

Modeling Recognizing Behavior of Radar High Resolution Range Profile Using Multi-Agent System

JIANSHENG FU

KUO LIAO

DAIYING ZHOU

WANLIN YANG

College of Electronic Engineering

University of Electronic Science and Technology of China

No.4, Section 2, North Jianshe Road, Chengdu

CHINA

fujiansheng2010@126.com liaokuo@ee.uestc.edu.cn daiyingzhou@163.com wlyang@uestc.edu.cn

Abstract: In an Automatic Target Recognition (ATR) system, target recognition-makers need assistance to determine which class a new High Resolution Range Profile (HRRP) belongs to. Note that the HRRP data can be obtained from an Open Database (ODB) freely, we present a new Multi-Agent System (MAS) model in which specialized intelligent agents, namely Individual Target Analyzing (ITA) agents, are designed to perform recognizing behaviour on behalf of their corresponding target classes, and then show their identity information and claims that Public Recognition Arbitrating (PRA) agent may adopt for HRRP analyzing and judging. In order to describe the details, we apply Generalized Discriminant Analysis (GDA) in the model, and accordingly, two new GDA variations come forth, called Distributed-GDA (D-GDA) and Synthetic-GDA (S-GDA) respectively. Generally, the traditional application of GDA is to emphasize the Common-Discrimination Information (C-DI) among all targets while D-GDA prefers to the Individual-Discrimination Information (I-DI) against other targets one by one, so their syntheses S-GDA can obtain more useful discrimination information than both of them. Experimental results for measured and simulated data show that GDA and D-GDA are complementary in many facets and can be considered as a feature extraction method couple. Furthermore, compared with GDA and D-GDA, the proposed S-GDA not only achieves better and better recognition performance with the number of targets increasing, but also is more robust to many challenges, such as noise disturbance, aspect variation, Small Sample Size (SSS) problem and etc. All these experimental results confirm the effectiveness of the MAS model proposed in this paper.

Key-Words: agent, target recognition, high resolution range profile, generalized discriminant analysis.

1 Introduction

We define an intelligent agent as a software and hardware system which has the smarts to assume responsibility for a specific task [1]. We can look at what a single intelligent agent must be capable of to help a user get her work done. But, just as we have companies where the unique talents of many people are combined to solve problems, we can have multiple intelligent agents working together towards their personal goals. In fact, this is a Multi-Agent System (MAS) [2], [3]. As the new development from artificial intelligence, Multi-Agent Technology (MAT) just appeared at the end of last century, but quickly received intensive attention from many scientific research communities [4]–[6], and has a wide foreground in many fields [7], [8]. Also MAT has been applied in ATR by three main performance forms as that the agent can correspond to either target models [9], [10], or image processing algorithms [11]–[13], and also can be used to explore the image features and desired objects in the image [14]–[16]. A terse introduction on using agent approach for image ana-

lyzing and recognizing was referred to in [17], which enumerates several successful applications worth paying attention to. Generally speaking, the application of MAT becomes more and more prevalent in ATR nevertheless it seldom appeared in radar HRRP target recognition.

A HRRP is the amplitude of the coherent summations of the complex time returns from target scatters in each range resolution cell, which represents the projection of the complex returned echoes from the target scattering centers onto the radar Line Of Sight (LOS) [18]. Among several kinds of wideband radar target signatures, HRRP is a promising signature and more easy to be acquired, but it is highly sensitive to time-shift and target-aspect variation, so how to extract robust and effective feature from the raw signal becomes a key problem. During past years, many researches confirmed that some physical structure signatures in HRRP, such as the even rank central moments [19], the phase [20], the amplitude fluctuation property [21], and especially the amplitude vector [22]–[26], are very helpful to recognition,

and accordingly, a number of statistical methods have been proposed for feature extraction and dimensionality reduction [23]–[29]. Although these variant methods may achieve good recognition performance in real applications sometimes, several challenges still exist, i.e., aspect variation, noise disturbance, Small Sample Size (SSS) problem [25], [28], etc. Furthermore, in terms of discrimination information, the capacity of every target is different, but the prevalent applications of these methods are designed to obtain the Common-Discrimination Information (C-DI) among all targets at the expense of the Individual-Discrimination Information (I-DI) between them [21]–[26], and as a result, they may lose the slight I-DI between similar targets. The definitions of C-DI and I-DI are made in Section 4, where there are some physical analysis and theoretical details about them.

Since C-DI and I-DI can be considered as the two aspects of discrimination information, in order to deal with both of them equally and synthetically, a new MAS model is presented in which two types of agent exist, that is, Individual Target Analyzing (ITA) agent and Public Recognition Arbitrating (PRA) agent. ITA agent performs recognizing behavior on behalf of her corresponding target class, and delivers her identity information and claim privately so that PRA agent may adopt for judging while other ITA agents can't obtain her private information. Also all ITA agents may cooperate for their common goal of sharing and depressing the calculation burden so as to obtain some useful information quickly. PRA agent not only acts as the direct superior of all ITA agents, but also makes some private analysis and estimation for the C-DI of all targets. In the MAS model proposed in this paper, ITA agents only emphasize their I-DI against other ITA agents one by one, while PRA agent regards both I-DI and C-DI. The last arbitration is made by PRA agent synthetically, and declared in public by ODB.

As an intelligent agent has many characters, i.e., autonomy, reactivity, adaptation, and so on [2], [3], therefore, many statistical discriminant analysis algorithms can be applied in this incompact MAS model. In order to show the detailed performance, we apply Generalized Discriminant Analysis (GDA) in the model [25], [29], and accordingly, two new variations of GDA, called Distributed-GDA (D-GDA) and Synthetic-GDA (S-GDA), are brought out. In addition, as one of the simplest and the most attractive pattern classification criterions, 1-Nearest Neighbor (1-NN) rule is used for template matching and image classifying [27], [28].

This paper is organized as follows. In Section 2, some synthetic functions are defined. In Section 3,

an overview of the model is given. In Section 4, we proceed to delve into the mathematical details about the model. In Section 5, we apply the model to a seven agent game corresponding to a seven simulating plane model to evaluate the recognition performances. Finally, some conclusions are made in Section 6.

2 Synthetic Functions and Analysis

Throughout this paper, we assume that the given training HRRP space $\{\mathbf{X}|\mathbf{x}_i, i=1, 2, \dots, M\}$ with M HRRPs, and each HRRP is represented as a n -dimensional vector. Let g be the total number of classes, $m_\xi (\xi=1, 2, \dots, g)$ be the number of the ξ^{th} class HRRPs, \mathbf{m} be the training HRRP number vector, and $\{\mathbf{X}_\xi|\mathbf{x}_{\xi,j}, j=1, 2, \dots, m_\xi\} (\xi=1, 2, \dots, g)$ denote the training HRRP subset of the ξ^{th} class, thus we have $M = \sum_{\xi=1}^g m_\xi$, $\mathbf{m} = [m_1 \ m_2 \ \dots \ m_g]$, and $\mathbf{X} = [\mathbf{X}_1 \ \mathbf{X}_2 \ \dots \ \mathbf{X}_g]$.

When an algorithm needs many complex formulas to demonstrate its detailed processing, usually, a single function can be used to represent it, which is called synthetic function in this paper.

2.1 Synthetic Function of Kernel Calculation

Given a $\theta \times \beta_H$ matrix \mathbf{H} and a $\theta \times \beta_W$ matrix \mathbf{W} , here $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \dots \ \mathbf{h}_{\beta_H}]$ and $\mathbf{W} = [\mathbf{w}_1 \ \mathbf{w}_2 \ \dots \ \mathbf{w}_{\beta_W}]$, the kernel matrix $\mathbf{K}_{\mathbf{H}, \mathbf{W}}$ is defined as

$$\mathbf{K}_{\mathbf{H}, \mathbf{W}} = \begin{bmatrix} k(\mathbf{h}_1, \mathbf{w}_1) & k(\mathbf{h}_1, \mathbf{w}_2) & \dots & k(\mathbf{h}_1, \mathbf{w}_{\beta_W}) \\ k(\mathbf{h}_2, \mathbf{w}_1) & k(\mathbf{h}_2, \mathbf{w}_2) & \dots & k(\mathbf{h}_2, \mathbf{w}_{\beta_W}) \\ \vdots & \vdots & \ddots & \vdots \\ k(\mathbf{h}_{\beta_H}, \mathbf{w}_1) & k(\mathbf{h}_{\beta_H}, \mathbf{w}_2) & \dots & k(\mathbf{h}_{\beta_H}, \mathbf{w}_{\beta_W}) \end{bmatrix} \triangleq \mathbb{k}(\mathbf{H}, \mathbf{W}), \quad (1)$$

where the kernel function $k(\mathbf{h}_i, \mathbf{w}_j)$ is defined by $k(\mathbf{h}_i, \mathbf{w}_j) = \langle \Phi(\mathbf{h}_i), \Phi(\mathbf{w}_j) \rangle$ corresponding to a given nonlinear mapping Φ , $\mathbb{k}(\mathbf{H}, \mathbf{W})$ is the synthetic function to obtain the kernel matrix $\mathbf{K}_{\mathbf{H}, \mathbf{W}}$ of \mathbf{H} by \mathbf{W} , and the symbol \mathbb{k} denotes the established kernel function. In this paper, we apply the Gaussian kernel function for kernel calculating:

$$k(\mathbf{h}_i, \mathbf{w}_j) = \exp\left(-\|\mathbf{h}_i - \mathbf{w}_j\|^2 / \sigma^2\right), \quad (2)$$

where σ^2 is equal to 0.5 in this paper.

2.2 Synthetic Function for GDA

As explicated in [29], the traditional GDA is a nonlinear extension of the classical Linear Discriminant Analysis (LDA) via kernel trick [27]. According to a variant of Fisher's kernel criterion [28], [29], it is aim to solve an optimization problem:

$$J(\mathbf{u}_{opt}) = \arg \max_{\mathbf{u}} \frac{\mathbf{u}^T (\mathbf{Q}\mathbf{W}\mathbf{Q})\mathbf{u}}{\mathbf{u}^T (\mathbf{Q}\mathbf{Q})\mathbf{u}}, \quad (3)$$

where the kernel symmetric matrix \mathbf{Q} is obtained by $\mathbf{Q} = \mathbf{K} - \mathbf{1}_M \mathbf{K} - \mathbf{K} \mathbf{1}_M + \mathbf{1}_M \mathbf{K} \mathbf{1}_M$, and the block diagonal matrix $\mathbf{W} = \text{diag}(\mathbf{1}_{m_1}, \mathbf{1}_{m_2}, \dots, \mathbf{1}_{m_o})$, here the mean value matrix $\mathbf{1}_n$ is defined as a $n \times n$ matrix with terms all equal to $1/n$, and the kernel matrix $\mathbf{K} = \mathbb{k}(\mathbf{X}, \mathbf{X})$.

Let us consider the coefficient vectors \mathbf{u}_i , which are sorted in descending order of their corresponding judgement values $J(\mathbf{u}_i)$. We select the first $g-1$ vectors as the Feature Extraction Subspace (FES) by

$$\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_{g-1}] \triangleq \mathbb{S}_{\text{GDA}}(\mathbf{K}, \mathbf{m}), \quad (4)$$

where the FES \mathbf{U} is a $M \times (g-1)$ matrix, and $\mathbb{S}_{\text{GDA}}(\mathbf{K}, \mathbf{m})$ is defined as the synthetic function for GDA to obtain the FES \mathbf{U} from the training HRRP space \mathbf{X} .

3 Overview of the Model

The agent's structure is shown in Fig. 1, which is explained as follows. The agent's perceptiveness apparatus obtains the original information (Such as HRRPs and Environment Information (EI)) from an ODB, and her communication apparatus can also receive some knowledge (such as Identity Estimating Difference (IED) value and algorithm instruction information) from other agents. All the information is deposited in her knowledge warehouse so that when she makes decisions in her decision model warehouse, she can distill the information from her knowledge warehouse expediently. Also she can deal with the information by the methods deposited in her history experience warehouse and algorithm rule warehouse.

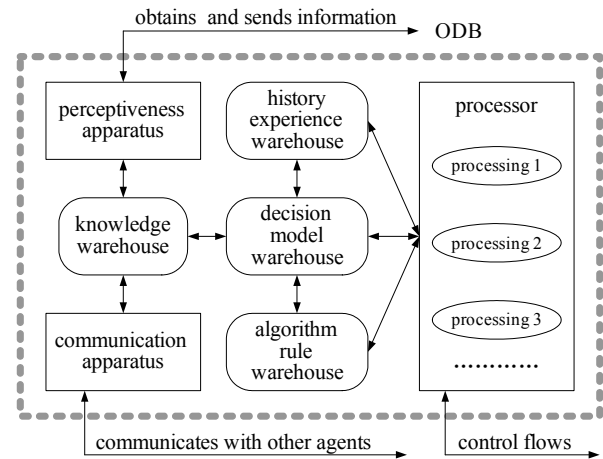


Fig. 1: Sketch map for a single agent's structure

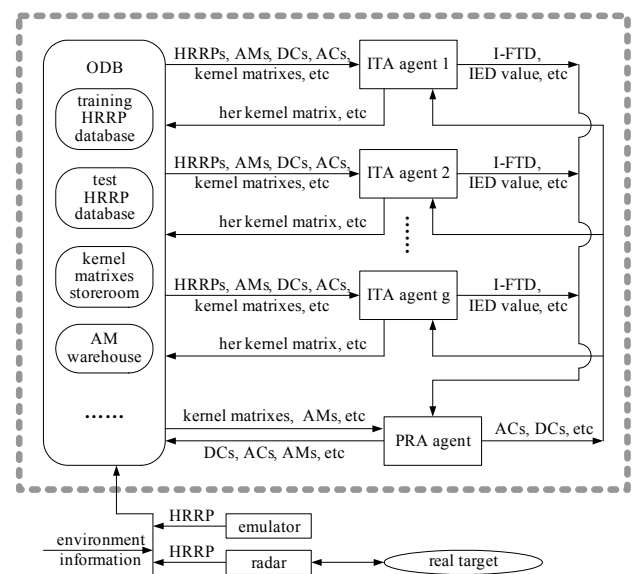


Fig. 2: Sketch map for the model's structure

house handily (such as calculating for IED value). The action is carried out mainly in processor, and perhaps some results are exported bringing some influence on the environment outside or other agents (such as judging and instructing).

For many geographical objects are not only defined by their visual characteristics but also the relationship with other objects, the agents in the model are designed to deal with both objects and algorithms. As shown in Fig. 2, two types of agent are afforded in the model, that is: ITA agent and PRA agent. Some suppositions are made as follows.

✚ Some profit is given to the possessor of a test HRRP. Here the possessor is defined as this ITA agent that the test HRRP belongs to her corresponding target class.

✚ Open Database (ODB) is exoteric to all agen-

ts. Every agent can obtain the information from ODB freely, but only special information can be kept in ODB according to her type.

✚ *ITA agent is self-serving but honest.* She tries to become the possessor of a test HRRP, but no rigged behaviors happen for her purpose.

✚ *PRA agent is self-giving and public.* She deals with the HRRPs and the claims of a test HRRP equitably, but partly believes the information provided by ITA agents.

Each class pattern in the ATR system is provided with an ITA agent. ITA agent can obtain HRRPs and kernel matrixes from ODB handily, extract her Individual-Feature Template Database (I-FTD) from the training HRRPs independently, keep her Feature Extraction Vector Gathering (FEVG) in her history experiment warehouse regularly, and deliver her IED value to PRA agent privately. However, as an individual, ITA agent has some self-serving characters. She never proclaims her private information by ODB so that other competitors can't obtain her secrets. Furthermore, she only performs recognizing behavior on behalf of her corresponding target class. In her opinion, there are only two possibilities about a test HRRP, that is: it belongs to her class or not. In order to become the possessor of a test HRRP, she may try her best to utilize all the methods which are kept in her algorithm rule warehouse or ODB Algorithm Model (AM) warehouse.

Although every ITA agent wants to become the possessor of a test HRRP, there is only one possessor according to a HRRP, therefore, the collision may come forth, which is solved solely by PRA agent. PRA agent acts as the direct superior of ITA agents, so the functions and powers in their hands are very different. In order to standardize the information obtained from ITA agents, PRA agent promulgates a series of commands to restrict ITA agents' behaviors. Usually, these commands are designed to instruct ITA agent for algorithm selecting and data normalizing, called Algorithm Commands (ACs) and Data Commands (DCs) respectively. Even though all ITA agents submit themselves to these commands, however, PRA agent still trusts ITA agents partly, and some estimation must be made by herself in private. After analyzing ITA agents' claims and her own estimation synthetically, PRA agent makes the final judgement in public, that is: declares its possessor.

Apparently, algorithm selection is of the most important in the model. It is nature for an intelligent agent to choice the methods which may be the most comfortable for her purpose due to her characters of autonomy, reactivity, adaptation, etc [2], [3]. If all ITA agents haven't a uniform algorithm criterion or each ITA agent has different algorithms for her I-

FTD and IED value, it will be very hard or even impossible for PRA agent to compare and analyze the information from ITA agents. Perhaps ITA agent may find some problems in ITA agent's strategies by utilizing other methods in private, and then she can communicate with PRA agent discussing and solving these problems, but the recognition information provided by her must accord with PRA agent's criterion.

4 Details of the Model

There are g ITA agents but only one PRA agent in the model. ITA agent acts as a selfish role while PRA agent as an arbitral role. Here we choose the α^{th} ($1 \leq \alpha \leq g$) ITA agent to describe ITA agent's processing, and the only one PRA agent is called PRA agent Θ . In order to analyze the model concretely, we suppose that PRA agent Θ utilizes GDA for feature extracting and 1-NN rule for template matching, and the corresponding ACs and DCs are sent to all ITA agents. Now we are concerned with one main challenge: what is the suitable information that autonomous agents can offer in target HRRP recognition?

4.1 Details of ITA Agent α

As described above, the primary intention that ITA agent α takes part in the recognition game is to become the possessor of a test HRRP, so she is mainly concerned about how she obtains her own I-FTD, FEVG, IED value, etc. She seems to arrive at the purpose of optimizing and maximizing her identity information by applying GDA against other ITA agents one by one. When a test HRRP appears in ODB, she begins to estimate its IED value which can be considered as her claim of possessing it. The total process is described in two main phases, and the data stream is showed in Fig. 3.

4.1.1 Training Phase

Apparently, before recognizing a test HRRP, ITA agent α needs make some discriminant analysis by herself. Although the training phase can be considered as the recognition preparation process, therefore, the operating time can be allowed at some degree subject to the practical demand, but a fact worth pointing out is that the training phase is vital and even crucial in pattern recognition. When obtaining the original data from ODB and accepting the commands from PRA agent, ITA agent α keeps her personal HRRPs \mathbf{X}_α in her history experience warehouse,

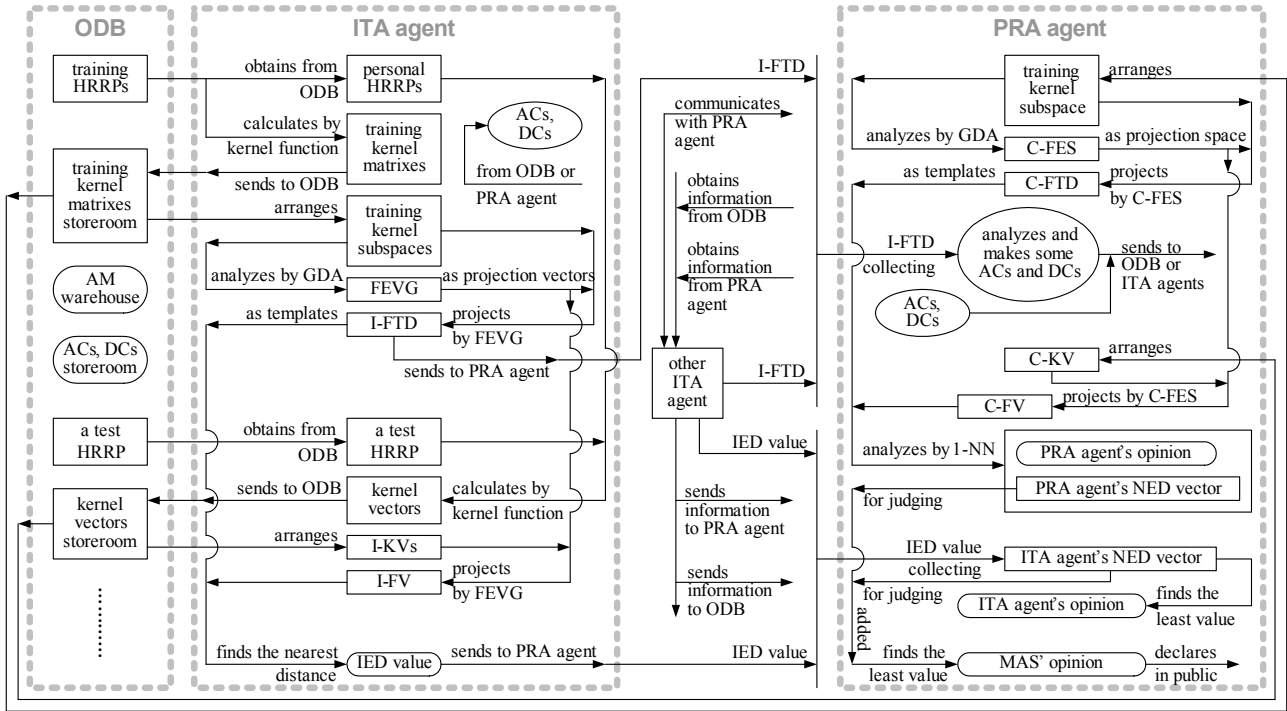


Fig. 3: Sketch map for the model's data stream

and cooperates with other ITA agents for calculating of the training kernel matrixes by

$$\begin{cases} \mathbf{K}_{\alpha,\xi} = \mathbb{k}(\mathbf{X}_\alpha, \mathbf{X}_\xi) \quad (\xi = 1, 2, \dots, g) \\ \mathbf{K}_\alpha = [\mathbf{K}_{\alpha,1} \quad \mathbf{K}_{\alpha,2} \quad \dots \quad \mathbf{K}_{\alpha,g}] \end{cases}, \quad (5)$$

where $\mathbf{K}_{\alpha,\xi}$ ($\xi = 1, 2, \dots, g$) is the kernel matrix calculated by ITA agent α , and \mathbf{K}_α denotes the total kernel matrix which is sent to ODB so that other ITA agents can obtain it. Also ITA agent α obtains other ITA agents' kernel matrixes \mathbf{K}_ξ ($\xi = 1, 2, \dots, g$) from ODB, and then some arrangement preparations are made by her as

$$\mathbf{Y}_{\alpha,\xi} = \begin{bmatrix} \mathbf{K}_{\alpha,\alpha} & \mathbf{K}_{\alpha,\xi} \\ \mathbf{K}_{\xi,\alpha} & \mathbf{K}_{\xi,\xi} \end{bmatrix} \quad \left(\begin{array}{l} \xi = 1, 2, \dots, g \\ \text{and } \xi \neq \alpha \end{array} \right), \quad (6)$$

where $\mathbf{Y}_{\alpha,\xi}$ is a $m_\alpha \times (m_\alpha + m_\xi)$ matrix, which denotes her training kernel subspace. We define $\mathbf{m}_{\alpha,\xi}$ as the corresponding training HRRP number vector, which is obtained by $\mathbf{m}_{\alpha,\xi} = [m_\alpha \quad m_\xi]$. Then she obtains her FEVG $\{\mathbf{u}_{\alpha,\xi}\}$ by

$$\mathbf{u}_{\alpha,\xi} = \mathbb{S}_{\text{GDA}}(\mathbf{Y}_{\alpha,\xi}, \mathbf{m}_{\alpha,\xi}) \quad \left(\begin{array}{l} \xi = 1, 2, \dots, g \\ \text{and } \xi \neq \alpha \end{array} \right). \quad (7)$$

Once ITA agent α obtains her FEVG¹, she calculates her I-FTD \mathbf{A}_α by

$$\begin{cases} \mathbf{b}_{\alpha,\xi} = [\mathbf{K}_{\alpha,\alpha} \quad \mathbf{K}_{\alpha,\xi}] \mathbf{u}_{\alpha,\xi} \\ \mathbf{A}_\alpha = [\mathbf{b}_{\alpha,1} \quad \mathbf{b}_{\alpha,2} \quad \dots \quad \mathbf{b}_{\alpha,g}] \\ \triangleq [\mathbf{a}_{\alpha,1} \quad \mathbf{a}_{\alpha,2} \quad \dots \quad \mathbf{a}_{\alpha,m_\alpha}]^T \end{cases} \quad \left(\begin{array}{l} \xi = 1, 2, \dots, g \\ \text{and } \xi \neq \alpha \end{array} \right), \quad (8)$$

where $\mathbf{b}_{\alpha,\xi}$ is a m_α -dimensional vector, $\mathbf{a}_{\alpha,j}$ is a $(g-1)$ -dimensional vector, and \mathbf{A}_α is a $m_\alpha \times (g-1)$ matrix which is sent to PRA agent Θ as her identity information. Obviously, each vector $\mathbf{a}_{\alpha,j}$ ($j = 1, 2, \dots, m_\alpha$) of \mathbf{A}_α can be considered as a constrigent image of a training HRRP, which is used as ITA agent α 's feature template in the

¹ The difference between FEVG and FES is that: FEVG is only a gathering in which each vector may have different dimensions while FES can be considered as a subspace in which all vectors have the same dimensions. For example, as the element of ITA agent α 's FEVG, $\mathbf{u}_{\alpha,\xi}$ ($\xi \neq \alpha$) is a $(m_\alpha + m_\xi)$ -dimensional vector.

upcoming recognition about a test HRRP.

4.1.2 Test Phase

In this phase, sometimes all ITA agents work together for their common goals, such as cooperating for the kernel vectors of a test HRRP, while in most time, they work privately and solely. For example, when a test HRRP \mathbf{e} appears in ODB, ITA agent α obtains it and begins to evaluate its IED value $\mu_{\alpha,e}$ in three steps as follows.

Step 1: Cooperating for kernel vectors

As shown in Fig. 3, in this step, all ITA agents cooperate as an alliance for \mathbf{e} 's kernel vectors by each ITA agent for one vector. ITA agent α obtains its kernel vector $\mathbf{k}_{\alpha,e}$ as that

$$\mathbf{k}_{\alpha,e} = \mathbb{k}(\mathbf{e}, \mathbf{X}_\alpha), \quad (9)$$

where $\mathbf{k}_{\alpha,e}$ is a m_α -dimensional vector, and denotes \mathbf{e} 's kernel vector calculated by ITA agent α . Then ITA agent α sends $\mathbf{k}_{\alpha,e}$ to ODB and obtains \mathbf{e} 's kernel vectors $\{\mathbf{k}_{\xi,e}, \xi = 1, 2, \dots, g\}$ from ODB synchronously. Now she obtains \mathbf{e} 's Individual-Kernel Vectors (I-KVs) $\mathbf{k}_{\alpha,\xi,e}$ by

$$\mathbf{k}_{\alpha,\xi,e} = \begin{bmatrix} \mathbf{k}_{\alpha,e} & \mathbf{k}_{\xi,e} \end{bmatrix} \begin{matrix} (\xi = 1, 2, \dots, g) \\ \text{and } \xi \neq \alpha \end{matrix}, \quad (10)$$

where $\mathbf{k}_{\alpha,\xi,e}$ can be considered as a permutation of \mathbf{e} 's kernel vectors, which is used for \mathbf{e} 's Individual-Feature Vector (I-FV) in the next step.

Step 2: Calculating of I-FV

According to (8), ITA agent α obtains \mathbf{e} 's I-FV $\mathbf{t}_{\alpha,e}$ by

$$\begin{cases} t_{\alpha,\xi,e} = \mathbf{k}_{\alpha,\xi,e} \mathbf{u}_{\alpha,\xi} \\ \mathbf{t}_{\alpha,e} = [t_{\alpha,1,e} \ t_{\alpha,2,e} \ \dots \ t_{\alpha,g,e}] \end{cases} \begin{matrix} (\xi = 1, 2, \dots, g) \\ \text{and } \xi \neq \alpha \end{matrix}, \quad (11)$$

where $\mathbf{t}_{\alpha,e}$ is a $(g-1)$ -dimensional vector, which can be considered as \mathbf{e} 's constrigent image estimated by ITA agent α and treated as an identification profile by PRA agent Θ .

Step 3: Calculating of IED value

According to PRA agent Θ 's ACs and DCs, ITA agent α evaluates the IED value $\mu_{\alpha,e}$ by

$$\mu_{\alpha,e} = \min_{j=1,2,\dots,m_\alpha} \|\mathbf{t}_{\alpha,e} - \mathbf{a}_{\alpha,j}\|, \quad (12)$$

where the IED value $\mu_{\alpha,e}$ can be considered as the possession claim of the test HRRP \mathbf{e} from ITA agent α , which is sent to PRA agent Θ for arbitration.

4.2 Details of PRA Agent Θ

The primary intention that PRA agent Θ takes part in the recognition game is to find the real possessor of a test HRRP as possible as she can. In order to improve the correct recognition rate, she not only makes her own analysis in private, but also analyzes the claims from ITA agents. The data stream is shown in Fig. 3, and the process is described in two main phases also.

4.2.1 Training Phase

As similar as ITA agent α does, before recognizing on a test HRRP, PRA agent Θ also needs some discriminant analysis. Her preparation for recognition can be described in two parts as follows.

Part 1: Analysis of Training HRRPs

This part can be considered as the traditional recognition process [27]–[29]. PRA agent Θ obtains the training kernel matrixes $\{\mathbf{K}_\xi, \xi = 1, 2, \dots, g\}$ from ODB, which are arranged by

$$\mathbf{K}_\Theta = [\mathbf{K}_1^T \ \mathbf{K}_2^T \ \dots \ \mathbf{K}_g^T]^T, \quad (13)$$

where \mathbf{K}_Θ is \mathbf{X} 's kernel matrix subspace, which is used to calculate the Common-FES (C-FES) \mathbf{U}_Θ and the Common-FTD (C-FTD) \mathbf{A}_Θ by

$$\begin{cases} \mathbf{U}_\Theta = \mathbb{S}_{\text{GDA}}(\mathbf{K}_\Theta, \mathbf{m}) \\ \mathbf{A}_{\Theta,\xi} = \mathbf{K}_\xi \mathbf{U}_\Theta \\ \mathbf{A}_\Theta = \mathbf{K}_\Theta \mathbf{U}_\Theta \end{cases} (\xi = 1, 2, \dots, g), \quad (14)$$

where $\{\mathbf{A}_{\Theta,\xi} | \mathbf{a}_{\Theta,\xi,j}, j = 1, 2, \dots, m_\xi\}$ denotes the ξ^{th} class' FTD estimated by PRA agent Θ .

Part 2: Supervision of ITA Agents

It is necessary for PRA agent Θ to supervise ITA agents' behaviors by all kind of ACs and DCs. As the ACs, GDA has been sent to all ITA agents before, and then PRA agent Θ receives the I-FTDs $\mathbf{A}_\xi (\xi = 1, 2, \dots, g)$ from all ITA agents as their identity information. In some sense, the I-FTDs can be

considered as HRRP's constringent profiles, so many discrimination methods, such as LDA and GDA, which are usually used in radar HRRP recognition, here can be used to analysis the identity information similarly. PRA agent Θ analyzes the I-FTDs in private so that she can obtain some personal discrimination information among all ITA agents, and then sets down some private ACs and DCs corresponding to each ITA agent respectively. In this paper, in order to compare with GDA, we suppose that PRA agent Θ discriminates them by 1-NN directly, and the corresponding ACs and DCs are sent to all ITA agents.

4.2.2 Test Phase

When a test HRRP \mathbf{e} appears in ODB, PRA agent Θ makes her personal evaluation rapidly, receives the claims from all ITA agents quickly, and soon declares the final arbitration in public. Therefore, there are three opinions about its possessor, which are described respectively in three parts as follows.

Part 1: Opinion of PRA Agent

For a test HRRP \mathbf{e} , according to the traditional GDA, its Common-Kernel Vector (C-KV) $\mathbf{k}_{\Theta, \mathbf{e}}$ is calculated by $\mathbf{k}_{\Theta, \mathbf{e}} = \mathbb{k}(\mathbf{e}, \mathbf{X})$. Here PRA agent Θ can obtain \mathbf{e} 's C-KV $\mathbf{k}_{\Theta, \mathbf{e}}$ from ODB as that

$$\mathbf{k}_{\Theta, \mathbf{e}} = [\mathbf{k}_{1, \mathbf{e}} \quad \mathbf{k}_{2, \mathbf{e}} \quad \cdots \quad \mathbf{k}_{g, \mathbf{e}}], \quad (15)$$

where $\mathbf{k}_{\xi, \mathbf{e}} (\xi = 1, 2, \dots, g)$ denotes \mathbf{e} 's ξ^{th} kernel vector calculated by ITA agent ξ as demonstrated by (9). Then \mathbf{e} 's Nearest Euclidean Distance (NED) vector $\mathbf{d}_{\Theta, \mathbf{e}}$ and attributive class $c_{\Theta, \mathbf{e}}$ are given by

$$\begin{cases} \mathbf{a}_{\Theta, \mathbf{e}} = \mathbf{k}_{\Theta, \mathbf{e}} \mathbf{U}_{\Theta} \\ d_{\Theta, \mathbf{e}, \xi} = \min_{j=1, 2, \dots, m_{\xi}} \|\mathbf{a}_{\Theta, \mathbf{e}} - \mathbf{a}_{\Theta, \xi, j}\| \\ \mathbf{d}_{\Theta, \mathbf{e}} = [d_{\Theta, \mathbf{e}, 1} \quad d_{\Theta, \mathbf{e}, 2} \quad \cdots \quad d_{\Theta, \mathbf{e}, g}] \\ c_{\Theta, \mathbf{e}} = \arg \min_{c=1, 2, \dots, g} d_{\Theta, \mathbf{e}, c} \end{cases} \quad (\xi = 1, 2, \dots, g), \quad (16)$$

where $\mathbf{a}_{\Theta, \mathbf{e}}$ is the Common-Feature Vector (C-FV) estimated by PRA agent Θ , and $c_{\Theta, \mathbf{e}}$ is PRA agent Θ 's personal opinion about the test HRRP \mathbf{e} , which denotes that \mathbf{e} belongs to ITA agent $c_{\Theta, \mathbf{e}}$.

Part 2: Opinion of ITA Agents

Also PRA agent Θ receives the personal claims from all ITA agents. She arbitrates these claims

$\mu_{\xi, \mathbf{e}} (\xi = 1, 2, \dots, g)$ by 1-NN rule to achieve the general opinion from all ITA agents. The attributive class $c_{\mu, \mathbf{e}}$ and the NED vector $\mathbf{d}_{\mu, \mathbf{e}}$ are estimated by

$$\begin{cases} c_{\mu, \mathbf{e}} = \arg \min_{c=1, 2, \dots, g} \mu_{c, \mathbf{e}} \\ \mathbf{d}_{\mu, \mathbf{e}} = [\mu_{1, \mathbf{e}} \quad \mu_{2, \mathbf{e}} \quad \cdots \quad \mu_{g, \mathbf{e}}] \end{cases}, \quad (17)$$

where $c_{\mu, \mathbf{e}}$ is the opinion of all ITA agents, which is made by PRA agent Θ in private, and denotes that the test HRRP \mathbf{e} belongs to ITA agent $c_{\mu, \mathbf{e}}$.

Part 3: Opinion of MAS

According to the description above, there are two opinions about a test HRRP, that is: PRA agent's personal opinion and ITA agents' claims. It is natural that PRA agent Θ not only regards her opinion but also the claims from all ITA agents. In order to deal with the two opinions equitably, a middle method is proposed to synthesize the last arbitration by

$$c_{s, \mathbf{e}} = \arg \min_{\xi=1, 2, \dots, g} (\mu_{\xi, \mathbf{e}} + d_{\Theta, \mathbf{e}, \xi}), \quad (18)$$

where $c_{s, \mathbf{e}}$ denotes that the test HRRP \mathbf{e} belongs to ITA agent $c_{s, \mathbf{e}}$, which is the last arbitration about \mathbf{e} 's possessor, and is considered as the opinion of MAS. Once PRA agent Θ obtains the synthetic opinion $c_{s, \mathbf{e}}$, she declares it by ODB, and then the recognition process of the test HRRP \mathbf{e} is over.

4.3 Algorithm Analysis

Let's analyze ITA agent α 's identity information \mathbf{A}_{α} , which is obtained by (8). Note that each element of matrix \mathbf{A}_{α} can be considered as the discrimination information against one of the other ITA agents while not against all of the others, we defined this kind of discrimination information as I-DI. Obviously, I-DI shows the individual differentiation of ITA agents, and each element denotes the optimal discrimination information between two ITA agents. However, compared with I-DI, the C-FTD \mathbf{A}_{Θ} , which is obtained by (14), can be considered as C-DI for each element of \mathbf{A}_{Θ} denotes the optimal discrimination information of one target against all the other targets, and accordingly, C-DI shows the common differentiation among all targets.

Mathematically, as illustrated by (14), (16), PRA agent's private opinion about a test HRRP is made by GDA and 1-NN rule, which is called GDA recognition process, while ITA agents' opinion is mainly

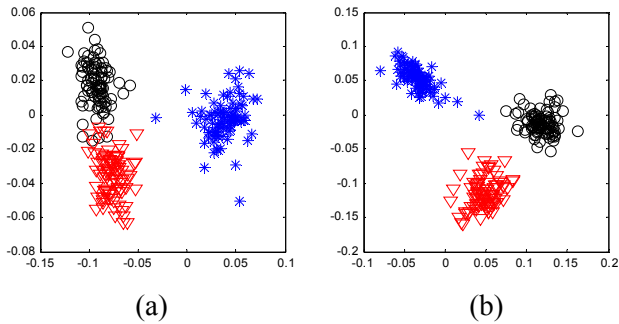


Fig. 4: Distribution of 270 test samples of three objects. (a) GDA-based subspace. (b) D-GDA-based subspace. (*: Cessna Citation, ∇ : Yark-42, \circ : An-26)

made by D-GDA and 1-NN rule in which GDA is distributed by ITA agents personally, so we define the whole ITA agents' processes as D-GDA recognition process. The total process of the model is made by the two recognition processes synthetically, which is called Synthesis GDA (S-GDA) recognition process in this paper. Although GDA and D-GDA have the same statistic thought of optimizing the kernel between-class scatter matrix from the kernel within-class scatter matrix as shown by (3), physically, GDA and D-GDA can be considered as the two aspects of the discriminant analysis, that is, GDA emphasizes the C-DI among all targets while D-GDA prefers to the I-DI between two targets. So GDA and D-GDA are complementary in some sense and can be considered as a feature extraction method couple. Since they have different emphasis aspects for feature extraction, they may have different recognition performances. Perhaps in some facets GDA may achieve some better performances than D-GDA does while in other facets D-GDA may achieve better performances than GDA does. If we synthesize the two analysis methods just as PRA agent Θ does above, we may achieve a better and more robust performance than both of them, so S-GDA comes forth.

In order to verify the different performances, as shown in Fig. 4, we compare the projections of test samples extracted by GDA and D-GDA respectively. The original HRRP data, including the 4th segment of An-26, the 4th segment of Cessna and the 2nd segment of Yark, is based on three real airplanes with each airplane 180 HRRPs. Each data set is divided into two equal subsets randomly. One is used as the training set and another is used as the test set. The data's detailed description can be referred to in [19]–[25]. From Fig. 4, we can see the different performances between GDA and D-GDA clearly.

Table 1: Parameters of planes and radar in the simulated experiments

radar parameters	center frequency		5520 MHz
	bandwidth		400 MHz
	sampling frequency		800 MHz
	PRF		1000 Hz
planes	length (m)	width (m)	scale
B-52	49.50	56.40	1:1
B-1B	44.80	23.80	1:1
Tu-16	33.80	33.00	1:1
Tornado	16.72	13.91	1:1
Mig-21	15.76	7.15	1:1
F-15	19.43	13.05	1:1
An-26	23.80	29.21	1:1

5 Experiments and Analysis

We simulate radar backscattering data of seven airplanes by a program [30], [31], and the parameters of targets and radar are shown in Table 1. As these seven airplanes are all symmetrical in horizontal, we only simulate azimuth $0^\circ \sim 180^\circ$ at interval 0.25° , and elevation angle 0° . Several experiments are conducted on these simulating aerial target datasets to show the model's effectiveness. As described above, we use ITA agent to perform recognizing behavior on behalf of her corresponding target class, therefore, there are seven ITA agents according to the seven airplanes. The recognition performance is evaluated by one computer as follows.

5.1 Experiment on Target Quantity

As shown in Fig. 5, this experiment is designed to obtain the average recognition performance of C_g^ξ , here C_g^ξ denotes the total possible combinations of ξ members from g members. For example, when we select 3 airplanes out of 7 airplanes, the number of the total possible combinations C_7^3 is 35. In this trial, each ITA agent has 360 HRRPs with elevation angle 0° and azimuth $0^\circ \sim 180^\circ$ at interval 0.25° . Each ITA agent's HRRPs are considered as a data set and divided into two equal subsets at azimuth interval 0.5° . One is used as the training set and another as the test set.

Fig. 5(a) shows the training time variation according to the number of ITA agents, which denotes that the training time of GDA is higher than D-GDA's. Furthermore, the difference between their training times becomes more and more obvious with the number of ITA agent increasing. If we analysis their space complexity [32], we may find that GDA needs much more EMS memory than D-GDA does.

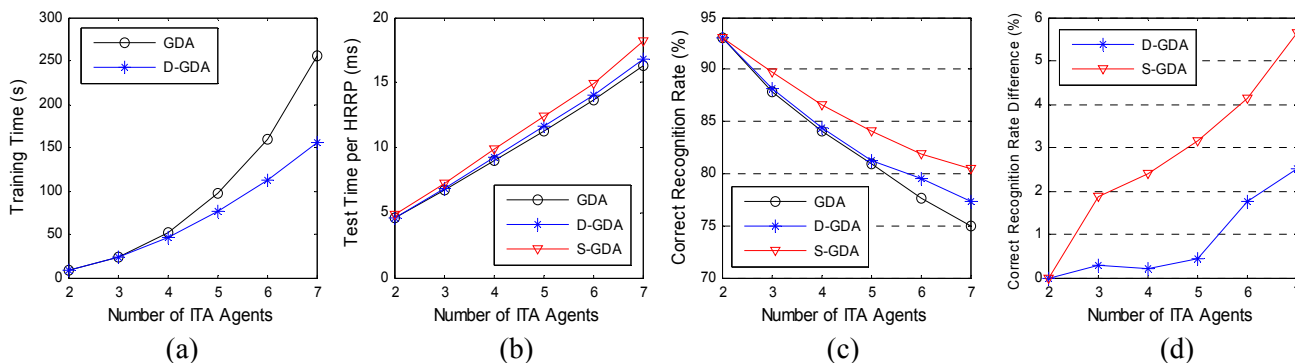


Fig. 5: Recognition performance versus number of ITA agents. (a) Training time versus number of ITA agents. (b) Test time per HRRP versus number of ITA agents. (c) Correct recognition rate versus number of ITA agents. (d) Correct recognition rate difference versus number of ITA agents.

Obviously, EMS memory is very important in radar HRRP recognition due to the huge storage requirement and computation burden which may lead to the program error ‘out of memory’. Let’s consider the test phases of the three recognition processes. Compared with GDA, D-GDA test phase has a same kernel calculation process, a same computation complexity of projection process, and a very similar template matching process which costs almost the same time as 1-NN does, so their recognition speeds is very similar as shown in Fig. 5(b). From Fig. 5(b), we also find that S-GDA’s recognition speed is acceptable comparing with GDA, which only slows 4~10 percent points than GDA does. Note that D-GDA process is just operated in one computer, if each ITA agent is afforded with one computer, without saying, the recognizing speed will advance sharply.

Let’s compare the correct recognition rates of the three discrimination processes. As shown in Fig. 5(c), the correct recognition rate of S-GDA is higher than D-GDA’s, and D-GDA’s is higher than GDA’s. Furthermore, with the number of ITA agents increasing, the difference of correct recognition rate between S-GDA and D-GDA, and the difference between D-GDA and GDA becomes more and more obvious as shown in Fig. 5(d). As described above, D-GDA emphases I-DI between targets while GDA prefers to C-DI among all targets, when the number of ITA agents is increasing, obviously, the C-DI among all ITA agents reduces more quickly than the I-DI between them, so D-GDA can perform a better recognition than GDA does. Also we can find that S-GDA obtains the best recognition performance, which confirms that GDA and D-GDA are complementary as a feature extraction method couple. As the synthesis of GDA and D-GDA, S-GDA can obtain more useful discrimination information than both of them, so it can achieve a better recognition.

When the number of ITA agent is 7, the correct recognition rate of S-GDA is still optimistic, about 80.6%, while GDA’s is only about 74.8% and D-GDA’s is about 77.4%. The recognition rate difference between S-GDA and GDA is near to 5.8%, and the difference between S-GDA and D-GDA is about 2.6%.

5.2 Experiment on Aspect Variation

Aspect variation is one of the main challenges in radar HRRP recognition, which change the distances between target scatters and radar receiver more or less, and as a result, the HRRP changes accordingly. When the distance change exceeds the range resolution cell, Range Cell Migration (RCM) appears, and the HRRP may change acutely or even can’t be recognized by the old FTG. Even though the distance fluctuation is within the range resolution cell, the change still can’t be overlooked. Furthermore, even there is no aspect flickering, the recognition performance still varies according to different azimuth sectors. Perhaps in some azimuth sectors the targets keep stable geometry shapes onto the radar LOS, and the corresponding HRRPs change little between two neighbor sampling points, so the recognition may achieve a good performance. But in other azimuth sectors, the shapes of targets onto the radar LOS may change sharply, and accordingly, the HRRPs may change acutely, so the recognition may achieve a bad performance.

As shown in Fig. 6(a), we test the seven airplanes’ recognition performances in 12 azimuth sectors. The sector serial number varies from 1 to 12, representing the azimuth sector 0~15°, 15~30°, 30~45°, 45~60°, 60~75°, 75~90°, 90~105°, 105~120°, 120~135°, 135~150°, 135~150°, 135~150°, 150~165° and 165~180° respectively. In each sector, the elev-

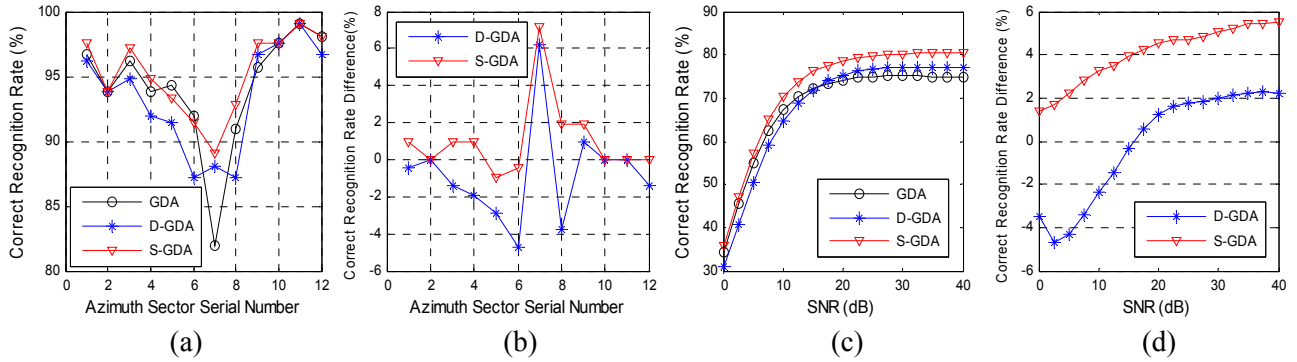


Fig. 6: Correct recognition rate versus azimuth sectors or SNR. (a) Correct recognition rate versus azimuth sector serial number. (b) Correct recognition rate difference versus azimuth sector serial number. (c) Correct recognition rate versus SNR. (d) Correct recognition rate difference versus SNR.

ation angle is settled at 0° , and each ITA agent's HRRPs are considered as a data set and divided into two equal subsets at azimuth interval 0.5° . One is used as the training set and another as the test set. Compared with GDA, the corresponding recognition rate differences are shown in Fig. 6(b). Firstly, we can see that the three recognition rate curves all fluctuate very much due to the high aspect sensitivity of HRRP, while S-GDA are more stable and robust than the others. Secondly, in terms of recognition performance, S-GDA apparently outperforms the other two methods, which reconfirms that GDA and D-GDA can be considered as a feature extraction method couple. Thirdly, as many statistical discriminant analysis algorithms suffered from the so-called SSS problem when the number of the samples is much smaller than the dimension of the sample space [25], [28], the three methods also suffer from this SSS problem, but S-GDA is apparently superior to the others, and GDA suffer most from it.

5.3 Experiment on Noise Disturbance

In some sense, HRRP can be considered as a function of target scatters, target distance, radar antenna gain, radar receiver gain, meteorology, etc, so the sources of noise are complex and difficult to analyze. Therefore, it is very important to build a robust and stable feature extraction method to noises. In this trial, we compare the average recognition rates of the seven airplanes under different SNR with azimuth varying from 0° to 180° at interval 0.25° . Each ITA agent's HRRPs are considered as a data set and divided into two equal subsets at azimuth interval 0.5° . One is used as the training set and another as the test set. As shown in Fig. 6(c), each curve denotes the average correct recognition rate with SNR varying from 0 to 40 dB at interval 2.5 dB, and each SNR

sampling point repeats 100. Compared with GDA, the corresponding recognition rate differences are shown in Fig. 6(d). Firstly, we can find that the average recognition rates of three methods are all improving with the SNR increasing. Secondly, S-GDA is more robust to noises than the others while keeping competitive performance with the other two methods almost in all SNR range, which reconfirms that GDA and D-GDA are complementary in some sense. Thirdly, compared with GDA, D-GDA performs a better recognition when $\text{SNR} > 15\text{dB}$ while a worse recognition when $\text{SNR} < 15\text{dB}$, which indicates that GDA performs better than D-GDA does in noise disturbance.

6 Conclusion

In this paper, we propose a new MAS model for radar HRRP recognition. In order to verify the effectiveness, we apply GDA in this model, and accordingly, two new variations of GDA come forth, namely D-GDA and S-GDA respectively. As described above, the traditional GDA emphasizes the C-DI among all targets while D-GDA prefers to the I-DI between all targets, so they are complementary and can be considered as a feature extraction method couple, which is confirmed by the experiments in Section 5. Since GDA and D-GDA are complementary in many facets, their syntheses S-GDA can obviously obtain more useful discrimination information, and as a result, it may perform better recognition than both of them. Experimental results based on measured and simulated data confirmed this conclusion. In the experiments, compared with GDA, D-GDA can keep competitive recognition performance and solve the SSS problem better while keeping more effective in data computation and storage. Furthermore, S-GDA apparently outperforms GDA and D-

GDA in almost all facets while keeping acceptable recognition speed. As application examples, these experimental results also reveal the high effectiveness of the MAS model proposed in this paper.

It is worth pointing out that MAT is very widely applied in many fields nevertheless it seldom appears in radar HRRP recognition. In this paper, we apply MAT in radar HRRP recognition and obtain an obvious effect. In order to obtain a more effective MAS model, future work will focus on the further study of agent and MAS' structures, which can adaptively represent different targets based on their different characters. Also we can apply other discrimination algorithms in this incompact MAS model to reconfirm its effectiveness.

References:

- [1] R. H. Bordini, M. Dastani, J. Dix, and A. E. F. Seghrouchni, *Multi-Agent Programming: Languages, Tools and Applications*, Springer, 2009.
- [2] M. d'Inverno, M. Luck, M. Fisher, and C. Preist, *Foundations and Applications of Multi-Agent Systems*, Springer, 2002.
- [3] J. Shamma, *Cooperative Control of Distributed Multi-Agent Systems*, Baker & Taylor Books, 2008.
- [4] N. Maudet, S. Parsons, and I. Rahwan, *Argumentation in Multi-Agent Systems*, Springer, 2007.
- [5] I. Rahwan, S. Parsons, and C. Reed, *Argumentation in Multi-Agent Systems*, Springer, 2008.
- [6] I. Rahwan, and P. Moraitis, *Argumentation in Multi-Agent Systems*, Springer, 2009.
- [7] Y. Demazeau, J. Pavón, J. M. Corchado, and J. Bajo, *7th International Conference on Practical Applications of Agents and Multi-Agent Systems (PAAMS 2009)*, Springer, 2009.
- [8] A. Håkansson, N. T. Nguyen, R. L. Hartung, R. J. Howlett, and L. C. Jain, *Agent and Multi-Agent Systems: Technologies and Applications*, Springer, 2009.
- [9] K. Yanai, An Image Understanding System for Various Images Based on Multi-Agent Architecture, *Proc. 3rd Int. Conf. Computational Intelligence and Multimedia Applications*, 1999, pp. 186-190.
- [10] U. Biiker, Cooperative Agents for Object Recognition, *Proc. 15th Int. Conf. Pattern Recognition*, Vol.4, 2000, pp. 157-160.
- [11] J.-M. Liu, Y.-Y. Tang, and Y.-C. Cao, An Evolutionary Autonomous Agents Approach to Image Feature Extraction, *IEEE Trans. Evolutionary Computation*, Vol.1, 1997, pp. 141-158.
- [12] A. Filippidis, L. C. Jain, and N. Martin, Fusion of Intelligent Agents for the Detection of Aircraft in SAR Images, *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol.22, 2000, pp. 378-384.
- [13] Q.-D. Du, and J. Liu, A New Neural Fusion Recognition Method With Multi-Agent, *Proc. 3rd Int. Conf. Intelligent Information Hiding and Multimedia Signal Processing*, Vol.1, 2007, pp. 127-130.
- [14] A. Choksuriwong, C. Rosenberger, and W. W. Smari, Multi-Agents System for Image Understanding, *Proc. Int. Conf. Integration of Knowledge Intensive Multi-Agent Systems*, 2005, pp. 149-154.
- [15] M. Simon, S. O'Hara, and P. Petrov, Statistical Fusion of Unmanned Aerial Vehicle Observations for Aided Target Recognition, *Proc. Int. Conf. Integration of Knowledge Intensive Multi-Agent Systems*, 2007, pp. 180-185.
- [16] P. Dasgupta, A Multiagent Swarming System for Distributed Automatic Target Recognition Using Unmanned Aerial Vehicles, *IEEE Trans. Systems, Man, and Cybernetics, Part A: Systems and Humans*, Vol.38, No.3, 2008, pp. 549-563.
- [17] H. S. Miu, K. S. Leung, and Y. Leung, An Evolutionary Multi-agent System for Object Recognition in Satellite Images, *Proc. Evolutionary Computation*, Vol.1, 2003, pp. 520-527.
- [18] L. Du, H.-W. Liu, Z. Bao, and M.-D. Xing, Radar HRRP Target Recognition Based on Higher Order Spectra, *IEEE Trans. Signal Processing*, Vol.53, No.7, 2005, pp. 2359-2368.
- [19] X.-K. Liu, M.-G. Gao, and X.-J. Fu, Application of HRRP Even Rank Central Moments Features in Satellite Target Recognition, *Proc. IET Int. Conf. Radar Systems*, 2007, pp. 1-4.
- [20] L. Du, H.-W. Liu, Z. Bao, and J.-Y. Zhang, Radar Automatic Target Recognition Using Complex High-Resolution Range Profiles, *IET Radar Sonar Navig.*, Vol.1, No.1, 2007, pp. 18-26.
- [21] L. Du, H.-W. Liu, and Z. Bao, Using the Amplitude Fluctuation Property of Target HRRP for Radar Automatic Target Recognition, *Proc. Int. Conf. Radar, CIE'06*, 2006, pp. 1-5.
- [22] B. Chen, H.-W. Liu, J. Chai, and Z. Bao, Large Margin Feature Weighting Method via Linear Programming, *IEEE Trans. Knowledge and Data Engineering*, Vol.21, No.10, 2009, pp. 1475-1488.
- [23] B. Chen, H.-W. Liu, and Z. Bao, PCA and Kernel PCA for Radar High Range Resolution Profiles Recognition, *Proc. IEEE Int. Conf. Radar*, 2005, pp. 528-533.

- [24] L. Du, H.-W. Liu, Z. Bao, J.-Y. Zhang, A Two-Distribution Compounded Statistical Model for Radar HRRP Target Recognition, *IEEE Trans. Signal Processing*, Vol.54, No.6, 2006, pp. 2226-2238.
- [25] H.-L. Liu, W.-L. Yang, Modified Generalized Discriminant Analysis for Radar HRRP Recognition, *Proc. Int. Conf. Microwave and Millimeter Wave Technology*, 2007, pp. 1-4.
- [26] Z.-W. Zhu, and J.-J. Zhou, Automatic Target Recognition Based on HRRP Using SKO-KPCA, *Proc. Int. Joint Conf. Computational Sciences and Optimization*, Vol.1, 2009, pp. 874-877.
- [27] K. Fukunaga, *Introduction to statistical pattern recognition (Second Edition)*, Academic Press, 1990.
- [28] E. A. Zoeller, *Pattern Recognition: Theory and Application*. Nova Science Publishers, 2008.
- [29] G. Baudat, and F. Anouar, Generalized Discriminant Analysis Using a Kernel Approach, *Neural Computation*, Vol.12, No.10, 2000, pp. 2385-2404.
- [30] S. A. Gorshkov, S. P. Leschenko, V. M. Orlenko, S. Y. Sedyshev, and Y. D. Shirman, *Radar Target Backscattering Simulation Software and User's Manual*, Artech House, 2002.
- [31] Y. D. Shirman, *Computer Simulation of Aerial Target Radar Scattering, Recognition, Detection, and Tracking*, Artech House, 2002.
- [32] M. A. Weiss, *Data Structures and Algorithm Analysis in C++ (3rd Edition)*, Addison Wesley, 2006.