzy clustering, an empirical data for basic concepts of statistics from university students will be discussed. Fuzzy clustering help constitute homogeneity groups. The results of this empirical analysis will provide utility of this integrated method and suggestions for statistics education.

2 Literature Review

Polytomous IRS and ISM are adopted to generate item hierarchy and concept tree. Fuzzy clustering is used to classify examinee. All the related literature are described and discussed as follows.

2.1 Dichotomous IRS

The purpose of IRS is to generate item hierarchy for dichotomous items [13]. With the cross table of response data between two dichotomous items, formula of IRS decides subordinate relationship and precondition among items. To take two dichotomous items for example, correct answer is recorded by 1 and wrong answer is 0. The cross table on response proportion for all examinee is shown as Table 1.

Table 1. Response between Two Items

		Item j 1	Item j 0	Total
Item i	1	p_{11}	p_{10}	$p_{1\bullet}$
	0	p_{01}	p_{00}	$p_{0\bullet}$
Total		$p_{\bullet 1}$	$p_{\bullet 0}$	1

In Table 1, it shows $p_{11}+p_{10}+p_{01}+p_{00}=1$. M. Takeya defined the ordering coefficient r_{ij}^* in order to determine whether item i is the precondition of item j. It is [13]

$$r_{ij}^* = 1 - \frac{p_{01}}{(p_{\bullet 1})(p_{0\bullet})} \quad (1)$$

r_{ij}^* means the degree for item i to be the precondition of item j. The threshold ε is to decide the precondition between items. It is

$$r_{ij} = \begin{cases} 1 & , \quad r_{ij}^* \geq \varepsilon \\ 0 & , \quad r_{ij}^* < \varepsilon \end{cases} \quad (2)$$

When it is $r_{ij}=1$, it means item i is the precondition of item j and there is a linkage $i \to j$ in the item hierarchy. Otherwise, When it is $r_{ij}=0$, item i is not the precondition of item j and there is no linkage from item i to j. It is suggested threshold $\varepsilon = .50$. An example of 7 dichotomous items with their ordering coefficient r_{ij}^* based on $\varepsilon = .50$ is shown as Table 2.

Coefficients r_{ij} of these items are shown in Table 3. Item hierarchy of these 7 items is depicted in Figure 1. As shown in Figure 1, the levels and preconditions among items with their correct ratio display their hierarchical structures. For example, it shows that item 1 is the precondition of item 4, 2, 6, 3.

Table 2. Coefficient r_{ij}^* of 7 Dichotomous Items

Items	1	2	3	4	5	6	7
1	---	1.0	1.0	1.0	-.13	1.0	.10
2	.14	---	.50	.10	.25	-.50	.10
3	.14	.50	---	.40	.25	1.0	.40
4	.36	.25	1.0	---	.13	1.0	.10
5	-.03	.40	.40	-.08	---	1.0	.28
6	.04	-.13	.25	.10	.16	---	.10
7	.04	.25	1.0	.10	.44	1.0	---

Table 3. Coefficient r_{ij} of 7 Dichotomous Items

Items	1	2	3	4	5	6	7
1	---	1	1	1	0	1	0
2	0	---	1	0	0	0	0
3	0	1	---	0	0	1	0
4	0	0	1	---	0	1	0
5	0	0	0	0	---	1	0
6	0	0	0	0	0	---	0
7	0	0	1	0	0	1	---

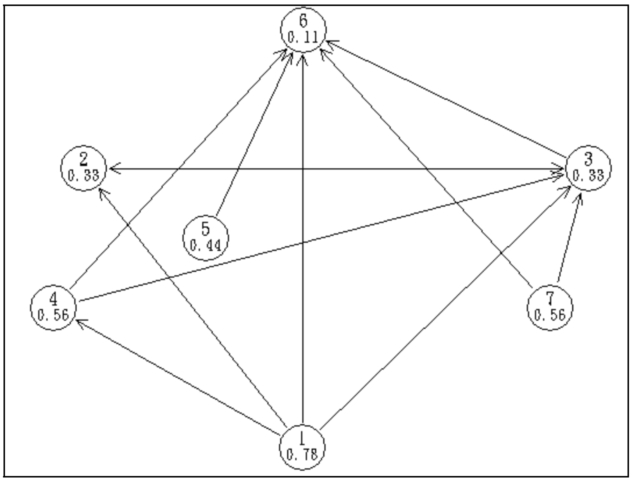

Fig 1. Item Hierarchy of 7 Dichotomous Items

2.2 Polytomous IRS

Because of the limitation of IRS, Lin, Bart and Huang had developed polytomous IRS and IRS is a special case of polytomous IRS [21]. Theoretical foundation of PIRS is as following steps.

(1) Item i and item j are two polytomous items. Assume the scoring categories of item i and item j be denoted by k ($k = 0,1,\cdots,C_i - 1$) and j ($l = 0,1,\cdots,C_j - 1$) respectively. The cross table of response proportion for all examinee is shown as Table 4. One is known that Table 4 is an extension of dichotomous items in Table 1.

Table 4. Response of Two Polytomous Items

Item i	Item j: $C_j - 1$	…	0	Total
$C_i - 1$	$p_{(C_i-1)(C_j-1)}$	…	$p_{(C_i-1)0}$	$p_{(C_i-1)\bullet}$
⋮	⋮	…	⋮	⋮
0	$p_{0(C_j-1)}$	…	p_{00}	$p_{0\bullet}$
Total	$p_{\bullet(C_j-1)}$	…	$p_{\bullet 0}$	1

(2) A is the set of response pairs which satisfy the condition for item i to be the precondition of item j. Set A is

$$A = \left\{ (k,l) \middle| \frac{k}{C_i - 1} < \frac{l}{C_j - 1} \right\} \tag{3}$$

(3) $\# A$ is the size of set A. Standardized ordering coefficient R_{ij}^* represents the degree for item i to be the precondition of item j. It is

$$R_{ij}^* = 1 - \left(\frac{1}{\# A}\right) \sum_k \sum_l \frac{p_{kl}}{(p_{\bullet k})(p_{l\bullet})} \ , \ \forall \frac{k}{C_i - 1} < \frac{l}{C_j - 1} \tag{4}$$

(4)Threshold ε is to generate coefficient R_{ij}, which reveals the precondition relationship. It is

$$R_{ij} = \begin{cases} 1 & , \ R_{ij}^* \ge \varepsilon \\ 0 & , \ R_{ij}^* < \varepsilon \end{cases} \tag{5}$$

$R_{ij} = 1$ means item i is the precondition of item j and it is shown as $i \to j$. Otherwise, $R_{ij} = 0$ means item i is not the precondition of item j and there is no linkage from i to j. One is concluded that IRS is a special case of PIRS when $C_i = C_j = 2$.

2.3 Dichotomous Ordering Theory

Ordering theory (OT) is mainly used to determinate the ordering relationship in psychometrics [6]. For two dichotomous item i and item j ($i \neq j$), right answer is represented by 1 and wrong is 0. Four response patterns, which are (1,1), (1,0), (0,1), (0,0), are considered. The response pattern (0,1), is called disconfirmatory pattern and it doesn't satisfy the condition for item i to be the precondition of item j [7] . A cross table based on the above four response pattern (1,1), (1,0), (0,1), (0,0) could be presented in Table 5. It is $n = n_{11} + n_{10} + n_{01} + n_{00}$ and the percentage of disconfirmatory pattern (0, 1) d is

$$d = n_{01}/n \, , \text{ where } \quad 0 \le (n_{01}/n) \le 1 \tag{6}$$

n_{01}/n is the percentage of disconfirmatory pattern (0, 1). The smallest the n_{01}/n is, the more probability item i is the precondition of item j. Tolerance level ε ($0 < \varepsilon < 1$) is to generate the relationship as follows. It is suggested that ε should be smaller than 0.2.

- If it is $n_{01}/n < \varepsilon$, item i is the precondition of item j. Then, item i could be linked forward to item j ($i \to j$). And item j belong to one higher level than item i.
- If it is $n_{01}/n \ge \varepsilon$, it means item i is not the precondition of item j. Then, there is no relationship between these two items.

Table 5. Cross Table for Item i and Item j

		Item j: 1	0	Total
Item i	1	n_{11}	n_{10}	$n_{1\bullet}$
	0	n_{01}	n_{00}	$n_{0\bullet}$
Total		$n_{\bullet 1}$	$n_{\bullet 0}$	n

OT has been widely adopted in educational measurement, logic thinking validation and psychological evaluation. Most literature indicates OT could construct visual hierarchy and it helps relationship be easily understood. However, one shortcoming of OT appears that it is only suitable for dichotomous items. This shortcoming hinders utility of OT.

2.4 Polytomous Ordering Theory

There exists limitation for dichotomous OT because it is only suitable for dichotomous items. Lin, Bart, and Huang developed polytomous ordering theory (polytomous OT). Cross table of response for two polytomous items is depicted in Table 6. Steps of polytomous OT are as follows.

Table 6. Cross Table for Two Polytomous Items

		Item j			Total
		C_j-1	..	0	
Item i	C_i-1	$n_{(C_i-1)(C_j-1)}$	..	$n_{(C_i-1)0}$	$n_{(C_i-1)\bullet}$
	$\vdots$	$\vdots$	..	$\vdots$	$\vdots$
	1	$n_{1(C_j-1)}$	..	n_{10}	$n_{1\bullet}$
	0	$n_{0(C_j-1)}$	..	n_{00}	$n_{0\bullet}$
Total		$n_{\bullet(C_j-1)}$	..	$n_{\bullet 0}$	n

(1) The scoring of items i and item j are k ($k=0,1...,C_i-1$) and l ($l=0,1...,C_j-1$) respectively. The cross table of response based on the two polytomous items could be displayed as Table 6.

(2) As to Table 6, for those response patterns with $\frac{k}{C_i-1}<\frac{l}{C_j-1}$, they don't satisfy the condition for item i to be the precondition of item.

(3) Normalized method for counting the frequencies of "item i is not the precondition of item j" is defined as follows.

$$n'=\sum_k\sum_l n_{kl}\ ,\ \forall\frac{k}{C_i-1}<\frac{l}{C_j-1} \qquad (7)$$

(4) Percentage of disconfirmatory patterns D is defined as follows.

$$D=n'/n\ \text{, where}\quad n'/n\in[0,1] \qquad (8)$$

n'/n indicates the measurement for item i to be the precondition of item j. The smaller the n'/n is, the higher that item i is the precondition of item j.

(5) Whether item i is the precondition of item j depends on tolerance level ε ($0<\varepsilon<1$). Ordering relation R_{ij}^{pot} is to determine the precondition as follows.

$$R_{ij}^{pot}=\begin{cases}1 & , \quad n'/n<\varepsilon \\ 0 & , \quad n'/n\geq\varepsilon\end{cases} \qquad (9)$$

In the above formula, it means

- $R_{ij}^{pot}=1$ if $n'/n<\varepsilon$ exists. Item i is the precondition of item j. Then, item i could be linked forward to item j ($i\to j$). And item j belong to one higher level than item i.
- $R_{ij}^{pot}=0$ if $n'/n\geq\varepsilon$ exists. Item i is not the precondition of item j. Then, there is no relationship between these two items.

All items constitute the matrix $R=\left(R_{ij}^{pot}\right)$. It reveals that dichotomous OT, which is $C_i=C_j=2$, is a special case of polytomous OT.

2.5 Interpretive Structural Modeling

The theoretical foundation of interpretive structural modeling (ISM) is based upon discrete mathematics and graph theory [29]. J. N. Warfield provided ISM and it is to arrange elements within a complex system in the form of hierarchical relation [24].

There are K elements within a set and the subordinate relationship among elements is known. In other words, the prerequisite relationship among elements must be acquired in advance. All the subordinate relationship could be expressed in the form of adjacent matrix $A=\left(a_{ij}\right)_{K\times K}$. If $a_{ij}=1$ exists, it means A_i is the precondition of A_j . Otherwise, $a_{ij}=0$ means A_i is not the precondition of A_j .

Boolean operation is used to construct hierarchical graph. The first step is to find transitive closure $\hat{A}=A\oplus A^2\oplus\cdots A^P$ and then reachability matrix is defined as $R=\hat{A}\oplus I=(A\oplus I)^P$. Based on the intersection of transitive closure $\hat{A}$ and reachability matrix R , the hierarchical graph of elements within matrix $A=\left(a_{ij}\right)_{K\times K}$ could be plotted.

An example of $A=\left(a_{ij}\right)_{7\times 7}$ is shown as follows. For instance, it shows that A_2 is the precondition of A_4 , A_6 and A_7 . It also indicates that A_7 is the precondition of A_1 , A_5 , A_3 and A_6 .

According to $A=\left(a_{ij}\right)_{7\times 7}$, the corresponding hierarchical graph is depicted in Fig. 2. It reveals there are 6 levels . From bottom to top, the elements within each level are $\{A_2\}$, $\{A_4\}$, $\{A_6, A_7\}$, $\{A_3\}$, $\{A_5\}$, $\{A_1\}$. Besides, the linkage among elements shows their prerequisite relationship. For example, A_2 is the precondition of A_4, A_6 and A_7.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

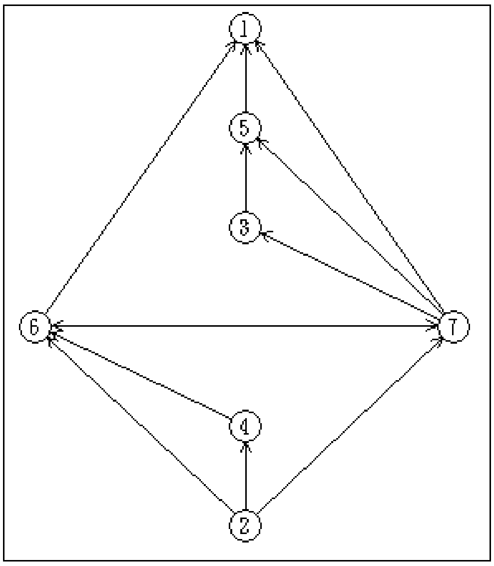

Fig. 2. Hierarchical Graph of Elements

2.6 Fuzzy Theory and Fuzzy Clustering

Fuzzy theory flourishes methodologies a lot in information science. It also improves data analysis n social science. Membership function $u_A(x)$ with $0 \le u_A(x) \le 1$ of fuzzy set A is to represent the degree for x to belong to the fuzzy set.

There are quite a few clustering algorithm and Bezdek innovated fuzzy clustering greatly because membership is considered into the objective function [8]. For a matrix $X = (x_{nm})_{N\times M}$ with N subjects and M variables, the membership matrix $U = (u_{cn})_{C\times N}$ and the cluster center matrix $V = (v_{cm})_{C\times M}$ are unknown under cluster number C. Objective function of optimization algorithm is defined as follows

$$J(U,V) = \sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})^q d^2(c,n) \tag{10}$$

where $d^2(c,n) = \sum_{m=1}^{M}(x_{nm} - v_{cm})^2$ and u_{cn}, v_{cm} are acquired by iteration with convergence. The maximum membership is to decide to which subjects belong.

Statistical clustering isn't really suitable for social science data because it is hard for any subject to belong to only one cluster exactly. In this situation, soft computing of fuzzy clustering seems more feasible and practical.

3 Research Design and Data

3.1 Empirical Data

Empirical data of statistics test for university students is analyzed by the integrated method in this study. The test is designed by the author and there are 51 sophomores, who are from Taiwan, take the statistics course and they must participate in the test. The assessment consists of 11 polytomous items which measure 5 basic statistics concepts. Concept attributes are depicted in Table 7. The item-attribute matrix is shown in Table 8. If one item exactly measures the concept, the value is 1; otherwise it is 0.

Table 7. Concept Attributes of Test

Concepts	Concept Attributes
1	Random Sampling
2	Definition of Sampling Distribution
3	Sampling Distribution of Mean
4	Central Limit Theorem
5	Sampling Distribution of Proportion

Table 8. Item-Attribute Matrix of Test

Items	Concepts				
	1	2	3	4	5
1	1	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	1	0	0
5	0	0	1	0	0
6	0	0	0	1	0
7	0	0	0	1	0
8	0	0	1	0	0
9	0	0	0	0	1
10	0	0	0	0	1
11	0	0	1	0	0

Each item could have its correct ratio by its mean divided by full score. The correct ratio of items is depicted in Fig. 3.

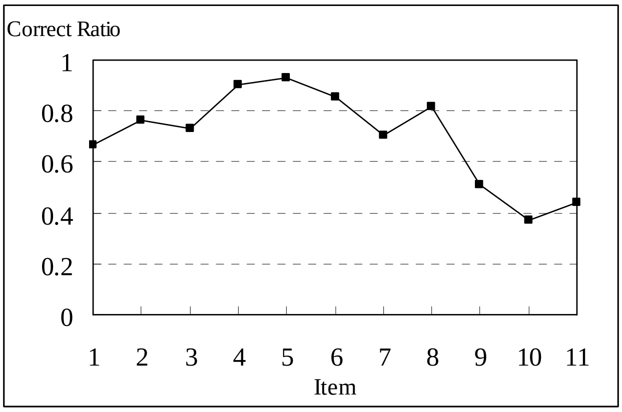

Fig. 3. Plot of Correct Ratio for Each Item

3.2 Process of Data Analysis

Response data matrix $X = (x_{nm})_{51\times 11}$ and item-attribute $Y = (y_{mk})_{11\times 5}$ are the source data. The concept score matrix is defined as $Z = (X)(Y) = (z_{nk})_{51\times 5}$. The process of data is depicted in Fig. 4.

As shown in Fig. 4., response data matrix is for fuzzy clustering to generate optimal number of cluster. Polytomous IRS is adopted to analyze item hierarchy. On the other hand, concept-score matrix is acquired by multiplication of response data matrix and item-attribute matrix. Matrix $R = \left(R_{ij}^{pot}\right)$ is acquired by polytomous OT from concept-score matrix. Finally, ISM is used to generate concept tree based on $R = \left(R_{ij}^{pot}\right)$.

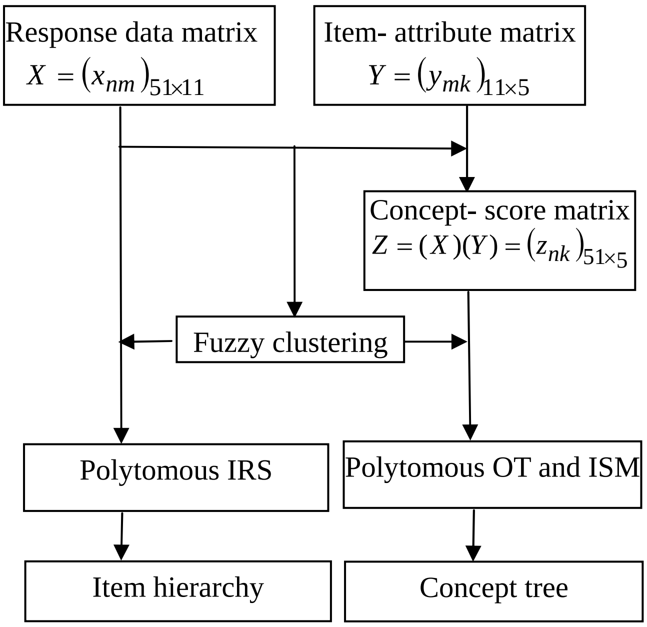

Fig. 4. Process of Algorithm for Data Analysis

4 Results

Results of the data analysis will display optimization of cluster, item hierarchy and concept tree.

4.1 Number of Cluster

Appropriate number of cluster must be decided in advance so that item hierarchy and concept tree could display features of knowledge structures for each cluster. Partition coefficient and partition entropy is to determine number of cluster. Calculations of partition coefficient $F(U;C)$ and partition entropy $H(U;C)$ are as follows. When the largest partition coefficient and the smallest partition entropy occur, it is the best number of cluster.

$$F(U;C) = \frac{1}{N}\sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})^q \tag{11}$$

$$H(U;C) = \frac{-1}{N}\sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})ln(u_{cn}), \quad \forall u_{cn} \neq 0 \tag{12}$$

As shown in Table 9, one is conclude that there exist the largest partition and the smallest partition entropy when the cluster number is 6. Consequently, the optimal number of cluster is 6.

Table 9. Partition Coefficient and Partition Entropy

Cluster Number	Partition Coefficient	Partition Entropy
2	.8213	.3977
3	.7527	.4452
4	.7842	.4307
5	.8180	.3914
6	.8335	.3810
7	.8164	.4141

Sample size of each cluster is displayed in Table 10. Sample size of each cluster is between 7 and 10. Besides, it also shows that cluster I has the most students. Cluster II and Cluster III has least students.

Table 10. Number of Students within Each Cluster

Cluster	Number of Students
I	10
II	7
III	7
IV	8
V	10
VI	9
Total	51

Correct ratio of items for each item is depicted in Table 11. According to the Table 11, one is concluded that cluster V has quite high correct ration on almost all items. However, for each item, its correct ratio vary a lot as compared with different cluster.

Table 11. Correct Ratio of Items for Each Cluster

Items	Cluster					
	I	II	III	IV	V	VI
1	0.30	0.31	0.74	0.77	0.90	0.92
2	0.03	0.85	0.98	0.94	0.99	0.88
3	0.59	0.02	0.69	0.98	0.91	0.99
4	0.85	0.91	0.86	0.91	0.94	0.96
5	0.94	1.00	0.77	0.95	1.00	0.91
6	0.78	0.77	0.85	0.99	0.84	0.88
7	0.59	0.99	0.04	0.96	0.90	0.66
8	0.89	0.85	0.53	0.67	0.94	0.95
9	0.31	0.53	0.03	0.02	0.99	0.99
10	0.01	0.87	0.10	0.11	0.51	0.70
11	0.45	0.13	0.46	0.44	0.98	0.02

4.2 Item Hierarchy Analysis

As shown from Figure 5 to Figure 10, item hierarchy based on PIRS of each cluster is depicted respectively. For each item, its correct ratio and linkage with other items are displayed. Linkage among items indicates the prerequisite relationship. Item hierarchy of each cluster represents its own features of knowledge structure. For instance, cluster I and cluster II are discussed as examples.

For the example of cluster I, correct ratio of item 5, 4, 8, 6 is higher than .80 and these four items are the preconditions of many other items. Furthermore, item 2 and item 10 own lowest correct ratio and it means students of cluster I need more understanding and comprehension on item 2 and item 10. For item 2, item 9, 4, 11, 6, 1 could be its prerequisite condition. Similarly, item 3 is the prerequisite condition of item 10. That is, students of cluster I must comprehend item 9, 4, 11, 6, 1 in advance of item 2. Furthermore, comprehension and understanding of item 3 is the prerequisite condition of item 10.

As to cluster II, correct ratio of item 8, 2, 5, 10, 7 and 4 is higher than .80 and these six items are the preconditions of many other items. Item 3 has the lowest correct ratio and it needs prerequisite understanding and comprehension from many other items. Well understanding of item 11, 9 and 1 could be the direct foundation of item 3. However, item 11 need the previous understanding from item 8, 2 and 10. Item 9 needs the previous understanding from item 8 and 5. Similarly, item 1 needs the previous comprehension from item 6 and 4.

Regarding cluster I and cluster II, correct ratio of items and prerequisite relationship among items are different. One is concluded that item hierarchy of 6 clusters differs. Therefore, it means that each cluster reveals its own features of knowledge structures. Design of remedial and adaptive instruction for each cluster could depend on its own item hierarchy.

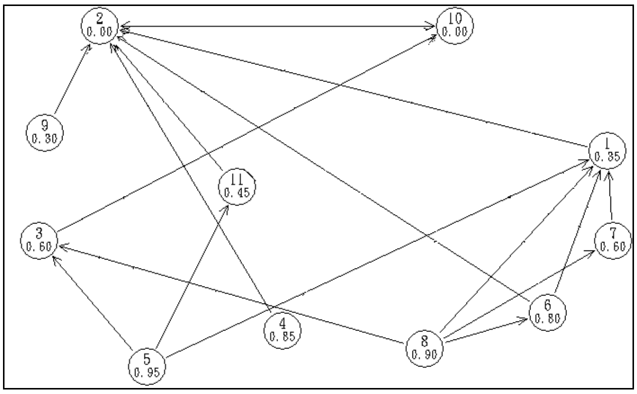

Fig. 5. Item Hierarchy of Cluster I

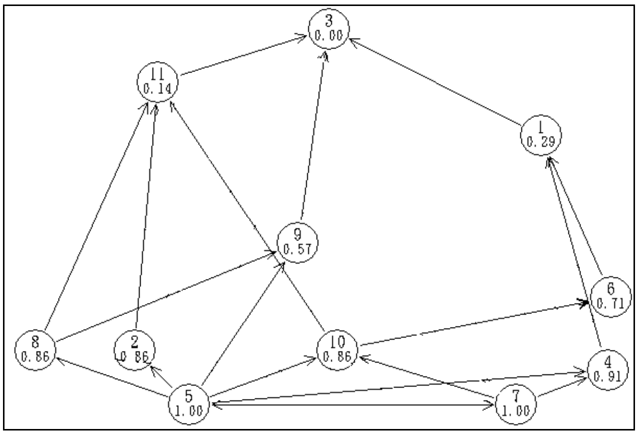

Fig. 6. Item Hierarchy e of Cluster II

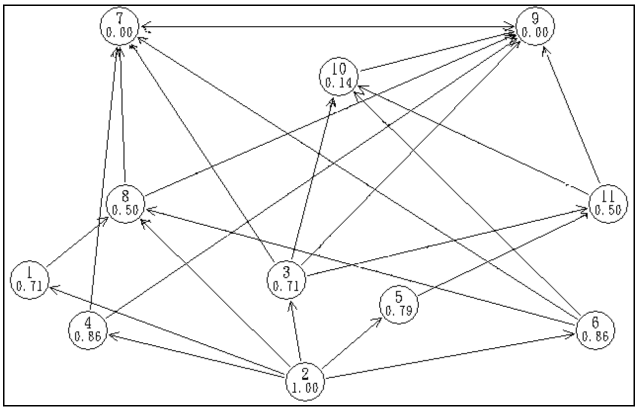

Fig. 7. Item Hierarchy of Cluster III

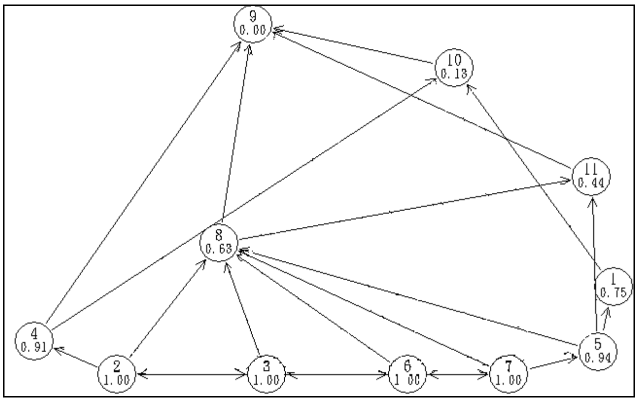

Fig. 8. Item Hierarchy of Cluster IV

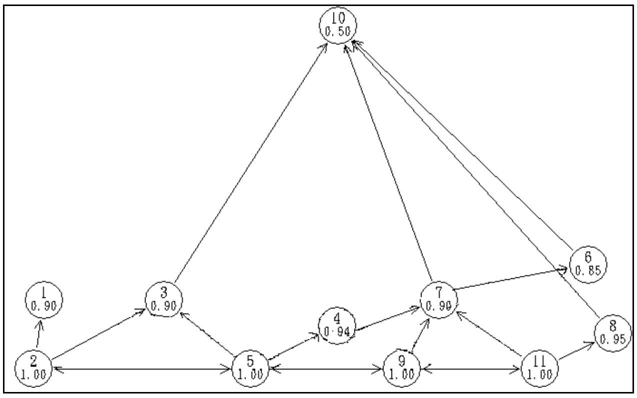

Fig. 9. Item Hierarchy of Cluster V

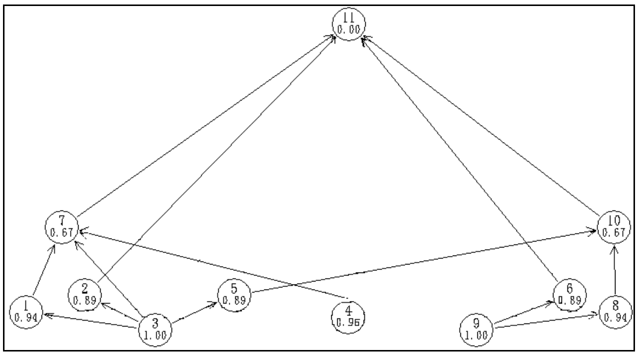

Fig. 10. Item Hierarchy of Cluster VI

4.3 Concept Tree Analysis

According to process of algorithm for data analysis in Fig. 4., concept-score matrix comes form $Z=(X)(Y)=\left(z_{nk}\right)_{51\times 5}$. Matrix $R=\left(R_{ij}^{pot}\right)$ is acquired by polytomous OT from concept-score matrix of each cluster. ISM is used to generate concept tree based on $R=\left(R_{ij}^{pot}\right)$ of each cluster. Therefore, concept tree for each cluster is shown as follows. Besides, the five concept attributes in Table 7 have been confirmed ordering from concept 1 to concept 5. Namely, based on viewpoints of statistics expert, concept 1 is the foundation of the other concepts and concept 5 need preconditions from the other concepts.

As shown from Fig. 11 to Fig. 16, concept tree vary greatly across different cluster. In comparison with each cluster, some findings could be concluded as follows.

Firstly, concept tree of cluster II and cluster III are similar and they look like concept tree of expert. Concept 1 and concept 2 are their common foundation. Concept 5 need precondition from other concepts. The only disparity occurs to the linkage between concept 3 and concept 4. In cluster II, concept 3 and concept 4 are independent. However, concept 3 could be the prerequisite of concept 4 in cluster III.

Secondly, cluster IV and cluster V have one unusual linkage from concept 2 to concept 1. It means there is incomplete learning or misconception on concept 1 and concept 2 for students of cluster IV and cluster V.

Thirdly, cluster VI has one uncommon linkage from concept 4 to concept 1. It reveals that there is learning deficient or misconception on concept 1 and concept 4 for students of cluster VI.

Finally, cluster I gather erroneous linkages of cluster IV, cluster V and cluster VI. It is because that there are two incorrect linkages. One is from concept 2 to concept 1. The other is from concept 4 to concept 1. These two linkages also occur in cluster IV, cluster V and cluster VI. Therefore, students of cluster I need the remedial instruction the most.

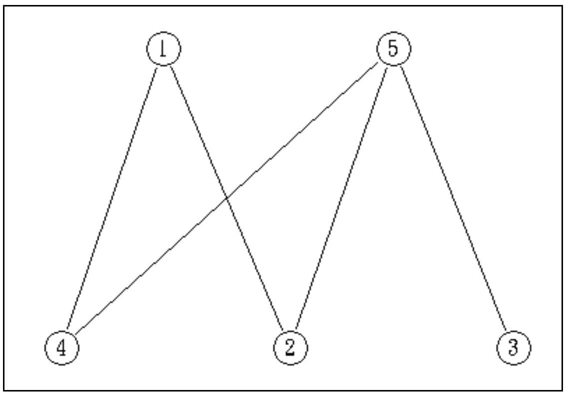

Fig. 11. Concept Tree of Cluster I

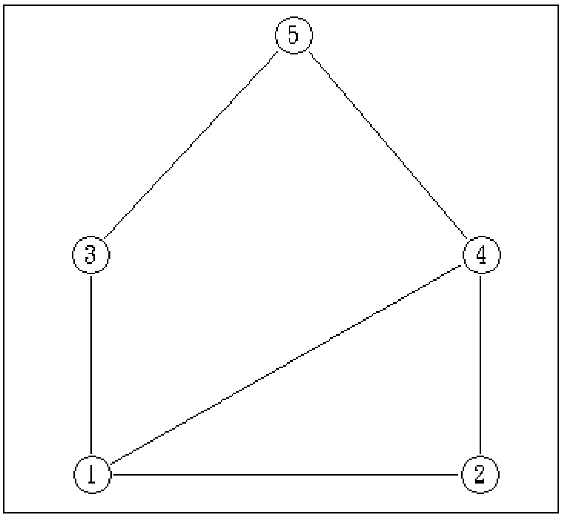

Fig. 12. Concept Tree of Cluster II

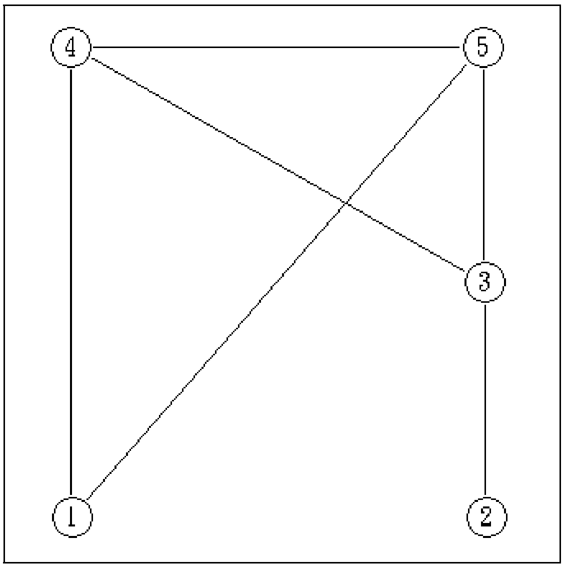

Fig. 13. Concept Tree of Cluster III

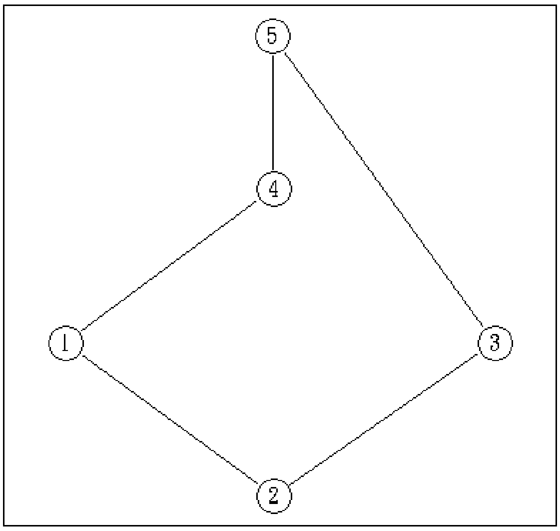

Fig. 14. Concept Tree of Cluster IV

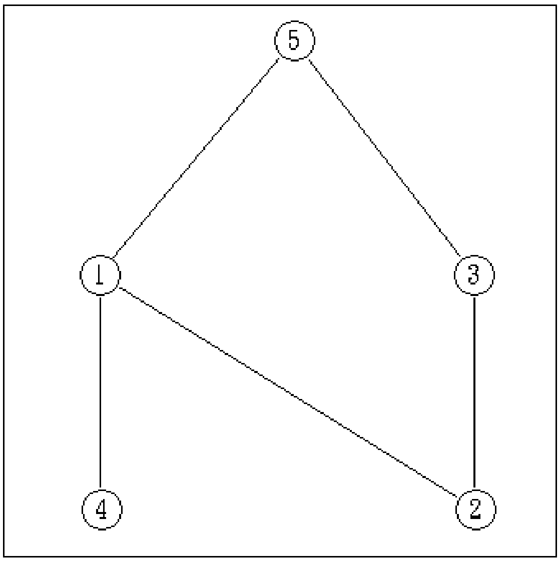

Fig. 15. Concept Tree of Cluster V

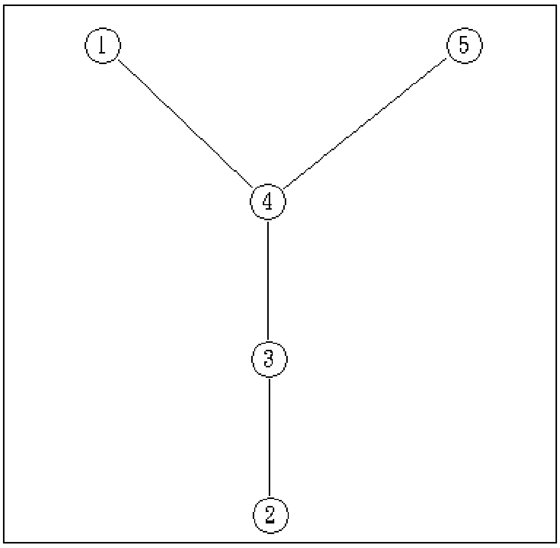

Fig. 16. Concept Tree of Cluster VI

5 Conclusions

An integrated method of item hierarchy and concept tree based on optimal clustering is discussed in this study. Item hierarchy and concept tree reveal knowledge structures in the form graphic hierarchy and linkage. Moreover, the representation of item hierarchy and concept tree is constructed adaptively because clustering improves homogeneity for each cluster. An empirical data of statistics test is analyzed and provide evidence for this integrated method. Some conclusions and recommendations are described as follows.

Firstly, graphic representation of knowledge structure is important information for assessment design and cognition diagnosis [5][23]. Item hierarchy could help teachers refine the assessment tool [20]. In this study, teachers could evaluate item design based on the item hierarchy. Furthermore, concept tree could help teachers understand cognitive deficiency of students.

Secondly, the assessment design and cognition diagnosis according to item hierarchy and concept tree could be more feasible and adaptive because fuzzy clustering provides homogeneity for each cluster so that students within the same cluster own similar knowledge structures. The analysis of empirical data from statistics test also shows that item hierarchy and concept tree vary among clusters. It means proficiency of students differs in clusters and they need adaptive and remedial instruction.

Thirdly, future study could extend clustering technique so that this clustering will be more robust. This integrated method could be applied in the other fields, such as mathematics and nature science. In addition, construction of intelligent internet system based on the integrated method is also prospective for further investigation [3][10][34].

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