A Study of two phases heat transport capacity in a Micro Heat Pipe

Cheng-Hsing Hsu¹, Kuang-Yuan Kung²*, Shu-Yu Hu¹, and Ching-Chuan Chang³

¹Department of Mechanical Engineering Chung-Yuan Christian University.
²*Department of Mechanical Engineering Nanya Institute of Technology.

Abstract: Present study modifies Cotter’s model by using the dimensionless liquid flow shape factor, \( K \), to predict the maximum heat transport capacity and to discuss the effects of contact angle. The results indicated that the dimensionless liquid flow shape factor, \( K \), decreases, the friction effects on the vapor-liquid interface flow, \( v_L \), increases, and the liquid flow influenced by the vapor flow also increases. The predicted maximum heat transport capacity agrees well with Babin’s experimental data of a copper-water micro heat pipe under \( v_L = 1 \) and contact angle, \( \alpha = 10^\circ \). In a micro heat pipe, the results indicated that both the maximum heat transport capacity and \( K \) increases with a increasing contact angle \( \alpha \).

Key-Words: Cotter’s model, Contact angle, Heat pipe, Heat transport capacity, Shape factor, Dimensionless

1 Introduction

Heat pipes are widely used in related heat transfer of micro heat pipe is the maximum heat transport capacity, which has been discussed by [7-12]. Faghri [10] theoretically and experimentally investigated both axially grooved heat pipes and flat miniature heat pipes. The maximum heat transfer capability in axially grooved heat pipes is subjected to the geometry of the heat pipes and the contact angle of liquid-solid interface.

The present paper deals with the numerical prediction of Micro heat pipes. It shows the volume flow rate, maximum heat transport capacity and input power of micro capillary grooves under the influence of the shearing stress of interface, contact angle and length. The numerical results are compared to the experimental data of Babin [1] and Ma et al. [6] model.

2 Theoretical Analysis

It is clearly that the vapor flow direction strongly impacts the fluid velocity and behavior of liquid surface[5]. But the shearing stress on the liquid-vapor interface was neglect in Cotter’s model [1], which results in a larger heat transfer than real situation. The fundamental assumptions are two dimensional Newtonian incompressible flow, fully developed laminar vapor flow and no slip condition on liquid-solid interface [6]. The momentum equation can be written as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\mu} \frac{dp}{dz}
\]

(1)

where \( u \) is axial velocity, \( z \) the corresponding axial coordinate, \( P \) the static pressure, \( \mu \) the dynamic
viscosity and $\theta$ groove angle. No slip conditions at the wall are assumed.

$\theta = 0, \quad u = 0$  \hspace{1cm} (2)

$\theta = 2\phi, \quad u = 0$  \hspace{1cm} (3)

In order to obtain a general solution, two boundary conditions in $r$ direction are assumed. A trapezoid shape in liquid region is formed when $r = r_2$, the liquid surface velocity is defined as $\bar{u}_{r_2}$ and zero liquid velocity at $r = r_1$. As $r_1 \to 0$, the solution relates to that obtained for a triangle groove, closed on these three sides.

Based on Ma et al. [6] dimensionless parameters as

$$u^* = \frac{u + r^2}{4\mu}(-dp/dz)$$  \hspace{1cm} (4)

$$x = \ln \frac{r}{r_1}$$  \hspace{1cm} (5)

and $\phi$ is half-channel angle. The geometry on the interface of liquid-vapor depends major on the shape of the liquid flow passage with low Bond number conditions [6]. They also describe radius as:

$$r_1 = r_{a1} \frac{\cos \phi}{\cos (\phi - \theta)}$$  \hspace{1cm} (11)

$$r_2 = r_{a2} \cos \alpha \cos (\phi - \theta) \frac{\sin^2 \phi - \cos^2 \alpha \sin \phi}{\cos (\alpha + \phi)}$$  \hspace{1cm} (12)

Fourier transformation method and superposition is applied to the Laplace governing equation (6), where the exact solution is subjected to fit the boundary conditions given in equations (7)-(10).

The analytical velocity $u$ can be shown as [6]

$$u = \left[ \frac{r_2^2}{4\mu}(-dp/dz) \right] \times$$

$$\left\{ \frac{2}{\pi} \sum_{n=1}^{\infty} \left[ 1 - (-1)^n \right] \frac{H_p \sin \frac{n\pi}{2}\phi \sin \frac{n\pi}{2}\theta}{2\phi} \sin \frac{n\pi}{2}\theta \sin \frac{n\pi}{2}\phi \right. \times$$

$$\left. \left[ \frac{1}{a} \frac{\sin \frac{n\pi}{2}\theta + \sin \frac{2\pi}{a}\phi}{\sin \frac{n\pi}{2}\theta} \right] \frac{1}{a} \frac{\sin \frac{n\pi}{2}\phi + \sin \frac{2\pi}{a}\theta}{\sin \frac{n\pi}{2}\phi} \sin \frac{n\pi}{2}\phi \sin \frac{n\pi}{2}\theta \right\}$$

Where

$$a = \ln \frac{r_2}{r_1}$$  \hspace{1cm} (14)

$$H_p = \frac{H_p}{4\mu}(-dp/dz) \times$$

$$\frac{r_2^2}{4\mu}(-dp/dz)$$  \hspace{1cm} (15)

and $\bar{u}_{r_2}$ is the average velocity of liquid surface at $r = r_2$. This value depends on both the liquid and vapor flow characteristics and properties. Due to an irregular cross-section is shown due to the fractional interaction of liquid flow and vapor flow. It is quite difficult to get a general solution of the average velocity of liquid surface $\bar{u}_{r_2}$. To avoid the rising difficulty of getting analytical solution, assume the liquid surface is free to the influence of frictional influences. The momentum equation can be written under these assumptions

$$\frac{d^2u_f}{dr^2} = \frac{dp}{d\theta} \frac{\mu}{\rho^2 d\theta^2}$$  \hspace{1cm} (16)

where $u_f$ is the free liquid surface velocity without the effects of vapor shearing stress and the corresponding boundary conditions are:

$$u_f = 0 \quad \text{at} \quad \theta = 0$$  \hspace{1cm} (17)

$$u_f = 0 \quad \text{at} \quad \theta = 2\phi$$  \hspace{1cm} (18)

Integrating the momentum equation, the solution of equation (16) with these boundary conditions is:
The average velocity of the liquid flow is preferred in the actual situation, and it is written as:

\[
\bar{u}_f = \frac{\int_0^{\phi} u_f d\theta}{\int_0^{\phi} d\theta}
\]  

Then the one dimensional average velocity of the free liquid surface can be obtained as:

\[
\bar{u}_L = \frac{1}{2\phi} \int_0^{\phi} \left( -\frac{1}{4\mu} \frac{dp}{dz} \right) d\theta
\]

\[
\times \left[ r_{n2} \left\{ \cos \alpha \cos(\phi-\theta) \right\} - \sin^2 \phi - \cos^2 \alpha \sin \phi \sin(\phi-\theta) \right]^{0.5} \cos(\alpha + \phi) \right\} \right] \times (4\phi - 2\phi^2) d\theta
\]  

(21)

Since the one dimensional assumption is insufficient for the actual case, a correction factor is necessary to modify it. This coefficient, \( K \), can be determined experimentally. Utilizing the experimental data of Ayyaswamy et al.\[4\], Ma et al.\[2\] presented that the coefficient \( K \), without notification, is simplified to be the value of 0.52 for a 60° channel angle. But the coefficient \( K \) should vary with the channel apex angle \( \theta \), by neglecting this effect Ma et al.\[2\] assumed the coefficient \( K \) to be a constant.

\[
\bar{u}'_f = K \bar{u}_f
\]  

(22)

where \( \bar{u}_f \) is defined in equation (21). Ma et al.\[3\] defined the actual velocity of the free liquid surface and a dimensionless liquid-vapor interface flow number \( L_v \) as:

\[
L_v = \frac{\Delta \bar{u}}{\bar{u}_f}
\]  

(23)

and

\[
\Delta \bar{u} = \bar{u}'_f - \bar{u}_L
\]  

(24)

and \( \bar{u}_f \) is the average velocity of the free liquid surface.

Ma et al.\[2\] demonstrates \( L_v \) including the correction factor \( K \) which is from Ayyaswamy\[4\] experimental data. But correction factor \( K \) cannot show both of the effects on the liquid-vapor interface and liquid-solid interface. Ma et al.\[2\] define a dimensionless liquid-vapor interface flow number \( L_v \) subjected to the frictional impacts on the liquid-vapor interface. The effects of shear stresses at the liquid surface are taken into consideration, which is due to frictional liquid-vapor interaction on the liquid flow. This paper is based upon Ma et al.\[2\] conclusion by adjusting \( L_v \) to study the impacts of the liquid-vapor interface on rheology.

The average velocity at \( r_2 \) is:

\[
\bar{u}_2 = \bar{u}_f \left( 1 - L_v \right)
\]  

(25)

combine equations (21) and (25), \( \bar{u}_2 \) can be shown as:

\[
- \frac{K(1-L_v)}{\frac{1}{2\phi} \int_0^{\phi} \left( -\frac{1}{4\mu} \frac{dp}{dz} \right) d\theta}
\]

\[
\times \left[ r_{n2} \left\{ \cos \alpha \cos(\phi-\theta) \right\} - \sin^2 \phi - \cos^2 \alpha \sin \phi \sin(\phi-\theta) \right]^{0.5} \cos(\alpha + \phi) \right\} \right] \times (4\phi - 2\phi^2) d\theta
\]  

(26)

From the equation (26), the average velocity of the liquid surface is related to \( \alpha, \phi, \theta, r_{n2}, dp/dz, K \) and \( L_v \). The flow direction of the liquid and the vapor velocity is values of the dimensionless vapor flow number, \( L_v \), which is well explained by Ma et al.\[2\].

In real situation, the value of \( L_v \) can be obtained by the method of Schlichting. The velocity distribution of the liquid fluid in a V groove at any position \( (r, \theta) \) can be evaluated in equation (13), while the value of \( \bar{u}_2 \) is obtained. Furthermore, The volume flow rate, \( Q \), through any crosssection can be expressed as:
3 Results and Discussion

The influence of shearing stress on liquid-vapor interface in a micro heat pipe is predicted. To verify the model predictions, results are compared with the experimental data reported by [1] and the numerical results by [9]. The frictional forces on the liquid-vapor interface could reduce the velocity of liquid flowing from condenser area toward evaporating area. Calculate the volume flow rate of liquid-wick in v-shape groove by analytical method and induce dimensionless parameter of flow factor $L_v$ to inspect the frictional force effects on rheology.

From equation (29), $K_1$ is proportional to volume flow rate $Q$. $Q$ is related to the velocity scale and cross section area $A$, meanwhile the frictional force on the liquid interface induced by the reverse direction between vapor flow and liquid flow, thereafter reduced the liquid velocity.

Operating temperature lies in 40°c – 70°c in Babin et al.[5], and the contact angle on the solid-liquid interface are in 10°c – 50°c. Perterson and Ma [8] presents the dimensionless vapor-liquid interface flow number as $L_v=0$; 0.5; 1; 1.5.

According to Babin[5] experimental data for a square heat pipe, the calculated result of Ha and Perterson [9] reports that the value of $K_v=0.94$, $\beta=1.1343$ and $L_v=12.7\text{mm}, \ L_o=31.6\text{mm}, \ L_e=12.7\text{mm}$.

This research tries to investigate the effects of shearing stress on liquid-vapor interface. Fig.1 presents the relationship between a dimensionless liquid flow factor $K_1$ and the contact angle of solid-liquid interface $L_v$, the $K_1$ changes along with the $L_v$ value. It shows that the reverse steam flowing do have the frictional influences. And it also demonstrated that $K_1$ increases along with the increasing contact angle of solid - fluid. Babin[5] obviously neglects the influence of contact angle by fixed value of $K_1=0.6$ and Cotter[1] simply put $K_1=0.5$. Use the analytical method in present study under the condition $L_v=0$ with contact angle of 10°, and the value of $K_1$ is 0.31, which is less than Cotter’s prediction.

Fig. 2 shows the variation of $K_1$ with the solid liquid contact angle $\alpha$ in a square groove for different values of $L_v$. It is observed that the lower value of $K_1$ will shown as increasing value of $L_v$ under same contact angle $\alpha$. Since the flow direction
of vapor is reversed to the flow direction of liquid, it
tends to induce a frictional force on the liquid-vapor
interface, and the effects of vapor flow reducing the
liquid flow by friction on interface is shown.
It is also shows that the value of \( K_1 \) increase with the
increasing value of contact angle \( \alpha \). The physical
dimensions of the triangular heat pipe are chosen as:
\( L_v = L_a = L_c = 5\, mm \), \( \beta = 1.433 \), \( K_v = 0.86 \). By
using the analytical method in present study, one can
get the values of \( K_1 \). The \( K_1 \) involves a correction
factor comparing to the \( K_1 \) in Ma et al. [10] researches, which is directly derived from Xu[3]. Ma
et al. [10] defines \( K_1 \) as
\[
K_1 = \frac{4\pi c}{K\beta^3}
\]
where
\[
c = 4\epsilon^3 \tan^2 \phi,
\]
\[
c_1 = \frac{1}{\tan \phi} + \phi - \frac{\pi}{2},
\]
\[
\beta = \sqrt{nc_1}
\]  
(31)

The dimensionless liquid flow shape factor, \( K_1 \),is
calculated to be \( K_1 = 0.135 \) by Ma et al.[6, 9], using
the parameter \( K \) induced by the influence on the
solid-liquid contact angle without the impacts of the shearing stress on liquid-vapor interface. Under the
condition of \( \alpha = 10^\circ \); \( L_v = 0 \), (i.e. without the
impacts of the shearing stress on liquid-vapor interface),we can get the value \( K_1 = 0.191 \), which is
lower than Cotter’s predicted \( K_1 \) value.
The maximum heat transport capacity is impacted by
the values of \( K_1 \). The maximum heat transport
capacity presents a similar tendency between Cotter’s prediction [1] and Babin’s[5] experimental data, which is only half the values of Cotter’s prediction.
Fig.3 to Fig.6 show the relationship between the
operational temperature and the maximum heat transfer rate when the round heat pipe is \( \psi = 30^\circ \). The
contact angles \( (\alpha) \) are respectively \( 10^\circ \), \( 20^\circ \), \( 30^\circ \)
and \( 40^\circ \). With changing of the operational temperature \( (T) \), the impacts of different \( L_v \) on the
maximum heat transfer rate are compared. From these
daigrams we learn that when \( L_v \) becomes greater, the
maximum heat transfer rate will decrease. Thereby
we find that the friction shear stress of air flow and
fluid flow will lower the maximum heat transfer rate.

Fig.7 to Fig.10 show the relationship between the
operational temperature and the maximum heat transfer rate when the round heat pipe is \( \psi = 30^\circ \). The
number of flow of the dimensionless liquid-air contact
surface is respectively \( L_v = 0, L_v = 0.5, L_v = 1 \)
and \( L_v = 1.5 \). With changing of the operational
temperature \( (T) \), the values of the maximum heat transfer rate at different contact angles \( (\alpha) \) are
compared. From observing these daigrams we learn that
the higher the contact angle, the greater the maximum heat transfer rate.
Fig.11 to Fig.16 show the relationship between the
operational temperature and the maximum heat transfer rate when the round heat pipe is \( \psi = 30^\circ \).
Respectively the operational temperatures are \( 40^\circ \),
\( 45^\circ \), \( 50^\circ \), \( 55^\circ \), \( 60^\circ \) and \( 65^\circ \). With changing of the
contact angles \( (\alpha) \), the values of the maximum heat transfer rate at different operational temperature \( (T) \)
are compared. From these daigrams we can tell that
the higher the operational temperature, the greater the
maximum heat transfer rate.
Comparing the outcomes above with Babin
experiment and Cotter prediction, we can see that as
the contact angles \( (\alpha) \) increases, the thermal
performance is enhanced; that as number of flow of the
dimensionless liquid-air contact surface \( L_v \)
increases, the thermal performance will be compromised; that as the operational temperature \( (T) \)
rises, the thermal performance will be improved.
Fig.17 presents the relationship between the operation
temperature and the maximum heat transport capacity
for a square micro heat pipe under different values of \( L_v \). The contact angle of solid-liquid interface
is \( 10^\circ \). The results presents comparison of the
maximum heat capacity with the numerical results
given by Cotter and the experimental results given by
Babin[5]. Fig.17 shows the maximum heat capacity
increase with increasing value of the contact angle of
the solid-liquid interface, but it decreased with higher
values of \( L_v \). The shearing stress induced by the
friction force on the liquid-vapor interface will reduce
the maximum heat transport capacity. In other word,
the reverse direction between the flow direction of
liquid and vapor will hinder the maximum heat transport capacity of the heat pipe.
Fig.18 to Fig.20 show the relationship between the
included angle \( (\psi) \) of the trough opening and the
maximum heat transfer rate when the operational
temperature of the round heat pipe is \( T = 40^\circ \). The
contact angles \( (\alpha) \) are respectively \( 20^\circ \), \( 30^\circ \) and
\( 40^\circ \).
Fig. 21 to Fig. 26 show the relationship between the included angle ($\psi$) of the trough opening and the maximum heat transfer rate when the contact angles of the round heat pipe is $\alpha = 10^\circ$. Respectively the operational temperatures are $40^\circ$, $45^\circ$, $50^\circ$, $55^\circ$, $60^\circ$ and $65^\circ$.

Fig. 27 to Fig. 29 presents the relationship between the operation temperature and the maximum heat transport capacity for a square micro heat pipe under different values of the contact angle on solid-liquid interface with $L_v = 0; 0.5; 1; 1.5$. It shows the maximum heat capacity increase with increasing value of the contact angle of the solid-liquid interface. The maximum heat transport capacity is coincide with the experimental results of Babin [5] under $L_v = 1.5 \cdot \alpha = 10^\circ$, and the same result of the onset of dryout is presented as the experimental data of Babin[5] under $\alpha = 20^\circ$. For circular heat pipe when $\psi = 13^\circ$ (groove angle $26^\circ$) may obtain the biggest thermal properties.

4 Conclusion

A mathematical model in square grooves with vapor flow crossing the liquid-vapor interface in micro-heat pipes is developed [10]. Under the same contact angle of liquid-solid interface and groove angle, the maximum heat transfer rate and $K_1$ will decrease with increasing values of $L_v$.

The fractional force on the liquid-vapor interface will slow down the liquid velocity and lower the maximum heat transfer rate and the values of $K_1$. Under the same dimensionless parameter of liquid-solid interface $L_v$ and groove angle, the maximum heat transfer rate and $K_1$ will increase with increasing values of contact angle $\alpha$.

Acknowledgement:
The present investigation was supported by National Science Council of Taiwan under granted NSC-97-2221-E-253-014.

References:

Fig. 1 Dependence of $K_1$ on the contact angle $\alpha$ for various $L_v$ with Square shape heat pipe.

Fig. 2 Dependence of $K_1$ on the contact angle $\alpha$ for various $L_v$ with equilateral triangle heat pipe.

Fig. 3 The maximum heat transport capacity versus operation temperature for various $L_v$ and $\alpha=10^\circ$.

Fig. 4 The maximum heat transport capacity versus operation temperature for various $L_v$ and $\alpha=20^\circ$.

Fig. 5 The maximum heat transport capacity versus operation temperature for various $L_v$ and $\alpha=30^\circ$. 
Fig. 6 The maximum heat transport capacity versus operation temperature for various $L_v$ and $\alpha=40^\circ$.

Fig. 7 The maximum heat transport capacity versus operation temperature for various $\alpha$ and $L_v=0$.

Fig. 8 The maximum heat transport capacity versus operation temperature for various $\alpha$ and $L_v=0.5$.

Fig. 9 The maximum heat transport capacity versus operation temperature for various $\alpha$ and $L_v=1$.

Fig. 10 The maximum heat transport capacity versus operation temperature for various $\alpha$ and $L_v=1.5$.

Fig. 11 The maximum heat transport capacity versus contact angle for various $L_v$ and $T=40^\circ$.
Fig. 12 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=45^\circ$.

Fig. 13 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=50^\circ$.

Fig. 14 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=55^\circ$.

Fig. 15 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=60^\circ$.

Fig. 16 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=65^\circ$.

Fig. 17 The maximum heat transport capacity versus channel angle for various $L_y$ and $\alpha =10^\circ$. 

Fig. 12 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=45^\circ$.

Fig. 13 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=50^\circ$.

Fig. 14 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=55^\circ$.

Fig. 15 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=60^\circ$.

Fig. 16 The maximum heat transport capacity versus contact angle for various $L_y$ and $T=65^\circ$.

Fig. 17 The maximum heat transport capacity versus channel angle for various $L_y$ and $\alpha =10^\circ$. 

ISSN: 1790-0832 1112 Issue 7, Volume 6, July 2009
Fig. 18 The maximum heat transport capacity versus channel angle for various $L_p$ and $\alpha=20^\circ$.

Fig. 19 The maximum heat transport capacity versus channel angle for various $L_p$ and $\alpha=30^\circ$.

Fig. 20 The maximum heat transport capacity versus channel angle for various $L_p$ and $\alpha=40^\circ$.

Fig. 21 The maximum heat transport capacity versus channel angle for various $L_p$ and $T=40^\circ$.

Fig. 22 The maximum heat transport capacity versus channel angle for various $L_p$ and $T=45^\circ$.

Fig. 23 The maximum heat transport capacity versus channel angle for various $L_p$ and $T=50^\circ$. 
**Fig. 24** The maximum heat transport capacity versus channel angle for various \( L_y \) and \( T=55^\circ \).

**Fig. 25** The maximum heat transport capacity versus channel angle for various \( L_y \) and \( T=60^\circ \).

**Fig. 26** The maximum heat transport capacity versus channel angle for various \( L_y \) and \( T=65^\circ \).

**Fig. 27** The maximum heat transport capacity versus channel angle for various \( T \) and \( L_y=0 \).

**Fig. 28** The maximum heat transport capacity versus channel angle for various operation temperature \( T \) and \( L_y=0.5 \).

**Fig. 29** The maximum heat transport capacity versus channel angle for various \( T \) and \( L_y=1 \).