Virtual Deformation of Soft Tissue using Bulk Variables

K. Sundaraj
University Malaysia Perlis
School of Mechatronic Engineering
02600 Jejawi - Perlis
MALAYSIA
kenneth@unimap.edu.my

Abstract: We present an alternative online simulation model for human tissue. Online simulation of human tissue deformation during surgical training or surgical assistance is becoming increasingly important within the medical community. Unfortunately, even classical simulation models find human tissue to be computationally too costly for online simulation. In this paper, we simplify the complex biomechanical nature of human tissue within reasonable limits to develop a mathematical model which can be used for online simulation. This simplification is based on two principles; volume conservation and Pascal's Principle. Volume conservation is inherent to many organs in the human body due to the high concentration of blood (almost incompressible liquid) in them. Given an externally applied force, we use Pascal's Principle to obtain the global deformation vector at each time-step during simulation.

Key–Words: Virtual Reality, Soft Tissue Simulation, Surgical Simulators.

1 Introduction

Human tissue is deformable. Hence, modeling this behavior is essential for applications that involve soft tissue simulation. In a medical simulator for example [1], human tissue is generally represented by a geometrical model and a physical model. Often, a combination of the two is referred to as a numerical model. In current surgical simulation systems, the numerical model may be subjected to real-time interactive deformations and haptic force feedback under different surgical tool based gestures [2]. So accurate models need to be designed to realize the consistency between them. In other words, there is a need to model the biomechanical nature of soft tissue by using mathematical equations so that consistent visual deformation and haptic force feedback can be provided interactively to the user.

Soft tissue, being complex physically and geometrically, is often divided into a set of smaller *elements* to facilitate analysis. Over the past fifteen years or so, many mathematical models that are based on the concept of discrete elements have been proposed for soft tissue. In practice, most of these models are simplified for soft tissue simulation. This results in a trade-off between physical accuracy and computation efficiency. Such an adaptive scheme gives us variations in the formulation of the above mentioned models. The main concerns regarding these physical models are described in the following sections.

1.1 Interactive Time

The first concern is the effective modeling of soft tissue to achieve an interactive-time surgical simulation. Up to now, several models have been suggested, but none of them has been satisfactory from the simulation point of view as yet. Since the simulated object itself is very complicated and computation resources are limited, it is natural to consider a trade-off between physical accuracy and computation efficiency. A compromise can be done by laying more emphasis on the areas of interest, for example, linear elasticity, local deformation or volume conservation. So this idea has to be implemented adaptively. This adaptation scheme is now the main topic of research in biomechanical models of soft tissue.

1.2 Numerical Stability

A second concern regarding soft tissue models is the numerical resolution scheme applied to solve these systems. Currently we find several methods; linear static, nonlinear static, linear dynamic and nonlinear dynamic, each being applied depending on the application and interactive-time requirements. For example, simulating cutting and tearing generally requires a dynamic model to accurately capture the viscoelastic properties of soft tissue when topology changes. On the other hand, simulating large deformations or stress-relaxation may only require at most a nonlinear static model owing to the well-damped nature of soft tissue. In either case, the main issue of interest is always stability and rapidity of the chosen scheme, during the entire history of load application.

1.3 Realism

Another concern regarding soft tissue models is realism. This corresponds to identifying the physical parameters of a model such that the behavior of the model is close to reality. This part is the most difficult as it requires a lot of expertise and experimentation. Expertise, usually from medical professionals, is required because soft tissue needs to be *alive* during experimentation so that the results obtained from them are valid and accurate.

In this work, we present another variation of a soft tissue simulation model based on bulk variables. This model is called the Volume Distribution Method (VDM) [3]. We are interested in simulating deformable objects which have an elastic shell as surface and are filled with an incompressible fluid. Many human organs have this characteristic, for example the human liver can be considered as such an object. It is composed of two major parts; an elastic skin called *Capsule of Glisson* as the surface and the *Parenchyma* which is the interior that is full of liquid ($\approx 95\%$ blood).

2 Previous Works [4]

It would be quite impossible to highlight all possible variations in soft tissue simulation models that have been used. These variations have been tailored for deformable objects and developed for a certain application. However, a survey on deformable models which are used on a virtual reality platform can be found in [5]. In this paper, the conclusion was that physically based models are more suitable for computer graphics simulation. Our work is more related to the formulation of *physically based models of soft tissue* using discrete elements. Within this context, so far finite elements and particle systems have been used as discrete elements. We discuss these models in the following section, highlighting the advantages and the disadvantages which has led us to propose an alternative physical model.

2.1 Particle Systems

The method of using a particle system network which consists of a mesh of nodes connected by elastic links to model soft objects has been applied by many authors in various fields. The mass-spring network is the most common technique used, i.e. the masses are the nodes and the springs are the edges of the mesh. This mass-spring network is used to discretize the equations of motion. The link connecting pairs of nodes allows the local description of elastic properties in soft tissue and the masses at the nodes give inertia properties. This particle system model is relatively easy to implement, computationally efficient (fast simulation) and numerically stable (no stiffness phenomena appears in deformable objects).

The mass-spring network is a simple physical model with a solid mathematical foundation and well-understood dynamics. Its computational burden is relatively small [4] and is thus suitable for interactive-time applications. Since the mass-spring network has a simple structure, many operations like large deformations and topology modifications can be simulated easily. Furthermore, as interactions in this model are local between nodes, parallel computations are possible. Hence, it is common that we find this model in many applications involving soft tissue.

Mass-spring networks has been widely used in 2D and 3D facial static and dynamic animation [6] [7]. It also has been used for cloth simulation, video games and animation movies. Several methods have been suggested to avoid numerical instability [8] [9]. A lot of research work has also been done on the mass-spring network to improve various aspects like adaptive refinement of the parameters [10] and controlling the isotropy or anisotropy of the material being simulated [11]. [12] developed a simple but efficient algorithm based on the mass-spring model for microsurgery simulation. This algorithm took advantage of the locality of the deformations to reduce calculations by using a *wave-propagation* technique that automatically halts computation when deformations become insignificant. Using this algorithm, they achieved an updating frequency of 30Hz for the deformations in a suturing vessel surgery, which is compatible with interactive-time graphic animation.

Unfortunately, this physical model has some drawbacks. When representing a volume using binary connectors, the model can lead to several problems. Certain constraints like volume conservation are not easily expressed in the model. Of course, more springs will improve connectivity and thus produce a better approximation of the volume. Thus, a volumetric object could perhaps be accurately modeled by an infinite amount of particles and springs, but this is clearly not an option computationally speaking. To remedy this problem, it has been proposed to add cross springs, thereby connecting opposing corners. However, this implies that the physical behavior of the object is intrinsically dependent on the connectivity of the springs. When aiming for physical realism, this is clearly a handicap. Alternatively [13] proposed the use of angular and torsion springs, but this again is another form of topological dependency. Also, proper values for the constants of the mass-spring network are not easily specified.

2.2 Finite Elements

The finite element method is a full continuum model. It gives the equilibrium of a body when subjected to external forces. This is obtained by minimizing the total potential energy of the system. In this method, soft tissue is represented by an elementary volume. The deformation of this volume is often expressed using the Green-Lagrange tensor which has the nice property of being invariant to rotation or translation. Since the internal stress of the volume is proportional to the deformation (or strain), we may obtain the forces at the nodes if the stress-strain relationship is known.

Several extensions to the FEM model has been done by several authors in various forms such that it can be applied to soft tissue simulation. The tensor-mass model discretizes the virtual organ with conformal tetrahedras [14]. It is found that when there is a topology modification procedure, the tensor-mass model can give more accurate results. The initially proposed tensor-mass model only accommodated small displacements, but [15] made modifications to the model for large displacements by using nonlinear strain tensors and anisotropic material laws. In the hybrid-elastic model [16], there is a combination of a quasi-static precomputed linear elastic model and a tensor-mass model. Thus, the hybrid-elastic model takes advantage of the good properties of the combined models. Another combination of models to produce a hybrid model system can be found in [17]. In [18] the hybrid-elastic model is used to simulate a *hepatectomy* surgical procedure.

On the whole, FEM has too heavy a computation burden to achieve accurate and interactive-time results. For complicated objects, under linear analysis, we may be interested only on the parts that can be seen (the surface). It is then possible to condense the matrix equation and apply some precomputation techniques to reduce computation [19]. In spite of this, we believe that FEM is not suitable for interactive-time applications for the present CPU capacity. But if there is no topology changes, it is possible to obtain real-time deformations by using precomputation [20]. Nevertheless this is limited to small deformations which can be a handicap for soft tissue simulations. But [21] have proposed an implementation of an explicit formulation of FEM taking into account large deformations and topology changes. They managed to perform cutting on a virtual human liver by considering human behavior and limiting stress conditions.

However, the finite element method (FEM) was originally intended for small deformations. Large deformations can be simulated but at the loss of accuracy. Furthermore, the physical behavior of soft tissue, particularly volume conservation, which is reflected in the choice of the deformation tensor and the stress-strain material tensor is arbitrary. It generally depends on the application intended for. Nevertheless, the finite element method has the best approximation of deformation of an elementary volume and as such produces the most realistic physical simulation for soft tissue.

3 Volume Distribution Method (VDM) [3]

VDM is a surface based method that allows the computation of a global deformation vector produced by an external load vector. It only requires the surface to be discretized with the inside being transparent to the model. The interior of the object is assumed to be filled by some incompressible fluid. This fluid acts as the medium that transfers the change in energy experienced by the deformable object due to a change in state from equilibrium.

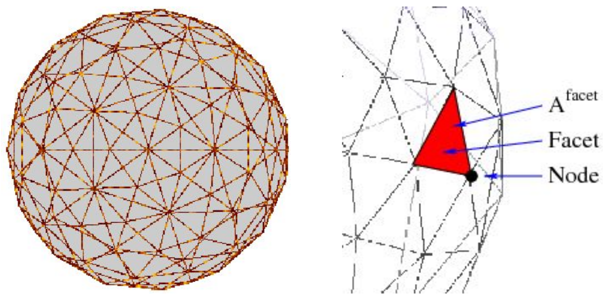


Figure 1: A deformable object with the surface discretized and a zoomed view of the surface. The interior of this object is filled with some incompressible fluid.

Consider a deformable object of volume V and surface area A that is represented by Q discrete surface elements of vectorial surface area $\boldsymbol{S}$ as shown in Fig. 1. Each element has M nodes and the entire surface has N nodes. Let each node i be shared by k neighboring elements and connected to j neighboring nodes. The following can be obtained,

$$\boldsymbol{\mathcal{A}}_i = \sum_k \frac{\boldsymbol{S}_k}{M} \tag{1}$$

$$\mathcal{V}_i = V \frac{\|\boldsymbol{\mathcal{A}}_i\|}{A} \tag{2}$$

where $\boldsymbol{\mathcal{A}}$ and $\mathcal{V}$ are the distributed area and volume respectively for each node. When a force is applied to a node on the surface, deformation is produced. However, this deformation is a result of the applied force F and skin tension T. They are given as,

$$F_i = B_i \frac{\Delta \mathcal{V}_i}{\mathcal{V}_i} \tag{3}$$

$$T_i = \sum_j B_{ij} \frac{\Delta \mathcal{V}_i - \Delta \mathcal{V}_j}{\mathcal{V}_i} \tag{4}$$

where B_i and B_{ij} are the modulus of elasticity and the connectivity modulus of elasticity constants of node i respectively. In the VDM model, stress is a function of strain and strain is related to a change in volume,

$$\epsilon_i = \sum_j \Delta \mathcal{V}_i - \Delta \mathcal{V}_j \tag{5}$$

$$\sigma_i = \frac{B_i}{\mathcal{V}_i} \epsilon_i \tag{6}$$

where ϵ and σ are the strain and stress respectively. Equilibrium of the system at each node is obtained when the external pressure E is equal to the internal pressure I. They are given as,

$$E_i = O_i + F_i + T_i \tag{7}$$

$$I_i = U_i + C_i + G_i \tag{8}$$

where O is the surrounding environmental pressure, U is the pressure of the incompressible fluid, C is the pressure when contact is applied and G is the pressure due to the effects of gravity. G is given as,

$$G_i = \rho_i g_i \delta_i \tag{9}$$

where ρ is the density of the incompressible fluid and δ is the measured hydrostatic distance of the node due to the contained fluid. By considering equilibrium of all nodes, the following is obtained using index notations,

$$\begin{aligned} B_i \frac{\Delta \mathcal{V}_i}{\mathcal{V}_i} &+ \sum_j B_{ij} \frac{\Delta \mathcal{V}_i - \Delta \mathcal{V}_j}{\mathcal{V}_i} - \Delta P_i \\ &= C_i + \rho_i g_i \delta_i \ \forall \ i = 1 \ldots N \end{aligned} \tag{10}$$

where $\Delta P = U - O$. By applying Pascal's Principle which gives constant change in pressure throughout the deformable object, the index i can be removed from ΔP_i. We can now add a boundary condition to our system. The incompressibility of the fluid imposes the constraint that the volume of the deformable object is maintained at all times. This can be stated as,

$$\sum_i^N \Delta \mathcal{V}_i = 0 \tag{11}$$

Since we are interested to obtain the global displacement vector, $\Delta \mathcal{V}$ can be rewritten as,

$$\Delta \mathcal{V}_i = \boldsymbol{\mathcal{A}}_i \boldsymbol{\Delta L}_i \tag{12}$$

where $\boldsymbol{\Delta L}$ is displacement vector of a node. We now have $3N + 1$ equations and $3N + 1$ unknowns; $\boldsymbol{\Delta L}_i$ for $i = 1 \ldots N$ and ΔP. These equations can be assembled in the following form,

$$\boldsymbol{K \Delta L} = \boldsymbol{R} \tag{13}$$

where $\boldsymbol{K}$ is the state matrix of the VDM assemblage, $\boldsymbol{\Delta L}$ is the global deformation vector and $\boldsymbol{R}$ is the load vector which consists of applied contact pressure $\boldsymbol{C}$ and the hydrostatic pressure $\boldsymbol{G}$ terms. During runtime, this equation is solved using standard numerical methods and the geometrical model is updated.

In our experiments, a nonlinear analysis was conducted whereby all nonlinear terms are updated at each time-step. This amounts to simulating large deformations and large strains. In this case, for large systems, a simple inversion or preconditioning of the state matrix at each time-step may be computationally expensive for interactive-time applications. However, the rapid increase in computational power has popularized iterative methods as a resolution scheme. We chose the Bi-Conjugate Gradient (BCG) iterative method as our optimal resolution scheme [22] for our experiments. This method is attractive for large sparse systems because only the nonzero terms of the state matrix is stored; hence minimal memory. For real-time solution of very large systems, even a solution in n iterations may be too expensive. However, in interactive-time applications the solution $\boldsymbol{\Delta L}$ only changes minimally from one time-step to another. Then, by using the previous result of the displacement vector as the starting guess for $\boldsymbol{\Delta L}$, we can achieve dramatic gains in speed after finding the first solution. The number of iterations needed to minimize the error below a certain tolerance is very much smaller than the value of N.

4 Simulation Results

In this section, we presents the simulation results of the VDM model. This simulation model was tested for anisotropic behavior, stress distribution and finally a comparison with a classical model like FEM was done.

4.1 Stress Distribution

To plot the stress distribution in our VDM model, we used the cube at rest as an example again (see Fig. 2).

This time, a force to compress the cube was applied and deformation was allowed in all directions without preference. Three test points were used to observe the magnitude of stress of the cube. The results are presented.

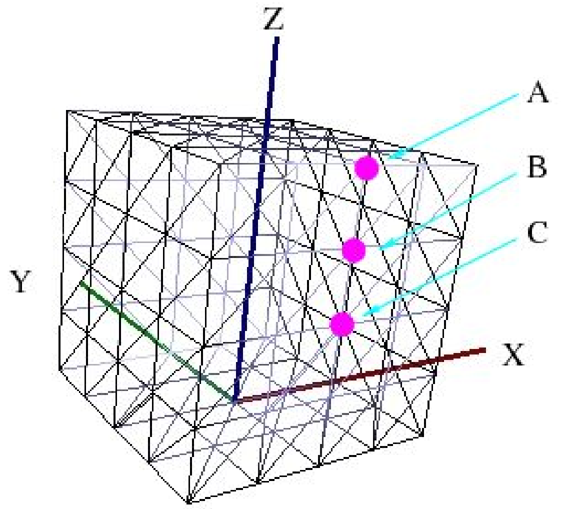
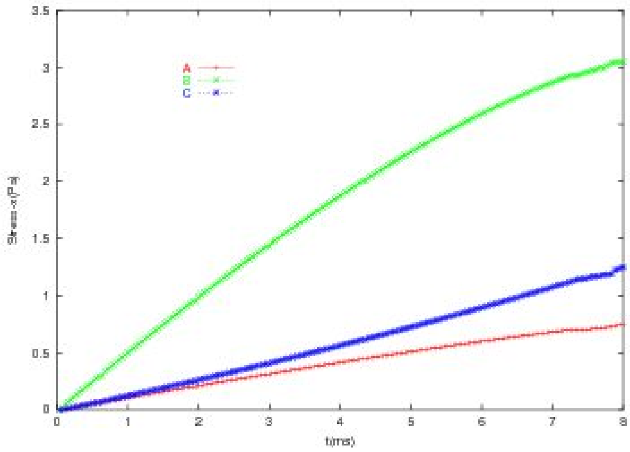
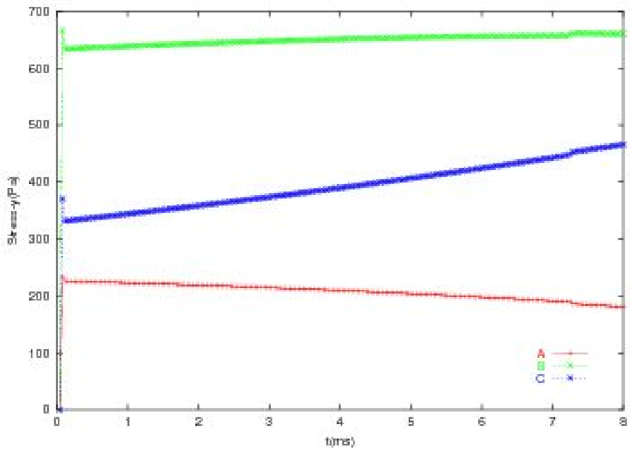


Figure 2: A test cube at rest was used as an example for the stress distribution test. 3 test points were chosen; A, B and C on the xz-plane.

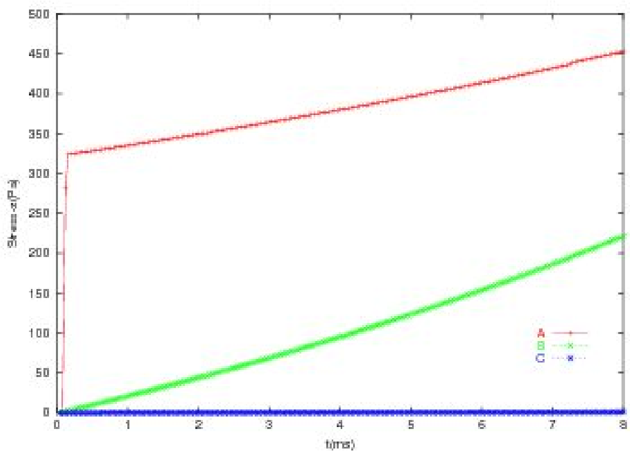
The results in Fig. 3 show that the stress experienced by the cube due to volumic tension is indeed dependent on the displacement and the distributed surface area. In other words, they are a function of volumic change. The stress in the x-direction is very minimal for all the test points because these points have displacement vectors with small x-components. On the other hand, since all the test points have significant displacement in the other directions, stress is observed to increase in the y and z directions. Point C has almost zero stress in the z-direction because during compression, this point has minimum distributed area in this direction. For the stress in the y-direction, point A has minimum stress. This is due to the fact that this point is constrained not to move in this direction. On the other hand, points B and C are displaced in the y-direction but point B has a higher stress which is due to the higher net volumic change experienced as compared to point C.



(a)



(b)



(c)

Figure 3: (a) Stress distribution along the x-direction. (b) Stress distribution along the y-direction. (c) Stress distribution along the z-direction.

4.2 Anisotropic Behavior

To test this behavior, we compared deformation curves of three points on a cube at rest that were placed on the xy, xz and yz planes respectively (see Fig. 4). In this test, a force was applied to compress the cube at rest. We first allowed deformation in all directions. We then changed the bulk modulus to observe the behavior of the cube. The results from the deformation curves show that the VDM model respects the imposed anisotropic constraints.

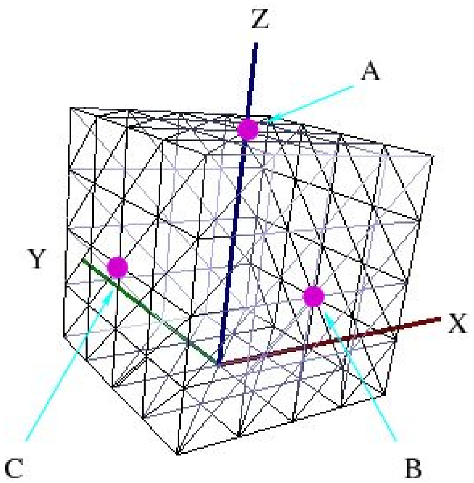


Figure 4: A test cube at rest was used as an example for the anisotropic behavior test. 3 test points were chosen; A on the xy-plane, B on the xz-plane and C on the yz-plane.

From the results shown in Fig. 6, we can see that the cube behaves differently when the bulk modulus is changed. In the first test, the displacement of points B and C coincide. This is because of the uniform bulk modulus. Hence, these points should move equally to maintain conservation of volume. In the second test, point B is on the xz-plane which has bulk modulus set to infinity. From the displacement curves, point B is seen to have nearly zero displacement. On the other hand, point C has a larger displacement vector in time. This is due to the constraint of conservation of volume. In the last test, deformation was preferred in the y-direction. This is observed in the displacement curve of point B by comparing with the first test. Point C however, has a smaller displacement vector to maintain volume conservation. We note that point A has a constant displacement curve in all the three tests. This is due to the constraint of type *contact* applied to this node. In conclusion, by changing the bulk modulus that is associated to the nodes, deformation can be preferred in a particular direction.

4.3 Comparison with FEM

To investigate the accuracy of the VDM model, we decided to compare it with a well known classical model like the FEM model. In these experiments, two beam mesh as shown in Fig. 5 with similar rigidity characteristics was used. One end of the beam was fixed and the other end was subjected to a displacement vector describing various types of large deformations. The final configuration of the beam was observed and the displacement vectors of several points along the beam was compared.

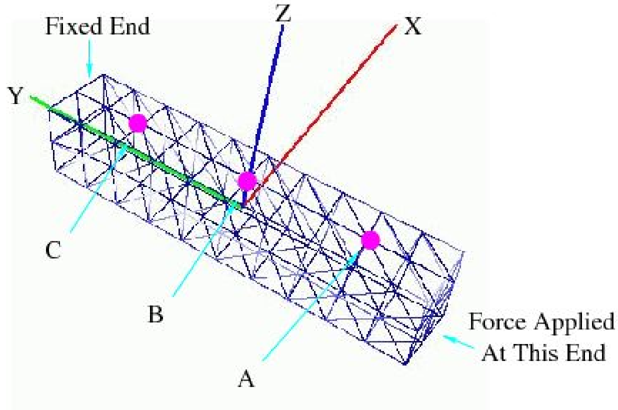
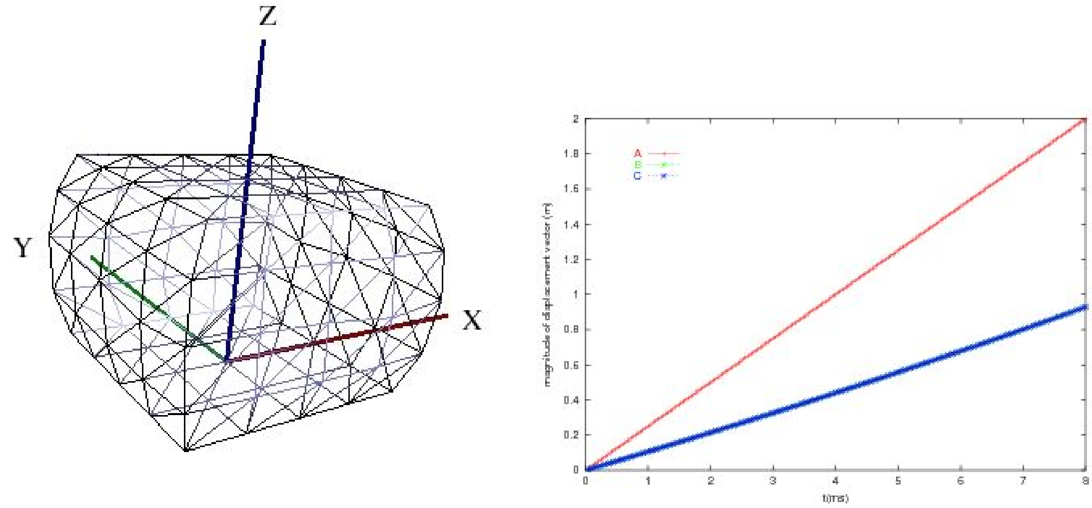


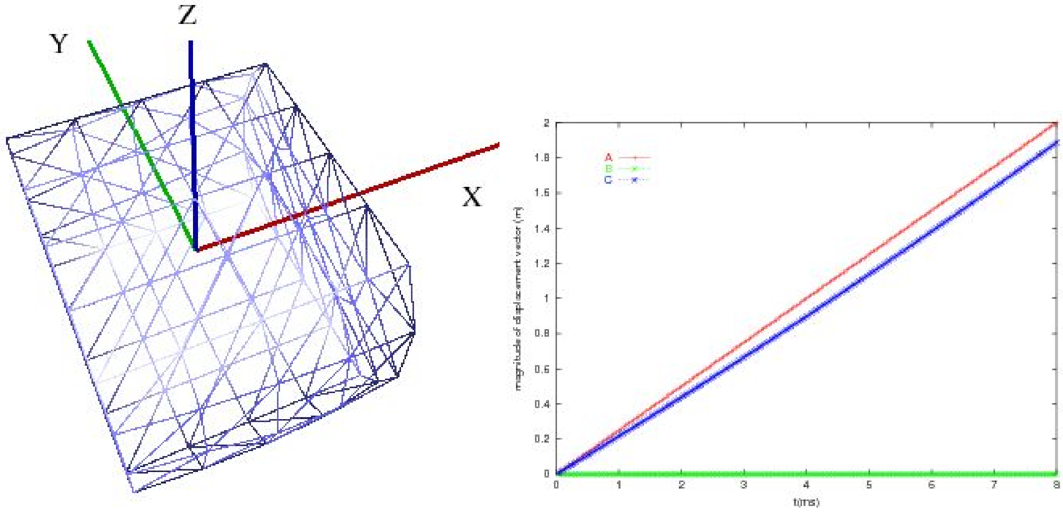
Figure 5: A test beam with one end fixed was used as an example for the comparison test. 3 test points were chosen; A, B and C along the y-axis.

From the results, we can see that there is a difference in the behavior of the nodes but the general shape of the beam seem to be identical. We observed the difference from one tine step to another. In the first test for stretching, a systematic increase of about 5% in the average error of the curves are observed for all the test points. When a force to bend the beam was applied, a constant systematic increase of about 2% in the average error is observed between all the test points. Twisting was applied in the last test where again a systematic increase of about 5% in the average error of the curves are observed for all the test points.

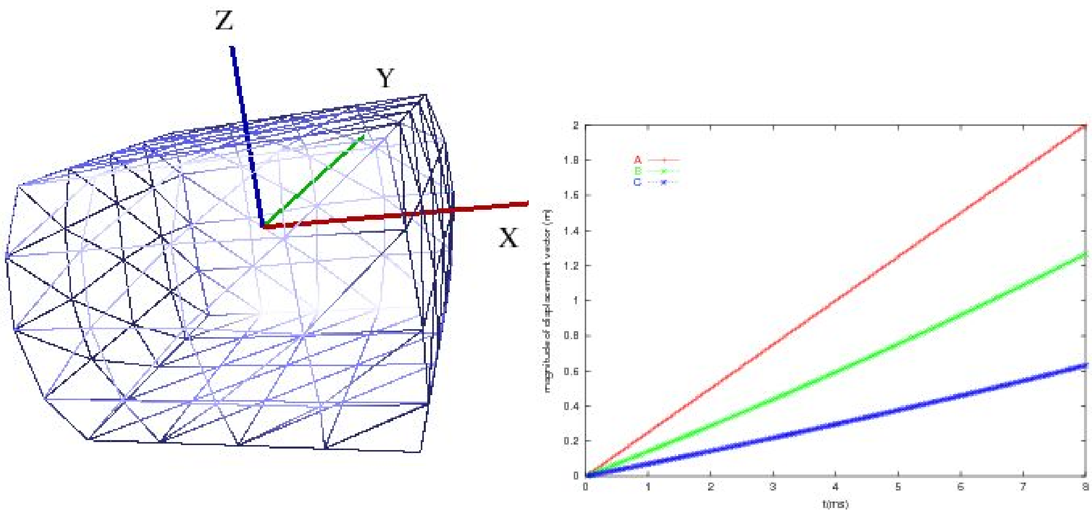
The difference between the models is nevertheless expected. FEM is a volumic model as compared to VDM which is surface based. Also, the physical parameters of FEM and VDM are not easily matched. The error in the *rigidity* constant is another source of error in the results. It is unclear how a deformable beam would behave under externally applied forces. But, we would like to note that realism was rather observed in the VDM model. At each time-step, we calculated the volume of the beam of the two models and found that volume conservation was rather observed in the VDM model. In conclusion, there is a difference between the two models, but if we would like to observe volume conservation, VDM seems to be a better choice.



(a)



(b)



(c)

Figure 6: (a) Compression with uniform bulk modulus. (b) The bulk modulus along the x-direction was set to infinity. (c) The bulk modulus of the cube was set such that deformation is preferred in the y-direction.

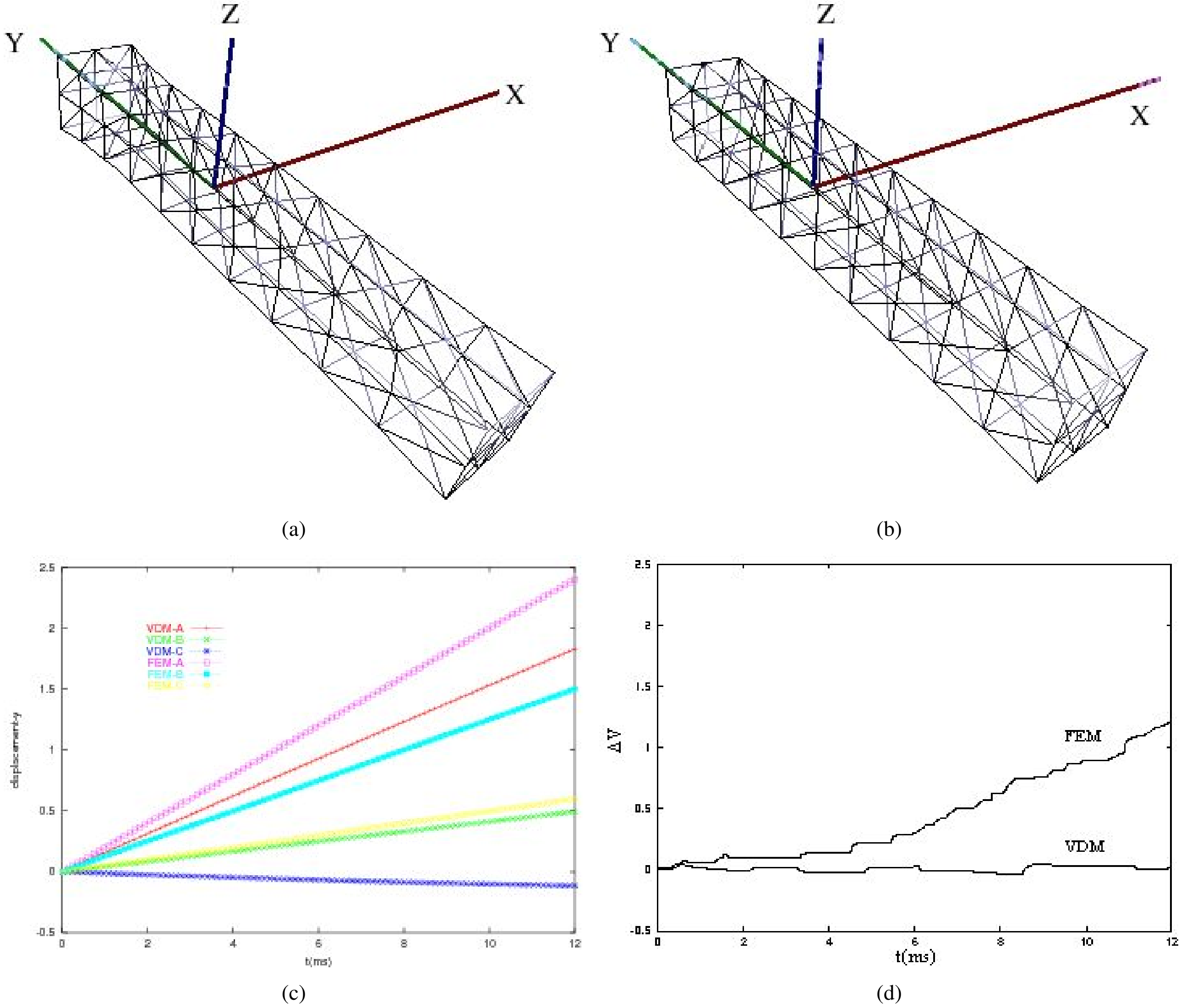


Figure 7: Stretching of a beam with one end fixed and a force applied along the y-axis at the other end. Final configuration of the beam with the, (a) VDM model and (b) FEM model. (c) The variation of the magnitude of the displacement vector of the 3 test points. (d) The variation in total object volume during stretching.

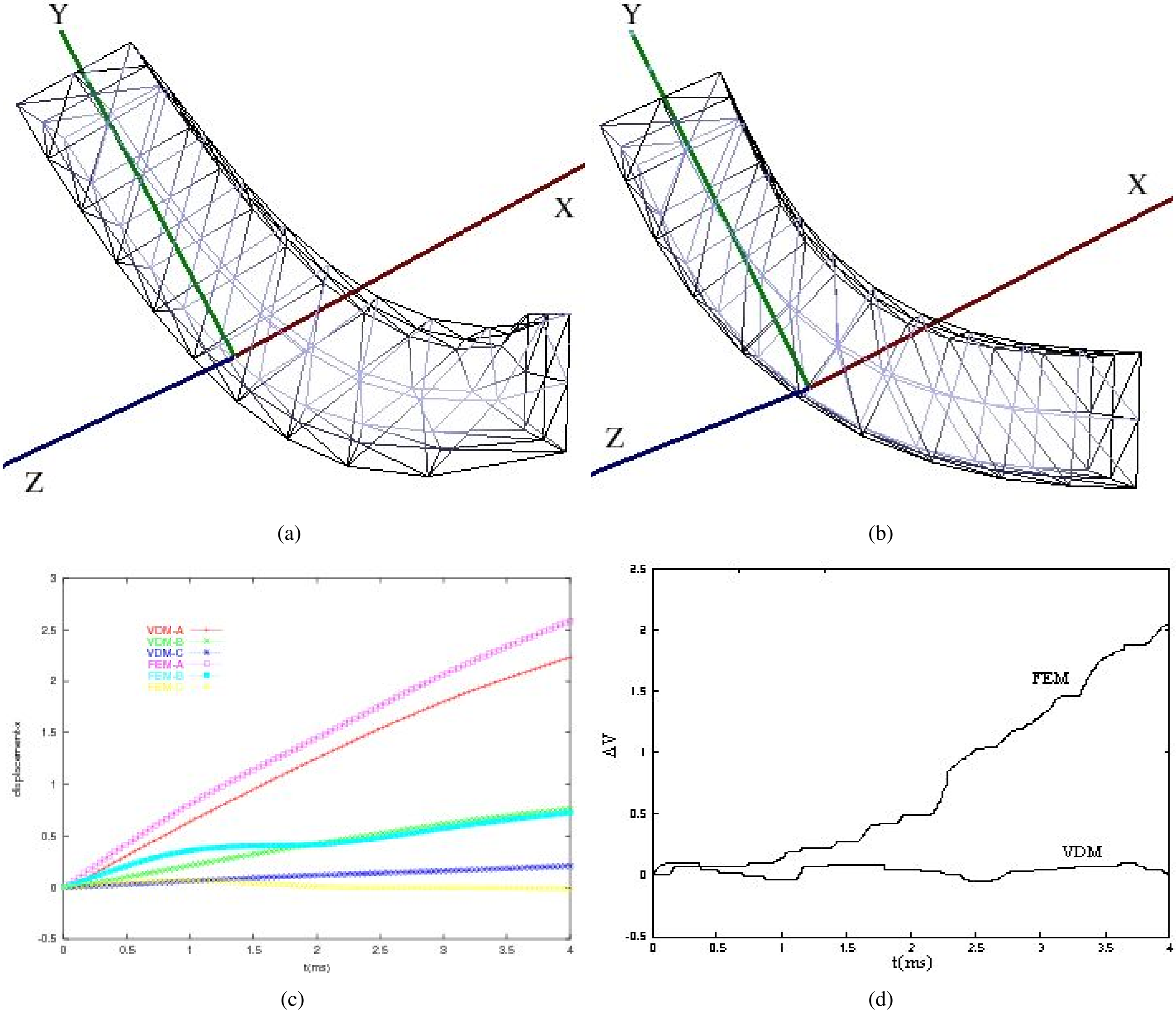
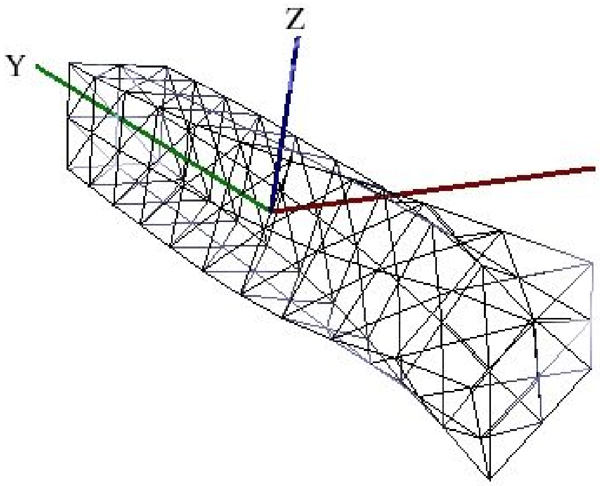
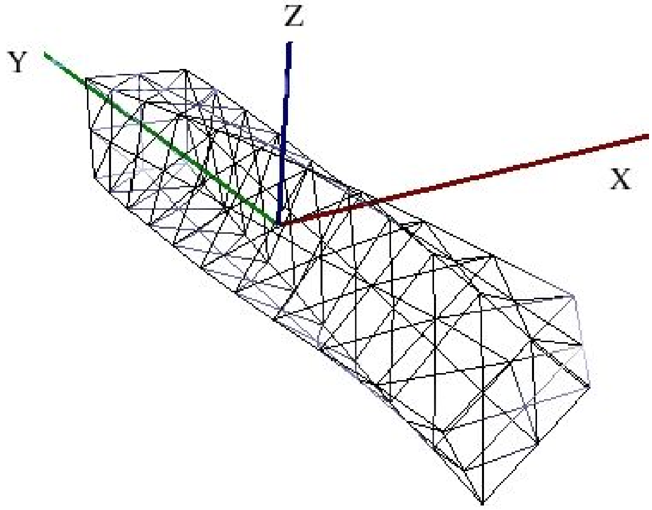


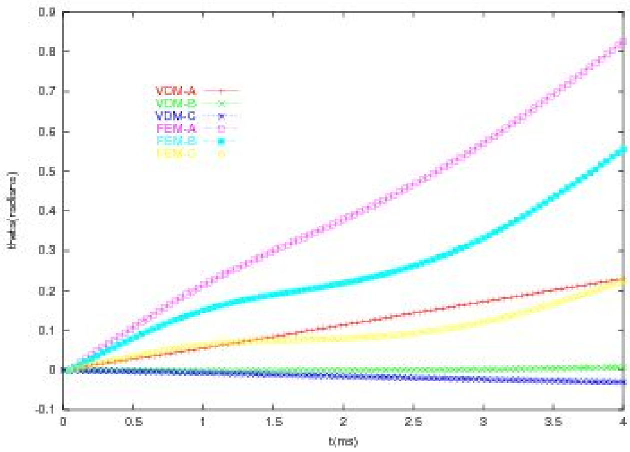
Figure 8: Bending of a beam with one end fixed and a force applied along the x-axis at the other end. Final configuration of the beam with the, (a) VDM model and (b) FEM model. (c) The variation of the displacement vector of the 3 test points. (d) The variation in total object volume during bending.



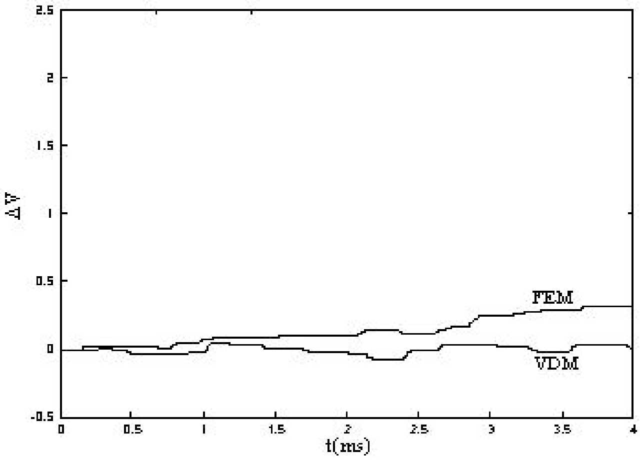
(a)



(b)



(c)



(d)

Figure 9: Twisting of a beam with one end fixed and a force applied to rotate along the xz-plane at the other end. Final configuration of the beam with the, (a) VDM model and (b) FEM model. (c) The variation of the displacement vector of the 3 test points. (d) The variation in total object volume during twisting.

5 Conclusion

An alternative soft tissue simulation model based on bulk variables like pressure, volume and modulus of elasticity has been presented. This model is surface based, hence it's complexity is in general lower than a volume based simulation model like FEM. A key feature of VDM is the absence of discretization of the interior. The physical properties of soft tissue in the VDM model depends on the organ being simulated. If for example a liver is being simulated, since it is 95% irrigated by blood, we require the density ρ and modulus of elasticity B of blood. Experiments to determine these parameters must be conducted. Simulation results show that the behavior of the VDM model follows a similar pattern like the FEM model for large deformations like stretching, bending and twisting but volume conservation was observed much more in VDM. In addition, these deformation patterns could be obtain much faster as VDM is computationally less costly. These results were obtained using a simple beam mesh to simulate soft tissue fibers.

References:

[1] V. Alves, J. Machado, A. Abelha, and J. Neves, "Agent based decision support systems in medicine," *WSEAS Transactions on Biology and Biomedicine*, vol. 2, no. 2, pp. 204–214, 2005.

[2] G. Jegajothi and S. Raghavan, "An overview and biological tissues characteristics using optical simulation method," *WSEAS Transactions on Biology and Biomedicine*, vol. 4, no. 1, pp. 7–14, 2007.

[3] K. Sundaraj, "Real-time dynamic simulation and 3D interaction of biological tissue: Application to medical simulators," Ph.D. dissertation, Institute Nationale Polytechnique de Grenoble, France, 2004.

[4] D. Aulignac, "Modelisation de l'interaction avec des objets dformables en temps-rel pour des simulateurs medicaux," Ph.D. dissertation, Institute Nationale Polytechnique de Grenoble, France, 2001.

[5] F. Gibson and B. Mirtich, "A survey of deformable models in computer graphics," Mitsubshi Electric Research Laboratory, Tech. Rep., 1997.

[6] K. Waters, "A muscle model for animating three dimensional facial expression," *Journal of Computer Graphics*, vol. 87, no. 1, pp. 17–24, 1987.

[7] D. Terzopoulos and K. Waters, "Physically-based facial modeling analysis and animation," *Journal of Visualization and Computer Animation*, vol. 1, no. 2, pp. 73–80, 1990.

[8] D. Baraff and A. Witkin, "Large steps in cloth simulation," *Journal of Computer Graphics*, vol. 98, no. 1, pp. 43–54, 1998.

[9] M. Desbrun, P. Schrder, and A. Barr, "Interactive animation of structured deformable objects," in *Proceedings of Graphics Interface*, 1999.

[10] D. Hutchinson, M. Preston, and T. Hewitt, "Adaptive refinement for mass/spring simulation," in *Proceedings of EACG Conference on Eurographics*, 1996.

[11] D. Bourguignon and M. P. Cani, "Controlling anisotropy in mass-spring systems," in *Proceedings of EACG Conference on Eurographics*, 2000.

[12] J. Brown and K. Montgomery, "A microsurgery simulation system," in *Proceedings of Medical Image Computing and Computer Assisted Intervention - MICCAI*, 2001.

[13] A. Deguet, A. Joukhadar, and C. Laugier, "Models and algorithms for the collision of rigid and deformable bodies," in *Proceedings of the Workshop on the Algorithmic Foundations of Robotics*, 1998.

[14] S. Cotin, "Modles anatomiques dformables en temps rel (in french)," Ph.D. dissertation, Universit de Nice, France, 1997.

[15] G. Picinbono, H. Delingette, and N. Ayache, "Nonlinear and anisotropic elastic soft tissue models for medical simulation," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2002.

[16] S. Cotin, H. Delingette, and N. Ayache, "Real-time elastic deformations of soft tissues for surgery simulation," *IEEE Transactions on Visualizations and Computer Graphics*, vol. 5, no. 1, pp. 62–73, 1999.

[17] D. C. Tseng and J. Y. Lin, "A hybrid physical deformation modeling for laparoscopic surgery simulation," in *Proceedings of IEEE International Conference of the Engineering on Medicine and Biology Society*, 2000.

[18] S. Cotin, H. Delingette, and N. Ayache, "A hybrid elastic model allowing real-time cutting, deformations and force-feedback for surgery training and simulation," *Journal of Visual Computer*, vol. 16, no. 8, pp. 437–452, 2000.

[19] M. Bro-Nielsen and S. Cotin, "Real-time volumetric deformable models for surgery simulation using finite elements and condensation," in *Proceedings of EACG Conference on Eurographics*, 1996.

[20] S. P. Dimaio and S. E. Salcudean, "Simulated interactive needle insertion," in *Proceedings of International Symposium on Haptic Interfaces for Virtual Environment and Teleoperator System*, 2002.

[21] C. Mendoza and C. Laugier, "Simulating soft tissue cutting using finite element models," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2003.

[22] P. Pao-La-Or, S. Sujitjorn, T. Kulworawanichpong, and S. Peaiyoung, "Studies of mechanical vibrations and current harmonics in induction motors using finite element method," *WSEAS Transactions on Systems*, vol. 7, no. 3, pp. 195–202, 2008.