Economic Production Quantity Model with Backordering, Rework and Machine Failure Taking Place in Stock Piling Time

FENG-TSUNG CHENG[a], HUEI-HSIN CHANG[b,*],
SINGA WANG CHIU[c]

[a]Department of Industrial Engineering and Systems Management,
Feng Chia University, Taichung 407,
TAIWAN
[b]Department of Department of Finance,
[c]Department of Business Administration,
Chaoyang University of Technology
168 Jifong East Road, Wufong, Taichung 413,
TAIWAN
[*]E-mail: chss@ms4.kntech.com.tw (HUEI-HSIN CHANG) http://www.cyut.edu.tw/

Abstract: - This paper investigates the optimal inventory replenishment policy for an economic production quantity (EPQ) model with backordering, rework and machine breakdown taking place in stock piling time. A prior paper by Chiu has examined the lot-sizing problem on an imperfect quality EPQ model. Due to another reliability factor - random machine breakdown seems to be inevitable in most real world manufacturing environments, and to deal with it the production planners must practically compute the mean time between failures (MTBF) and establish a robust production plan accordingly in terms of the optimal replenishment lot size that minimizes total production-inventory costs for such an unreliable system. This study extends Chiu's work and incorporates a machine breakdown taking place in the stock piling stage into his model. The effects of random machine failure on optimal run time and on the long-run average costs are examined in this paper. Mathematical modeling and cost analysis are employed. The renewal reward theorem is utilized to cope with variable cycle length. Convexity of the long-run average cost function is proved and an optimal lot-size that minimizes the expected overall costs for such an imperfect system is derived. Numerical example is given to demonstrate its practical usage. Managers in the field can adopt this run time decision to establish their own robust production plan accordingly.

Key-Words: - Optimization, Run time, Breakdown, Lot size, Production, Backorder, Rework, Inventory

1 Introduction

One implicit assumption of conventional economic production quantity model is that the produced items are always of perfect quality. However, in real-life manufacturing systems, due to many unpredictable factors, generation of random defective items seems to be inevitable. For this reason, many studies have been carried out to address the imperfect quality item issues in EPQ model [1-8]. Lee and Rosenblatt [2] studied an EPQ model with joint determination of production cycle time and inspection schedules, they derived a relationship that can be used to determine the effectiveness of maintenance by inspection. Cheung and Hausman [6] developed an analytical model of preventive maintenance (PM) and safety stock (SS) strategies in a production environment subject to random machine breakdowns. They illustrated the trade-off between investing in the two options (PM and SS) and provided optimality conditions under which either one or both strategies should be implemented to minimize the associated cost function. Both the deterministic and exponential repair time distributions are analyzed in detail in their study. Boone et al. [8] investigated the impact of imperfect processes on the production run time. They built a model in an attempt to provide managers with guidelines to choose the appropriate production run times to cope with both the defective items and stoppages occurring due to machine breakdowns. Despite the simplicity of EPQ model, it is still the basis for the analyses of more complex systems [9-17].

The imperfect quality items produced in some circumstances can be reworked and repaired. For example, the printed circuit board assembly (PCBA) in PCBA production or the plastic goods in plastic injection molding process, etc., hence, the long-run average production-inventory costs can be reduced significantly [18-24]. Hayek and Salameh [18] examined an EPQ model that all defective items produced are repairable. They derived an optimal operating policy for such an imperfect EPQ model under the effect of reworking of defective items. Chiu [19] studied optimal lot size for an imperfect quality finite production rate model with rework and backlogging. Jamal et al. [20] investigated optimal production batch size with rework process at a single-stage manufacturing system. Due to excess demands, stock-out situations may arise occasionally. Sometimes, shortages are permitted and they are backordered and satisfied in the very next replenishment. Hence, the total production-inventory costs can be reduced substantially [18,19,22].

In the real-life production management, random machine failure is another common reliability factor that troubles production practitioners most. To be able to effectively control and manage the disruption caused by the random breakdown in order to minimize overall production costs becomes a critical task to most of the production planners. Hence, determination of optimal inventory replenishment policy for production systems subject to machine failures has received extensive attention from researchers in past decades [25-34]. Groenevelt et al. [25] proposed two inventory control policies to deal with machine failures. One assumes that production of the interrupted lot is not resumed (called no resumption-NR policy) after a breakdown. The other policy considers that production of the interrupted lot will be immediately resumed (called abort/ resume-AR policy) after the breakdown is fixed and if the current on-hand inventory falls below a certain threshold level. Repair time is assumed to be negligible, and effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions were investigated. Makis and Fung [26] investigated effects of machine failures on the optimal lot size as well as on optimal number of inspections. Formulas for the long-run expected average cost per unit time was obtained. Then the optimal production/inspection policy that minimizes the expected average costs was derived. Giri and Dohi [28] developed the exact formulation of stochastic EMQ model for an unreliable production system. Their EMQ model is formulated based on the net present value (NPV) approach and by taking limitation on the discount rate the traditional long-run average cost model is obtained. They also provided criteria for the existence and uniqueness of the optimal production time and computational results showing that the optimal decision based on the NPV approach is superior to that based on the long-run average cost approach. Chiu et al. [29] considered the optimal run time for EPQ model with scrap, rework and random breakdown. They proved theorems on conditional convexity of the integrated cost function and on bounds of the production run time. An optimal run time was located by the use of the bisection method based on the intermediate value theorem.

This paper incorporates a machine breakdown factor into model studied by Chiu [19] and studies its effect on the optimal inventory replenishment policy as well as on the long-run production-inventory costs. Because little attention was paid to the area of imperfect quality EPQ model with backlogging, rework and machine breakdown taking place in stock piling time; this paper intends to bridge the gap.

2 Problem Formulation

The following describes our proposed imperfect quality EPQ model. Consider a manufactured item's production rate is P per year and its annual demand rate is λ, where the value of P is much larger than that of λ. Manufacturing process may randomly produce x portion of defective items at a production rate d, where $d=Px$. All produced items are screened and the inspection cost per item is included in the unit production cost C. Assuming that the production rate of perfect quality items must always be greater than or equal to the sum of the demand rate λ and the defective rate d. Hence, the following condition must hold: $(P-d-\lambda)>=0$ or $(1-x-\lambda/P)>=0$.

A θ portion of the imperfect quality items is scrap and they are discarded when the regular production ends. The other $(1-\theta)$ portion of imperfect quality items is reworked at a rate of P_1 immediately after the regular process. Stock-out situation is allowed. Shortages are backordered and satisfied by the immediate next replenishment. Further, according to the mean time between failures (MTBF) data, a machine failure may take place randomly in the stock-piling time (refer to Figure 1), and an abort/ resume inventory control policy is adopted in this study. Under such a policy, when a machine failure occurs, machine is under corrective maintenance immediately. A constant repair time is assumed and the interrupted lot will be resumed right after the restoration of machine.

In this study, we assume that probability of more than one machine breakdown occurrences in a production cycle is very small due to a very good preventive maintenance schedule. However, if it happens, the safety stock will be used to satisfy the demand during machine repairing time. Therefore, multiple machine failures are assumed to have insignificant effect on the proposed model. Here, the purpose of using safety stock is to simplify our on-hand inventory analysis when multiple breakdowns occur.

Figure 1 depicts the on-hand inventory level of perfect quality items in the proposed EPQ.

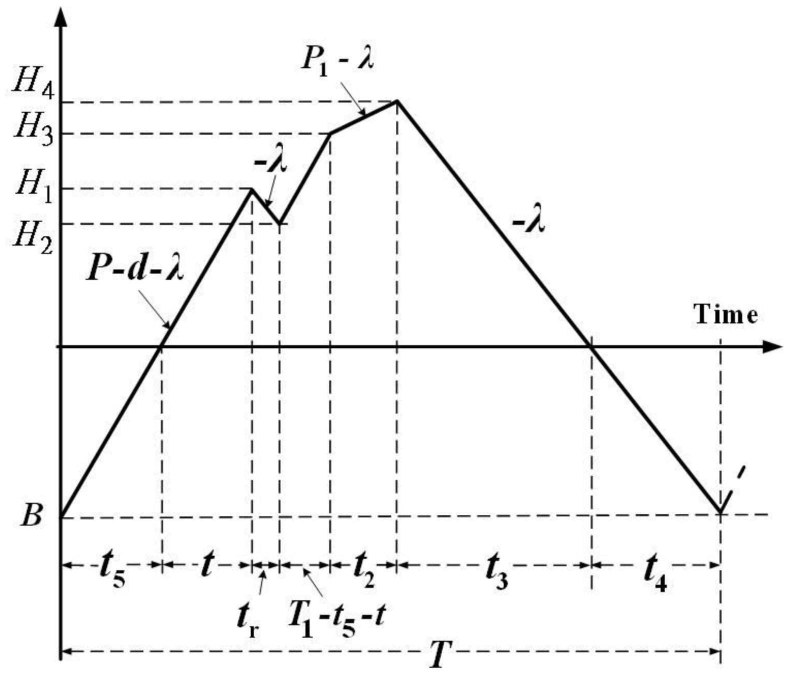

Fig. 1: On-hand inventory of perfect quality items in EPQ model with backordering, rework and machine failure taking place in stock piling time

Cost variables considered in the proposed EPQ model include: setup cost K, unit holding cost h, unit production cost C, repair cost for each defective item reworked C_R, disposal cost per scrap item C_S, unit shortage/backorder cost b, unit holding cost per reworked item h_1, and the cost for repairing/restoring failure machine M. Additional parameters used are listed below.

T = the production cycle length,
Q = production lot size for each cycle,
B = the maximum backorder level allowed for each cycle,
H_1 = level of on-hand inventory when machine breakdown occurs,
H_2 = level of on-hand inventory when machine is repaired and restored,
H_3 = level of on-hand inventory when regular production process ends,
H_4 = the maximum level of perfect quality inventory when rework finishes,
T_1 = production run time to be determined by the proposed study,
t = production time before a random breakdown occurs,
t_2 = time needed to rework the defective items,
t_r = time required for repairing and restoring the machine,
t_5 = time required for filling the backorder quantity B,
t_3 = time required for depleting all available perfect quality on-hand items,
t_4 = shortage permitted time,
$TC(T_1,B)$ = total production-inventory costs per cycle,
$TCU(T_1,B)$ = total production-inventory costs per unit time (e.g. annual),
$E[TCU(T_1,B)]$ = the expected total production-inventory costs per unit time.

Let maximum machine repair time be a constant t_r and $t_r = g$. In this study, it is conservatively assumed that if a failure of a machine cannot be fixed within a certain allowable amount of time, then a spare machine will be in place to avoid further delay of production. The following derivation procedure is similar to what was used by past studies [18-19].

Refer to Figure 1, one can obtain the following: the cycle length T; production run time T_1; time for reworking defective items t_2; time required for depleting all available on-hand items t_3; shortage allowed time t_4, time t_5 for refilling the maximum backordering quantity B, and the on-hand inventory levels of H_1, H_2, H_3 and H_4.

$$T = T_1 + t_2 + t_3 + t_4 + t_r \tag{1}$$

$$T_1 = Q / P \tag{2}$$

$$t_2 = \frac{d \cdot T_1 (1-\theta)}{P_1} \tag{3}$$

$$t_3 = H_4 / \lambda \tag{4}$$

$$t_4 = B / \lambda \tag{5}$$

$$t_5 = \frac{B}{P - d - \lambda} \tag{6}$$

$$H_1 = (P - d - \lambda) t \tag{7}$$

$$H_2 = H_1 - t_r \lambda = H_1 - g\lambda \tag{8}$$

$$H_3 = H_2 + (P - d - \lambda) \cdot (T_1 - t_5 - t) \tag{9}$$

$$H_4 = H_3 + (P_1 - \lambda) t_2 \tag{10}$$

where $d=Px$ and let g be the constant machine repair time, i.e. $t_r = g$. The level of on-hand defective items for the proposed imperfect quality EPQ model is illustrated in Figure 2.

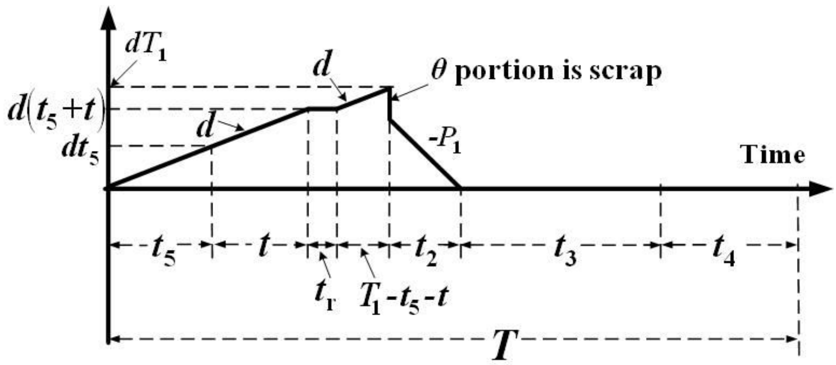

Fig. 2: On-hand inventory of defective items in EPQ model with backordering, rework and machine failure taking place in stock piling time

Total imperfect quality items produced during the production run time T_1 are:

$$d \cdot T_1 = x \cdot Q \tag{11}$$

Figure 3 depicts the on-hand inventory level of scrap items produced during the regular production process. The total scrap items produced during T_1 are as follows.

$$\theta \cdot (d \cdot T_1) = \theta \cdot (P \cdot x \cdot T_1) = \theta \cdot (x \cdot Q) \tag{12}$$

Total production-inventory cost per cycle $TC(T_1,B)$ is:

$$\begin{aligned} TC(T_1,B) &= K + M + C \cdot (PT_1) + C_R \cdot [PT_1 \cdot x \cdot (1-\theta)] + C_S \cdot (PT_1 \cdot x \cdot \theta) \\ &+ h\left[\begin{array}{l} \frac{H_1(t)}{2} + \frac{H_1+H_2}{2}(t_r) + \frac{H_2+H_3}{2}(T_1 - t_5 - t) \\ + \frac{H_3+H_4}{2}(t_2) + \frac{H_4(t_3)}{2} \end{array}\right] \\ &+ h\left[\frac{d(t_5+t)}{2}(t_5+t) + (t_5+t)t_r + \frac{(t_5+t)+dT_1}{2}(T_1 - t_5 - t)\right] \\ &+ h_1\left[\frac{P_1 t_2}{2}(t_2)\right] + b\left[\frac{B}{2}(t_5) + \frac{B}{2}(t_4)\right] \end{aligned} \tag{13}$$

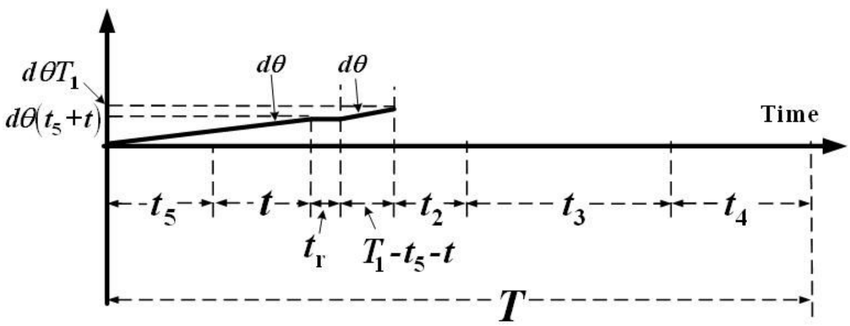

Fig. 3: On-hand inventory level of scrap items in EPQ model with backordering, rework and machine failure taking place in stock piling time

By substituting all variables in Equations (1) to (12) in Equation (13), the production-inventory cost function $TC(T_1,B)$ becomes:

$$TC(T_1,B)=C\cdot P\cdot T_1+K+M+C_R\cdot T_1\cdot P\cdot x\cdot(1-\theta)+C_S T_1 Px\cdot\theta$$
$$+\frac{h}{2}\left\{\frac{P^2}{\lambda}\left(1-2x\theta+x^2\theta^2\right)T_1^2+\frac{(1-x)B^2}{\lambda\left(1-x-\frac{\lambda}{P}\right)}-PT_1^2+2Px\theta T_1^2\right\}$$
$$+\frac{b(1-x)B^2}{2\lambda(1-x-\frac{\lambda}{P})}+\frac{P^2x^2(1-\theta)^2T_1^2}{2P_1}\left(h_1-h\right)-h\frac{P}{\lambda}T_1B+h\frac{Px\theta}{\lambda}T_1B$$
$$+\frac{hg}{(1-x-\frac{\lambda}{P})}B-hPgT_1+hPgx\theta T_1+hPgt+\frac{hg^2\lambda}{\left(1-x-\frac{\lambda}{P}\right)} \tag{14}$$

Owing to the defective, scrap, and machine failure rates are random (in this study, we assume uniformly distributed defective rate and the Poisson distributed breakdown rate, with mean equals to β per unit time); production cycle length is not constant. Therefore, to take randomness of defective, scrap, and breakdown rates into account, one can employ renewal reward theorem [30,35] in the production-inventory cost analysis to cope with the variable cycle length and use the integration of $TC(T_1,B)$ to deal with the random breakdown happening in stock-piling time.

Hence, the long-run average production-inventory costs per unit time $E[TCU(T_1,B)]$ can be calculated as follows.

$$E\left[TCU\left(T_1,B\right)\right]=\frac{E\left[\int_0^{T_1-t_5}TC\left(T_1,B\right)\cdot f\left(t\right)dt\right]}{E\left[T\right]}=\frac{E\left[\int_0^{T_1-t_5}TC\left(T_1,B\right)\cdot\left(\beta e^{-\beta t}\right)dt\right]}{\left[T_1P\left(1-\theta E[x]\right)/\lambda\right]\cdot\left(1-e^{-\beta(T_1-t_5)}\right)} \tag{15}$$

Substituting all related parameters from Equations (1) to (14) in the numerator of Equation (15), one has the following:

$$E\left[\int_0^{T_1-t_5}TC\left(T_1,B\right)f\left(t\right)dt\right]$$
$$=-hPgT_1+h\left(g^2\lambda+gB\right)E\left[\frac{1}{1-x-\lambda/P}\right]+\left[1-e^{-\beta(T_1-t_5)}\right]\cdot$$
$$\left\{\begin{array}{l}K+M+P\cdot T_1\cdot\left[C+C_RE[x](1-\theta)+C_SE[x]\theta\right]\\+hPg\theta T_1E[x]+hPg/\beta-hPT_1B\left(1-\theta E[x]\right)/\lambda\\+\frac{h}{2}\left[\begin{array}{l}P^2\left(1-2\theta E[x]+\theta^2\left(E[x]\right)^2\right)T_1^2/\lambda\\-PT_1^2+2P\theta T_1^2E[x]\end{array}\right]\\+\frac{B^2\left(b+h\right)}{2\lambda}E\left[\frac{1-x}{1-x-\lambda/P}\right]+\frac{P^2\left(1-\theta\right)^2T_1^2\left(E[x]\right)^2\left[h_1-h\right]}{2P_1}\end{array}\right\} \tag{16}$$

After further derivations, one should be able to obtain the $E[TCU(T_1,B)]$ as follows.

$$E\left[TCU\left(T_1,B\right)\right]=\frac{h\lambda\left(g^2\lambda+gB\right)}{T_1P(1-e^{-\beta(T_1-t_5)})\left(1-\theta E[x]\right)}E\left[\frac{1}{1-x-\lambda/P}\right]-\frac{hg\lambda}{(1-e^{-\beta(T_1-t_5)})\left(1-\theta E[x]\right)}$$
$$+\left\{\begin{array}{l}\frac{\lambda\left(K+M\right)}{T_1P\left(1-\theta E[x]\right)}+\frac{\lambda\cdot\left[C+C_RE[x](1-\theta)+C_SE[x]\theta\right]}{\left(1-\theta E[x]\right)}\\+\frac{hg\lambda}{T_1\beta\left(1-\theta E[x]\right)}-hB+hg\theta\lambda\frac{E[x]}{1-\theta E[x]}\\+\frac{h}{2}\left[PT_1\left(1-\theta E[x]\right)-\frac{T_1\lambda}{1-\theta E[x]}+2\theta T_1\lambda\frac{E[x]}{1-\theta E[x]}\right]\\+\frac{B^2}{2PT_1\left(1-\theta E[x]\right)}\left(b+h\right)E\left[\frac{1-x}{1-x-\lambda/P}\right]\\+\frac{PT_1\lambda\left(1-\theta\right)^2\left(E[x]\right)^2}{2P_1\left(1-\theta E[x]\right)}\left[h_1-h\right]\end{array}\right\} \tag{17}$$

Let $t_1'=\left(T_1-t_5\right);\ E_0=\frac{1}{1-\theta E[x]};\ E_1=\frac{E[x]}{1-\theta E[x]};$

$$E_2=\frac{\left(E[x]\right)^2}{1-\theta E[x]};\ E_3=\frac{1}{1-\theta E[x]}E\left[\frac{1-x}{1-x-\lambda/P}\right]; \tag{18}$$
$$E_4=\frac{1}{1-\theta E[x]}E\left[\frac{1}{1-x-\lambda/P}\right];\ E_5=1-\theta E[x]$$

Then Eq. (17) becomes:

$$E\left[TCU\left(T_1,B\right)\right]=\frac{h\lambda\left(g^2\lambda+gB\right)}{T_1P(1-e^{-\beta t_1'})}E_4-\frac{hg\lambda}{(1-e^{-\beta t_1'})}E_0$$
$$+\frac{\lambda\left(K+M\right)}{T_1P}E_0+\lambda\cdot\left[CE_0+C_R(1-\theta)E_1+C_S\theta E_1\right] \tag{19}$$
$$+\frac{hg\lambda E_0}{T_1\beta}-hB+hg\theta\lambda E_1+\frac{hT_1}{2}\left(PE_5-\lambda E_0+2\theta\lambda E_1\right)$$
$$+\frac{B^2}{2PT_1}\left(b+h\right)E_3+\frac{PT_1\lambda\left(1-\theta\right)^2}{2P_1}\left[h_1-h\right]E_2$$

3 The Optimal Replenishment Policy

To find the optimal replenishment policy for the proposed imperfect quality EPQ model, one should first prove the convexity of the long-run average cost function $E[TCU(T_1,B)]$. Hessian matrix equations [35] can be employed to verify the existence of the following for the proof of convexity.

$$\begin{bmatrix}T_1 & B\end{bmatrix}\begin{pmatrix}\frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1^2} & \frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B}\\ \frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial T_1\partial B} & \frac{\partial^2E\left[TCU\left(T_1,B\right)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix}>0 \tag{20}$$

$E[TCU(T_1,B)]$ is strictly convex only if equation

(20) is satisfied, for all T_1 and B different from zero:

$$[T_1 \quad B]\cdot\begin{pmatrix} \frac{\partial^2 E\left[TCU(T_1,B)\right]}{\partial T_1^2} & \frac{\partial^2 E\left[TCU(T_1,B)\right]}{\partial T_1 \partial B} \\ \frac{\partial^2 E\left[TCU(T_1,B)\right]}{\partial T_1 \partial B} & \frac{\partial^2 E\left[TCU(T_1,B)\right]}{\partial B^2} \end{pmatrix}\cdot\begin{bmatrix} T_1 \\ B \end{bmatrix} \quad (21)$$

$$=\frac{2(K+M)\lambda}{T_1 P}E_0+\frac{2hg\lambda}{T_1\beta}E_0+\frac{2hg^2\lambda^2E_4}{T_1P(1-e^{-\beta t_1'})}>0$$

Equation (21) is resulting positive because all parameters are positive. Hence, *E[TCU(*T_1*,B)]* is a strictly convex function. It follows that for the optimal uptime T_1 and maximal backorder level *B,* one can differentiate *E[TCU(*T_1*,B)]* with respect to T_1 and with respect to B, and solve linear systems of equations (22) and (23) by setting these partial derivatives equal to zero.

$$\frac{\partial E\left[TCU(T_1,B)\right]}{\partial T_1}=\begin{Bmatrix} -\frac{\lambda(K+M)}{T_1^2P}E_0+\frac{h}{2}\left(PE_5-\lambda E_0+2\theta\lambda E_1\right) \\ -\frac{B^2}{2PT_1^2}(b+h)E_3-\frac{h\lambda\left(g^2\lambda+gB\right)}{T_1^2P(1-e^{-\beta t_1'})}E_4 \\ +\frac{P\lambda(1-\theta)^2}{2P_1}[h_1-h]E_2-\frac{hg\lambda}{T_1^2\beta}E_0 \end{Bmatrix}$$

(22)

$$\frac{\partial E\left[TCU(T_1,B)\right]}{\partial B}=\frac{B}{T_1P}(b+h)E_3-h+\frac{hg\lambda}{T_1P(1-e^{-\beta t_1'})}E_4 \quad (23)$$

Equation (24) can be rewritten as:

$$\therefore\ B^*=\left(\frac{h}{b+h}\right)\left(\frac{1}{E_3}\right)\cdot\left[PT_1-\frac{g\lambda E_4}{(1-e^{-\beta t_1'})}\right] \quad (24)$$

With further derivations, equation (22) becomes:

$$\frac{1}{T_1^2}\left[\frac{\lambda(K+M)}{P}E_0+\frac{B^2}{2P}(b+h)E_3+\frac{hg\lambda}{\beta}E_0+\frac{h\lambda\left(g^2\lambda+gB\right)}{P(1-e^{-\beta t_1'})}E_4\right]$$
$$=\frac{h}{2}\left(P\cdot E_5-\lambda\cdot E_0+2\theta\lambda\cdot E_1\right)+\frac{P\lambda(1-\theta)^2}{2P_1}\left(h_1-h\right)E_2$$

(25)

Substituting Equation (24) in Equation (25), one has:

$$\frac{1}{T_1^2}\begin{bmatrix} \frac{\lambda(K+M)}{P}E_0+\frac{hg\lambda}{\beta}\cdot E_0+\frac{hg^2\lambda^2}{P(1-e^{-\beta t_1'})}E_4 \\ +\frac{1}{2P}(b+h)E_3\left[\frac{hPT_1}{(b+h)E_3}-\frac{hg\lambda E_4}{(b+h)E_3(1-e^{-\beta t_1'})}\right]^2 \\ +\frac{hg\lambda}{P(1-e^{-\beta t_1'})}E_4\left[\frac{hPT_1}{(b+h)E_3}-\frac{hg\lambda E_4}{(b+h)E_3(1-e^{-\beta t_1'})}\right] \end{bmatrix} \quad (26)$$
$$=\frac{h}{2}\left(P\cdot E_5-\lambda\cdot E_0+2\theta\lambda\cdot E_1\right)+\frac{P\lambda(1-\theta)^2}{2P_1}\left(h_1-h\right)E_2$$

or

$$T_1^2=\frac{\frac{1}{P}\begin{bmatrix} 2\lambda(K+M)E_0-\frac{h^2g^2\lambda^2E_4^2}{(b+h)E_3(1-e^{-\beta t_1'})^2} \\ +\frac{2hg^2\lambda^2E_4}{(1-e^{-\beta t_1'})}+\frac{2Phg\lambda E_0}{\beta} \end{bmatrix}}{P\begin{Bmatrix} h\left(E_5-\frac{\lambda}{P}E_0+\frac{2\theta\lambda}{P}E_1\right) \\ +\frac{\lambda(1-\theta)^2}{P_1}\left(h_1-h\right)E_2-\frac{h^2}{(b+h)E_3} \end{Bmatrix}} \quad (27)$$

Finally the optimal replenishment policy in terms of production run time and level of backordering can be obtained as follows:

$$T_1^*=\frac{1}{P}\sqrt{\frac{2\lambda(K+M)E_0-\frac{h^2g^2\lambda^2E_4^2}{(b+h)E_3(1-e^{-\beta t_1'})^2}+\frac{2hg^2\lambda^2}{(1-e^{-\beta t_1'})}E_4+\frac{2Phg\lambda}{\beta}\cdot E_0}{h\left(E_5-\frac{\lambda}{P}\cdot E_0+\frac{2\theta\lambda}{P}\cdot E_1\right)+\frac{\lambda(1-\theta)^2}{P_1}[h_1-h]E_2-\frac{h^2}{(b+h)E_3}}} \quad (28)$$

$$B^*=\left(\frac{h}{b+h}\right)\left(\frac{1}{E_3}\right)\cdot\left[PT_1^*-\frac{g\lambda E_4}{(1-e^{-\beta t_1'})}\right] \quad (29)$$

It is noted that practitioner should check both the numerator and denominator of Eq. (28) are positive before adopting this optimal replenishment run time in real life usage. Proof of positive of denominator can be found in Appendix of [24]. One notes that verification of B^* to be a positive number is required when put it in use.

From Equations (2) and (28), one can also obtain the optimal lot-size Q^* as follows:

$$Q^*=\frac{1}{P}\sqrt{\frac{2\lambda(K+M)E_0-\frac{h^2g^2\lambda^2E_4^2}{(b+h)E_3(1-e^{-\beta t_1'})^2}+\frac{2hg^2\lambda^2}{(1-e^{-\beta t_1'})}E_4+\frac{2Phg\lambda}{\beta}\cdot E_0}{h\left(E_5-\frac{\lambda}{P}\cdot E_0+\frac{2\theta\lambda}{P}\cdot E_1\right)+\frac{\lambda(1-\theta)^2}{P_1}[h_1-h]E_2-\frac{h^2}{(b+h)E_3}}} \quad (30)$$

3.1 Solution Verification

If the machine failure is not a factor to be considered, then the repairing cost and time for the failure machine, M=0 and g=0, Equations (29) and (30) become the same equations as were given by [19]:

$$Q^* = \sqrt{\frac{2K\lambda}{\left[\begin{array}{l} h\left(1-\frac{\lambda}{p}\right)+\frac{\lambda(1-\theta)^2}{P_1}(h_1-h)E[x]^2-\frac{h^2\{1-\theta E[x]\}^2}{(b+h)E\left(\frac{1-x}{1-x-\lambda/P}\right)} \\ -2h\theta\left(1-\frac{\lambda}{p}\right)E[x]+h\theta^2E[x]^2 \end{array}\right]}} \tag{31}$$

$$B^* = \left(\frac{h}{b+h}\right)\left(\frac{1-E[x]\theta}{E\left[\frac{1-x}{1-x-\lambda/P}\right]}\right)Q^* \tag{32}$$

Further, if the regular production process produces no defective items (i.e. x=0) then Equations (31) and (32) become the same equations as were presented by the classic EPQ model with shortages permitted and backordered [33,36]:

$$\text{If } x=0, \text{ then } Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}}\cdot\sqrt{\frac{b+h}{b}} \tag{33}$$

$$\text{and } B^* = \left[\frac{h}{(b+h)}\left(1-\frac{\lambda}{P}\right)\right]\cdot Q^* \tag{34}$$

4 Numerical Example

Consider the annual demand of a manufactured item is 3,600 units and it can be produced at a production rate of 9,000 units per year. The percentage x of imperfect quality items produced follows a uniform distribution over the interval [0, 0.2].

A portion θ= 0.2 of defective items is scrap. The rate of rework process is P_1=600 units per year. Other parameters are summarized as follows.

C = \$1 per item,
C_R = \$0.5 for each item reworked,
C_S = \$0.3 disposal cost for each scrap item,
K = \$450 for each production run,
h = \$0.6 per item per unit time,
h_1 = \$0.8 per item per unit time,
b = \$0.2 per shortage item backordered per unit time,
g = 0.018 years, time needed to repair and restore the machine,
M = \$500 repair cost for restoration of machine failure.

Applying Equations (28) to (30), one obtains the optimal production run time T_1* =0.8794 years, the optimal backorder quantity B* =3,146, and optimal lot-size Q* =7,915. By using Equations (18) and (19), one obtains the optimal long-run expected cost $E[TCU(T_1^*,B^*)]$= \$4,680.

Figure 4 depicts variation of the defective rate x and scrap rate θ effects on the optimal production run time T_1*. It indicates that as x increases, the value of optimal production run time T_1* decreases slightly, and as scrap rate θ increases, T_1* increases significantly.

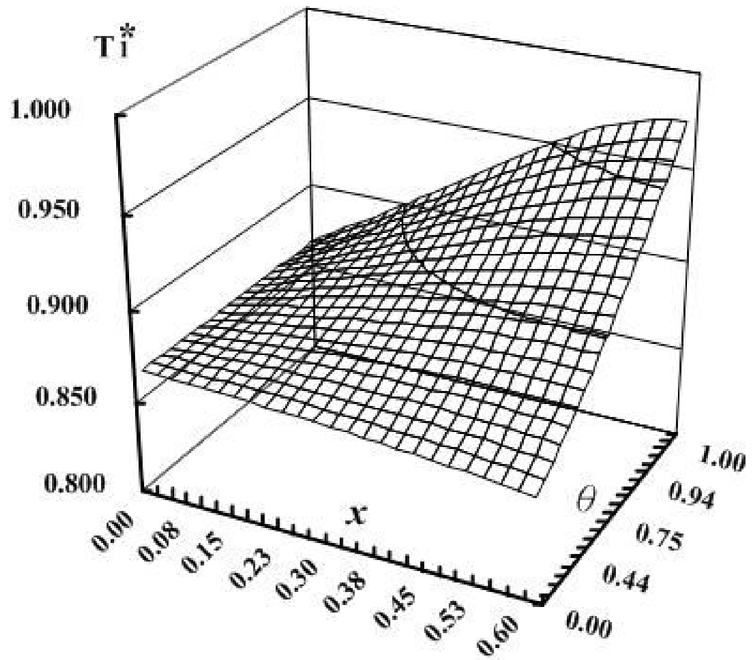

Fig. 4: Behavior of the optimal production run time with respect to the defective rate x and the scrap rate θ

Variation of the defective rate x and the scrap rate θ effects on the optimal backorder quantity B* is illustrated in Figure 5. It shows that as x increases, the value of optimal backorder quantity B* decreases significantly; and as the scrap rate θ increases, the value of the optimal backorder quantity B^* decreases slightly. Figure 6 depicts behavior of $E[TCU(T_1^*,B^*)]$ with respect to the defective rate x and the scrap rate θ. It shows that for different defective rate x values, as x increases, the overall production-inventory costs $E[TCU(T_1^*,B^*)]$ increases significantly; and as scrap rate θ increases, $E[TCU(T_1^*,B^*)]$ increases slightly.

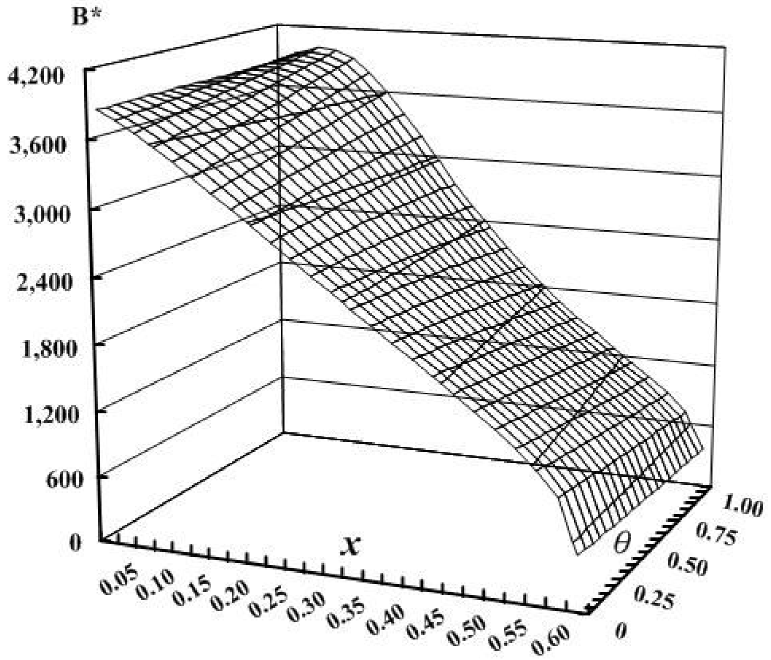

Fig. 5: Variation of the defective rate x and scrap rate θ effects on the optimal backordering quantity B^*

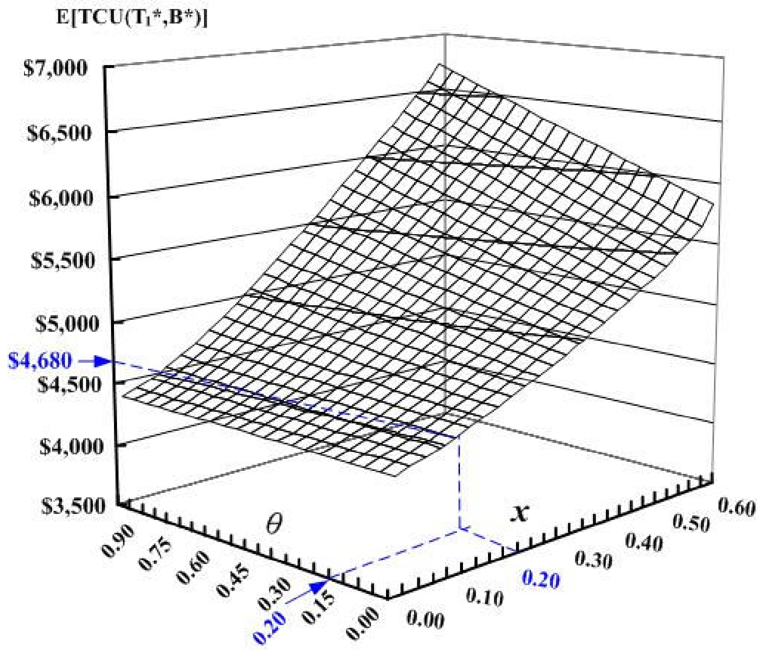

Fig. 6: Behavior of $E[TCU(T_1^*,B^*)]$ with respect to the scrap rate θ and the defective rate x

Figure 7 illustrates the convexity of the long-run expected production-inventory costs $E[TCU(Q,B)]$ with respect to the level of backorder B and the replenishment lot-size Q.

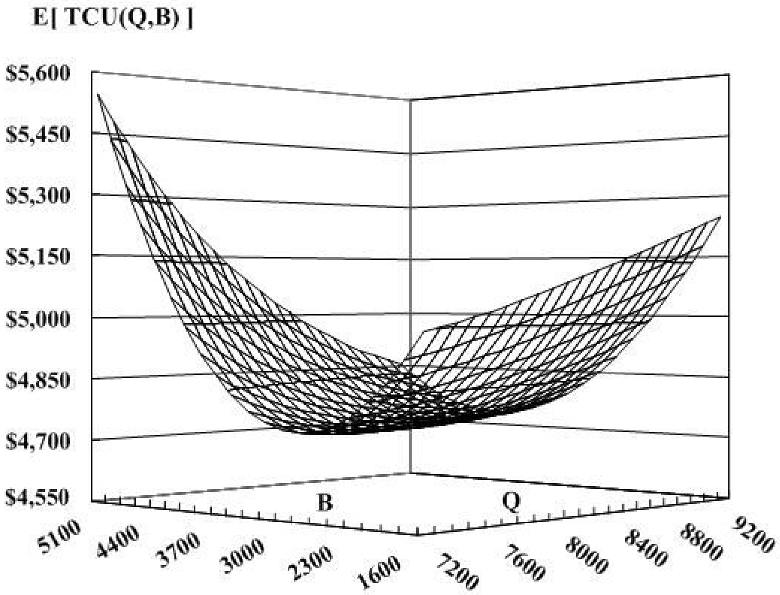

Fig. 7: Convexity of the expected cost function E[TCU(Q,B)] for the proposed EPQ model with backordering, rework and machine failure taking place in stock piling time

5 Concluding Remarks

When dealing with an unreliable production system with backordering, rework, and machine breakdown taking place in stock piling time, if the results of the present study are not available, one can only use the lot-size solutions given by [19] and obtain run time T_1 = 0.5834 years and B = 2131 units. Plugging T_1 and B into equation (17) of this paper, one obtains $E[TCU(T_1,B)]$=$4,757. In comparison with the optimal $E[TCU(T_1^*,B^*)]$=$4,680, the difference is $77. If we exclude $4268 (i.e. the sum of costs for rework, disposal, and variable manufacturing cost) from the $E[TCU(T_1^*,B^*)]$; as a result, $E[TCU(T_1,B)]$ =$4,757 is 18.69% more on sum of holding and setup costs than that obtained by using the optimal inventory replenishment policy of this study.

With an in-depth investigation on such a real life unreliable manufacturing system, optimal operating disciplines and related facts of the system can now be revealed. Practitioners and managers in the field can adopt the research results of this study as their inventory replenishment decisions to establish their own robust production plan accordingly.

For future research, to investigate the effect of random machine failure taking place in the backorder refilling time and with an abort/resume inventory control policy on the optimal lot-size decision, will be an interesting topic. Also, decisions on maximum repairing time of the machine to be allowed as well as acquisition of the spare machine may also have important effects on the replenishment run time studied.

Acknowledgements

Authors thank National Science Council of Taiwan for supporting this research under Grant Number: NSC 97-2410-H-324-013-MY2. Authors also would like to express their sincere appreciation to the anonymous reviewers for their valuable comments and suggestions to the earlier version of manuscript.

References:

[1] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production processes. *IIE Transactions*, Vol. 18, 1986, pp.48-55.

[2] H.L. Lee and M.J. Rosenblatt, Simultaneous determination of production cycle and inspection schedules in a production system. *Management Science*, Vol. 33, 1987, pp.1125-1136.

[3] T. Bielecki and P.R. Kumar, Optimality of zero-inventory policies for unreliable production facility, *Operations Research*, Vol. 36, 1988, pp.532-541.

[4] X. Zhang and Y. Gerchak, Joint lot sizing and inspection policy in an EOQ model with random yield. *IIE Transactions*, Vol. 22, 1990, pp.41-47.

[5] T.C.E. Cheng, An economic order quantity

model with demand-dependent unit production cost and imperfect production processes. *IIE Transaction*, Vol. 23, 1991, pp.23-28.

[6] K.L. Cheung and W.H. Hausman, Joint determination of preventive maintenance and safety stocks in an unreliable production environment. *Naval Research Logistics*, Vol. 44(3), 1997, pp.257-272.

[7] T.K. Das and S. Sarkar, Optimal preventive maintenance in a production inventory system. *IIE Transactions*, Vol.31, 1999, pp.537-551.

[8] T. Boone, R. Ganeshan, Y. Guo and J.K. Ord, The impact of imperfect processes on production run times. *Decision Sciences,* 31(4), 2000, pp.773-785.

[9] S. Osteryoung, E. Nosari, D. McCarty, W.J. Reinhart, Use of the EOQ model for inventory analysis. *Production and Inventory Management*, Vol. 27, 1986, pp.39-45.

[10] Y-S.P. Chiu, C-K. Ting, C-B. Cheng, and F-Y. Lin, Alternative approach for realistic lot-size problem with random scrap rate and backlogging, *WSEAS Transactions on Mathematics*, Vol. 6 (6), 2007, pp.736-740

[11] S.W. Chiu, Optimization problem for EMQ model with backlog level constraint. *WSEAS Transactions on Information Science & Applications*, Vol. 4 (4), 2007, pp.687-692.

[12] S. Nahmias, Production and Operations Analysis, McGraw-Hill Co. Inc., New York, 2009.

[13] Y-S.P. Chiu, F-T. Cheng, and C-K. Ting, Algebraic methods for optimizing EPQ model with rework and scrap. *WSEAS Transactions on Systems*, Vol. 6 (11), 2007, pp.1319-1323.

[14] P.H. Zipkin, *Foundations of Inventory Management*. McGraw-Hill, New York, 2000.

[15] S.W. Chiu, J-C. Yang, S-Y. C. Kuo, Manufacturing lot sizing with backordering, scrap, and random breakdown occurring in inventory-stacking period. *WSEAS Trans. on Mathematics*, Vol. 7 (4), 2008, pp.183-194.

[16] Y-S.P. Chiu and S.W. Chiu, Determining the materials procurement policy based on the economic order/production models with backlogging permitted. *International Journal of Advanced Manufacturing Technology*, Vol. 30 (1-2), 2006, pp.156-165.

[17] Y-S.P. Chiu, S-S. Wang, C-K. Ting, H-J. Chuang, Y-L. Lien, Optimal run time for EMQ model with backordering, failure-in-rework and breakdown happening in stock-piling time. *WSEAS Transactions on Information Science & Applications*, Vol. 5 (4), 2008, pp.475-486.

[18] P.A. Hayek and M.K. Salameh, Production lot sizing with the reworking of imperfect quality items produced. *Production Planning and Control*, Vol. 12, 2001, pp.584-590.

[19] Y-S.P. Chiu, Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Engineering Optimization*, Vol.35 (4), 2003, pp.427-437.

[20] A.M.M. Jamal, B.R. Sarker and S. Mondal, Optimal manufacturing batch size with rework process at a single-stage production system. *Computers and Industrial Engineering*, Vol. 47, 2004, pp.77-89.

[21] S.W. Chiu, C-K. Ting, Y-S.P. Chiu, Optimal order policy for EOQ model under shortage level constraint. *International Journal for Engineering Modelling*, Vol. 18 (1-2), 2005, pp. 41-46

[22] S.W. Chiu and Y-S.P. Chiu, Mathematical modeling for production system with backlogging and failure in repair. *Journal of Scientific & Industrial Research*, Vol. 65(6), 2006, pp.499-506.

[23] Y-S.P. Chiu, K-K. Chen, C-K. Ting, V. Chiu, Economic lot sizing with imperfect rework derived without derivatives. *International Journal for Engineering Modelling*, Vol. 20, 2007, pp.61-65

[24] H-D. Lin, Y-S.P. Chiu, C-K. Ting, A note on optimal replenishment policy for imperfect quality EMQ model with rework and backlogging. *Computers and Mathematics with Applications,* Vol.56 (11), 2008, pp.2819-2824.

[25] H. Groenevelt, L. Pintelon and A. Seidmann, Production lot sizing with machine breakdowns. *Management Science*, Vol. 38, 1992, pp.104-123.

[26] V. Makis and J. Fung, An EMQ model with inspections and random machine failures. *Journal of Operational Research Society*, Vol. 49, 1998, pp.66-76.

[27] H. Kuhn, A dynamic lot sizing model with exponential machine breakdowns. *European Journal of Operational Research*, Vol. 100, 1997, pp.514-536.

[28] B.C. Giri and T. Dohi, Exact formulation of stochastic EMQ model for an unreliable production system. *Journal of the Operational Research Society*, Vol. 56(5), 2005, pp.563-575.

[29] S.W. Chiu, S.L. Wang and Y-S.P. Chiu, Determining the optimal run time for EPQ model with scrap, rework, and stochastic

breakdowns. *European Journal of Operational Research*, Vol. 180, 2007, pp.664-676.

[30] G.C. Lin and D.E. Kroll, Economic lot sizing for an imperfect production system subject to random breakdown. *Engineering Optimization,* Vol. 38(1), 2006, pp.73-92.

[31] Y-S.P. Chiu, C-Y. Tseng, W-C. Liu, C-K. Ting, Economic manufacturing quantity model with imperfect rework and random breakdown under abort/resume policy. *P I Mech E Part B: Journal of Engineering Manufacture*, Vol. 223 (2), 2009, pp.183-194.

[32] Y-S.P. Chiu, S.W. Chiu, and H-C. Chao, Effect of shortage level constraint on finite production rate model with rework. *Journal of Scientific & Industrial Research*, Vol. 67 (2), 2008, pp.112-116.

[33] F.S. Hillier and G.J. Lieberman, *Introduction to Operations Research*, McGraw Hill, New York, 2001.

[34] Y-S.P. Chiu, C-K. Ting, A note on determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns. *European Journal of Operational Research*, Vol. 201 (2), 2010, pp.641-643

[35] R.L. Rardin, *Optimization in Operations Research*. Prentice-Hall, New Jersey, 1998.

[36] E.A. Silver, D.F. Pyke and R. Peterson, *Inventory Management and Production Planning and Scheduling,* John Wiley & Sons, New York, 1998.