

# Resolution of Resource Conflict Between the Different Two Projects in MPL-CCPM Representation

MUNENORI KASAHARA

Faculty of Management and  
Information Systems Engineering,  
Nagaoka University of Technology  
Nagaoka, Niigata 940-2188  
JAPAN  
munenori333@gmail.com

HIROTAKA TAKAHASHI

Department of Humanities,  
Yamanashi Eiwa College  
Kofu, Yamanashi 400-8555  
JAPAN  
hirotaka@y-eiwa.ac.jp

HIROYUKI GOTO

Department of Management and  
Information Systems Science,  
Nagaoka University of Technology  
Nagaoka, Niigata 940-2188  
JAPAN  
hgoto@kjs.nagaokaut.ac.jp

*Abstract:* Max-Plus-Linear (MPL) system is known as a solution for scheduling problems of a class of discrete event systems. Typical systems include: manufacturing system and project management, etc. In such systems, the initial schedule is frequently changed due to unpredictable disturbances. Critical Chain Project Management (CCPM) method is a management method that can shorten the time required to complete a project based on the Theory Of Constraints (TOC). Therefore, in our previous papers [1] [2], we proposed a method for applying the concepts of the CCPM to MPL representation in order to control the occurrence of an undesirable state change. The CCPM method also provides a solution for the problem of the resource conflict. However, we did not discuss the resource conflict for simplicity. In this paper, we propose a method for discovering resource conflict between the different two projects in the MPL-CCPM representation. We also discuss how to shift the resource in order to resolve the conflict.

*Key-Words:* Max-Plus Algebra, Max-Plus-Linear system, Discrete Event Systems, Critical Chain Project Management, Resource Conflict, State-Space Representation, Scheduling.

## 1 Introduction

We focus on discrete event systems with a structure of parallel processing, synchronization and non-concurrency. Typical examples of this kind of system include: manufacturing systems, transportation systems [3] [4] and project management, etc. It is known that the behavior of this kind of system can be described using max-plus algebra [5] [6], a subclass of Dioid algebra [7]. In this kind of systems, the initial schedule is frequently changed due to unpredictable disturbances.

In this context, we examine a method of controlling the occurrence of an undesirable state change in those systems. The state change we mention here means a significant change of the schedule of tasks from the initial one. In general, not only giving buffers to the system but monitoring and controlling the tasks in a wider range are effective for controlling such changes.

Focusing on Max Plus Linear (MPL) discrete event systems, there are several researches which consider uncertainty of the execution time of tasks [8]. However, if the relevant parameter contains stochastic variations, there is a strong non-linearity in the states of the system. Thus, it is difficult to handle large-scale

systems.

On the other hand, the method based on CCPM method (e.g. [9] and the reference therein) has an advantage in that it can handle such systems easily. The CCPM method has been found to be an effective tool to protect projects from delays. The CCPM method is an outgrowth of the theory of constraints (TOC), developed by Goldratt [10] in 1990, for scheduling and management of manufacturing. In the CCPM method, an empirical value is used to obtain an estimate of the process duration for each process. Moreover, the CCPM method provides a method for determining places locations at which time buffers should be inserted in order to prevent unplanned delays in completing the project. This is because the method does not consider the change of execution points of individual tasks, and a buffer is incorporated into the cluster of tasks. Therefore, in our previous papers [1] [2], we proposed a method for applying the concepts of the feeding buffer and the project buffer in MPL-CCPM discrete event systems. Moreover, we proposed a method of monitoring the buffer consumption with fever chart [9] to accomplish projects, which is a buffer management policy for CCPM-MPL representation [11].

The CCPM method also provides a solution for the problem of the resource conflict. However, in our previous papers [1] [2] [11], we did not consider this problem for simplicity. Recently the international conference on Management, Marketing and Finances 2010, we proposed a method for discovering resource conflict between the two different projects in the MPL-CCPM representation [12]. In this paper, we extend this method [12] and also discuss how to shift the resource in order to resolve the conflict.

This paper is organised as follows. In Section 2, we give an overview of the Max-Plus algebra and the MPL discrete event systems. In Section 3, we overview the concept of the CCPM. In Section 4, we describe the methods for applying the concepts of the feeding and the project buffers in the MPL-CCPM discrete event systems. In Section 5, we propose a method for discovering the resource conflict between the different two projects in the MPL-CCPM representation. Moreover, we also discuss a method for rescheduling in order to resolve the resource conflict. In Section 6, a simple model and numerical examples are presented. Finally, in Section 7, we summarize and conclude our work.

## 2 Max-Plus Linear System

In this section, we briefly review the Max-Plus Algebra and the MPL discrete event systems, both of which would be the essential backgrounds throughout this paper.

### 2.1 Max-Plus Algebra

Max-Plus algebra is an algebraic system that is suitable for describing a certain class of discrete event systems. In a field  $\mathcal{D} = \mathbf{R} \cup \{-\infty\}$ , where  $\mathbf{R}$  is the real field, the operators for addition and multiplication are defined as:

$$x \oplus y = \max(x, y), \quad (1)$$

$$x \otimes y = x + y. \quad (2)$$

The symbol  $\otimes$  corresponds to the multiplication in conventional algebra, and we often suppress this notification when no confusion is likely to arise. For instance, we simply write  $xy$  as the simplified expression of  $x \otimes y$ . These hold the commutative, associative and distributive laws. By definition, the unit elements for these operators are given by  $\varepsilon (= -\infty)$  and  $e (= 0)$ , respectively. The following relationships are satisfied for an arbitrary  $x \in \mathcal{D}$ :

$$\varepsilon \oplus x = x \oplus \varepsilon = x, \quad (3)$$

$$e \otimes x = x \otimes e = x. \quad (4)$$

Further, the following two operators are defined for subsequent discussions.

$$x \wedge y = \min(x, y), \quad (5)$$

$$x \setminus y = -x + y. \quad (6)$$

Denote the unit element of  $\wedge$  by  $\bar{\varepsilon} (= +\infty)$ . Moreover, we adopt the following axiom :

$$\varepsilon \otimes \bar{\varepsilon} = \bar{\varepsilon} \otimes \varepsilon = \varepsilon. \quad (7)$$

An operator for the powers of real numbers  $\alpha$  is introduced as:

$$x^{\otimes \alpha} = \alpha \times x \text{ for } \alpha \in \mathbf{R}. \quad (8)$$

If  $m \leq n$ , operators for multiple numbers are:

$$\bigoplus_{k=m}^n x_k = \max(x_m, \dots, x_n), \quad (9)$$

$$\bigwedge_{k=m}^n x_k = \min(x_m, \dots, x_n). \quad (10)$$

For matrices  $\mathbf{X} \in \mathcal{D}^{m \times n}$ ,  $[\mathbf{X}]_{ij}$  express the  $(i, j)$ -th element of  $\mathbf{X}$ , and  $\mathbf{X}^T$  is the transpose matrix of  $\mathbf{X}$ . For  $\mathbf{X}, \mathbf{Y} \in \mathcal{D}^{m \times n}$ ,

$$[\mathbf{X} \oplus \mathbf{Y}]_{ij} = \max([\mathbf{X}]_{ij}, [\mathbf{Y}]_{ij}), \quad (11)$$

$$[\mathbf{X} \wedge \mathbf{Y}]_{ij} = \min([\mathbf{X}]_{ij}, [\mathbf{Y}]_{ij}). \quad (12)$$

If  $\mathbf{X} \in \mathcal{D}^{m \times l}$ ,  $\mathbf{Y} \in \mathcal{D}^{l \times p}$ ,

$$\begin{aligned} [\mathbf{X} \otimes \mathbf{Y}]_{ij} &= \bigoplus_{k=1}^l ([\mathbf{X}]_{ik} \otimes [\mathbf{Y}]_{kj}) \\ &= \max_{k=1, \dots, l} ([\mathbf{X}]_{ik} + [\mathbf{Y}]_{kj}), \end{aligned} \quad (13)$$

$$\begin{aligned} [\mathbf{X} \odot \mathbf{Y}]_{ij} &= \bigwedge_{k=1}^l ([\mathbf{X}]_{ik} \setminus [\mathbf{Y}]_{kj}) \\ &= \min_{k=1, \dots, l} (-[\mathbf{X}]_{ik} + [\mathbf{Y}]_{kj}), \end{aligned} \quad (14)$$

where the priority of operators  $\otimes, \setminus$  and  $\odot$  are higher than that of operators  $\oplus$  and  $\wedge$ . Unit elements for matrices are denoted as  $\varepsilon_{mn}$ .  $\varepsilon_{mn}$  is a matrix whose all elements are  $\varepsilon$  in  $\varepsilon_{mn} \in \mathcal{D}^{m \times n}$ , and  $e_m$  is a matrix whose diagonal elements are  $e$  and off-diagonal elements are  $\varepsilon$  in  $e_m \in \mathcal{D}^{m \times m}$ . In  $x, y \in \mathcal{D}^m$ , if  $[x]_i \leq [y]_i$  holds for all  $i$  ( $1 \leq i \leq m$ ), then we simply write  $x \leq y$ .

The operators introduced above also have other interesting and attractive properties that are not used in this paper. See [5], [6], or [7] for additional details.

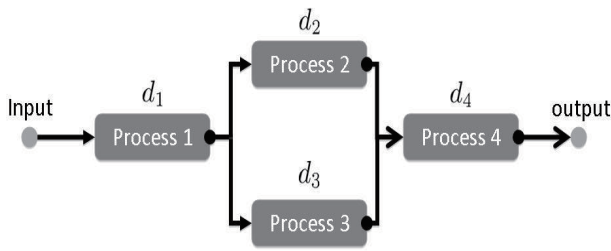


Figure 1: The manufacturing sequence of a simple production system.

## 2.2 Max-Plus Linear System

The Max Plus Linear (MPL) discrete event system [13] [14] [15] is defined as a system whose behaviour can be described in linear form in Max-Plus Algebra, it is similar to the state-space equations in modern control theory :

$$\mathbf{x}(k) = \mathbf{A}(k)\mathbf{x}(k - 1) \oplus \mathbf{B}\mathbf{u}(k), \quad (15)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k), \quad (16)$$

where  $k$  is called the event counter that represents the number of event occurrences from the initial state.  $\mathbf{x}(k) \in \mathcal{D}^n$ ,  $\mathbf{u}(k) \in \mathcal{D}^p$  and  $\mathbf{y}(k) \in \mathcal{D}^q$  are the states, input and output variables, respectively.  $n, p$  and  $q$  are the corresponding dimensions.  $\mathbf{A} \in \mathcal{D}^{n \times n}$ ,  $\mathbf{B} \in \mathcal{D}^{p \times n}$  and  $\mathbf{C} \in \mathcal{D}^{q \times n}$  are called the system input and output matrices respectively. Let us consider a simple production system shown in Fig.1 as an example. The processing times in sequences No.1-4 are denoted as  $d_1, d_2, d_3$  and  $d_4$ , respectively. It is assumed that each process has the following specific constraints:

- While the machines are in operation, they cannot initiate the processing of subsequent parts.
- Process Nos.2 through 4, which have precedence constraints, cannot begin until the parts manufactured in the preceding processes have been received.
- Process No.1, which has an external input, cannot begin processing until it receives the corresponding materials.
- If the machine is empty, processing starts as soon as all required materials from the preceding processes and external inputs become available.

For the  $k$ -th job, suppose  $u(k), \mathbf{x}(k)$  and  $y(k)$  represent the feeding, processing starting and complete times, respectively. We then find the following

relations.

$$x_1(k) = \max\{u(k), x_1(k - 1) + d_1\}, \quad (17)$$

$$x_2(k) = \max\{x_2(k - 1) + d_2, x_1(k) + d_1\}, \quad (18)$$

$$x_3(k) = \max\{x_3(k - 1) + d_3, x_1(k) + d_1\}, \quad (19)$$

$$x_4(k) = \max\{x_4(k - 1) + d_4, x_2(k) + d_2, x_3(k) + d_3\}, \quad (20)$$

$$y(k) = x_4(k) + d_4. \quad (21)$$

It can be found that the Eqs.(17)-(21) can be described in the forms of Eqs.(15) and (16) , where

$$\mathbf{A} = \begin{pmatrix} d_1 & \varepsilon & \varepsilon & \varepsilon \\ d_1^2 & d_2 & \varepsilon & \varepsilon \\ d_1^2 & \varepsilon & d_3 & \varepsilon \\ a_{41} & d_2^2 & d_3^2 & d_4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} e \\ d_1 \\ b_4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \varepsilon \\ \varepsilon \\ d_4 \end{pmatrix}^T, \quad (22)$$

$$a_{41} = d_1^2(d_2 \oplus d_3) \text{ and } b_4 = d_1(d_2 \oplus d_3).$$

Since there is a term  $x_i(k)$  on the right hand side of Eqs.(18)-(20), they must be transformed into equations without the term  $x_i(k)$ . This implies that they are expressed in the form of Eq.(15). Thus, there has been few discussions about the domain, where equations are described as a general form of the system matrix and input/output matrices. Therefore, we briefly review a general formulation of the MPL equations for systems with precedence constraints or synchronizations that were developed in [13].

## 2.3 Max-Plus Representation

In this subsection, we briefly review the process for deriving the max-plus linear representation for a certain class of discrete event systems that were developed in [13]. We assume that the relevant constraints are imposed on the focused system in the following way:

- The number of processes is  $n$ , the number of external inputs is  $p$  and the number of external outputs is  $q$ .
- All processes are used only once for single batch.
- The subsequent batch cannot start processing while the process is at work with the current one.
- Processes that have precedence constraints cannot start processing until they have received all the required parts from the preceding processes.
- For processes that have external inputs, processing cannot start until all the required materials have arrived.
- The processing starts as soon as all conditions above stipulated are satisfied.

For the  $k$ -th job in process  $i$  ( $1 \leq i \leq n$ ), let  $d_i(k) (\geq 0)$ ,  $[\mathbf{x}(k)]_i$ ,  $[\mathbf{u}(k)]_i$  and  $[\mathbf{y}(k)]_i$  be the processing time, the starting time, external input time and external output time for each process. Matrices  $\mathbf{A}_k^0$ ,  $\mathbf{F}_k$ ,  $\mathbf{B}^0$  and  $\mathbf{C}_k^0$  are introduced for representing the structure of systems as follows.

$$[\mathbf{A}_k^0]_{ij} = \begin{cases} d_i(k): & \text{if } i = j. \\ \varepsilon & : \text{otherwise.} \end{cases} \quad (23)$$

$$[\mathbf{F}_k]_{ij} = \begin{cases} d_j(k): & \text{if process } i \text{ has} \\ & \text{a preceding process } j. \\ \varepsilon & : \text{if process } i \text{ does not have} \\ & \text{a preceding process } j. \end{cases} \quad (24)$$

$$[\mathbf{B}^0]_{ij} = \begin{cases} e & : \text{if process } i \text{ has} \\ & \text{an external input } j. \\ \varepsilon & : \text{if process } i \text{ dose not have} \\ & \text{an external input } j. \end{cases} \quad (25)$$

$$[\mathbf{C}_k^0]_{ij} = \begin{cases} d_j(k): & \text{if process } j \text{ has} \\ & \text{an external output } i. \\ \varepsilon & : \text{if process } j \text{ dose not have} \\ & \text{an external output } i. \end{cases} \quad (26)$$

$\mathbf{F}_k$  is referred to the adjacency matrix.

For the  $k$ -th job in process  $i$  ( $1 \leq i \leq n$ ),  $[\mathbf{A}_k^0 \mathbf{x}(k-1)]_i$  gives the complete time,  $[\mathbf{F}_k \mathbf{x}(k)]_i$  states the latest time among complete times in the preceding processes, and  $[\mathbf{B}^0 \mathbf{u}(k)]_i$  is equal to the latest feeding time from external inputs. Furthermore,  $[\mathbf{C}_k^0 \mathbf{x}(k)]_i$  represents the latest time among the complete times in processes attached to the corresponding output.

The earliest starting time is defined as the minimum value on which the corresponding process can start processing immediately. Utilising the above discussions, the earliest starting times of any of the processes are given by [13]:

$$\mathbf{x}_E(k) = \mathbf{F}_k^* \mathbf{A}_{k-1}^0 \mathbf{x}(k-1) \oplus \mathbf{F}_k^* \mathbf{B}^0 \mathbf{u}(k), \quad (27)$$

where

$$\mathbf{F}_k^* = \mathbf{e}_n \oplus \mathbf{F}_k \oplus \mathbf{F}_k^2 \oplus \dots \oplus \mathbf{F}_k^{l-1}, \quad \mathbf{F}_k^l = \varepsilon_{nn}. \quad (28)$$

An instance  $l$  ( $1 \leq l \leq n$ ) depends on the precedence-relations of the systems. The corresponding output times are given by:

$$\mathbf{y}_E(k) = \mathbf{C}_k^0 \mathbf{x}(k). \quad (29)$$

Furthermore, the latest starting time is defined as the maximum value for which the same output time by the earliest time is accomplished. The latest starting times of any of the processes are given by [13]:

$$\mathbf{x}_L(k) = (\mathbf{A}_k^0 \mathbf{F}_k^*)^T \odot \mathbf{x}(k+1) \wedge (\mathbf{C}_k^0 \mathbf{F}_k^*)^T \odot \mathbf{y}(k). \quad (30)$$

The latest feeding times are also given by:

$$\mathbf{u}_L(k) = (\mathbf{B}^0)^T \odot \mathbf{x}_L(k). \quad (31)$$

A critical path is defined as the processes whose total floats are zero. Moreover, the total float is defined as a total sum of the float times of the corresponding processes. It is also stated as the difference between two primary times, one of which is the minimum value among the latest starting times of the succeeding processes, by which the output time is unchanged, and the other of which is the complete time in the corresponding process caused by the earliest starting time. The total floats regarding all processes are obtained as:

$$\mathbf{w}(k) = \mathbf{x}_L(k) - \mathbf{x}_E(k). \quad (32)$$

The critical path is determined by a set of numbers  $\alpha$  that satisfy:

$$\{\alpha \mid [\mathbf{w}(k)]_\alpha = 0\}. \quad (33)$$

### 3 Critical Chain Project Management

We briefly review the concept of the critical chain project management (CCPM) method. Projects often exceed their initial planned schedule. This is usually due to unforeseen uncertainties related to external factors. To resolve this dilemma, the CCPM method is often considered (e.g. [9] and the references therein). The CCPM addresses several shortcomings of the Program Evaluation and Review Technique (PERT), the most widely used tool for project management. The PERT is based on identifying a critical path, which is the longest chain of linked processes in the entire project. Focusing only on the longest chain of processes may result in several problems, such as multitasking. The CCPM method instead asserts that, in addition to process dependencies, a good project management should address resource constraints only if they are absolutely required. The CCPM method provides a method for determining places locations at which time buffers should be inserted in order to prevent unplanned delays in completing the project. In the PERT, each process in the project consists of a set of four times: the earliest start time, the earliest output time, the latest start time, and the latest output time. Since these times are made known to everyone involved in the project, they can be closely monitored. The difference between the earliest and the latest start times is equivalent to the slack. Processes on the critical path do not have any slack

time and should be given significant attention. In estimating the process duration, there is a tendency to use optimistic estimates, which include significant safety margins, in order to ensure the completion in the specified time frame. This is often referred to as the 90% estimate. In the CCPM method, an empirical value is used to obtain an estimate of the Aggressive But Possible (ABP) time for each process, and this value is used for the process duration. In the present paper, we use  $ABP = HP^{\otimes \frac{1}{3}}$ , where Highly Possible (HP) is the time to completion with a probability of 90%.

The next step is to determine a buffer to encapsulate the uncertainty of task durations. This is referred to as the project buffer, and it absorbs variations in the critical path. A project buffer is embedded between the final process on the critical path and the external output. A feeding buffer is inserted before a process that joins the critical path but is not on the critical path. The role of the feeding buffer is to protect the critical chain from variations in non-critical paths of tasks, i.e., to help maintain the critical path as the critical path.

Finally, the critical path is monitored by closely monitoring the rate at which the project buffer is consumed [9] [11].

The CCPM method also provides a solution for the problem of the resource conflict. There are two cases : the resource conflict in same project and the resource conflict in different projects. In the former case, the policy to resolve the resource conflict is as follows : by the definition of CCPM method [9], we should not delay the project. Thus, when the resource in the same project is conflict, we should assign the resource to the process on the critical path. In the latter case, the policy to resolve the resource conflict is as follows : by the definition of CCPM method [9], we should not delay the project which has a high priority. Thus, when the resource between two projects is conflict, we should assign the resource to the process on the project which has a high priority.

As the first step, in this paper, we consider the case of the resource conflict in different two projects.

## 4 Application of a CCPM Based Framework on MPL System

In this section, we briefly review how to determine the position and the size of the Project Buffer (PB) and Feeding Buffer (FB). Many more details of the method for applying the concepts of CCPM on MPL systems were discussed in [1]. In this section, we do not consider the resource conflict.

Before the discussion about determination of the position and size of buffer, we introduce the following

two vectors :

$$[v]_i = \begin{cases} e & : i \in \beta \\ \varepsilon & : i \notin \beta \end{cases}, [w]_i = \begin{cases} e & : i \in \alpha \\ \varepsilon & : i \notin \alpha \end{cases}, \quad (34)$$

where  $\beta$  is a set of processes on the non-critical path. Moreover, the adjacency matrix  $F_k$  is decomposed using the following formula:

$$F_k = F_{\alpha\alpha} \oplus F_{\alpha\beta} \oplus F_{\beta\alpha} \oplus F_{\beta\beta}, \quad (35)$$

where

$$F_{\alpha\alpha} \equiv \text{diag}(w)^T \otimes F_k \otimes \text{diag}(w), \quad (36)$$

$$F_{\alpha\beta} \equiv \text{diag}(v)^T \otimes F_k \otimes \text{diag}(w), \quad (37)$$

$$F_{\beta\alpha} \equiv \text{diag}(w)^T \otimes F_k \otimes \text{diag}(v), \quad (38)$$

$$F_{\beta\beta} \equiv \text{diag}(v)^T \otimes F_k \otimes \text{diag}(v). \quad (39)$$

$F_{\alpha\alpha}$  means the adjacency matrix that shows the transition from the process on the critical path onto the process on the critical path. Similarly,  $F_{\alpha\beta}$  means the adjacency matrix that shows the transition from the process on the critical path onto the process on the non-critical path.  $F_{\beta\alpha}$  is the adjacency matrix that shows the transition from the process on the non-critical path onto the process on the critical path.  $F_{\beta\beta}$  is also the adjacency matrix that shows the transition from the process on the non-critical path onto the process on the non-critical path.

### 4.1 Position and Size of Project Buffer

In order to prevent delay of the project, the PB is embedded. The PB is embedded between the final process on the critical path and the external output, and its position is determined by using the output matrix  $[C_k^0]_{ij}$ . If the element  $[C_k^0]_{i\alpha}$  has a finite value, process  $i$  has an external output, then we embed the PB after process  $i$ . The size of PB is estimated following formula,

$$PB = y_E^{\otimes \frac{1}{3}}, \quad (40)$$

where  $y_E = (y_E^T \otimes y_E)^{\otimes \frac{1}{2}}$ . This method is based on "cut and paste method" or "50% of the chain" [9].

### 4.2 Position and Size of Feeding Buffer

In order to protect a critical path from delay of a non-critical path, the FB is embedded. From the discussion in Section 3, the FB is embedded in the juncture between non-critical path and critical path. The position of FB is determined by :

$$[v']_\gamma = [F_{\beta\alpha}^T \otimes w]_\gamma. \quad (41)$$

If the  $\gamma$ -th element of Eq.(41) has a finite value, a FB is embedded after process  $\gamma$ .

Next, we discuss how to determine the size of the FB. We basically consider the size of FB to sum up the processing times of processes on the non-critical path: from the fork between the critical path and the non-critical path to the joint between the critical path and the non-critical path. The size of FB is estimated by [1]:

$$FB = \left[ \text{diag}(\mathbf{v}') \otimes \tilde{\mathbf{F}}_{\beta\beta}^* \otimes \mathbf{v} \right]^{\otimes \frac{1}{3}}, \quad (42)$$

$$(43)$$

where

$$\tilde{\mathbf{F}}_{\beta\beta} = \text{diag}(\mathbf{j}^\#) \otimes \mathbf{F}_{\beta\beta}, \quad (44)$$

$$\mathbf{j} = [\mathbf{F}_{\beta\beta} \otimes \mathbf{v}']^{\otimes \frac{1}{2}}. \quad (45)$$

An operator  $\#$  for a vector  $\mathbf{p}$  defined as :

$$\{[\mathbf{p}]_i\}^\# = \begin{cases} \varepsilon : \text{if } i\text{-th element is a finite value.} \\ e : \text{if } i\text{-th element is } \varepsilon. \end{cases} \quad (46)$$

$\tilde{\mathbf{F}}_{\beta\beta}$  means that only the transition from the process where FB is embedded to the process in  $\beta$  has been deleted. Moreover, if the  $\gamma$ -th element of  $\mathbf{j}$  has finite value, then the process  $\gamma$  is on the non-critical path and the process  $\gamma$  also has FB in preceding process.

## 5 Resolution of Resource Conflict

### 5.1 Discovery of the Resource Conflict

In this subsection, we propose a method for discovering the resource conflict between the different two projects in the MPL-CCPM representation. We also discuss how to calculate the amount of shift in order to resolve resource conflict.

We discuss how to discover the resource conflict between the project A and the project B. We assume that we use the same resource, if the process number between the project A and the project B is same.

For the process  $i$  in the project A, let  $[\mathbf{x}_A^-]_i$  and  $[\mathbf{x}_A^+]_i$  be the earliest starting time and the earliest complete times, respectively. Similarly, for the process  $i$  in the project B, let  $[\mathbf{x}_B^-]_i$  and  $[\mathbf{x}_B^+]_i$  be the earliest starting time and the earliest complete times, respectively.

Fig.2, Fig.3, Fig.4, and Fig.5 show the Gantt charts of the relation to the process  $i$  between the project A and the project B. In the cases of Fig.2 and Fig.3, you can find that the process  $i$  between the project A and the project B are conflict.

Firstly, we consider the case of Fig.2. In the case of Fig.2, the process  $i$  in the project A starts earlier

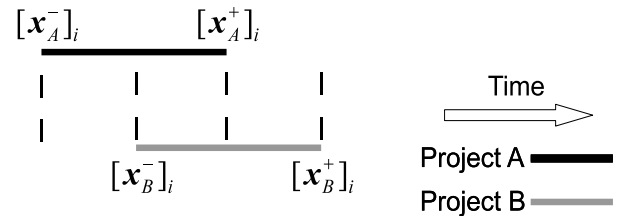


Figure 2: The relation to the process  $i$  between the project A and the project B as the function of time. The process  $i$  between the project A and the project B is conflict.

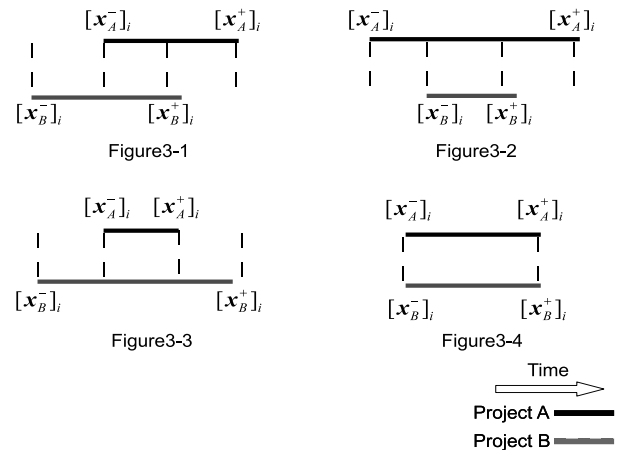


Figure 3: The same figure of Fig.2. All cases when the process  $i$  between the project A and the project B is conflict are showed.

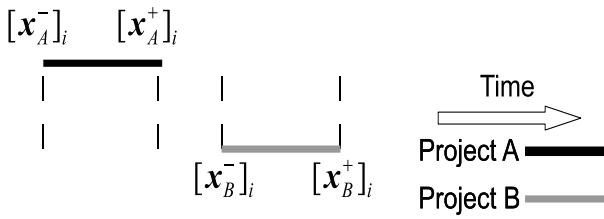


Figure 4: The same figure of Fig.2. But the process  $i$  between the project A and the project B is not conflict.

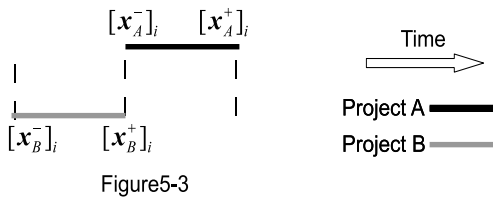
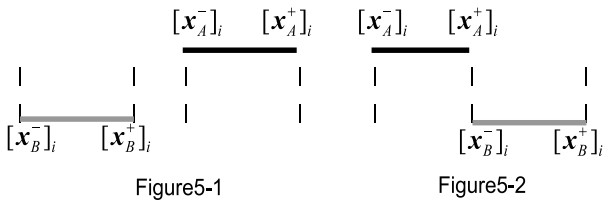


Figure 5: The same figure of Fig.2. All cases when the process  $i$  between the project A and the project B is not conflict are showed.

than the process  $i$  in the project B, and the process  $i$  in the project B starts before the end of the process  $i$  in the project A. Then, we can find the following relations immediately :

$$[\mathbf{x}_A^+]_i > [\mathbf{x}_B^-]_i, \quad [\mathbf{x}_B^+]_i > [\mathbf{x}_A^-]_i. \quad (47)$$

Then, we can obtain :

$$[\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i > 0, \quad [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i > 0. \quad (48)$$

Similarly, in the cases of Fig.3, we can obtain the same relation as Eq.(48). Note that, in the lower right case of Fig.3 (Figure 3-4), we can obtain the following relations:

$$[\mathbf{x}_A^-]_i = [\mathbf{x}_B^-]_i, \quad [\mathbf{x}_A^+]_i = [\mathbf{x}_B^+]_i, \quad (49)$$

$$[\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i = [\mathbf{x}_A^+]_i - [\mathbf{x}_A^-]_i = d_A > 0, \quad (50)$$

$$[\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i = [\mathbf{x}_A^+]_i - [\mathbf{x}_A^-]_i = d_A > 0. \quad (51)$$

Then, even if the lower right case of Fig.3 (Figure 3-4), we find that the relation of Eq.(48) is satisfied.

Next, we consider the case of Fig.4 and Fig.5. In cases of Fig.4 and Fig.5, you can find that the process  $i$  between the project A and the project B are not conflict. In the case of Fig.4, the process  $i$  in the project A starts earlier than the process  $i$  in the project B, and the process  $i$  in the project B starts after the end of the process  $i$  in the project A. Then, we can find the following relations immediately :

$$[\mathbf{x}_A^+]_i < [\mathbf{x}_B^-]_i, \quad [\mathbf{x}_B^+]_i > [\mathbf{x}_A^-]_i. \quad (52)$$

Then, we can obtain :

$$[\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i < 0, \quad [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i > 0. \quad (53)$$

In the upper left case of Fig.5 (Figure 5-1), on the contrary, we obtain :

$$[\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i > 0, \quad [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i < 0. \quad (54)$$

Moreover, in the upper right case of Fig.5 (Figure 5-2), we can find the following relations:

$$[\mathbf{x}_A^+]_i = [\mathbf{x}_B^-]_i, \quad [\mathbf{x}_B^+]_i > [\mathbf{x}_A^-]_i. \quad (55)$$

Then, we can obtain :

$$[\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i = 0, \quad [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i > 0. \quad (56)$$

Similarly, in the lower left case of Fig.5 (Figure 5-3), we obtain : Then, we can obtain :

$$[\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i > 0, \quad [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i = 0. \quad (57)$$

From above discussion, if the process  $i$  between the project A and the project B is conflict, we find that the following relation must be satisfied :

$$\min([\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i, [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i) > 0. \quad (58)$$

On the other hand, if the process  $i$  between the project A and the project B is not conflict, from Eqs.(54), (56) and (57), we find that the following relation must be satisfied :

$$\min([\mathbf{x}_A^+]_i - [\mathbf{x}_B^-]_i, [\mathbf{x}_B^+]_i - [\mathbf{x}_A^-]_i) \leq 0. \quad (59)$$

Using the Max-Plus expression, the left hand side of Eqs.(58) and (59) can be written by :

$$\left( \text{diag}(\mathbf{x}_B^-) \odot \mathbf{x}_A^+ \right) \wedge \left( \text{diag}(\mathbf{x}_A^-) \odot \mathbf{x}_B^+ \right). \quad (60)$$

In conclusion, we show that in order to discover the resource conflict between the project A and the project B, we should examine the following quantity:

$$\mathbf{c} = \left( \text{diag}(\mathbf{x}_B^-) \odot \mathbf{x}_A^+ \right) \wedge \left( \text{diag}(\mathbf{x}_A^-) \odot \mathbf{x}_B^+ \right). \quad (61)$$

If  $[\mathbf{c}]_i > 0$ , the resource conflict is occurred in process  $i$  between the project A and the project B.

## 5.2 Shift of the Resource

In this subsection, we discuss how to shift the resource in order to resolve resource conflict between different two projects.

In order to resolve the resource conflict, we need to shift the resource in the projects. Before the discussion about the shift, based on the CCPM policy discussed in Section 3, we should determine which projects have a high priority. If the resource between two different projects is conflict, the policy to resolve the resource conflict is as follows : by the definition of CCPM method [9], we should not delay the project which has a high priority. Thus, when the resource between two projects is conflict, we should assign the resource to the project which has a high priority.

We are reminded that we use the same resource, if the process number between the project A and the project B is same. Now, we assume that the resource  $s$  between project A and B is conflict : i.e.  $[c]_s > 0$ . We also assume that the project A has a higher priority than that of project B.

From the above discussion, because we should not delay the project A, we should assign the resource  $s$  to the project A which has a high priority. Therefore, after the shift, the process  $s$  in the project A must start earlier than that in the project B. In this paper, for simplicity, after the shift, we assume that when the process  $s$  in the project A is completed, the process  $s$  in the project B starts immediately. In this case, in order to resolve resource conflict, we should shift the process  $s$  in the project B backward. Then, as you may notice from Figs.2 and 3, we should set the amount of shift as  $[x_A^+]_s - [x_B^-]_s$ . Using the Max-Plus expression, we can get a general formulation to calculate the amount of shift  $[d'_m]_i$  in order to resolve resource conflict is given by :

$$[d'_m]_i = [\text{diag}(\mathbf{x}_B^-) \odot \mathbf{x}_A^+]_i. \quad (62)$$

Next, we discuss how to shift the resource. If we shift only process  $s$  in the project B backward, the precedence-constrain may be unsatisfied between the process  $s$  and the downstream processes in the project B. Therefore, we need to shift the process  $s$  to satisfy the precedence-constrain between the process  $s$  and the downstream processes in the project B.

In order to satisfy the precedence-constrain between the process  $s$  and the downstream processes in the project B, in this paper, we shift the process  $s$  in the project B using the forward MPL state-space representation [16].

The complete time of all processes in the project B after the shift of the process  $s$  in the project B is

given by [16]:

$$\mathbf{x}_{B \text{ new}}^+ = (\mathbf{A}_{kB}^0 \mathbf{F}_{0B})^* \otimes (\text{diag}(\mathbf{x}_B^+) \mathbf{d}'_{mB}). \quad (63)$$

where

$$[d'_{mB}]_i = \begin{cases} [d'_m]_i & : i \in \mu, [c_i] > 0. \\ e & : i \in \mu, [c_i] \leq 0. \\ \varepsilon & : i \notin \mu. \end{cases} \quad (64)$$

$[d'_{mB}]_i$  shows the amount of shift of the process  $i$  in the project B and  $\mu$  is the set of the process in the project B which is not a high priority project.

Note that the definition of  $\mathbf{F}_{0B}$  in Eq.(63) is a bit different from that of Eq.(24). The definition of  $\mathbf{F}_{0B}$  is as follows:

$$[\mathbf{F}_{0B}]_{ij} = \begin{cases} e & : \text{if process } i \text{ in the project B has} \\ & \text{a preceding process } j \text{ in the project B,} \\ \varepsilon & : \text{if process } i \text{ in the project B does not} \\ & \text{have a preceding process } j. \end{cases} \quad (65)$$

## 6 Numerical Example

In this section, a simple model and numerical example are presented to promote a better understanding of the proposed method in the section 5.

Fig.6 shows simple two projects, called A and B, which have with one-input, one-output, two processes. Fig.7 shows the Gantt chart of these projects.

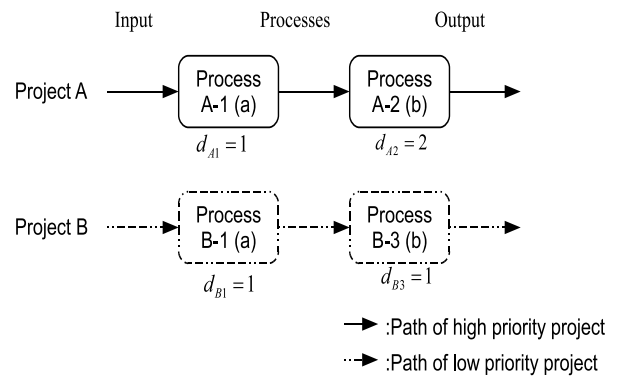


Figure 6: The two projects with resource conflict.

We assume that the project A has higher priority than that of project B. We can calculate the starting times  $\mathbf{x}_A^-$  and the complete times  $\mathbf{x}_A^+$  in the project A :

$$\mathbf{x}_A^- = (e \ 1 \ \varepsilon)^T, \mathbf{x}_A^+ = (1 \ \varepsilon \ 3)^T. \quad (66)$$



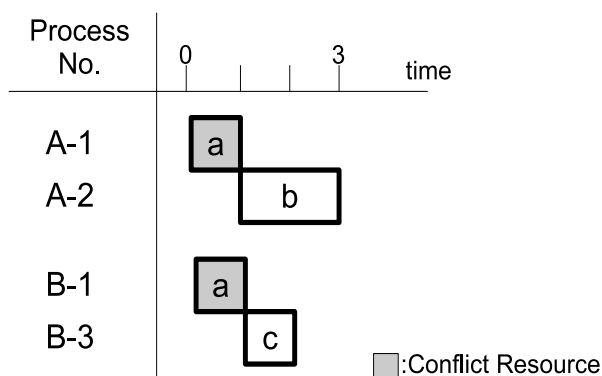


Figure 7: The Gantt chart of the two projects shown in Fig.6.

Similarly, we can also calculate  $x_B^-$  and  $x_B^+$  in the project B :

$$x_B^- = (e \ \varepsilon \ 1)^T, \quad x_B^+ = (1 \ \varepsilon \ 2)^T. \quad (67)$$

Using Eq.(61), the resource conflict between the project A and the project B can be discovered. Eq.(61) are calculate using Eqs. (66) and (67) :

$$c = (1 \ \varepsilon \ \varepsilon)^T. \quad (68)$$

Because of  $[c]_1 > 0$ , we can find that the resource "a" of the process A-1 in project A is in conflict with the resource "a" of the process B-1 in project B.

Using Eq.(62), we can calculate the amount of the shift :

$$d'_m = (1 \ \varepsilon \ \varepsilon)^T. \quad (69)$$

Then, in order to resolve the resource conflict, we can find that we should shift the process B-1 backward and the amount of shift is 1.

Next, we shift the resource in order to resolve resource conflict between two projects.

From Eq.(23) and Eq.(65), we set  $A_B^0$  and  $F_{0B}$  as follows :

$$A_B^0 = \text{diag}(1, \varepsilon, 1), \quad (70)$$

$$F_{0B} = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ e & \varepsilon & \varepsilon \end{pmatrix}. \quad (71)$$

Moreover, using Eq.(69), we can calculate  $d'_{mB}$  from Eq.(64) as follows :

$$d'_{mB} = (1 \ \varepsilon \ \varepsilon)^T. \quad (72)$$

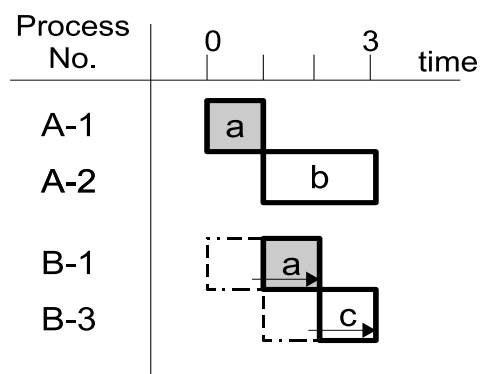


Figure 8: The Gantt chart of the two projects after the shift.

Then, the new earliest complete times  $x_{B \text{ new}}^+$ , which is shifted the processes B-1 backward to satisfy the precedence-constrain between the process B-1 and the downstream process B-3 in the project B, is calculated by Eq.(63):

$$x_{B \text{ new}}^+ = (2 \ \varepsilon \ 3)^T. \quad (73)$$

From the earliest complete time  $x_B^+$  in Eq.(67) and the new earliest complete time  $x_{B \text{ new}}^+$  in Eq.(73), we find that not only the process B-1 but also the process B-3 is shifted by 1. Then, we can confirm that the influence of the shift of the process B-1 is propagated to the process B-3 which is downstream of process B-1.

Based on the new earliest complete time  $x_{B \text{ new}}^+$  in Eq.(73), we draw the Gantt chart in Fig.8. In Fig.8, we can also find that the influence of the shift of the process B-1 is propagated to process B-3. Then, the precedence-constrain between the processes in the project B is satisfied. Also, we can confirm that we resolve resource conflict between two projects.

## 7 Conclusion

In this paper, we have reviewed a method for applying the concept of the feeding and project buffers in the CCPM-MPL method in order to address the project even if an undesirable state change has occurred.

The CCPM method also provides a solution for the problem of the resource conflict. However, in our previous paper [1] [2] [11], we did not discuss the resource conflict for simplicity. Therefore, in this paper, we have proposed a method for discovering resource conflict between the different two projects in the MPL-CCPM representation. We have also discussed how to shift the resource in order to resolve the conflict.

As a result, we have realized the method for discovering the resource conflict and shifting the resource has been useful. It has also been useful for rescheduling in order to resolve of the resource conflict.

However, for simplicity, we did not discuss the case of the resource conflict in the same project. We need to discuss a method for resolving the resource conflict in the same project. These extensions will be discussed elsewhere.

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