## A Fuzzy AHP Application on Evaluation of High-Yield Bond Investment

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*Abstract:* - The returns and risks of high-yield bond (HYB) lie between the stocks and Treasury bonds. In view of investment opportunities and the rate of return, the advantages of HYB are both lower risks and higher shares. Therefore, HYB has become one of important components in the portfolios. The purpose of this study is to identify critical factors related to the selection of HYB. Primary criteria to evaluate HYB selection are obtained by the literatures survey and applying fuzzy Delphi method (FDM), and then fuzzy analytic hierarchy process (FAHP) is employed to calculate the weights of these criteria, so as to build the fuzzy multi-criteria model of HYB investment. The results indicate a greatest weight on the dimension of economic environment, and three critical evaluation criteria related to HYB selection are: (1) spread versus Treasury, (2) bond callability, and (3) default rate indicator.

*Key-Words:* - High-Yield Bond, Portfolio Management, Credit Rating, Financial Failure, Analytic Hierarchy Process (AHP), Fuzzy Delphi Method (FDM), Fuzzy Analytic Hierarchy Process (FAHP).

## **1** Introduction

Facing the shuddering global decline in economy, the universal interest rates are also decreasing, and the outcome is the shrinkage of interest gains as well. In addition, worsened by some non-economic factors, it leads to the stock market plunge in numerous nations. The risk of the stock market has driven the investors to pause and ponder. Hence, for diversification of risks, the portfolio which is able to lead to both acceptable risk tolerance and consistent return becomes the investors' favor.

While high-yield bond (HYB) continuously grows up in size and expands, the practitioners including the investors, the analysts, and the portfolio managers want to evaluate the risk and the return performance of HYB and use appropriate indicators to effectively select the bonds with worthiness. And the researchers try to figure out which of the factors can be used to discriminate good bonds from bad ones such that they can objectively evaluate and predict the solvency of bonds or bond issuers to raise the rate of return.

Nevertheless, previous studies evaluated HYB selection with either financial statistics or macroeconomic variables, and almost none of them took a collection of factors with more complete aspects in account to evaluate HYB selection.

Accordingly, the aim of this paper is to find out a more complete and concerned collection of explanatory variables and identify critical factors of HYB selection from the collection.

In our research, the survey of studies relative to

HYB selection including HYB evaluation, credit rating, and financial failure is made, and the collection of variables are divided into four groups to serve as preliminary evaluation dimensions. They include characteristics of bonds, corporate non-financial factors, corporate financial factors, and economic environment factors.

A questionnaire investigation with two stages is conducted in this study. Potential explanatory variables related to HYB selections are obtained from literature survey and collected to form the first-stage questionnaire. The variables with more concerns by experts' consensus serve as primary evaluation criteria of HYB investment. At second stage, pair-comparison of all criteria is made, and the weight of each individual criterion is calculated.

This paper is organized as follows: the first section is introduction, and literature review is presented in Section 2. Methodology and the analysis of results are illustrated in Section 3, and finally the conclusions are provided in Section 4.

## 2 Literature Review

High-yield bonds are corporate bonds that have been assigned a bond rating as non-investment or speculative grade by major credit rating agencies such as Moody's Investors Service and Standard & Poor's Rating Services.

Rating agencies analyze the capacity for payment of interest and principal by the issuing organization and the specific issue to determine the credit rating. Standard & Poor's as well as Moody's credit rating scales are listed in Table 1.

Table 1. Standard & Poor's and Mo	ody's
Dating Casles	

Ratin	g Scales	
	Standard &	Moody's
	Poor's	
Investment grade	AAA	Aaa
	AA	Aa
	А	А
	BBB	Baa
Speculative grade	BB	Ba
	В	В
	CCC	Caa
	CC	Ca
	С	С
	D	-

When the bond issuers are in higher default risk with higher probability of interest payment and principal in arrears, they have to pay the higher rate of interest as a risk premium to attract the investors. Hence, the high-yield bonds are also referred to as speculative bonds and junk bonds.

#### 2.1 Relative Studies of High-Yield Bond

Most researches related to high-yield bonds focus on the connection of the default risk with financial statistics and macroeconomic environment variables.

Fabozzi [11] considered the following risks should be taken into account when bonds investment: interest rate risk, reinvestment risk, call risk, default risk, inflation risk, foreign exchange rate risk, liquidity risk, volatility risk, and risk of risk.

Huffman and Ward [21] took companies rated in default by Moody's and Standard & Poor's Corporation as the samples and adopted the Logit regression method in four models to conduct the research. Their results indicated a great correlation between key financial variables and the default. Especially growth in assets, change in liquidity, collateralizable assets, and operating profit margin were more important variables in explaining default.

Fridson and Garrman [13] studied the determinants of spreads on new HYBs, and they divided the explanatory variables into two groups as company-specific variables and environmental variables. Company-specific variables were credit-risk rating, seniority, term, callability, zero-coupon status, float, 144a status, whether the bond was from a first-time issuer, and the type of underwriter. Environmental variables included spread versus Treasuries, BB-B spread, yield curve, default rate, volume of initial public offerings, forward calendar, high-yield-bond mutual fund flows, the cash position of high-yield-bond mutual funds, changes in interest rates, and recent high-yield returns.

Moeller and Molina [27] applied an econometric method, proportional hazard model, to analyze how the defaults of high-yield bonds evolve over time. The variables included in model were convertibility, coupon rate, the size of the issue, maturity, credit rating, current assets / current liabilities, earnings before interest and taxes margin (EBITM), fixed assets ratio, returns on assets (ROA), net sales / total assets, total debt / total assets, total assets, spread versus Treasury. Their results indicated that the default risk increased significantly about four years after issuance, especially for nonconvertible bonds.

Altman and Bana [3] tracked the performance of high-yield bonds over 1971 through 2002, and their findings pointed out the correlation of the default with macroeconomic activity, bankruptcy, credit rating change, bond aging, and spreads versus Treasury.

Grammenos and Arkoulis [16] employed variables including spread between yield on HYB and yield on Treasury with the same maturity, credit rating, callability, maturity, float, default rate, security status, 144A status, and one financial leverage (long-term debt / total shareholder's equity) in the regression analysis, and their empirical results suggested that credit rating was the major determinant of pricing high yield bond issues.

Bondt and Ibáñez [7] examined the development of the high-yield corporate bond market in the U.S., UK, and the euro area. Their results suggested that the adoption of high-yield bonds was significantly affected by industrial production index and a number of macroeconomic variables including leveraged buy-outs, stock market index, mergers and acquisitions, and spread between the yield on speculative grade and BBB-rated bonds.

Grammenos et al. [15] included a set of microeconomic, macroeconomic, and industry related factors in their pricing model. Their results were in favor of the following variables including credit rating, term-to-maturity, changes in earnings, the yield on 10-year Treasury bonds, and the yield on the Merrill Lynch single-B index.

#### 2.2 Relative Studies of Credit Ratings

Credit rating means the rating of credit status and solvency. The statistical methods are applied to set rating criteria by quantifying credit attributes of rated objects and calculating their rating grade. And the rating results are provided to the issuers, investors, and stakeholders. Specifically, the purpose of credit rating is to measure the credit risk rather than other investment risks.

The Top-Down approach is adopted in the process of rating by both S&P's and Moody's Corporation. For example, the Moody's Corporation first evaluates the nation risk of the bond issuers to determine their upper bound of the rating grade, and then extends to the industry they belong to whose attributes include industry history, market maturity, business cycle, size, development, competition, and limitation. And the business risk of the bond issuers is measured by both the quantitative and qualitative approaches.

The qualitative analysis focuses on the data of the issuing company itself and the content relative to the issue contract, such as the company's regulations, past record of interest payment, capability of the authority, the morality, the market share, the position in industry, the impact by business cycle, the industrial perspective, value of the mortgages, and the ways of repayment. As to the quantitative analysis, the issuer's operational condition and its solvency are introduced, such as asset protection, financial resource, profitability. The S&P's Corporation evaluates the ordinary industries by two kinds of risks as business risks (industry properties) and financial risks (financial properties). The business risks include the position of the issuing company (indicators such as market segmentation, technique, and efficiency), administrative capacity, and competitiveness. And the financial risks include financial policy, profitability, capital structure, cash flow, and financial flexibility.

Horrigan [18] applied a multiple regression model to build up the bonds evaluation model, and classified the financial ratios into four groups: long-term and short-term capital turnover, long-term payment ratio, and profitability to serve as the evaluation variables. The empirical results indicated an accuracy rate of 58% at the prediction of Moody's credit ratings while an accuracy rate of 52% at the prediction of S&P's credit ratings.

Pinches and Mingo [30] employed the factor analysis to introduce seven factors which were earnings stability, company size, financial leverage, long-term capital intensity, rate of returns, debt stability, and short-term capital intensity into the discrimination model. The results presented an accuracy rate of 69.7% for original samples and an accuracy rate of 64.58% for validation samples.

Goh and Ederinton [14] explored the reaction of stock market on the change duration of bonds upgrading and downgrading. The results proved a correlation between stock market and ratings change, that is, a stronger impact on the stock market by the magnitude of downgrading at low-ratings than at high-ratings.

Syau et al. [36] proposed a fuzzy financial grading model with the parameters related to corporate financial conditions. The variables they used were divided into four categories: ability to pay (solvency), financial structure, earning ability, and management ability. Within the category of solvency, there were two variables: quick ratio (current assets investor / current liabilities) and current ratio (current assets / current liabilities). As to the category of financial structure, debt-equity ratio (total debt / equity) and fixed long-term turnover (fixed assets + long term investment / equity + long term liabilities) were included.

The category of earning ability was comprised of three variables: expense ratio (expenses / sales), profit margin (net income / sales), and return on equity (ROE) (net income / equity). Within the category of management ability, it contained three variables: inventory turnover (sales / inventory), receivables turnover (sales / receivables), and total assets turnover (sales / total assets).

Alfonso [1] considered six variables appear to

be the most relevant to determining a country's credit rating: gross domestic product (GDP) per capita, external debt, level of economic development, default history, real growth rate, and inflation rate.

The evidence obtained from the research of Grunert et al. [17] suggested that the combined use of financial and non-financial factors leads to a more accurate prediction of future default events than the single use of each of these factors. Six financial factors they used were logarithm of total assets, equity-to-assets ratio, current ratio, cash flow-to-net liabilities, capital intensity ratio, and return on assets (ROA). And non-financial factors were comprised of two variables: management quality and market position.

Amira [4] examined the determinants of sovereign Eurobonds yield spreads with macroeconomic and security specific variables, and the results suggested that maturity, the issue size, and gross fees are positively related to the yield spread, while credit rating and the number of managers decreased the yield spread.

#### **2.3 Relative Studies of Financial Failure**

Since Beaver [6] had initiated the prediction of financial failure by financial ratios, many scholars made their efforts to build the prediction model of financial failure in order to find out a fitter model to predict financial failure. Ohlson [29] analyzed the properties of 105 bankrupt companies by the dimensions of financial ratios, company size, financial structure, operational performance, and liquidity.

Flagg et al. [12] considered four potential failure events including reductions in dividends, "going concern" qualified opinions, troubled debt restructurings, and violations of debt covenants, and six financial ratios which were current ratio, cash flows / total assets, total debt / total assets, net earnings / total assets, retained earnings / total assets, and log of total assets, to predict bankruptcy. The results suggested two events (reductions in dividends and "going concern" qualified opinions) and four ratios (current ratio, total debt / total assets, net earnings / total assets, retained earnings / total assets) were significant factors.

Jones and Hensher [24] predicted firm financial distress with a mixed Logit model including the following variables: new economy sector, finance sector, resources sector, net operating cash flow / total assets, cash flow cover, cash resources / total assets, total debt / gross operating cash flow, total debt / total equity, working capital / total assets, and sales / total assets.

Sun [35] collected the following variables to predict bankruptcy: net income / total assets, current assets / sales, current assets / current liabilities, current assets / total assets, cash / total assets, long-term debt / total assets, natural logarithm of sales, and so on.

#### 2.4 Relative Studies of Fuzzy Delphi Method and Fuzzy Analytic Hierarchy Process

Delphi method is a technique for structuring an effective group communication process by providing feedback of contributions of information and assessment of group judgments to enable individuals to re-evaluate their judgments. Since its development in the 1960s at Rand Corporation, Delphi method has been widely used in various fields.

In order to deal with the fuzziness of human participants' judgments in traditional Delphi method, Ishikawa et al. [23] introduced fuzzy set theory proposed by Zadeh [38] into the Delphi method to improve time-consuming problems such as the convergence of experts' options presented by Hwang and Lin [22]. Fuzzy set theory is increasingly applied in many researches such as by Caballero et al. [9] and by Lin et al. [26].

Analytic hierarchy process (AHP) method was developed by Saaty [33]. It is a powerful method in solving complex decision problems. AHP method assists the analysts to organize critical aspects of a problem into a hierarchical structure similar to a family tree. By reducing complex decisions to a series of simple pair-wise comparisons and rankings, then synthesizing the results, AHP method not only facilitates arriving at the best decision, but also provides a clear rationale for the choices made.

Suh [34] proposed two-phased DSS using the concept of AHP method to assist financial managers in evaluating proposals for strategic and long-range planning. Wu et al. [37] employed experimental simulation to generate the weights of the judgers and compared the aggregation methods in the analytic hierarchy process (AHP) approach.

Hence, AHP approach has been widely applied in various relative fields to solve the decision-making problems with multiple hierarchies under the situation of uncertainty. Antón et al. [5] and Oddershede et al. [28] also employ the AHP method to solve their decision-making problems.

Nevertheless, due to the defect of traditional AHP application presented by Buckley [8] such as the characteristics of subjectiveness, fuzziness, and imprecision, some researchers such as Ruining and Xiaoyan [32] incorporated fuzzy theory into AHP method to improve its application.

Thus, fuzzy analytic hierarchy process (FAHP) method is adopted increasingly by researchers. Hsieh et al. [19] employed fuzzy analytic hierarchy process (FAHP) method to solve the problem of planning and design tenders selection in public office building. And FAHP method was also applied in the research of Chen et al. [10] to evaluate expatriate assignments.

## 3 Methodology and the Analysis of Results

In this study, due to fuzziness existed in decision-making process of evaluating HYB selection by human experts, we decide to adopt fuzzy Delphi method (FDM) to generate primary evaluation criteria of HYB selection, and employ fuzzy analytic hierarchy process (FAHP) method to calculate the weight of each individual criterion so as to establish the fuzzy multi-criteria model of HYB selection.

Relative to the selection of explanatory variables, Altman [2] selected the variables according to (1) adopted by general literatures, (2) variables related to the purpose of the conducted study. Successive researches mostly followed Altman's rules. Our study also conforms to these rules and combines these two methodologies, FDM and FAHP.

Explanatory variables directly related to our study's purpose are included through the survey of relative literatures about high-yield bonds investment. Then we compare the above selected variables with those variables used in relative studies of credit ratings and financial failures respectively in pairs to obtain potential evaluation criteria with significant importance.

Two-stage questionnaire investigation is conducted in this study by fuzzy Delphi method (FDM) and to select the experts with the experience of international bonds investment from the trust companies, the bonds agencies, relative divisions of banks, and the stock underwriters to form the experts group as the questionnaire subjects.

At first stage, the questionnaire is designed in a fuzzy semantic differential scale to provide for chosen experts to rate the importance of explanatory variables in the form of triangular fuzzy numbers. And potential evaluation criteria of HYB investment are obtained when reaching a consensus in determining the importance of those variables.

At second stage, the statistic results are provided to these experts, and pair-comparison of selected evaluation criteria is made, then the weight of each individual criterion is calculated by fuzzy analytic hierarchy process (FAHP) method. Hence, a fuzzy multi-criteria model of HYB investment is established through the process of two-stage treatment. The detail of the process is illustrated as follows.

## 3.1 Choosing the Experts

This study focuses on the analysis of evaluation criteria of HYB selection. Thus the experts chosen are the professionals in the fields related to our research with the experience of international bonds investment from the trust companies, the bonds agencies, relative divisions of banks, and the stock underwriters.

Besides, these experts should have at least 5 years of working experience with the market investment experience, and their positions are at least the rank of assistant managers.

Robbins [31] considered that the number of experts between 5 and 7 was appropriate, and therefore we select seven experts. Moreover, in order to ensure the consistency of data collection, the identical experts group is adopted at both first stage and second stage as the questionnaire subjects.

## 3.2 Determining the Evaluation Criteria

The fuzzy Delphi method (FDM) is employed to explore the important criteria of HYB selection, and the process is listed as follows.

#### **Step 1.** Building the Evaluation Criteria

At first stage, through the literature survey of high-yield bonds, credit ratings, and prediction model of financial failures, we obtain four primary dimensions: "characteristics of bonds", "corporate non-financial factors", "corporate financial factors", and "economic environment factors".

At this stage, 5 bond characteristics, 6 corporate non-financial factors, 25 corporate financial factors, and 11 economic environment factors are totally chosen, and corporate financial factors are further classified into six categories according to the classification of domestic Prospectus as solvency, operational efficiency, profitability, financial structure, growth potential, and cash flow.

#### Step 2. Collecting the Experts' Opinions

Selected experts are asked to answer the questionnaire in a 9-point fuzzy semantic differential scale of "absolutely important", "very important", "pretty important", "quite important", "no comment", "fairly unimportant", "quite unimportant", "very unimportant", and "absolutely unimportant".

And selected experts assign a relative importance to every collected variable with respect to four dimensions of bonds characteristics, corporate non-financial and financial factors, and economic environment factors in order to identify critical factors as the evaluation criteria of HYB selection.

#### Step 3. Applying the Fuzzy Delphi Method to Select Important Evaluation Criteria

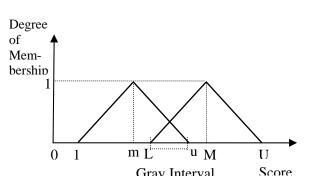
1. Establishing the Triangular Fuzzy Function All experts' estimations gathered by prior step are used to establish the triangular fuzzy function of each individual criterion through the process of fuzzy Delphi method (FDM) proposed by Ishikawa et al. [23]. The process of application is as follows:

- (1) The elements of evaluation set are determined by expert questionnaires of high-yield bonds selection. Given a score of 100 and 0 to the traditional binary logics of "absolutely important" and "absolutely unimportant" respectively, the other elements of evaluation set are quantified objectively through the treatment of fuzzy Delphi method.
- (2) The questionnaires are designed for the elements of evaluation set other than "absolutely important" and "absolutely unimportant", and selected experts are invited to fill the quantitative score interval of every element in the evaluation set. The maximum of interval value is the experts' most optimistic cognition of the quantitative score for the element, and the minimum of interval value is the experts' most conservative cognition of the quantitative score for the element.
- (3) Solving the minimum L, geometric mean M, and the maximum U of all experts' most optimistic cognition score for each individual element, along with the minimum l, geometric mean m, and the maximum u of all experts' most conservative cognition score for each individual element, respectively.

Triangular fuzzy number  $A = (L, M, U)_{L-R}$  of all experts' most optimistic cognition for each individual element and triangular fuzzy number a = (1, m, m)u)<sub>L-R</sub> of all experts' most conservative cognition for each individual element are established respectively and illustrated in Figure 1.

2. Analyzing the Value of Triangular Fuzzy Function

To organize and analyze the expert questionnaires collected at first stage, triangular fuzzy function with respect to every potential variable is established as shown in Table 2.



Gray Interval

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Fig. 1. Triangular fuzzy number of the most optimistic cognition and the most conservative cognition

3. Selecting Important Evaluation Criteria

When selecting the evaluation criteria, it is generally considered important if relative importance is greater than 80%. Hence, we calculate the median of gray interval for every potential variable and take 80 as the threshold to filter out those variables with the score of less than 80 on the median of gray interval. Thus, important criteria consistently agreed by selected experts are accordingly obtained.

According to the above filtering treatment, totally thirty variables are eliminated, and one involved dimension, non-financial factors, are also dropped. Seventeen important variables are obtained to serve as primary evaluation criteria of HYB selection. They are listed as follows.

- (1) Characteristics of bonds: bond liquidity, change in credit rating, and bond callability.
- (2) Financial factors: (a) solvency: current ratio, quick ratio, and interest expense rate, (b) profitability: earnings before interest and taxes (EBIT) margin, net profit margin, and return on net worth, (c) financial structure: current assets / total assets ratio, retained earnings / total assets ratio, and total debt / total assets ratio, (d) growth potential: asset growth rate, and (e) cash flow: cash flow / total debt ratio.
- (3) Economic environmental factors: default rate indicator, real interest rate change, and spreads versus Treasury.

## **3.3** Applying the Fuzzy Analytic Hierarchy **Process (FAHP) Method**

We apply the fuzzy analytic hierarchy process (FAHP) method to calculate the weight of each individual dimension (criterion) and subcriterion of HYB selection. The process is listed as follows.

			The Most		The Most	The
Dimensions		Potential	Conservative	Gray	Optimistic	Median
		Variables	Cognition	Interval	Cognition	of Gray
			(min, med, max)		(min, med, max)	Interval
Characteristics		Liquidity	(71,82.23,92)	(92,80)	(80,92.09,100)	86
of Bonds		Duration	(60,70.65,85)	(85,70)	(70,80.05,92)	77.5
		Change in Credit Rating	(71,81.83,92)	(92,80)	(80,92.09,100)	86
		Callability	(68,84.68,92)	(92,81)	(81,95.87,100)	86.5
		Maturity	(55,65.23,85)	(85,68)	(68,75.68,92)	76.5
No	n-financial	Company Size	(55,65.04,85)	(85,65)	(65,75,92)	75
Factors		Mortgage Assets	(51,67.39,82)	(82,60)	(60,78,92)	71
		Volatility of Current Assets	(48,71.72,85)	(85,62)	(62,81.95,92)	73.5
		Operating Profit Margin	(60,75.09,85)	(85,70)	(70,85.97,100)	77.5
		Issuer or Administrator	(45,68,85)	(85,55)	(55,78.54,92)	70
	,	Company Seniority	(45,55.61,71)	(71,55)	(55,67.25,83)	63
	Solvency	Current Ratio	(71,80.32,92)	(92,80)	(80,90.17,100)	86
		Quick Ratio	(69,78.37,92)	(92,80)	(80,87.90,100)	86
		Operating Capital Ratio	(61,72.08,85)	(85,70)	(70,82.42,92)	77.5
		Interest Expense Rate	(68,73.46,80)	(80,80)	(80,85.06,92)	80
-	Operating	Receivables Turnover	(55,73.72,85)	(85,65)	(65,82.73,92)	75
F	Efficiency	Fixed Assets Turnover	(55,66.45,80)	(80,65)	(65,77.26,90)	72.5
i		Total Assets Turnover	(55,66.45,80)	(80,65)	(65,77.26,90)	72.5
n		Equity Turnover	(55,68.50,80)	(80,65)	(65,79.35,90)	72.5
a	Profitability	Gross Profit Margin	(55,64.41,81)	(81,65)	(65,74.89,90)	73
n	-	Operating Profit Margin	(60,71.20,81)	(81,70)	(70,82.82,92)	75.5
c :		EBIT Margin	(68,73.59,81)	(81,81)	(81,84.90,90)	81
i		Net Profit Margin	(69,74.39,82)	(82,81)	(81,86.71,90)	81.5
$\begin{bmatrix} a \\ 1 \end{bmatrix}$		Return on Equity (ROE)	(61,68.18,82)	(82,70)	(70,78.06,90)	76
1	Financial Structure	Return on Assets (ROA)	(61,67.70,71)	(71,70)	(70,78.86,83)	70.5
F		Return on Net Worth	(68,73.43,81)	(81,80)	(80,85.06,92)	80.5
a		Current Assets / Total Assets Ratio	(68,72.35,85)	(85,80)	(80,82.68,92)	82.5
c c		Retained Earnings / Total Assets Ratio		(92,70)	(75,83.96,100)	81
t		Total Debt / Total Assets Ratio	(68,79.33,92)	(92,81)	(81,90.00,100)	86.5
0		ROE / Total Debt Ratio	(55,75.82,92)	(92,65)	(65,86.12,100)	78.5
r		Fixed Assets Ratio	(55,68.89,85)	(85,65)	(65,78.87,92)	75
s	Growth	Asset Growth Rate	(68,75.16,91)	(91,80)	(80,86.87,100)	85.5
	Potential	Net Income Margin	(55,69.60,85)	(85,65)	(65,78.67,92)	75
	Cash Flow	Cash Flow / Total Debts Ratio	(68,79.16,92)	(92,80)	(80,90.17,100)	86
	Cush i low	Cash Flow / Sales Ratio	(55,72.92,92)	(92,65)	(65,82.16,100)	78.5
		Cash Flow / Total Assets Ratio	(55,68.11,92)	(92,65)	(65,78.33,100)	78.5
Fcc	onomic	Default Rate Indicator	(60,75.83,92)	(92,03) (92,70)	(70,85.75,100)	81
	nvironmental	Inflation Rate	(60,72,82)	(92,70) (82,70)	(70,82.62,92)	76
	actors	Leading Index	(45,68.87,85)	(82,70) (85,55)	(55,79.44,100)	70
10		Mutual Fund Flow	(45,57.46,68)	(68,55)	(55,67.86,81)	61.5
		GDP	(40,60.05,78)	(78,54)	(54,72.30,92)	66
		Stock Index	(30,55.18,82)	(82,45)	(45,67.74,90)	63.5
		Real Interest Rate Change	(70,79.67,85)	(85,80)	(80,86.62,92)	82.5
		January Effect	(18,44.28,61)	(61,40)	(40,60.13,70)	50.5
		Zero Coupon Status	(47,59.97,82)	(82,59)	(59,70.09,90)	70.5
		BB-B Spreads	(45,67.33,91)	(91,55)	(55,78.28,100)	73
		Spreads versus Treasury	(69,75.95,91)	(91,80)	(80,86.49,100)	85.5
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<b>Table 2.</b> The Triang	ular Fuzzy Function	on with Respect to	Every Potential Variable
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#### Step 1. Building the Hierarchical Structure

The hierarchical structure is described as follows. The goal is placed at the top of hierarchy, and the general criteria (dimensions) are placed at second level. The subcriteria subordinated to every general criterion (dimension) are placed at third level, and the rest of hierarchy is organized analogically.

In our case, the ultimate goal at the top level is "evaluation of HYB selection", and there are three general criteria (dimensions), "characteristics of bonds", "financial factors", and "economic environmental factors" at second level. As to each individual criterion, there are subordinate subcriteria listed at third level. For example, five subcriteria including "solvency", "profitability", "financial structure", "growth potential", and "cash flow" are subordinated to the general criterion "financial factors".

Moreover, there may be sub-subcriteria subordinated to certain subcriterion, and they will be listed at fourth level. In our case, evaluation criteria "current ratio", "quick ratio", and "interest expense rate" subordinated to the subcriterion "solvency" are sub-subcriteria listed at fourth level. The detail of hierarchical structure is illustrated as Figure 2.

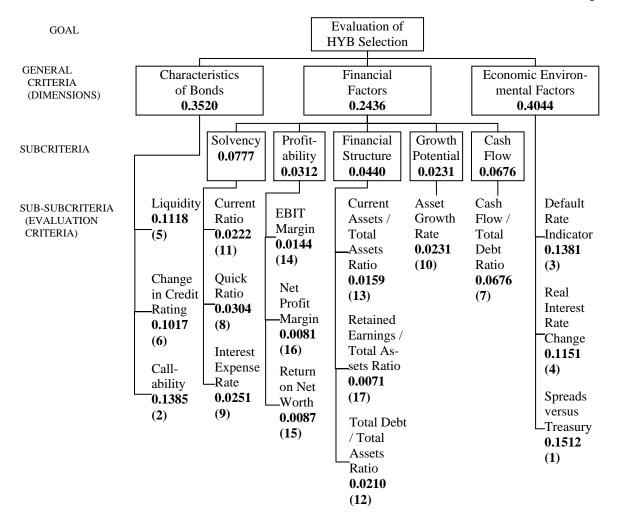


Fig. 2. Hierarchy Structure for Evaluation Criteria of HYB Selection

**Step 2.** Building the Pair-wise Comparison Matrix By the second questionnaires collected from selected experts, we obtain the relative importance of paired evaluation factors at level n+1 under the evaluation of criteria at level n by experts' opinions, and the pair-wise comparison matrix is accordingly built.

**Step 3.** Calculating Triangular Fuzzy Numbers Concerning the relative importance of each individual evaluation factor in pair-wise comparison matrix, triangular fuzzy number is calculated to integrate all experts' opinions. It can be used to present the fuzziness of all experts' opinions with respect to the relative importance of paired factors.

$$\widetilde{\alpha}_{ij} = \left( \alpha_{ij}, \beta_{ij}, \delta_{ij} \right)_{L-R}$$

Where

 $\tilde{\alpha}_{ii}$ : Triangular fuzzy number

 $\alpha_{ii}$ : The minimum of the j-th subcriterion sub-

ordinated to the i-th general criterion

- $\beta_{ij}$ : The geometric mean of the j-th subcriterion
- subordinated to the i-th general criterion  $\delta_{ii}$ : The maximum of the j-th subcriterion sub-

ordinated to the i-th general criterion

L-R: Fuzzy interval of triangular fuzzy numbers

# Step 4. Building the Fuzzy Positive Reciprocal Matrix

After triangular fuzzy numbers are solved to represent the fuzziness of experts' opinions, the fuzzy positive reciprocal matrix A can be further established.

$$A = \begin{bmatrix} \tilde{\alpha}_{ij} \end{bmatrix}$$
$$\tilde{\alpha}_{ij} = \begin{bmatrix} \alpha_{ij}, \beta_{ij}, \delta_{ij} \end{bmatrix}$$

**Step 5.** Calculating the Fuzzy Weights of Fuzzy Positive Reciprocal Matrix

In our study, the method developed by Buckley [8] and improved by Hsu [20] is employed to calculate the fuzzy weights. This method is based on the experts' precise value and synthesizes the experts' opinions with the geometric mean instead of the fuzzy numbers input directly by experts.

Thus, not only the consistency but also the concept of normalization is easily achieved. Through the following formulas, the positive reciprocal geometric mean  $Z_i$  of triangular fuzzy numbers and the fuzzy weight  $\overline{W_i}$  can be obtained.

$$Z_{i} = [\widetilde{\alpha}_{i1} \otimes ... \otimes \widetilde{\alpha}_{in}]^{1/n}, \forall_{i}$$

$$\overline{W_{i}} = Z_{i} \otimes (Z_{1} \oplus ... \oplus Z_{n})^{-1}$$
(1)

$$\widetilde{\alpha}_{1} \otimes \widetilde{\alpha}_{2} \cong \left(\alpha_{1} \times \alpha_{2}, \beta_{1} \times \beta_{2}, \delta_{1} \times \delta_{2}\right)$$
(2)

$$\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 \cong \left(\alpha_1 + \alpha_2, \beta_1 + \beta_2, \delta_1 + \delta_2\right) \tag{3}$$

$$Z_1^{-1} = \left(\delta_1^{-1}, \beta_1^{-1}, \alpha_1^{-1}\right)_{L-R}$$
(4)

$$\widetilde{\alpha}_{1}^{\frac{1}{n}} = \left(\alpha_{1}^{\frac{1}{n}}, \beta_{1}^{\frac{1}{n}}, \delta_{1}^{\frac{1}{n}}\right)$$
(5)

#### Step 6. Defuzzification

Since the weights of all evaluation criteria are fuzzy values, it is necessary to compute a non-fuzzy value by the process of defuzzification. In our study, the Centroid method is employed to defuzzy because of two reasons: (1) the Centroid method is widely used in relative literatures such as Klir's and Yuan's [25], and (2) the solution can be figured out quite quickly. Through the following formulas, the defuzzified weight  $W_i$  can be obtained.

$$W_i = \frac{W_{\alpha i} + W_{\beta i} + W_{\delta i}}{3} \tag{6}$$

 $W_{\alpha i}$ : The right-end value of the fuzzy weight

 $W_{\beta_i}$ : The value of the fuzzy weight with the

degree of membership as 1

 $W_{\delta i}$ : The left-end value of the fuzzy weight

#### Step 7. Normalization

In order to effectively compare the relative importance among evaluation criteria, we normalize the obtained weights using the following formula.

$$NW_i = \frac{W_i}{\sum_{i=1}^{i=n} W_i}$$
(7)

#### Step 8. Synthesis of Hierarchy

The weight of each individual evaluation criterion at bottom level can be obtained by the implementation of step 1 through step 7. And the weights of criteria or subcriteria at upper level are the synthesis of the weights of their subordinations applying the following formula. Hence, the weights of all criteria at every level of hierarchy can be obtained.

$$NW_k = NW_i \times NW_{ip} \tag{8}$$

## **3.4** The Empirical Results

In this research, we apply the FAHP method to calculate the relative importance of criteria and subcriteria on the evaluation of HYB selection. The weights of all criteria and subcriteria along with the ranks of evaluation criteria at the bottom level are calculated and presented in Table 3, and they are also displayed in Figure 2.

Where the obtained weights are the decimals below each individual criterion and subcriterion, and the rank of every evaluation criterion at the bottom level is the number in parentheses below the weight. Accordingly, further explanations are discussed as follows.

#### 3.4.1 Comparison of weights among dimensions

Concerning four dimensions related to the investment of high-yield bonds, there is a greatest weight of 0.4044 on the dimension "economic environmental factors", and the second-greatest weight is on the dimension "characteristics of bonds" of 0.3521. The third-greatest weight is on the dimension "financial factors" of 0.2436, whereas the dimension "non-financial factors" is excluded.

Table 3. The weight and the Rank of Each Individual Criterion						
General Criteria (Dimensions)	Weights of General Criteria	Subcriteria	Weights of Subcriteria	Sub-Subcriteria (Evaluation Criteria)	Weights of Sub- Subcriteria	Rank
· · · · · · · · · · · · · · · · · · ·				Liquidity	0.1118	5
Characteris- tics of Bonds	0.3520			Change in Credit Rating	0.1017	6
ues of bolius				Callability	0.1385	2
		Solvency	0.0777	Current Ratio	0.0222	11
				Quick Ratio	0.0304	8
				Interest Expense Rate	0.0251	9
	0.2436	Profitability	0.0312	Earnings before Interest and Taxes (EBIT) Margin	0.0144	14
				Net Profit Margin	0.0081	16
				Return on Net Worth	0.0087	15
Financial Eactors		Financial Structure	0.0440	Current Assets / Total Assets Ratio	0.0159	13
Factors				Retained Earnings / Total Assets Ratio	0.0071	17
				Total Debt / Total Assets Ratio	0.0210	12
		Growth Potential	0.0231	Asset Growth Rate	0.0231	10
		Cash Flow	0.0676	Cash Flow / Total Debt Ratio	0.0676	7
Economic				Default Rate Indicator	0.1381	3
Environ- mental	0.4044			Real Interest Rate Change	0.1151	4
Factors				Spreads versus Treasury	0.1512	1

**Table 3.** The Weight and the Rank of Each Individual Criterion

The obtained results indicate that selected experts generally consider the dimension "economic environmental factors" a greatest impact on selection of high-yield bonds, and the reason is that the default and the return included in this dimension are considerably important factors related to the evaluation of HYB investments and supported by the most literatures.

Moreover, because of the similarity in nature to equity securities, high-yield bonds are easily influenced by the change of economic environments, and the investigated results by Moody's Invest Service during 1990-1991 also supported a high correlation between the economic environment performance and the default rate of HYB.

The dimension "characteristics of bonds" obtains a second-greatest weight, and it indicates that selected experts lay much stress on the fundamental aspect of HYB selection.

The third-greatest importance is laid on the dimension "financial factors". It is considered by selected experts that the information provided by this dimension is the past message without time effect, and hence the result reflects the least importance on the dimension "financial factors" among all dimensions when evaluation of HYB selection.

As to the exclusion of the dimension "non-financial factors", it indicates that there is a considerable disagreement among experts' opinions on the relative importance of this dimension.

# 3.4.2 Comparison of Criteria or Subcriteria within Every Dimension

1. Analysis of the Characteristics of Bonds

The results indicate that the relative importance of evaluation criteria within this dimension in descending order are "bonds callability" with a weight of 0.1385, "bonds liquidity" with a weight of 0.1118, and "change in credit rating" with a weight of 0.1017.

The greatest weight is laid on the evaluation criterion "bonds callability" within this dimension, and it means that a relatively greatest impact on the HYB selection is callability. According to the definition of HYB, high-yield bonds are corporate bonds with higher default risk, and hence the bond callability at maturity without the defaults is considerably emphasized when the investors evaluate

#### the HYB selection.

#### 2. Analysis of the Financial Factors

Among five subcriteria within this dimension, their relative importance in descending order are "solvency" with a weight of 0.0776, "cash flow" with a weight of 0.0676, "financial structure" with a weight of 0.0440, "profitability" with a weight of 0.0312, and "growth potential" with a weight of 0.0231.

There is a greatest importance on the subcriterion "solvency" within this dimension, and it is sensitive to the events such as increase in debt or shortage of current capital. Those events will probably lead to the default, for example, no payment of the interest or the principal. And a little difference of 0.01 between the weight of "solvency" and the weight of "cash flow" suggests that cash flow is also similarly important.

With respect to "solvency", the relative importance of subordinated evaluation criteria in descending order are "quick ratio" with a weight of 0.0304, "interest expense rate" with a weight of 0.0251, and "current ratio" with a weight of 0.0222. It indicates a relatively greatest importance on the evaluation criterion "quick ratio", and the "quick ratio" reflects the short-term solvency of a company.

With respect to "profitability", the relative importance of subordinated evaluation criteria in descending order are "earnings before interest and taxes (EBIT) margin" with a weight of 0.0144, "return on net worth" with a weight of 0.0087, and "net profit margin" with a weight of 0.0081. There is a relatively greatest importance on the evaluation criterion "EBIT margin".

At long run, the profitability is the capability of sustainable operation of companies, and a direct link to the profitability is a stable rise in incomes and earnings, and hence the evaluation criterion "EBIT margin" obtains a relatively greatest concern.

With respect to "financial structure", the relative importance of subordinated evaluation criteria in descending order are "total debt / total assets ratio" with a weight of 0.0210, "current assets / total assets ratio" with a weight of 0.0159, and "retained earnings / total assets ratio" with a weight of 0.0071. A relatively greatest importance is laid on the evaluation criterion "total debt / total assets ratio".

With respect to "growth potential", the most important factor is the evaluation criterion "asset growth rate" with a weight of 0.0231. It means the company operates in good status and possibly increases in profit when the asset growth rate rises, and hence the probability of bankruptcy and the default risk are correspondingly lower.

With respect to "cash flow", the most important factor is "cash flow / total debt ratio" with a weight of 0.0676. Beaver [6] also pointed out that the cash flow / total debt ratio was the most significant variable to predict the financial failure of companies.

3. Analysis of the Economic Environmental Factors The results indicate that the relative importance of subordinated evaluation criteria in descending order are "spreads versus Treasury" with a weight of 0.1512, "default rate indicator" with a weight of 0.1380, and "real interest rate change" with a weight of 0.1151.

A relatively greatest importance is laid on the evaluation criterion "spreads versus Treasury".

#### **3.4.3** Comprehensive Analysis

Besides, we make a comprehensive analysis and comparison among seventeen evaluation criteria.

The results demonstrate that four evaluation criteria with more importance in descending order are "spreads versus Treasury" (0.1512), "bonds callability" (0.1385), "default rate indicator" (0.1380), and "real interest rate change" (0.1151).

And four evaluation criteria with less importance in ascending order are "retained earnings / total assets ratio" (0.0071), "net profit margin" (0.0081), "return on net worth" (0.0087), and "earnings before interest and taxes margin" (0.0144).

## **4** Conclusions and Future Research

According to observations from the results, a certain extent of difference is existed rather than equivalence on the weights of evaluation criteria and also dimensions. Thus, it is necessary to clarify the relative importance of these evaluation criteria and dimensions in order to identify critical factors of HYB selection.

The analysis of the relative importance of evaluation criteria and dimensions is described as follows.

- 1. The relative importance of evaluation dimensions in descending order is "economic environmental factors", "characteristics of bonds", and "financial factors".
- 2. Within the dimension "characteristics of bonds", the relative importance of subordinated evaluation criteria in descending order are "bonds callability", "bonds liquidity", and "change in

credit rating".

- 3. Within the dimension "financial factors", the relative importance of subordinated evaluation criteria in descending order are "solvency", "cash flow", "financial structure", "profitability", and "growth potential".
- (1) As to the subcriterion "solvency", the relative importance of subordinated evaluation criteria in descending order are "quick ratio", "interest expense rate", and "current ratio".
- (2) As to the subcriterion "profitability", the relative importance of subordinated evaluation criteria in descending order are "earnings before interest and taxes (EBIT) margin", "return on net worth", and "net profit margin".
- (3) As to the subcriterion "financial structure", the relative importance of subordinated evaluation criteria in descending order are "total debt / total assets ratio", "current assets / total assets ratio", and "retained earnings / total assets ratio".
- (4) As to the subcriterion "growth potential", the most important factor is "asset growth rate".
- (5) As to the subcriterion "cash flow", the most important factor is "cash flow / total debt ratio".
- 4. Within the dimension "economic environmental factors", the relative importance of subordinated evaluation criteria in descending order are "spreads versus Treasury", "default rate indicator", and "real interest rate change".
- 5. Obtained from the comprehensive analysis, three evaluation criteria with more relative importance are "spreads versus Treasury", "bonds callability", and "default rate indicator".

This study contributes to extract critical factors related to more complete dimensions rather than only financial ones on the selection of HYB and to estimate the relative importance of these factors in the experts' views. It can be used to facilitate the decision-making process of evaluation of HYB investments. Especially under a pessimistic atmosphere of falling stock markets, selection and inclusion of profitable HYB can balance the risk and the return to raise the performance of portfolio management.

Our results can be referred and extended in the future to develop more in-depth researches. Many fuzzy multi-attribute decision-making methods, like fuzzy DEA, fuzzy TOPSIS, and fuzzy ANP, can be used to build different evaluation models and then their results can be analyzed and compared.

Moreover, the same researches can be conducted in different countries, and some interesting results may be observed. Critical factors obtained from this research can be used to facilitate the generation of populations with more fitness and the formation of an optimal solution of HYB selection when applying GA in the portfolio management.

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