Fuzzy Covering and Partitioning Problems Based on the Expert Valuations: Application in Optimal Choice of Candidates

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Abstract: A new approach in the discrete optimization (partitioning and covering) problems is presented based on the expert knowledge presentations. An a priori uncertain information on the alternatives is given by some probability distribution and an a priori certain information on the knowledge competitions is given by some weights. A new criterion is introduced for a minimal fuzzy covering or partitioning problem which is a minimal value of average misbelief in possible alternatives. A bicriterial problem is obtained using the new criterion and the criterion of minimization of average price of the covering or partitioning. The proposed approach is illustrated by an example for the partitioning problem.

Key–Words: Minimal fuzzy covering or partitioning, Minimal compatibility level, Positive and negative discrimination measures, Average misbelief criterion, Bicriterial problem

1 Introduction

Optimization and decision-making problems are traditionally handled by either the deterministic or the probabilistic approach. The former provides an approximate solution, completely ignoring uncertainty, while the latter assumes that any uncertainty can be represented as a probability distribution. Of course, both approaches only partially capture reality uncertainty that indeed exists but not in the form of known probability distributions.

The existing literature clearly supports the notion of using the fuzzy set theory and soft-computing techniques to further expand the human capability in making optimal decisions involving non-probabilistic uncertainty [3], [4], [9], [14]–[20], [23]–[37], [40]–[44].

In the Preface of the Journal of Fuzzy Optimization and Decision Making (vol. I, 2002, pp. 11–12) Professor L. A. Zadeh had said: “My 1970 paper with R.E. Bellman, ‘Decision-Making in a Fuzzy Environment' was intended to suggest a framework based on the theory of fuzzy sets for dealing with imprecision and partial truth in optimization and decision analysis. In the intervening years, a voluminous literature on applications of fuzzy logic to decision analysis has come into existence."

In particular, when constructing decision-making systems [3], [9], [14]–[25], [29]–[36], [42]–[44]. the use of fuzzy set theory is rather effective, since information fuzziness is a typical property of any system of this kind. Frequently, the main material for the construction of such systems is expert knowledge and representations. The use of such systems containing subjective, fuzzy uncertainty [11] leads to natural generalizations of the above-mentioned problems in the form of fuzzy optimization problems.

Fuzzy programming problems has been discussed widely in literature [3], [9], [14], [19], [20], [24], [25], [28], [31], [32], [33], [35], [36], [38] and applied in such various disciplines as operations research, economic management, business administration, engineering and so on. Liu B. (Liu, [19] 2002) presents a brief review on fuzzy programming models, and classifies them into three broad classes: expected value models, chance-constrained programming and chance-dependent programming.

Our further study belongs to the first class, where we used the instrument of fuzzy statistics and fuzzy set theory for our investigation.

In this paper we continue the investigation of a discrete fuzzy optimization problem, presented in i.e. a minimal set covering and partitioning problems with expert data. The obtained bicriterial optimization problem is a specific compromised approach between expert and objective methods of optimization.

2 Preliminary Concepts

2.1 Classical set covering problem

Partitioning, covering and packing problems serve as a mathematical model for many theoretical and applied problems such as the coloring of graphs, construction of perfect codes and minimal disjunctive normal forms, drawing up of block-diagrams, information search, drawing up of traffic schedules, administrative division into zones and so on [2], [7], [8], [12].

Let us introduce some basic notions [2], [12]. Suppose that we are given the finite set $R = \{r_1, \ldots, r_m\}$ and the family of its subsets $S = \{S_1, \ldots, S_n\}$. Let $S' = \{S_{j_1}, \ldots, S_{j_p}\}$, $1 \le p \le n$, be some subfamily of the family S. If each element r_i is contained in at most (at least) one of the sets S_j belong to S', then S' is called a packing (covering) of the set R. A covering which is simultaneously a packing is called a partitioning of the set R. Let $A = \|a_{ij}\|_{m\times n}$ be an incidence matrix of elements R and subsets S_j: $a_{ij} = 1$ if $r_i \in S_j$, and $a_{ij} = 0$ if $r_i \notin S_j$. Each subfamily S' of the family S is given by means of the characteristic vector which has the component $x_j = 1$ if the subset S_j is contained in S', and $x_j = 0$ otherwise. If to each $S_j \in S$ we assign a (positive) price c_j, then partitioning, covering and packing problems take the form

$$1)\ \min_{A\overline{x}=\overline{e}}(\overline{c},\overline{x}); \quad 2)\ \min_{A\overline{x}\ge\overline{e}}(\overline{c},\overline{x}); \quad 3)\ \max_{A\overline{x}\le\overline{e}}(\overline{c},\overline{x});$$

here $\overline{c} = (c_1, \ldots, c_n)$ is the price vector, $\overline{x} = (x_1, \ldots, x_n)$ is the vector with components 0 and 1, and $\overline{e}$ is the vector consisting of 1's. Note that in many interesting problems $c_j = 1, j = 1, \ldots, n$ (such is, for instance, the problem on finding a minimal dominating set in the graph), but this does not simplify the solution process of these problems.

2.2 On the most typical value (MTV) of a compatibility function with respect to a probabilistic or fuzzy measure

If data are represented in intervals, their distribution is obscure, they overlap and are described or obtained by an individual expert (insufficient expert data), then they are considered to be of combined nature. In that case, along with probabilistic-statistical uncertainty, there arises the so-called possibilistic uncertainty produced by an individual (expert) and demanding the application of fuzzy analysis methods. In such situations only probabilistic-possibilistic analysis can provide satisfactory results by using the fuzzy methods to be discussed below.

In describing such data functionally, in many real situations the property of additivity remains unrevealed for a measurable representation of a set and this creates an additional restriction. Hence, to study subjective insufficient expert data it is frequently better to use monotone estimators instead of additive ones.

We introduce the definition of a fuzzy measure (Sugeno, [39]) adapted to the case of a finite referential.

Definition 1 Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set and g a set function

$$g : \mathcal{P}(X) \to [0, 1],$$

where $\mathcal{P}$ is the power set of X. We will say g is a fuzzy measure on X if it satisfies:
(i) $g(\varnothing) = 0$; $g(X) = 1$.
(ii) $\forall A, B \subseteq X$, if $A \subseteq B$, then $g(A) \le g(B)$.

A fuzzy measure is a normalized and monotone set function. it can be considered as as extension of the probability concept, where additivity is replaced by the weaker condition of monotonicity.

Let us, for example, consider three typical symptoms x_1, x_2 ,x_3, which indicate some illness y. Let an expert (physician) provide objective-subjective data using his/her wide experience and medical records of patients (another expert would certainly provide different data).

Assume that we have the following information: 80% of patients with illness y exhibit the symptoms x_1 and x_2, 20% of them have the symptoms x_1 and x_3. This information can be written using the monotone instead of the additive, measure g defined on the subsets of the set $X = \{x_1, x_2, x_3\}$ (Table 1).

Table 1: Distribution table showing the dual measures g and g^*

$A \subseteq X$	g	g^*
$\{x_1\}$	0	1
$\{x_2\}$	0	0.8
$\{x_3\}$	0	0.2
$\{x_1, x_2\}$	0.8	1
$\{x_1, x_3\}$	0.2	1
$\{x_2, x_3\}$	0	1
$\{x_1, x_2, x_3\}$	0	1

g^* is called the dual measure of g defined by $g^*(A) = 1 - g(\overline{A})$. Note that g^* contains the same information as g but is written in a different way.

Non-additive but monotone measures were first used in fuzzy analysis in the 80s by M. Sugeno [39].

The fuzzy integral is a functional which assigns some number or a compatibility value to each fuzzy subset when the fuzzy measure is already fixed. As known [39], the concept of a fuzzy integral makes it possible to condense information provided by a compatibility function and a fuzzy measure. Having the fuzzy measure determined, we can estimate a fuzzy subset by the most typical compatibility value (MTV). The MTV is essentially different in content and significance from a probabilistic average even when a probabilistic measure is used instead of a fuzzy measure. The pre-image of the MTV with respect to a compatibility function distinguishes from the universe the most typical representative values of the considered fuzzy subset.

As already known, fuzzy averages differ both in form and content from probabilistic-statistical averages and other numerical characteristics such as *mode* and *median*. Nevertheless, in some cases "nonfuzzy" (objective) and "fuzzy" (subjective) averages coincide [34], [39]. For a given set of fuzzy subsets with compatibility function values from the interval [0; 1], the fuzzy average determines the most typical representative compatibility value ME – Monotone Expectation.

The following fuzzy integral (based on the Choquet operator [6]) is the monotone expectation, which was defined by Bolaňos et.al. [5]:

2.2.1 Fuzzy measure and a monotone expectation

Definition 2 ([5]) Let g be a fuzzy measure on X and $h : X \to \mathbb{R}_0^+$ a non-negative function. The monotone expectation of h with respect to g is

$$E_g(h) = \int_0^{+\infty} g(H_\alpha)\, d\alpha,$$

where $H_\alpha = \{x \in X \mid h(x) \ge \alpha\}$.

The monotone expectation always exists and it is finite for each g and h. It is obvious that $E_g(\cdot)$ is a generalization of the mathematical expectation: that is what it becomes when the used fuzzy measure is a probability measure, that is,

$$E_P(h) = \int_X h\, dP,$$

where P denotes a probability measure, E_P – mathematical expectation.

Some of the most important properties of the monotone expectation are see in [24]-[31], [39], [44].

Since the monotone expectation is a generalization of the mathematical expectation, it can be questioned whether the former possesses some weaker property in relation to additivity than the latter. The following proposition gives an expression of the monotone expectation that permits us to analyse that question.

Proposition 1 ([5]) If the values of a non-negative function h are ordered as

$$h(x_1) \leq h(x_2) \leq \cdots \leq h(x_n),$$

then the monotone expectation of h with respect to a fuzzy measure g can be written as

$$E_g(h) = \sum_{i=1}^{n} h(x_i)(g(A_i) - g(A_{i+1})),$$

where $A_i = \{x_i, x_{i+1}, \ldots, x_n\}$, $i = 1, \ldots, n$, $g(A_{n+1}) = 0$.

Thus, the monotone expectation is an additive functional for functions ordered equally.

We can also notice that $E_g(h)$ is an average of the h function values weighted by

$$p_i = g(A_i) - g(A_{i+1}), \;\; i = 1, \ldots, n, \;\; p_n = g(A_n).$$

As

$$\sum_{i=1}^{n} p_i = g(A_1) = g(X) = 1 \quad \text{and} \quad p_i \geq 0,$$
$$i = 1, \ldots, n,$$

the values p_i can be interpreted as the values of a probability function. Then $E_g(h)$ is equivalent to the mathematical expectation of h with respect to that probability distribution.

The values p_i depend on the fuzzy measure g and the sets A_i, which depend on h only in the order determined by its values. So we can say:

Proposition 2 The monotone expectation of a non-negative function h with respect to a fuzzy measure g coincides with the mathematical expectation of h with respect to a probability that depends only on g and the ordering of the values of h.

2.2.2 Fussy measure and the fuzzy expected value (FEV)

In this section, we discuss the main estimators of fuzzy statistics: the fuzzy expected value (FEV) of the population. The FEV determines MTV for a compatibility function.

Let h be a compatibility function of some fuzzy subset of X, $h : X \to [0, 1]$ be an F-measurable function.

Definition 3 ([16]) The FEV of the compatibility function h with respect to the fuzzy measure g is Sugeno's integral over X:

$$FEV(h) = \int_X h \circ g(\cdot) \equiv \sup_{\alpha \in [0,1]} \{\alpha \wedge g(H_\alpha)\},$$

where $\wedge$ denotes a minimum of two arguments.

It clearly follows that the FEV somehow "averages" the values of the compatibility function h not in the sense of a statistical average but by cutting subsets of the α level, whose values of a fuzzy measure g are either sufficiently "high" or sufficiently "low".

Thus the FEV gives a concrete value of the compatibility function h, this value being the most typical characteristic of all possible values with respect to the fuzzy measure g, obtained by cutting off the "upper" and "lower" strips on the graph of $g(H_\alpha)$.

Thus the information carried by h and g gets condensed in the FEV which is the most typical value of all compatibility values.

Consider the situation where $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set.

Proposition 3 ([39]) If the values of a compatibility function h are ordered as

$$h(x_1) \leq h(x_2) \leq \cdots \leq h(x_n),$$

then the FEV of h with respect to a fuzzy measure g can be written as

$$FEV = \max_i \{h(x_i) \wedge g(A_i)\} = \min_i \{h(x_i) \vee g(A_i)\},$$

where $A_i = \{x_i, x_{i+1}, \ldots, x_n\}$, $i = 1, \ldots, n$, and where $\vee$ is a maximum of two arguments.

3 Aggregation by the monotone expectation in the set Covering and Partitioning Problems

Our further consideration concerns minimal fuzzy covering and partitioning problems. Let $\widetilde{S} = \{\widetilde{S}_1, \widetilde{S}_2, \ldots, \widetilde{S}_n\}$ be some family of fuzzy subsets on R. Denote the compatibility level $\mu_{\widetilde{S}_j}(r_i) \equiv b_{ij}$ for $r_i \in R$, $j = 1, 2, \ldots, n$. In constructing $\widetilde{S}_j$, we use certain subjective expert estimates: $\mu_{\widetilde{S}_j}(r_i) > 0$ means that element r_i will be covered by a fuzzy set $\widetilde{S}_j$ with some positive level, even if this level is small.

Definition 4 Any subfamily $\widetilde{S}' = \{\widetilde{S}_{j_k}\} \subset \widetilde{S}$, $k = 1, \ldots, p$, $1 \leq p \leq n$, of fuzzy subsets is called a fuzzy covering (fuzzy partitioning) of the set R if for each r_i there exists (there exists only one) fuzzy subset $\widetilde{S}_{j_k} \subset \widetilde{S}'$ such that $\mu_{\widetilde{S}_{j_k}}(r_i) > \alpha$, where $0 \leq \alpha < 1$ is called the minimal compatibility level, by which an element r_i is covered by a fuzzy set $\widetilde{S}_{j_k}$.

So we have defined the values of elements of an incidence matrix:

$$a_{ik} = 1 \text{ if } \mu_{\widetilde{S}_{j_k}}(r_i) > \alpha \text{ and } a_{ik} = 0 \text{ otherwise.}$$

It is clear that if $\alpha = 0$, then we receive a classical case.

If to each $\widetilde{S}_j \in \widetilde{S}$ we assign a (positive) price c_j, then the fuzzy covering (fuzzy partitioning)problem is formulated as follows: find a fuzzy covering (fuzzy partitioning) $\widetilde{S}'$ of the set R having the least price with the least misbelief in subjective data. Thus under an optimal fuzzy covering (fuzzy partitioning) we understand a covering (partitioning)defined by means of two criteria: 1) minimization of a covering (partitioning) average price with probability uncertainty produced by a priori probability distribution on prices; 2) minimization of average misbelief in fuzzy uncertainty produced by the assumption of fuzzy covering (fuzzy partitioning). We obtain a bicriterion discrete optimization problem. Note that if under $\widetilde{S}$ we understand the classical covering (partitioning), then this problem can be reduced to the well known covering (partitioning) problem [7].

Suppose we are given some fuzzy set on $\mathbb{R}_0^+$ with the definition: "a large ratio" := $(L-R)$ with a nondecreasing compatibility function $\mu_{L-R} : \mathbb{R}_0^+ \to [0, 1]$.

Analogously [23], we introduce the notation of positive and negative discriminations:

$$p_{ij} = \frac{1}{n-1} \sum_{\substack{k=1 \\ k \neq j}}^{n} \mu_{L-R}\Big(\frac{b_{ij}}{b_{ik}}\Big),$$
$$n_{ij} = \frac{1}{n-1} \sum_{\substack{k=1 \\ k \neq j}}^{n} \mu_{L-R}\Big(\frac{b_{ik}}{b_{ij}}\Big), \qquad (1)$$
$$\begin{cases} i = 1, \ldots, m, \\ j = 1, \ldots, n, \end{cases}$$

where a heuristic explanation of the positive (p_{ij}) and the negative (n_{ij}) discrimination measure is that p_{ij} represents the accumulated belief that the element $\widetilde{S}_j$ is more indicative (in the sense of covering or partitioning) of an element r_i than anyone of the remaining elements r_l $(l = 1, \ldots, m,\ l \neq i)$, while n_{ij} represents the belief that an element $\widetilde{S}_j$ is more indicative of not an element r_i, but of other elements r_l $(l = 1, \ldots, m,\ l \neq i)$ with respect to belief covering (partitioning) levels b_{ij} of the fuzzy subsets $\widetilde{S}_j$, $j = 1, \ldots, n$.

Suppose we are given some weights' distribution $\begin{pmatrix} r_1 & r_1 & \ldots & r_m \\ w_1 & w_2 & \ldots & w_m \end{pmatrix}$, $\sum\limits_{i=1}^{M} w_j = 1$, on the set R, which indicates an a priori information about the preferences on choice of elements r_i.

Let two fuzzy sets be given on $[0; 1]$: one defined as "large" with some nondecreasing compatibility functions $\mu_{\text{large}} : [0; 1] \to [0; 1]$, and the other defined as "small" with some nonincreasing compatibility functions $\mu_{\text{small}} : [0; 1] \to [0; 1]$. We introduce the following values

$$\pi_j = \sum_{i=1}^{m} p_{ij} w_i, \quad v_j = \sum_{i=1}^{m} n_{ij} w_i, \qquad (2)$$
$$j = 1, \ldots, n,$$

where π_j and v_j are called weighted average positive and the negative discrimination measures of the covering (partitioning), respectively, for elements $\widetilde{S}_j$, $j = 1, \ldots, n$.

Now, on the set $\{\widetilde{S}_1, \ldots, \widetilde{S}_n\}$ we construct a misbelief distribution of a covering (partitioning), where both the positive and the negative discrimination measure (π_j, v_j) are taken into account;

$$\delta_j = \nu\mu_{\text{small}}(\pi_j) + (1-\nu)\mu_{\text{large}}(v_j), \quad j = 1, \ldots, n, \tag{3}$$

where ν, $0 < \nu < 1$, is a weighted parameter which indicates preference of positive or negative discriminations.

The information content of δ_j is as follows: a covering (partitioning) level of misbelief in "the acceptance" of an element $\widetilde{S}_j$.

Let $\widetilde{S}' = \{\widetilde{S}_{j_k}\}$, $k = 1, 2, \ldots, p$; $1 \le p \le n$, be some fuzzy covering (partitioning). It can be characterized by the binary vector $x_{\widetilde{S}'} = (x_1, \ldots, x_n)$, where

$$x_i = \begin{cases} 1, & \text{if the fuzzy subset } \widetilde{S}_i \text{ is} \\ & \quad \text{contained in } \widetilde{S}', \\ 0, & \text{otherwise.} \end{cases}$$

Let us consider the misbelief distribution on $x_{\widetilde{S}'}$

$$\widetilde{S}\,' \Leftrightarrow \begin{pmatrix} x_1, \ldots, x_n \\ \delta_1, \ldots, \delta_n \end{pmatrix}.$$

We say that the values x_j are chosen with some a priori information and we can consider some probability distribution on $x_{\widetilde{S}'}$:

$$P_{\widetilde{S}'} = \begin{pmatrix} x_1, \ldots, x_n \\ p_1, \ldots, p_n \end{pmatrix}.$$

Thus for each fuzzy covering (partitioning) $\widetilde{S}\,'$ we have constructed the fuzzy misbelief distribution $(\delta_1, \ldots, \delta_n)$ on $x_{\widetilde{S}'}$ and the probability distribution $P_{\widetilde{S}'}$. Applying the method of fuzzy statistics which was presented in Subsection 2.2.1 [5], [34], [37], [30], [39], we define a fuzzy average value of $\widetilde{S}\,'$ as a monotone expectation [6] [34], (which here coincides with mathematical expectation)

$$E_p(\widetilde{S}\,') \overset{def}{\equiv} \int_0^1 P_{\widetilde{S}\,'}(\mu_{\widetilde{S}\,'} \ge \alpha)d\alpha = = \sum_{j=1}^{n} x_j\, \delta_j p_j. \tag{4}$$

Note that the value $E_p(\widetilde{S}\,')$ is an average measure of misbelief in a fuzzy covering (partitioning). Minimizing the average misbelief in the fuzzy covering (partitioning) $\widetilde{S}\,'$, we obtain the criterion

$$\sum_{j=1}^{n} x_j\, \delta_j{}' \to \min, \tag{5}$$

where $\delta' = (\delta_1 p_1, \ldots, \delta_n p_n)$.

Finally, the minimal fuzzy covering problem (partitioning) is reduced to a bicriterial problem of the type (minsum-minsum) [12] for an ordinary covering (partitioning) with the target functions

$$\begin{aligned} & f_1 = \sum_{j=1}^{n} c'_j\, x_j \to \min \\ & \quad \text{(minimization of an average price)} \\ & \quad (c'_j = c_j p_j), \\ & f_2 = \sum_{j=1}^{n} \delta_j{}'\, x_j \to \min \\ & \quad \text{(minimization of an average misbelief).} \end{aligned} \tag{6}$$

If X is the set of all Boolean vectors satisfying the conditions of the fuzzy covering problem, then by considering the scalar optimization problem

$$\lambda f_1 + (1-\lambda) f_2 \to \min, \quad (x_1, \ldots, x_n) \in X, \quad \lambda \in (0,1), \tag{7}$$

where

$$X = \{x_{\widetilde{S}'} \in \{0,1\}^n \mid \widetilde{S}' \subset \widetilde{S},\ \widetilde{S}' \text{ is the covering}\} \equiv \equiv \{\bar{x} \in \{0,1\}^n \mid A\bar{x} \ge \bar{e}\}$$

and λ is a weighted parameter, we can find, in the general case, some Pareto optima [12].

An aggregation by the FEV in Minimal Fuzzy Covering Problem is the problem of our future investigation.

4 Example: Application in the Optimal Choice of Candidates

As an application let us consider an example based on the problem from [7] called the problem of a choice of translators. Suppose some company needs to hire translators from Polish, French, German, Greek, Italian, Spanish, Russian, Chinese, Portuguese and Japanese into English and there are sixteen applicants A, B, C, D, . . . , P. It is assumed that each candidate knows only some subset from the above-mentioned set of languages and demands the definite salary. We introduce, in the problem, a new element of estimation of their knowledge of languages and admit that they should be examined in the above-mentioned languages. The results of examinations (which usually includes expert valuations) are normalized and represented in the form of numbers b_{ij} which determine the level of knowledge of the j-th candidate with respect to the i-th language. Information on the candidates, estimates of language knowledge and salaries demanded by the candidates are represented in the form of Table 2, where numbers a_{ij} are given in the upper part of each cell, and numbers b_{ij} in the lower part ($a_{ij} = 1$ if $\mu_{\widetilde{S}_j}(r_i) = b_{ij} > \alpha = 0,5$ and $a_{ij} = 0$ otherwise; that means that the minimal compatibility level or determined minimal level of language knowledge competitions is equal to 0,5).

To construct a target function which guarantees minimal misbelief, we introduce the following concrete compatibility functions (any other choice of acceptable functions does not influence the final decisions [10]):

$$
\begin{aligned}
\mu_{L-R}(x) : &= \frac{x}{x+2}, \\
\mu_{\text{large}}(x) : &= x, \quad 0 \le x \le 1, \\
\mu_{\text{small}}(x) : &= 1 - x, \quad 0 \le x \le 1.
\end{aligned}
$$

Positive and negative discrimination measures (1) and their average values are calculated. On the set $\{x_1, \ldots, x_n\}$ the uniform probability distribution was taken in the role of a priori information measure, because we did not have any initial information about the candidates. The parameter ν is equal to 0,5. The uniform weights $w_i = \frac{1}{n}$ were taken in the role of a priori information measure, because there was no preference for any language.

Table 2: Data base of language knowledge and demanded salaries of the candidates A, B, C, D, . . . , P

Translator / Language	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	3100	1000	3500	4000	4000	4500	900	1600	800	1100	1200	3500	4500	4600	2500	3400
Chinese	0 / 0	0 / 0	1 / 0.6	0 / 0	1 / 0.7	1 / 0.8	0 / 0	0 / 0	0 / 0	0 / 0	1 / 0.8	0 / 0.2	0 / 0.5	1 / 0.9	0 / 0.5	0 / 0
German	1 / 0.9	1 / 0.8	1 / 0.6	0 / 0.4	0 / 0	1 / 0.7	0 / 0	0 / 0	0 / 0	0 / 0.2	0 / 0.3	1 / 0.9	0 / 0	1 / 0.9	0 / 0	0 / 0.5
Greek	0 / 0	0 / 0	0 / 0	1 / 0.8	0 / 0	1 / 0.6	1 / 0.7	0 / 0	0 / 0	0 / 0	0 / 0	1 / 0.6	1 / 0.7	0 / 0	1 / 0.7	0 / 0
French	1 / 0.7	1 / 0.7	0 / 0	0 / 0	0 / 0	1 / 0.9	0 / 0	0 / 0.4	0 / 0	0 / 0.1	0 / 0	1 / 0.8	0 / 0	1 / 0.7	0 / 0	0 / 0
Italian	0 / 0	0 / 0	0 / 0	0 / 0	1 / 0.9	0 / 0	0 / 0	1 / 0.8	1 / 0.7	0 / 0	0 / 0.2	1 / 0.6	0 / 0.4	1 / 0.6	0 / 0	0 / 0.5
Japan	0 / 0	0 / 0	0 / 0	1 / 0.9	1 / 0.7	0 / 0.4	0 / 0	0 / 0	0 / 0	0 / 0	1 / 0.9	0 / 0.4	1 / 0.9	0 / 0	1 / 0.9	0 / 0
Polish	1 / 0.8	0 / 0.2	0 / 0	1 / 0.7	1 / 0.9	0 / 0.4	1 / 0.8	1 / 0.9	0 / 0.3	0 / 0.4	0 / 0.2	0 / 0.5	1 / 0.8	0 / 0.5	1 / 0.9	1 / 0.6
Portuguese	1 / 0.6	0 / 0	1 / 0.7	0 / 0	1 / 0.8	0 / 0	0 / 0	0 / 0.2	0 / 0	1 / 0.9	0 / 0	0 / 0	0 / 0	1 / 0.8	1 / 0.7	1 / 0.9
Russian	0 / 0	0 / 0.1	1 / 0.8	1 / 0.7	0 / 0	0 / 0	0 / 0.1	0 / 0	1 / 0.9	0 / 0	0 / 0	0 / 0	0 / 0.4	1 / 0.6	0 / 0	1 / 0.6
Spanish	0 / 0.3	0 / 0	1 / 0.8	0 / 0.1	0 / 0	0 / 0.3	0 / 0	0 / 0	0 / 0.4	1 / 0.8	0 / 0.3	0 / 0.4	1 / 0.7	0 / 0	0 / 0	1 / 0.8

Finally, a fuzzy distribution of misbelief for the candidates is written in the form

$$\delta_1 = 0,484, \quad \delta_2 = 0,550, \quad \delta_3 = 0,452,$$
$$\delta_4 = 0,483, \quad \delta_5 = 0,448, \quad \delta_6 = 0,483,$$
$$\delta_7 = 0,549, \quad \delta_8 = 0,548, \quad \delta_9 = 0,548,$$
$$\delta_{10} = 0,548, \quad \delta_{11} = 0,549, \quad \delta_{12} = 0,483,$$
$$\delta_{13} = 0,483, \quad \delta_{14} = 0,416, \quad \delta_{15} = 0,482,$$
$$\delta_{16} = 0,484.$$

After passing to dimensionless values of target functions we obtain

$$\begin{aligned}\overline{f}_1 = {} & 0,620x_1 + 0,200x_2 + 0,700x_3 + \\ & + 0,800x_4 + 0,800x_5 + 0,900x_6 + \\ & + 0,180x_7 + 0,320x_8 + 0,160x_9 + \\ & + 0,220x_{10} + 0,240x_{11} + 0,700x_{12} + \\ & + 0,900x_{13} + 0,920x_{14} + 0,500x_{15} + \\ & + 0,680x_{16}, \\ \overline{f}_2 = {} & 0,269x_1 + 0,306x_2 + 0,251x_3 + \\ & + 0,268x_4 + 0,249x_5 + 0,269x_6 + \\ & + 0,305x_7 + 0,305x_8 + 0,305x_9 + \\ & + 0,305x_{10} + 0,305x_{11} + 0,269x_{12} + \\ & + 0,268x_{13} + 0,231x_{14} + 0,268x_{15} + \\ & + 0,269x_{16}.\end{aligned}$$

Note that for f_1 an optimal solution is the candidates B, G, I, J, K should be hired, and for f_2 the candidates M, N. For $\lambda = 0,5$, by scalarized linear convolution, criterion (7) (if both criteria are assumed equivalent) gives the solution M, N. To solve the scalar problem on a minimal covering we use an algorithm of search tree type from [7].

During the research a software has been developed. The software consists of two basic modules: the first module is responsible for reducing minimal fuzzy covering to classical covering problem and the second module is responsible for solving classical covering problem using both exact (search tree type algorithm [7]) and approximate (greedy algorithm) methods. The software is in the processes of development and is considered to be the basis for decision making support systems software.

5 Conclusion

We apply the methods of fuzzy statistics [23], [34], [30] to the considered discrete optimization problems with fuzzy data. In an appropriate manner we introduce the definitions of positive and negative discriminations of expert knowledge of optimization problem parameters, i.e. parameters of possible solutions and alternatives (candidates). We thereby determine a fuzzy distribution of misbelief on the set of alternatives. As a result we obtain a bicriterial discrete optimization problem which is solved by the method of linear convolution of criteria. The scalar problem is solved by the search tree algorithm from [7]. The obtained bicriterial optimization problem is a specific compromised approach between the expert (fuzzy) and the objective (probabilistic) method of optimization of decision-making, where both the minimization of average misbelief in alternatives and the minimization of an average price for alternatives are taken into account. The constructed approach (minimal fuzzy covering or partitioning) to the solution of the discrete optimization problem with data of combined (expert-objective) nature can be regarded as more trustworthy from the standpoint of application than the classical optimization methods.

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