# Rescue Plan, Bank Interest Margin and Future Promised Lending: An Option-Pricing Model 

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#### Abstract

This paper examines a bank rescue plan for future lending. We demonstrate that an increase in the loans guaranteed by the government or in bank responsible for the first stake of any losses results in an increased interest margin. Eventually, the plan will be lifted when bank becomes healthy. The bank will keep its promise to increase its future lending at a reduced margin.


Key-Words: Bank Rescue Plane, Interest Margin, Future Lending

## 1. Introduction

There is a recent Britain's bank

[^0]rescue plan: "To ... prod the bank into lending more, the government will guarantee assets on RBS's [Royal Bank of Scotland's] books with a value of $£ 302$ billion ...
about a quarter of its total risk-weighted balance-sheet. RBS will be responsible for the first $£$ 19.5 billion of any losses, and the state for most of the rest. ... It [RBS] also promises to lend £50 billion more in the next two years, ..." (Economist, February $28^{\text {th }} 2009$, p. 56)

In this paper, we develop an option-pricing model of bank behavior that is used to study bank interest margin with bank rescues. ${ }^{1}$ We address questions such as: Can bank loans guaranteed by the government lead to high bank interest margin? Can bank responsible for the first stake of any losses lead to high bank margin? What are the most likely effects of the rescue plan on bank future lending?

The answer to the first question is yes. With government help, the bank now has a margin with a less risk base. One way the bank may attempt to augment its margin is by decreasing its lending amount at an increased loan rate. With regard to the second question, the answer is again yes. With bank rescues, the bank now has a margin with a

[^1]subsidy. The bank's attempt follows a similar argument as in the case of loan guarantee. With regard to the third question, we argue that the rescue plan will be lifted eventually at least in the longer run when the bank becomes healthy, and then the bank can keep its promise to increase its future lending at a reduced margin.

The banking industry is experiencing a renewed focus on retail banking, a trend often attributed to the stability and profitability of retail activities (Hirtle and Stiroh, 2007). Banks in retail activities are institutions that engage in two distinct types of bank activities, taking deposits and making loans. Taking deposits involves issuing claims that are risk-free and demandable, that is, claims that can redeemed for a fixed value at any time. Making loans involves the acquisition of costly information regard opaque borrowers, and extending credit based on this information. Risk-assets and risk-free deposits can differ due to accounting conventions with respect to the timing and magnitude of revenues and costs, and are not necessarily based on identical proposals. Risks then demonstrate the potential adverse impact on profitability of
possible uncertainty.

Our primary emphasis is the selection of the bank's optimal interest margin in the retail banking. The interest margin is defined as the difference between the rate of interest the bank charges borrowers and the rate the bank pays to depositors. In particular, a bank interest margin is managed through a "cost of goods sold" approach in which risk-free deposits are the "materials" and loans and investment are the "work in process" (see Finn and Frederick, 1992). As the interest margin is so important to bank profits and risks, we use the approach by providing a model of bank behavior that integrates the risk considerations of portfolio-theoretic approach with the market conditions and loan rate-setting behavior mode of the firm-theoretic approach.

Ho and Saunders (1981), McShane and Sharpe (1985), and Allen (1988) have provides models of bank interest margin determination based on the bid-ask spread setting of Stoll (1978). Zarruk and Madura (1992), and Wong (1997) also provide firm-theoretic models to explain bank interest margin behavior. Unlike previous formulations, the
model developed here assumes an integrated portfolio-firm-theoretic setting in which the bank is subject to prevailing secure plan. The principal advantage of this integrated is the explicit treatment of uncertainty which has long played a prominent role in discussions of bank behavior. In addition, the effect of bank margin behavior is that liquidity considerations, which have also played a prominent role in discussions of bank behavior, are incorporated. In light of previous work, the purpose of this paper is to develop an integrated portfolio-firm-theoretic model of bank behavior to answer the three questions mentioned as above.

This paper is organized as follows. Section 2 develops the basic framework of the model. Section 3 derives the solution of the model and the comparative static analysis. The final section concludes.

## 2. The Model

We analyze bank interest margins in a single period model. At the start of the period, the bank raises $D$ in deposits and $K$ in equity capital. The bank provides
depositors with a market rate of return equal to the risk-free rate, $R_{D} . \quad K$ is tied by regulation to be a fixed proportion, $q$, of the bank's deposits, $K \geq q D$. Following Zarruk and Madura (1992), we assume that the required capital-to-deposits ratio, $q$, is an increasing function of the amount of the loans, $L$, held by the bank at the start of the period, $\partial q / \partial L=q^{\prime}>0$. Thus, when the capital constraint is binding, the bank's liquidity constraint is given by

$$
\begin{equation*}
L=D+K=K\left(\frac{1}{q}+1\right) \tag{1}
\end{equation*}
$$

The loan demand faced by the bank is specified as $L\left(R_{L}\right)$. We assume that the bank has some market power in lending (see Zarruk and Madura, 1992), which implies that loan demand is a downward-sloping function of loan rate, $\partial L / \partial R_{L}<0$. Empirical evidence by Hancock (1986) supports the presence of rate-setting behavior in loan markets.

In a bank rescue plan, the government will guarantee a part of loan repayments on the bank's books with a value of $\theta\left(1+R_{L}\right) L$, $0<\theta<1$. The bank will be responsible for the first portion of
any losses, and the government for most of the rest. ${ }^{2}$ At the end of the period, an audit takes place to determine the bank's asset portfolio and assess its current market value under the rescue plan. To gain the essence of the rescue plan, the bank's objective is to set $R_{L}$ to maximize the Black-Scholes (1973) formulas defined in terms of profits. Furthermore, the profit function is a function of the expected return when a part of loan repayments guaranteed by the government and the first portion of any losses paid by the bank.

First, in the formula, the value of the bank's equity can be viewed as a call option on its non-guaranteed loan repayments, $V=(1-\theta)\left(1+R_{L}\right) L$. The strike price of the call option is the book value of the bank's net liabilities defined as the difference between the total promised interest payments to depositors and the amount of repayments from the risk-free guaranteed loans, $Z=\left(1+R_{D}\right) D-\theta\left(1+R_{L}\right) L \quad$, respectively. ${ }^{3}$ With this approach,

[^2]the market value of $V$ follows a geometric Brownian motion. The dynamic processes of $V$ and $Z$ follow as below, respectively:
$$
\frac{d V}{V}=\mu d t+\sigma d W
$$
\[

$$
\begin{equation*}
\frac{d Z}{Z}=\delta d t \tag{3}
\end{equation*}
$$

\]

where $\mu$ and $\sigma$ are the deterministic drift and the deterministic volatility of $V$, respectively. $\delta$ is the spread between $R$ and $R_{D}$, where $R$ can be defined as the security-market interest rate, an opportunity cost of liquidity management in the liquidity constraint. A standard Wiener process is $W . \quad t$ is the length of the single period.

The first part of the bank's profits will then be given by the Black and Scholes (1973) formula for the call option:

$$
\begin{equation*}
S_{1}=V N\left(d_{1}\right)-Z e^{-\delta} N\left(d_{2}\right) \tag{4}
\end{equation*}
$$

where
changes in the parameters of the model. Mullins and Pyle (1994) use the same approach. However, if the customer acceptance is explicitly considered, the administrative costs are required (see Asosheha, Bagherpour, and Yahyapour, 2008)
$d_{1}=\frac{1}{\sigma}\left(\ln \frac{V}{Z}+\delta+\frac{1}{2} \sigma^{2}\right)$
$d_{2}=d_{1}-\sigma$
$N(\cdot)=$ the cumulative standard normal distribution.

Second, we define $S_{2}$ as the Black and Scholes’ (1973) value of the put option, written on $V$ and with a strike price equal to $Z$, which the insurer (the government) has effectively written to the bank's equity holders. The reason is that if $V$ is less than $Z$, the insurer pays out $Z-V$. However, the bank is requested to pay a part of any losses first under the rescue plan. The second part of the bank's profits will then given by the option-pricing formula for the put option:

$$
\begin{equation*}
S_{2}=\alpha\left[Z e^{-\delta} N\left(-d_{2}\right)-V N\left(-d_{1}\right)\right] \tag{5}
\end{equation*}
$$

where

$$
0<\alpha<1
$$

With this setting, the market value of equity $S$ will then be given by the Black and Scholes' (1973) formulas for the call option minus the put option:

$$
\begin{equation*}
S=S_{1}-S_{2} \tag{6}
\end{equation*}
$$

## 3. Solutions and Results

Partially differentiating equation (6) with respect to $R_{L}$, the first-order condition is given by

$$
\begin{align*}
\frac{\partial S}{\partial R_{L}}= & (1-\alpha)\left(\frac{\partial V}{\partial R_{L}} N\left(d_{1}\right)+V \frac{\partial N}{\partial d_{1}} \frac{\partial d_{1}}{\partial R_{L}}\right. \\
& \left.-\frac{\partial Z}{\partial R_{L}} e^{-\delta} N\left(d_{2}\right)-Z e^{-\delta} \frac{\partial N}{\partial d_{2}} \frac{\partial d_{2}}{\partial R_{L}}\right) \\
& +\alpha\left(\frac{\partial V}{\partial R_{L}}-\frac{\partial Z}{\partial R_{L}} e^{-\delta}\right)=0 \tag{7}
\end{align*}
$$

where

$$
\begin{aligned}
& V \frac{\partial N}{\partial d_{1}} \frac{\partial d_{1}}{\partial R_{L}}=Z e^{-\delta} \frac{\partial N}{\partial d_{2}} \frac{\partial d_{2}}{\partial R_{L}} \\
& \begin{aligned}
& \frac{\partial V}{\partial R_{L}}=(1-\theta)\left[L+\left(1+R_{L}\right) \frac{\partial L}{\partial R_{L}}\right]<0 \\
& \frac{\partial Z}{\partial R_{L}}=-\frac{\left(1+R_{D}\right) K q^{\prime}}{q^{2}} \frac{\partial L}{\partial R_{L}} \\
& \quad-\theta\left[L+\left(1+R_{L}\right) \frac{\partial L}{\partial R_{L}}\right]
\end{aligned}
\end{aligned}
$$

Consider next the impact on the bank's loan rate (and thus on the bank's margin) from changes in the government's guarantee and the bank's responsibility. First, implicit differentiation of equation (7) with respect to $\theta$ yields:

$$
\begin{equation*}
\frac{\partial R_{L}}{\partial \theta}=-\frac{\partial^{2} S}{\partial R_{L} \partial \theta} / \frac{\partial^{2} S}{\partial R_{L}^{2}} \tag{8}
\end{equation*}
$$

where

$$
\begin{aligned}
\frac{\partial^{2} S}{\partial R_{L} \partial \theta}=(1-\alpha)\left[\frac{\partial^{2} V}{\partial R_{L} \partial \theta}\right. & N\left(d_{1}\right) \\
& -\frac{\partial^{2} Z}{\partial R_{L} \partial \theta} e^{-\delta} N\left(d_{2}\right) \\
& \left.+\frac{\partial V}{\partial R_{L}}\left(\frac{\partial N}{\partial d_{1}}-\frac{N\left(d_{1}\right)}{N\left(d_{2}\right)} \frac{\partial N}{\partial d_{2}}\right) \frac{\partial d_{1}}{\partial \theta}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\partial^{2} V}{\partial R_{L} \partial \theta}=\frac{\partial^{2} Z}{\partial R_{L} \partial \theta}=-\left[L+\left(1+R_{L}\right) \frac{\partial L}{\partial R_{L}}\right]>0 \\
& \frac{\partial d_{1}}{\partial \theta}=-\frac{\left(1+R_{L}\right) L}{\sigma}\left(\frac{1}{V}-\frac{1}{Z}\right)<0
\end{aligned}
$$

Before proceeding with the analysis of the comparative static results in equation (8), we define the term
$\left(\partial N / \partial d_{1}\right)-\left(N\left(d_{1}\right) / N\left(d_{2}\right)\right)\left(\partial N / \partial d_{2}\right)$ as the risk elasticity effect that reflects the bank's underlying risk in the call-put options. The sign of this term can be equivalent to the sign of the difference between the
following two terms. First, $1 / b_{1}=\left(N\left(d_{1}\right) / d_{1}\right) /\left(\partial N\left(d_{1}\right) / \partial d_{1}\right)$ represents the reciprocal risk-adjusted factor elasticity of non-guaranteed loan repayments. Second, $1 / b_{2}=\left(N\left(d_{2}\right) / d_{2}\right) /\left(\partial N\left(d_{2}\right) / \partial d_{2}\right)$ represents that elasticity of net-obligation payments. When there is $b_{1}>b_{2}$, a conventional explanation of the negative risk elasticity effect, it demonstrates that the bank has an increasing risk magnitude for $S$. In this case, we say that the bank operates its liquidity management under greater risk since $b_{1}$ is more sensitive than $b_{2}$. When there is $b_{1}<b_{2}$, we say that the bank encounters less risk.

When the bank encounters greater risk, an increase in the guarantee loans on the bank's books by the government increases the bank's loan rate (the bank's margin). As the bank's loans are increasingly guaranteed by the government, the bank now has a return with a lower default base. One way the bank may take advantage of the rescue plan to augment its returns by decreasing its loan portfolio at an increased loan rate. If loan demand is relatively rate-elastic, a less loan portfolio is possible at an increased margin.

As stated earlier, a purpose of the rescue plan is to prod the bank into future lending more. It is reasonable to believe that generous subsidies by the rescue plan is a short-term help. Eventually, the help will be gradually or completely lifted. Thus, setting up such the particular bank rescue plan will be a good move for the loan market.

Second, implicit differentiation of equation (7) with respect to $\alpha$ yields:

$$
\begin{equation*}
\frac{\partial R_{L}}{\partial \alpha}=-\frac{\partial^{2} S}{\partial R_{L} \partial \alpha} / \frac{\partial^{2} S}{\partial R_{L}^{2}} \tag{9}
\end{equation*}
$$

where

$$
\begin{array}{r}
\frac{\partial^{2} S}{\partial R_{L} \partial \alpha}=-\left\{\frac{\left(1+R_{D}\right) K q^{\prime}}{q^{2}} \frac{\partial L}{\partial R_{L}} e^{-\delta} N\left(-d_{2}\right)\right. \\
+\left[L+\left(1+R_{L}\right) \frac{\partial L}{\partial R_{L}}\right] \\
\left.\quad \times\left[\theta e^{-\delta} N\left(-d_{2}\right)+(1-\theta) N\left(-d_{1}\right)\right]\right\}>0
\end{array}
$$

Equation (9) demonstrates that an increase in the bank's responsibility for the first portion of any losses increases the bank's margin. Basically, increases the bank's responsibility discourage its lending activity. In an imperfect
loan market, the bank decreases the amount of loans at an increased loan rate and thus an increased margin. However, a bank rescue plan is a possible "bad bank" solution, which would be a step toward patching up the financial system. The total payments to the bank's losses may be reduced even though the bank is forced to increase its responsibility by decreasing subsidies. Thus, setting up such the rescue plan will be an increased move for bank lending activities.

## 4. Conclusion

The results imply that changes in the bank's rescue plan, such as loan guarantee and bank responsibility for losses, have a direct effect on the bank's optimal interest margin. In particular, increases in both the parameters increase the bank's margin with decreasing its lending activities. However, a bank rescue plan is a short-term plan, which is a possible "bad bank" solution. This solution is expected to be a step toward patching up the lending environment. Setting up such the rescue plan will be an increased move for bank lending activities, at least in the longer run.

Of course, it is recognized that
bank may be reluctant to sell their loans and their most toxic securities under this particular rescue plan. The policy effectiveness of the rescue plan may be reevaluated. Such concerns are beyond the scope of this paper and so not addressed here. What this paper does demonstrate, however, is that banks may take this opportunity to increase its profits currently and increase their lending activities in the near future.

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## References

[1]. Allen, L. (1988), "The Determinants of Bank Interest Margins: A Note," Journal of Financial and Quantitative Analysis, 23, 2, 231-235.
[2]. Asosheha, A., S. Bagherpour, and N. Yahyapour (2008) "Extended Acceptance Models for Recommender System Adaption, Case of Retail and Banking Service in Iran," WSEAS Transactions on Business and Economics, 5, 5, 189-200.
[3]. Black, F., and M. Scholes (1973)
"The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81, 2, 637-659.
[4]. Economist (2009) "Bank Rescues: When Will We See Your Like Again?" Economist, February $28^{\text {th }}, 56$.
[5]. Finn, W. T., and J. B. Frederick (1992) "Managing the Margin," ABA Banking Journal, 84, 4, 50-54.
[6]. Hancock, D. (1986) "A Model of the Financial Firm with Imperfect Asset and Deposit Elasticities," Journal of Banking and Finance, 10, 1, 37-54.
[7]. Hirtle, B., and K. J. Stiroh (2007), "The Return to Retail and the Performance of US Banks," Journal of Banking and Finance, 31, 4, 1101-1133.
[8]. Ho, T. S. Y., and A. Saunders (1981) "The Determinants of Bank Interest Margins: Theory and Empirical Evidence," Journal of Financial and Quantitative Analysis, 16, 4, 581-600.
[9]. Lee, C. Y., and J. H. Cheng (2008) "A Fuzzy AHP Application on Evaluation of High-Yield Bond Investment,"

WSEAS Transactions on Information Science and Applications, 6, 5, 1044-1056.
[10]. McShane, R. W., and I. G. Sharpe (1985) "A Time Series/Cross Section Analysis of the Determinants of Australian Trading Bank Loan/Deposit Interest Margin," Journal of Banking and Finance, 9, 1, 115-136.
[11]. Mullins, H. M., and D. H. Pyle (1994) "Liquidation Costs and Risk-Based Bank Capital," Journal of Banking and Finance, 18, 1, 113-138.
[12]. Shih, M. L., S. H. Lin, S. H. Hsiao, L. M. Huang, C. C. Chiu, and K. Y. Chen (2009) "The Study of the Correlation among Personality Traits, Leadership Competence and Organizational Performance," WSEAS Transactions on Business and Economics, 1, 6, 11-20.
[13]. Stoll, H. R. (1978) "The Supply of Dealer Services in Security Markets," Journal of Finance, 33, 4, 1133-1151.
[14]. Wong, K. P. (1997) "On the Determinants of Bank Interest Margins under Credit and Interest Rate Risks," Journal of Banking and Finance, 21, 2, 251-271.
[15]. Zarruk, E. R., and J. Madura (1992) "Optimal Bank Interest Margin under Capital Regulation and Deposit Insurance," Journal of Financial and Quantitative Analysis, 27, 1, 143-149.


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[^1]:    ${ }^{1}$ Our model cannot be used to discuss a farmer-oriented financial institution (see Shih, Lin, Hsiao, Huang, Chiu, and Chen, 2009)

[^2]:    ${ }^{2}$ Results to be derived from our model do not extend to the care where the rescue plan is related to high-yield bond (see Lee and Cheng, 2008)
    ${ }^{3}$ The administrative costs of loans and deposits and the fixed costs are omitted for simplicity because they will have the same qualitative effect on the optimal loan rate

