A Default Risk Model with Multi-Loan Lending Operations: A System of Integral Scale with Scope

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Abstract: Recent research on multi-loan lending operations has remained largely silent on the question of what ties together the retail banking functions of scale and scope lending. Our main point is that in a sense, they are just two different manifestations of banking expansion. This is especially true to the extent that banks are heavily involved in consolidation-based lending. In a multi-loan call pricing model, changes in lending scope have direct effects on the bank's interest margin related to lending scale and thus bank profits and risks. Comparative static analysis shows that scale and scope lending are complements. However, this commingling is not guaranteed to produce higher return and greater safety for the bank. As a result, bank equity return and default risk are overvalued when the integral role on the inclusion of more realistic loan portfolio management in the consolidated banking is ignored.

Keywords: Bank Interest Margin, Default Risk, Scale, Scope

1 Introduction

In their paper on "Should Banks Be Diversified? Evidence from Individual Bank Loan Portfolios," Acharya, Hasan, and Saunders (2006) address three fundamental questions. Should banks be focused or diversified? Does the extent of focus or diversification affect the quality of their loan portfolio? And, does diversification lead to greater safety for banks? The answers show that loan portfolio diversification and performance are inconsistent with the maximization of shareholder value. In other words, loan portfolio diversification is not guaranteed to produce superior return performance and/or greater safety for banks.

Their paper does not further explain, however, the consolidated or integral role of loan portfolios played by lending scope (diversification) in the relationship of lending scale (focus). There are several reasons why the diversification with focus issue is important in the context of banking firms. First, the Riegle-Neal Act of 1994 permitted interstate banking and branching and undoubtedly escalated consolidation in banking, particularly over the last decade as bank consolidations reached record numbers and unit banking approaches extinction (Clarke (2004)). A great deal of

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analysis has been devoted to understanding the scope or scale effects of bank deregulation on bank performance.[1] While much has been learned from this work, with few exceptions it has not addressed a fundamental question of whether or not there is an integral between scope and scale.

Second, discerning the integral structure of scale and scope in the banking industry is an important issue. The existence of significant returns to scope would imply that the bank may need to continue providing a new service investment in order to produce all services at the highest possible return. Significant returns to scale would indicate that the bank can increase its returns by increasing the size of current operation for any one service. Gomes and Livdan (2004) explain that diversification related to scope issue allows a mature, slow-growing firm to explore better productive opportunities while taking advantage of synergies. Their explanation, however, is still limited to an explicit treatment of diversification. It is clear that changes in a large bank's loan portfolio diversification may affect its returns and risks by influencing its lending scale in operations management.

In addition, this integral role of scope on scale related to large-bank returns and risks is also substantial due to the pivotal role of large institutions in shaping the structure of the banking industry. If scope on increasing returns to scale is closely related to high-return and low-risk operations of financial services, the implication is that the post-deregulatory movement will result in a highly concentrated banking industry dominated by a relatively small number of institutions. Conversely, if scope on decreasing returns to scale occurs, it is likely that the organizational structure of the industry would be less concentrated, with a large number of banks offering services to the public. Without further research on the scope issue on changing returns scale, no inferences about deregulation and related policy implications to banking industry structure could be made.

In this paper, we are interested in how the bank's equity return and its associated default risk vary with the integral effects of lending scope on lending scale in loan portfolio management. Much of the literature follows Merton's (1974) model by explicitly linking the risk of a firm's default process to the variability in the firm's asset value and viewing the market value of the firm's equity as the standard call option on the market value of the firm's asset with strike price equal to the promised payment of its liabilities. The former can be motivated based on a dynamic volatility argument in the spirit of Vassalou and Xing (2004), while the latter can be motivated based on a market-value equity argument in the spirit of Black and Scholes (1973).

The purpose of this paper is to apply Merton (1974) and set up an option-based model to determine the bank's optimal interest margin defined as the difference between the rates of interest the bank charges borrowers and the rate the bank pays to depositors. Margin earnings in the return to retail banking typically account for a significant portion of bank profits (Hirtle and Stiroh (2007)). Since the margin is

[1] For example, Berger and DeYoung (2001), and Saunders and Wilson (2001) use aggregated measures of bank diversification to examine geographical diversification. As pointed out by Bos and Kolari (2005), numerous studies on large U. S. banks in the 1980s and 1990s summarize that scale economies are found for banks between \$1 billion and \$15 billion in assets with diseconomies thereafter. Berger, Demsetz, and Strahan (1999) find that consolidation in the financial services industry has been consistent with greater diversification of risks on average but with little or no cost efficiency improvements.

so important to bank profitability and default risk, the issues how it is optimally determined and how it adjusts to changes in the integral scale with scope deserve closer scrutiny.

In practice, bank interest margin management is done through a "cost of goods sold" approach in which deposits are the "material", operations are the administrative costs, and loans are the "work in process" (Finn and Frederick (1992)). Specifically, the administrative costs and the work in process are designed to capture in a minimalist fashion the integral structure of scale and scope lending of the bank. In our model, the bank's loan portfolio is diversified for two reasons. First, loan portfolio diversification allows the bank to take advantage of returns to scope by eliminating redundancies across different lending activities. Second, loan portfolio diversification allows the bank to explore attractive new investment opportunities. We formalize this concept by assuming that the integral effect on the bank's lending scale from changes in scope. This paper follows this conceptual framework by providing a call-option-pricing, firm-theoretic model of bank behavior to study the determination of optimal bank interest margin and its associated default risk.

Our model features a multi-loan bank embarking on a guest for the concepts of scale and scope economies that yields insights into multi-loan market structures. Specifically, scale is present because the bank is assumed to have some market power in lending (see Kashyap, Rajan, and Stein (2002)). The assumption of market power is only to limit the scale of lending activities in the rate variation setting. Scope arises because the bank is assumed to have incentives to diversify its loan portfolio (see Acharya, Hasan, and Saunders (2006)). The assumption of loan portfolio diversification is only to limit the number of loans in the quantity variation setting. By doing so, we can focus on the integral substitutes or complements that arise from the inter-relations among loans providing.

In this paper, we show that an increase in the number of loans increases the default risk in the bank's equity through increasing the loan amount at a reduced interest margin. This result implies that scale and scope in lending are complements rather than substitutes. This complementarily further suggests that the integral returns of scale and scope are not guaranteed to produce superior return performance and greater safety for the bank. Our result is largely supported by the empirical evidence of Acharya, Hasan, and Saunders (2006) concerning loan portfolio diversification and margin determination behavior. One immediate application of our suggestion from a protection-oriented policy is an inevitably created social cost from too-big-to-fail decisions since large banks get hurt from diversifying scope with scale of their loan portfolios.

The remaining parts of this paper are organized as follows. Section II provides a literature review of bank scale and scope of lending. Section III lays out the basic option-based model of a banking firm with an integral scale-scope structure of lending. Section IV characterizes the optimal bank margin and the default risk in equity return, and develops the comparative static properties of the model. The final section concludes the paper.

2 Related Literatures

Our theory of integral scale-scope structure of bank lending is related to three strands of the literature. The first is the literature on bank interest margin determination, in which Ho and Saunders (1981), Allen (1988), and Angbazo (1997) are major contributors.

They have provided models of bank interest margins based on the bid-ask spread model of Stoll (1978). The model set up by Ho and Saunders (1981) is that banks are viewed as risk-averse dealers in loan and deposit market where loan requests and deposit funds arrive monsynchronously during a period horizon. Bank interest margins are shown to be fees charges by banks for the liquidity providing. Ho and Saunders' (1981) model is further extended to explain cross-elastic of demand between bank loans (Allen (1988)) and default risk (Angbazo (1997)). The model developed by Zarruk and Madura (1992) assumes a setting in which the bank is subject to prevailing capital regulation and deposit insurance. While we also examine bank interest margin, our focus on the integral returns to scale and scope to account for default risk takes our analysis in a different direction.

The second strand is the literature on scale and scope economies related to cost and profit efficiency in banking. The majority of the US literature on scale economies in banking markets has analyzed the cost structures and found that scale economies are usually exhausted somewhere around \$100-\$500 million asset size.[2] Results for scope economies (i.e., joint production of outputs) are mixed, with most authors concluding that banks do not gain efficiencies from providing multiple financial services to the public (Bos and Kolari (2005)). Moreover, recent research has expanded the analyses to consider both cost and profit efficiency.[3] The primary difference between our model and these papers is that we consider the impact on the bank's interest margin (and thus on the bank's equity return) from changes in the integral composition and size of loan portfolio. The presence or absence of complementarily among loan portfolio in liquidity providing related to margin determination become a crucial matter, which has no counterpart in the scalar setting.

The third strand is the literature on returns to scale and scope of lending related to risk states in banking. Berg, Førsund, and Jansen (1992), Mester (1996a, b), and Berger and DeYoung (1997) have expanded the analyses of scale and scope economies to consider related risk variables. In general, these studies have confirmed Berger and Humphrey's result that cost and profit frontier inefficiencies outweigh output inefficiencies associated with scale and scope economies by a considerable margin. Alternatively, we consider the impact on the default risk in the bank's equity return from changes in scope lending through adjusting scale lending. This allows to developing a model of bank behavior that integrates the risk considerations of the portfolio-theoretic approach with market conditions, cost considerations, and margin determining behavioral model of the firm-theoretic approach.

Our work embarks on a quest for multi-loan and cost concepts that yield insights into multi-loan market structures. These insights are important aspects of returns to scale and scope of lending activities since an individual bank behavior may arise from the inter-relations between scale-oriented focused lending and scope-oriented diversified lending. What distinguishes our work from this literature in our focus on the commingling of the assessment of returns to scale and scope when the bank's interest margin and its associated default risk are optimally determined.

3 Model Setup

[2] For example, see surveys in Berger and Humphrey (1997), Berger and Strahan (1998), and Berger, Demsetz, and Strahan (1999).

[3] For example, see Berger and Humphrey (1997), Berger and Mester (1997), and others.

Consider a banking firm involving a multiplicity of loans, for example, real estate loans, consumer loans, and business loans, represented by the vector $L = (L_1, \cdots, L_n)$ [4]. The most fundamental characterization of the loan relationships is provided by the Cournot-Nash behavioral mode in a rate variation model. Note that changes in vector L imply that the model will incorporate two distinct loan market frictions: there needs to be changes in the size of $L_i, i = 1, \cdots, n,$ as well as changes in the number of $L_i, i = 1, \cdots, m,$ where $m \neq n$. The former can be motivated based on a scale-size argument, while the latter can be motivated based on a scope-composition argument. These different operations work together in a system to affect bank profits and risks.

In a single-period call-option pricing model, $t \in [0, 1]$, the bank makes term loans L at $t = 0$ which mature and are paid off at $t = 1$. The one-period interest rates on individual loans of the bank's risky-asset portfolio $L = (L_1, \cdots, L_n)$ are denoted by the vector $R_L = (R_1, \cdots, R_n)$, respectively. We assume that the bank has some market power in individual lending (see Wong(1997), and Cosimano and McDonald (1998)) which implies that $L_i(R_i)$ where $\partial L_i / \partial R_i < 0$ and $i = 1, \cdots, n$. We note that individual loan demand is expressed as a function of only its loan rate due to the assumption of the Cournot-Nash behavioral mode in the rate variation setting as mentioned earlier. In addition to term loans, the bank can also hold an amount B of liquid assets, for example, central bank reserves or Treasury bills, on its balance sheet at $t \in [0, 1]$. These assets earn the security-market interest rate of R_B.[5]

The total assets to be financed at $t = 0$ are $L + B$. They are financed partly demandable deposits. At $t = 0$, the bank accepts D dollars of deposits and provides depositors with a rate of return equal to the market rate R_D. We assume that this amount D is exogenously determined. In addition to deposits, the bank can also issue claims in the public market at $t = 0$, denoted by K. The claims mature at $t = 1$ and can be thought of as bank equity. The bank's equity capital K is assumed to be fixed over the planning horizon $t \in [0, 1]$ and tied by regulations to be a fixed proportion q of its deposits, $K \geq qD$, where q is defined as the capital-to-deposits ratio. This ratio captures a risk-based system of capital standards.

To show how our cost attributes are related to integral system of scale and scope, we describe the cost-minimizing structure which is considered. The cost function in bank operations management in general includes interest costs of deposits, administrative costs of assets and liabilities, and fixed costs. To gain the essence of lending scale and scope, let us consider a linear cost structure setting due to Kihlstrom and Levhair (1977). They make a fundamental assumption about a "linear service technology". Using this additive assumption allows us separating interest costs from administrative costs. In our model, the administrative cost of deposits and the fixed costs are omitted

[4] Different loan acceptance in retail banking is critical to operations management (see Asosheha, Bagherpour, and Yahyapour (2008). However, our focus is the lending cost issue related to scale and scope, so this abstraction is sufficient (see Kashyap, Rajan, and Stein (2002)).

[5] R_B set in our model does not extend to the case of the high-yield bond investment discussed in Lee and Cheng (2008). For simplicity, the bank is assumed to be a rate taker in the security market for liquidity management purposes.

for simplicity because they will be have the same qualitative effect on the optimal bank decisions as the administrative cost of loans since, overall, the asset and liability sides of bank operations are dichotomized (see Slovin and Sushka (1983)).

Given a vector of parametric loan-administrated input price $w > 0$, we can then define the multi-loan administrative cost function:

$$C(L, w) = \min_{w}\{\sum w_i x_i \mid x_1, \cdots x_n, L_1, \cdots L_n \in T\} = wx^*(L, w),$$

where production frontier $T = \{(x, L) \mid L$ can be produced from x with $D + K\}$, and $x^*(L, w)$ is some vector of input levels that minimizes the administrative loan cost of producing L at input prices w. We further embark on our quest for a multi-loan cost concept that yields an insight into multi-loan portfolio structure analogous to those average cost offers us in the conceptual homogeneous-loan case. To do this, we express the value of the bank's risky assets at $t \in [0, 1]$ as follows:

$$V(R_i)\begin{cases} = \sum(1 + R_i)L_i = V^0 \\ \qquad\qquad \textit{if no loan losses} \\ < V^0 \\ \qquad\qquad \textit{if loan losses} \end{cases} \quad (1)$$

For simplicity, we further assume n identical loan amount granted by the bank that $\sum(1 + R_i)L_i = n(1 + R_i)L_i$ where L_i can be treated as a representative loan i. Similarly, the multi-loan administrative cost function can be simplified as a form of $C(nL_i)$. A simple extension is therefore to use the aggregate measure of loans, nL_i, which allows the discussion of scale lending denoted by ΔL_i and scope lending denoted by Δn.[6] The total cost structure with minimization setting is the linear treatment of $(1 + R_D)D + C(nL_i)$.

The value of the bank's equity return at $t = 1$ is the residual value of the bank after meeting all obligations:

$$S = S(\max[0, n(1 + R_i)L_i + (1 + R_B)B - (1 + R_D)D - C(nL_i)]) \quad (2)$$

subject to $nL_i + B = D + K = K(\frac{1}{q} + 1)$

where $C'(nL_i) > 0$ and $C''(nL_i) > 0$ reflect its strict convexity and satisfy the Inada conditions. Both the terms of $n(1 + R_i)L_i$ and $C(nL_i)$ in objective S can express the integral system of scale and scope in lending operations management. The balance sheet constraint of equation (2) captures the bank's operations management in lending since the total assets on the left-hand side are financed by demandable deposits and equity capital on the right-hand side. Further, the required capital-to-deposits ratio q is assumed to be an increasing function of the amount of the loans nL_i held by the bank at $t = 0$, $\partial q / \partial(nL_i) > 0$ (see Zarruk and Madura (1992)).

In this paper, we assume that the bank sets the optimal loan rate to maximize its market value of equity defined as in equation (2). To do this, we follow Merton's (1974) model by explicitly linking the risk of the bank's default process to the variability in the bank's risky-loan value and viewing the

[6] As pointed out by Kim (1986), an aggregate measure of loans may introduce an estimation bias. However, the details of what drive disaggregated loan demands are unimportant for our purposes, so this simple theoretical reduced-form approach is sufficient and adding this complexity affects none of the qualitative results.

market value of bank equity as the call option on the market value of the bank's loan repayment V with strike price equal to the promised book payment of its net obligations Z.

As noted earlier, the bank has two types of investment opportunities: one instantaneously risky and the other riskless. The vector of instantaneous net returns on the two opportunities follows the dynamics:

$$\begin{pmatrix} dV = \mu V dt + \sigma V dW \\ dZ = \delta Z dt \end{pmatrix} \qquad (3)$$

where

$V = n(1+R_i)L_i$,

$\mu =$ the deterministic drift coefficient illustrating V,

$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$, the volatility matrix where subscripts 1 and 2 indicate V and Z, respectively,

$W =$ a standard Wiener process,

$$Z = \frac{(1+R_D)K}{q} + C(nL_i) - (1+R_B)[K(\frac{1}{q}+1) - nL_i],$$

and

$\delta = R_B - R_D$.

Given the conditions of vector (3), the expected value in equation (2) with the call option pricing when default only occurs at maturity can be expressed as $\hat{S} = \hat{S}(\max[0, V - Z])$. Based on the risk-neutral valuation argument, the call option pricing is the value of this discounted at the riskless spread rate δ, that is, $S = e^{-\delta}\hat{S}$. Specifically, $\ln V$ has the probability distribution expressed as the form of $\ln V$ obeying the normal distribution with mean $\ln V + (\mu - (\sigma^2/2))$ and standard deviation σ.

In light of previous work, evaluating $e^{-\delta}\hat{S}(\max[0, V - Z])$ is an application of integral calculus.[7] With this approach, the market value of objective (2) is given by Black and Scholes (1973) and Merton (1974) , and Lin, Chang, and Lin (2009, a, b) for the call option pricing:

$$\underset{R_i}{Max}\, S = VN(d_1) - Ze^{-\delta}N(d_2) \qquad (4)$$

where $d_1 = (\ln(V/Z) + \delta + (\sigma^2/2))/\sigma$, $d_2 = d_1 - \sigma$, and $N(\cdot)$ is the cumulative density function of the standard normal distribution. Furthermore, the market value of the bank's equity is assumed to be strictly increasing at R_i if there exists a $\Delta > 0$ such that $S(R_i^0) > S(R_i^1)$ for all R_i^0 and R_i^1 with $R_i - \Delta > R_i^1 > R_i^0 > R_i + \Delta$. $S(\cdot)$ is said to be globally increasing if the above relation holds for all $R_i > 0$. A globally increasing objective function renders our analysis widely applicable as it allows us to present the objective function of any call-option-pricing contingent-claim-maximizing agent (as in , for example, Mullins and Pyle (1994), and Vassalou and Xing (2004), and Lin, Chang, and Lin (2009b)).

Using information about objective (4), we apply Vassalou and Xing (2004) and formulate the default probability in the bank's equity returns as follows. The default probability is the probability when V is less than Z. In other words, in a single-period horizon setting $t \in [0,1]$,

$$\begin{aligned} P_{def}(t=0) &= prob(V(t=1) \\ &\quad \le Z(t=0) \mid V(t=0)) \\ &= prob(\ln V(t=1) \\ &\quad \le \ln Z(t=0) \mid V(t=0)) \end{aligned} \qquad (5)$$

[7] See Hull (1993, p.224).

Since the value of the loan repayment follows the geometric Brownian motion of vector (3), the value of the repayment at $t \in [0,1]$ is given by:

$$\ln V(t=1) = \ln V(t=0) + (\mu - \sigma^2/2) + \sigma\varepsilon(t=1) \quad (6)$$

where $\varepsilon(t=1)$ is defined as the difference $W(t=1)$ and $W(t=0)$ and $\varepsilon(t=1)$ follows a normal distribution of $N(0,1)$. Therefore, equation (5) can be rewritten as:

$$\begin{aligned} P_{def}(t=0) &= prob(\ln V(t=0) - \ln Z(t=0) + (\mu - \sigma^2/2) + \sigma\varepsilon(t=1) \le 0) \\ &= prob(-\ln(V(t=0)/Z(t=0) + (\mu - \sigma^2/2))/\sigma \ge \varepsilon(t=1)) \end{aligned} \quad (7)$$

We can then define the distance to default d_3 as follows:

$$d_3(t=0) = \frac{\ln(V(t=0)/Z(t=0) + (\mu - \sigma^2/2)}{\sigma} \quad (8)$$

d_3 tells us by how many standards the log of this ratio needs to deviate from its mean in order for default to occur. We then use the theoretical distribution implied by Merton's (1974) model, which is the normal distribution. In this case, the theoretical probability of default will be given by:

$$P_{def} = N(-d_3) = N\left(-\frac{\ln(V/Z) + \mu - \sigma^2/2}{\sigma}\right) \quad (9)$$

4 Solutions and Comparative Static Results

Solving the first order condition yields:

$$\frac{\partial S}{\partial R_i} = \frac{\partial V}{\partial R_i} N(d_1) + V \frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial R_i} - \frac{\partial Z}{\partial R_i} e^{-\delta} N(d_2) - Ze^{-\delta} \frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial R_i} = 0 \quad (10)$$

Equilibrium condition (10) can be simplified by the following calculation.

$$\begin{aligned} d_2^2 &= d_1^2 + \sigma^2 - 2d_1\sigma \\ &= d_1^2 - 2(\ln(V/Z) + \delta) \end{aligned} \quad (11)$$

Following Hull (1993), we use the numerical procedures to directly evaluate $N(d_2)$. One such approximation is

$$N(d_2) = 1 - (a_1 k + a_2 k^2 + a_3 k^3)\frac{\partial N(d_2)}{\partial d_2} \quad (12)$$

where $k = 1/(1 + \alpha d_2)$, $\alpha = 0.33267$, $a_1 = 0.436183$, $a_2 = -0.1201676$, $a_3 = 0.937280$, and $\partial N(d_2)/\partial d_2 = e^{(-d_2^2/2)}/\sqrt{2\pi} > 0$. We can restate the following term as:

$$\begin{aligned} \frac{\partial N(d_2)}{\partial d_2} &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(d_1^2 - 2(\ln(V/Z)+\delta)} \\ &= \frac{\partial N(d_1)}{\partial d_1}\frac{V}{Ze^{-\delta}} \end{aligned} \quad (13)$$

Further, we can have

$$V\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial R_i} - Ze^{-\delta}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial R_i} = (V\frac{\partial N(d_1)}{\partial d_1} - Ze^{-\delta}\frac{\partial N(d_1)}{\partial d_1}\frac{V}{Ze^{-\delta}})\frac{\partial d_1}{\partial R_i} = 0 \quad (14)$$

where $\partial d_2 / \partial R_i = \partial d_1 / \partial R_i \neq 0$. Imposing condition (14) on the first order condition, we have the simplified form of equilibrium condition in the following:

$$\frac{\partial S}{\partial R_i} = \frac{\partial V}{\partial R_i}N(d_1) - \frac{\partial Z}{\partial R_i}e^{-\delta}N(d_2) = 0 \quad (15)$$

where

$$\frac{\partial V}{\partial R_i} = n(L_i + (1+R_i)\frac{\partial L_i}{\partial R_i}) < 0$$

$$\frac{\partial Z}{\partial R_i} = [\frac{(R_B - R_D)Kq'}{q^2} + C' + (1+R_B)]n\frac{\partial L_i}{\partial R_i} < 0$$

The second-order condition for a maximum of objective (4) is $\partial^2 S / \partial R_i^2 < 0$. The term $(L_i + (1+R_i)(\partial L_i / \partial R_i))$ of $\partial V / \partial R_i$ can be interpreted as the interest rate elasticity of loan demand, which implies that this elasticity is negative. That is, the bank operates on the elastic portion of its loan demand curve, just as a monopolistic firm does. Thus, we have $\partial V / \partial R_i < 0$. The term $\partial Z / \partial R_i$ is negative in sign since it is intuitive that $R_B - R_D$ is positive. Condition (15) then implies that the bank set a Cournot-Nash type of the optimal loan rate in operations management where $(\partial V / \partial R_i)N(d_1) = (\partial Z / \partial R_i)e^{-\delta}N(d_2)$. We can further substitute the optimal loan rate to obtain the default probability in equation (9) staying on the maximization optimization.

Having examined the solution to the bank's optimization problem, in this section we consider the effect on the optimal loan rate (and thus the optimal bank interest margin, from changes in the number of loans in the loan portfolio. Moreover, the result of this section is also needed for the default probability evaluation presented later.

Implicit differentiation of equation (15) with respect to n yields:

$$\frac{\partial R_i}{\partial n} = -\frac{\partial^2 S}{\partial R_i \partial n} / \frac{\partial^2 S}{\partial R_i^2} \quad (16)$$

where

$$\frac{\partial^2 S}{\partial R_i \partial n} = [\frac{\partial^2 V}{\partial R_i \partial n}N(d_1) - \frac{\partial^2 Z}{\partial R_i \partial n}e^{-\delta}N(d_2)] + \frac{\partial V}{\partial R_i}\frac{\partial N(d_1)}{\partial d_1}(1 - \frac{VN(d_1)}{Ze^{-\delta}N(d_2)})\frac{\partial d_1}{\partial n}$$

$$\frac{\partial^2 V}{\partial R_i \partial n} = L_i + (1+R_i)\frac{\partial L}{\partial R_i} < 0$$

$$\frac{\partial^2 Z}{\partial R_i \partial n} = -\frac{2n(R_B - R_D)K(q')^2}{q^3}\frac{\partial L_i}{\partial R_i} + [\frac{(R_B - R_D)Kq'}{q^2} + (1+R_B) + nC'']\frac{\partial L_i}{\partial R_i}$$

$$\frac{\partial d_1}{\partial n} = \frac{1}{n}(\frac{n}{V}\frac{\partial V}{\partial n} - \frac{n}{Z}\frac{\partial Z}{\partial n})$$

$$\frac{\partial V}{\partial n} = (1+R_i)L_i > 0$$

$$\frac{\partial Z}{\partial n} = [\frac{(R_B - R_D)Kq'}{q^2} + C' + (1+R_B)]L_i > 0$$

Before proceeding with the analysis of equation (16), we demonstrate that the first term $[\cdot]$ on the right-hand side of $\partial^2 S / \partial R_i \partial n$ can be interpreted as the mean equity effect, while the second term can be interpreted as the variance or "risk" effect.

The mean equity effect captures the change in $\partial S / \partial R_i$ due to an increase in n, holding the variance effect constant. As noted earlier, the number of loans n reflects the diversified concept of scope operations. When $\partial^2 Z / \partial R_i \partial n$ is negative, it will reinforce the term $\partial^2 V / \partial R_i \partial n$, or when $\partial^2 Z / \partial R_i \partial n$ is positive, it will be in general insufficient to offset the term $\partial^2 V / \partial R_i \partial n$. The mean profit effect is negative because an increase in the economies of scope by increasing n makes loan portfolio less costly to grant. In response to this, the bank has an incentive to increase the amount of loans it grants by charging a lower R_i, ceteris paribus.

The variance effect implies the change in $\partial S / \partial R_i$ due to an increase in n of every possible mean equity state. As usual, the sign of this variance effect is indeterminate. However, the term associated with $\partial d_1 / \partial n$ of $\partial^2 S / \partial R_i \partial n$ in equation (16) provides us with a hunch that the variance effect should be negative since the loan-portfolio elasticity term $(n / V)(\partial V / \partial n)$ is less significant that the net-obligation elasticity term $(n / Z)(\partial Z / \partial n)$. Hence, we have $\partial d_1 / \partial n < 0$. The rationale is that as scope operations through increasing the number of loans increase, the bank has an incentive to diversify the loan portfolio it grants by charging a lower R_i. Since the variance effect reinforces the mean equity effect to give an overall negative response of R_i to an increase in n, we establish the following proposition.

Proposition 1: An increase in the scope diversification operations increases the scale focus operations in bank lending at the decreasing optimal bank interest margin.

Intuitively, when the bank takes advantage of scope diversification operations by increasing an attractive new lending opportunity, it must now provide a return to a larger lending base. One way the bank may attempt to augment its total returns is by shifting its investments to its individual loan and away from the liquid asset. If current loan demand L_i is relatively rate-elastic, a larger L_i reflected by scale focus operations is possible at a reduced margin. Proposition 1 allows us to explain the changes in the integral system of scope operations from scale operations due to a change in the bank's optimal interest margin.

Next, we consider the impact on the bank's default risk in equity return from changes in the number of loans in its loan portfolio. To show this result, we differentiate equation (9) evaluated at the optimal loan rate with respect to n:

$$\frac{dP_{def}}{dn} = \frac{\partial P_{def}}{\partial n} + \frac{\partial P_{def}}{\partial R_i}\frac{\partial R_i}{\partial n} \tag{17}$$

where

$$\frac{\partial P_{def}}{\partial n} = -\frac{\partial N(d_3)}{\partial d_3}\frac{\partial d_3}{dn} > 0$$

$$\frac{\partial d_3}{\partial n} = \frac{\partial d_1}{dn} < 0$$

$$\frac{\partial P_{def}}{\partial R_i} = -\frac{\partial N(d_3)}{\partial d_3}\frac{\partial d_3}{\partial R_i}$$

$$\frac{\partial d_3}{\partial R_i} = \frac{1}{\sigma R_i}\left(\frac{R_i}{V}\frac{\partial V}{\partial R_i} - \frac{R_i}{Z}\frac{\partial Z}{\partial R_i}\right)$$

The first term on the right-hand side of equation (17) can be explained as the direct effect, while the second cross term can be explained as the indirect effect through the optimal loan rate adjustment. The direct effect implies the change in P_{def} due to an increase in n, holding the optimal loan rate constant. This direct effect is positive in sign because an increase in scope n makes loans less administrative costs to grant and makes new loans more attractive to invest. In response to this, the bank's default in equity return increases with nL_i due to an increase in the bank's equity return, ceteris paribus.

The indirect effect can be decomposed into two terms. First, this indirect effect demonstrates the impact on the default risk from changes in the optimal loan rate. This partial effect is positive since the negative loan-repayment elasticity of loan rate is in general more significant than the negative net-obligation elasticity. Second, this indirect effect also captures the impact on the optimal loan rate from changes in the number of loans. This partial effect is negative in sign as known from equation (16). The second term in equation (17) provides a clue that the indirect effect is negative.

Overall, the sign of the total effect is indeterminate sign of the overall effect is indeterminate. However, the direct effect is generally insufficient to be offset by the indirect effect. If this is the case, equation (17) gives an overall positive response of P_{def} to an increase in n. We can establish the following proposition.

Proposition 2: An increase in the scope diversification operations increases the default risk in the bank's equity return through increasing the scale focus operations in lending.

Intuitively, as the banks use the scope operations to reduce the multi-loan administrative cost, it must now provide a return to a less administrative cost base and a more attractive new investment base. One way the bank may attempt to augment its total returns is by increase its scale economies. If loan demand is relatively rate-elastic, a larger L_i is possible at a reduced margin and sequentially an increased default probability in equity return.

5 Conclusion

In this paper, we have developed an option-based, firm-theoretic model to study the optimal bank interest margins and the default risk in equity returns of a multi-loan bank. We use the model to show how scale and scope operations jointly determine the optimal margin and its associated default risk decisions. This allows to focusing on a phenomenon that arises from the impact on the bank's margin featuring scale focus operations from changes in the number of loans featuring scope diversification operations. We find that the optimal bank interest margin is negatively related to the number of loans in the bank's loan portfolio, and the default risk in the bank's equity return is positively related to the number of loans. In other words, the impact on lending scale is positively related to lending scope that we say scale and scope operations are complements. Accordingly, we can argue that diversification is not guaranteed to produce superior equity performance and greater safety for the bank.

Of course, in a state where multi-loan revenue and cost functions possess no natural salary quantity over which revenues and costs may be average, other composite varieties would affects the integral system of scope and scale operations in lending activities. For example, another important way in which the magnitude of a bank's operations may change is through variation in the size of one loan holding the quantities of other loans constant. This would be a more complicated integral system of scope and scale operations. Such concerns are beyond the focus of this paper and so are not addressed here. What this paper does demonstrate, however, is the important, however, is the important role played by the integral system of scope and scale in multi-loan lending operations management affecting bank margin determination and default risk assessment.

Acknowledgement : The second author's research has been supported partially by a research grant from the National Science Council (97-2118-M-032-003).

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