# Stroke Order Computer-based Assessment with Fuzzy Measure Scoring

GUEY-SHYA CHEN, YU-DU JHENG, HSU-CHAN YAO, HSIANG-CHUAN LIU Graduate Institute of Educational Measurement, Department of Bioinformatics Taichung University, Asia University No. 140, Ming-Sheng Rd., Taichung, 40306, TAIWAN

grace@mail.ntcu.edu.tw, doora0622@yahoo.com.tw, doremiyao@yahoo.com.tw, lhc@asia.edu.tw

Abstract: - The purpose of this research is to develop a computer-based assessment for stroke order and a novel algorithm based on Choquet integral with fuzzy measure for scoring stroke order of Chinese Character writing. The learning of stroke order is very important for a fixed stroke writing character system therefore this research based on the theory of "five indexing system of Chinese characters" and combine the features of strokes develops a stroke order assessment system to real-time evaluate learners' stroke order and score their character writings. Except using traditional additive scoring method, a non-additive scoring algorithm based on Choquet integral with  $\lambda$  -measure and L -measure was proposed. A real data set with 121 samples was examined and experiment results show that the performances of this novel scoring model based on Choquet integral with both  $\lambda$  -measure and L -measure outperform the performance of traditional scoring approach.

*Key-Words:* - stroke order, computer-based assessment, five indexing systems of Chinese characters,  $\lambda$  -measure, L -measure, Choquet integral

## **1** Introduction

There are two main writing systems in the world, one is Phono-semantic compound characters system, for example Chinese and the other is Pin-ying characters system, for example English. A word is composed of several alphabets in Pin-ying system and is composed of many strokes in Phono-semantic compound system. There is no stroke order problem in Pin-ying system but this problem is very essential for a fixed stroke writing character. Therefore, the learning of stroke order is very important for a fixed stroke writing character system, for example Chinese characters and there is an average of 12 strokes in each traditional Chinese character. A more complicated example, the character<sup>Γ</sup> , is composed of 28 strokes.

Many teaching tools for Chinese characters writing are developed [1], [2] but few researchers have interest in developing system for the assessment of writing Chinese characters. In a Chinese character writing class in elementary school, because each Chinese character has its own individual stroke order, teachers must teach by writing each stroke on the black board so that the students may follow them. To ensure that the students memorize each Chinese character, there is some homework after class for practice. Then the teacher will collect the homework from the students and grade it to complete the teaching of Chinese character writing.

It is a difficult work for a teacher to access the students' stroke orders of their Chinese character writing. Even when the number of students studied in classes are reducing, it is still a major problem for Chinese teachers. The main reason is that stroke orders aren't like other subjects, by handing in homework or test papers the result is clear. It is hard to evaluate students' stroke orders unless teachers are at students' side observing the whole writing process. If teachers only look at students' homework book, the mistakes that teachers can see are only "shape" mistakes, not "stroke order" mistakes.

Recent years as computer technology progresses fast, personal computers and the Internet are closely linked to human lives. As someone puts, "information technology is like the air we breathe, surrounding us". Technology has indeed greatly influenced our education as well in the process. From the early stages of computer aiding teaching, to the present, education combined with information technology into learning, this technology has become an efficient tool for teaching as a teacher and learning as a student. Therefore, how to use PC technology to help teachers improve their teaching and assess

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students' exercises and performance is the most important mission we face now.

To accomplish mission, we developed a computer-based assessment for stroke order [3] and a scoring model based on Choquet integral with fuzzy measure is proposed [4]. Instead of typing on a keyboard, students can input the characters through a hand writing computer input device. Then the system records the movement of each stroke and adapt to the writing speed for different students. By breaking each character into different strokes and then extracting important features of each stroke, an algorithm for recognizing stroke order and 4 scoring methods are developed.

It is complex to score the stroke order because many various latent factors are embedded in the interaction between strokes for each character. A traditional linear additive scoring model is simple but it does not fit the need. It does not consider different characters have the same partial part, for example characters " $\Delta$ " and " $\Delta$ " both have the same " $\lambda$ " part. Therefore a fuzzy scoring model based on Choquet Integral is proposed in this research. Four different scoring methods are examined with a real data set.

This paper was organized as followings. Feature extraction and stoke order recognition algorithm were introduced in section 2. In section 3, fuzzy measures were reviewed and Choquet integral was described in section 4. Four scoring methods were described in section 5. Experiment and result were described in section 6 and final section is for conclusions.

## 2 Stroke Order Recognition and Feature Extraction

The system diagram of stroke order computer-based assessment is shown in Figure 1. The stroke order recognition algorithm is based on both the theory of "five indexing system of Chinese characters" proposed by Chen Lifu [5], and the rules of stroke order published by the Ministry of Education in Taiwan [6].



### Figure 1 System diagram of stroke order assessment 2.1 Five Indexing System of Chinese Characters [3], [5]

The theory of 'five indexing systems of Chinese characters', which uses quadrant as a way to distinguish different strokes is shown in Figure 2. There are only four quadrants in this approach so it may lead to unreliable outputs. In this research, we use slope to analyze strokes characteristics.



Figure 2 Basic stroke diagram of 'five indexing systems of Chinese characters'

### **2.2 Basic Stroke Detection** [3]

Based on "five indexing system of Chinese characters", we use slope instead of quadrants in this research. The system takes dia stroke and na stroke as na stroke because of the similarity between the two slopes and it omits hua stroke because of its specialty. The six categories of basic strokes used in Chinese characters writing assessment system are shown in Table1.

number	stroke name	shape	corresponding angle
1	héng	1	0°~ 25°
2	tī		25°~ 80°
3	gōu	>	90°~ 150°
4	piĕ	ノ	195°~ 255°
5	shù	1	265°~ 275°
6	nà	$\mathbf{\lambda}$	295°~ 335°

Table 1 Six basic strokes in the system

Athough diă stroke and nà stroke are considered as the same category, the system will still category them as diă stroke or nà stroke in the output result according to their length. Basically, every stroke can be broken up into these six basic strokes categories this research has proposed.

### 2.3 Compound Stroke Processing

After distinguishing the six basic strokes, the system uses combination to deal with the other complex strokes. Example: When the system detects stroke "L ", it automatically breaks it as two of the basic strokes shù and héng, therefore naming it shù-héng. Using this method, we can break up the 23 compound strokes, announced by the ministry of education, into all of the six basic strokes. All the compound strokes and their corresponding basic strokes are shown in table 2.

**2.4 Overlapping/Complementary Interaction** There are some overlapping interactions among tradition Chinese characters, for example "大" and "天" both two characters have the same part of "大". And there are many same parts which are "交", "木", "人" among characters "校", "佼", "休".

The writing order of character "川" is from left-hand side to the right-hand side. The order of "仁" is from left-hand side to the right-hand side then from top to bottom. There are many writing order rules for traditional Chinese characters. For example from outside to inside for character "月", stroke shù after stroke héng for character " $\pm$ ", stroke nà after stroke piě for character " $\lambda$ ". There are some latent complementary interactions within ordering rules among characters.

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nuumbe	shape	compound stroke	example
r			•
1	٢	héng-gōu	他、宅
2	フ	héng-piĕ	又、反
3	7	héng-shù	丑、真
4	乙	héng- piĕ- héng-gōu	乞、乙
5	フ	héng-shù-gōu	勾、兩
6	۲	héng-shù-héng	凹、朵
7	2	héng-shù-nà-gōu	九、飛
8	3	héng- piĕ-héng-shù-gōu	艿、乃
9	$\Box$	shù-héng	繼、框
10	ł	shù-tī	辰、長
11	5	shù-gōu	水、小
12	J	shù-piĕ	月、麻
13	L L	shù-héng-gōu	比、匕
14	4	shù-héng-shù	吳、誤
15	5	shù-héng- shù-gōu	弟、号
16	L	piĕ-tī	絲、級
17	J	piĕ-héng	碌、錄
18	く	piĕ-nà	巢、女
19	`	nà	衣、裏
20		nà-nà	不、美
21	Ś	nà-héng-gōu	心、芯
22	)	nà-shù-gōu	狐、狗
23	7	nà-nà-gōu	找、戈

Table 2 23 Compound strokes

## **3 Fuzzy Measure** [7], [8],[9]

Doing an overall evaluation with several attributions that have latency interactions between them, it is an easy way to apply a traditional additive measure model but its performance is poor. We can use fuzzy integral instead. Note that a proper monotonicity measure is chosen before we use. We will introduce  $\lambda$ -measure, *L*-measure, and the category of fuzzy measure in the following.

#### **Definition 1: Fuzzy Measure**

A fuzzy measure  $\mu$  on a finite set X is a set function  $g_{\mu}: 2^{X} \rightarrow [0,1]$  satisfying the following axioms: (i) boundary conditions

$$g_{\mu}(\phi) = 0, g_{\mu}(X) = 1$$
 (1)

(ii) monotonicity

$$A \subseteq B \Longrightarrow g_{\mu}(A) \le g_{\mu}(B) \tag{2}$$

## **Definition 2: Singleton Measure**

The singleton measure s of a fuzzy measure  $\mu$  on a finite set X is a function  $s: X \rightarrow [0,1]$  satisfying

$$s(x) = \mu(\lbrace x \rbrace), x \in X$$
(3)

#### **3.1** $\lambda$ -measure [10]

For given singleton measure s, let  $\lambda \in (-1, \infty)$ , a  $\lambda$ -measure,  $g_{\lambda}$  is a fuzzy measure

on a finite set X , |X| = n , satisfying

(i) 
$$A, B \in 2^{X}, A \cap B = \phi, A \cup B \neq X$$
  

$$\Rightarrow g_{\lambda} (A \cup B) = g_{\lambda} (A) + g_{\lambda} (B) + \lambda g_{\lambda} (A) g_{\lambda} (B)$$
(4)

(ii) 
$$\prod_{i=1}^{n} \left[ 1 + \lambda s(x_i) \right] = \lambda + 1 > 0, s(x_i) = g_{\lambda}(\{x_i\})$$
(5)

Note that since  $\lambda \in (-1, \infty)$ , the above equation of variable  $\lambda$  with degree n has only one solution, and the solution is always not a closed form.

#### **3.2** *L*-measure [11], [12]

For given singleton measure s, A L -measure,  $g_L$ , is a fuzzy measure on a finite set X , |X| = n , satisfying

(i) 
$$L \in [0, \infty)$$
  
(ii)  $s(x) = g_L(\lbrace x \rbrace), x \in X$  (6)  
(iii)  $\forall A \subset X, n - |A| + (|A| - 1)L > 0$   
 $\Rightarrow g_L(A) = \max_{x \in A} [s(x)] +$ 

$$\frac{(|A|-1)L\sum_{x\in A}s(x)}{\left[n-|A|+(|A|-1)L\right]\sum_{x\in X}s(x)}\left[1-\max_{x\in A}\left[s(x)\right]\right]$$
(7)

Note that Since  $L \in [0,\infty)$ , *L*-measure have infinitely many solutions with closed form in the equation (7)

And when 
$$L = 0$$
,  
 $\Rightarrow g_L(A) = \max_{x \in A} [s(x)]$ 
(8)

In the equation (8) L -measure is just a P -measure.

#### 3.3 The Category of Fuzzy Measure

Let  $g_{\mu}$  be a fuzzy measure on  $(X, 2^{X})$  and there are four kinds of additive measure for  $g_{\mu}$ .

(i) additive measure

If 
$$\forall A, B \in 2^{X} \text{ and } A \cap B = \phi,$$
  
 $\Rightarrow g(A \cup B) = g(A) + g(B).$  (9)  
 $g_{\mu}$  is an additive measure on  $(X, 2^{X}).$ 

(ii) superadditive measure  
If 
$$\forall A, B \in 2^{X}, A \neq \phi, B \neq \phi \text{ and}$$
  
 $A \cap B = \phi, A \cup B \neq X,$   
 $\Rightarrow g(A \cup B) > g(A) + g(B).$  (10)  
 $a$  is an superadditive measure on  $(X, 2^{X})$ 

 $g_{\mu}$  is an superadditive measure on  $(X, 2^{x})$ .

- (iii) subadditive measure If  $\forall A, B \in 2^X, A \neq \phi, B \neq \phi \text{ and}$   $A \cap B = \phi, A \cup B \neq X,$   $\ni g(A \cup B) < g(A) + g(B).$  (11)
  - $g_{\mu}$  is an subadditive measure on  $(X, 2^{X})$ .
- (iv) mixture fuzzy measure
   If g<sub>µ</sub> does not belong (i), (ii), or (iii), it is called mixture measure.

## **4** Scoring Model for Stroke Order

There are four scoring models for stroke order discussed in this paper. The first two methods are traditional additive scoring models and the others are based on fuzzy scoring model.

### 4.1 Traditional Additive Scoring Model

The first method is generally used in a traditional paper-pen based character writing test. If all the strokes in a character are correct, give one point otherwise there is zero point. For example, there are four characters "小", "生", "同", "找" in a writing test and 4-point is obtained if a student can write the stroke order of these four characters correctly.

Because there are different number of strokes in each Chinese character a polytomous model is used for scoring each character. There are 3-stroke for "小", 5-stroke for "生", 6-stroke for "同", and 7-stroke for "找" and 21-point is obtained if a student can write the stroke order of these four characters correctly. But a student write "小"and "同"correctly and write 2-stroke of "生" and 3-stroke of "找" correctly, 14-point is obtained (3+2+6+3=14).

## 4.2 Fuzzy Scoring Model Based on Choquet Integral

Let  $\mu$  be a fuzzy measure on a finite set X. The Choquet integral [13] of  $f: X \to R_+$  with respect to  $\mu$  is denoted by

$$\int_{C} f d\mu = \sum_{j=1}^{n} \left[ f\left(x_{(j)}\right) - f\left(x_{(j-1)}\right) \right] g_{\mu}\left(A_{(j)}\right)$$
(12)

Where  $f(x_{(0)}) = 0$ ,  $f(x_{(j)})$  indicates that the indices have been permuted so that

$$0 \le f\left(x_{(1)}\right) \le f\left(x_{(2)}\right) \le \dots \le f\left(x_{(n)}\right) \tag{13}$$

$$A_{(i)} = \left\{ x_{(i)}, x_{(i+1)}, \dots, x_{(n)} \right\}, A_{(n+1)} = \phi$$
(14)

And

(i) if  $\mu = \lambda$ , Equation (12) is a Choquet integral base on  $\lambda$ -measure scoring model.

(ii) if  $\mu = L$ , Equation (12) is a Choquet integral base on *L*-measure scoring model.

The following example is used to explain our proposed scoring method based on Choquet integral with L-measure. We suppose that

 $f(x_1) = 2, f(x_2) = 5, f(x_3) = 7, f(x_4) = 9$ , are known and their weight measure value are as followed.

$$g({x_1}) = 0.6$$
,  $g({x_2}) = 0.5$ ,  $g({x_3}) = 0.4$ ,  
 $g({x_4}) = 0.3$ 

And from the definition of fuzzy measure, it is known

$$g(\phi) = 0 , g(\{x_1, x_2, x_3, x_4\}) = 1$$
  
Considering fuzzy measure of a join event, we obtain  
 $A_1 = \{x_1, x_2, x_3, x_4\}, g_L(A_1) = 1,$   
 $A_2 = \{x_2, x_3, x_4\}, g_L(A_2) = 0.8030,$   
 $A_3 = \{x_3, x_4\}, g_L(A_3) = 0.5667,$   
 $A_4 = \{x_4\}, g_L(A_4) = 0.3$ 

Therefore the value of Choquet integral is  $C_{L}(f) = f(x_{1})g_{L}(A_{1}) + (f(x_{2}) - f(x_{1}))g_{L}(A_{2}) + (f(x_{3}) - f(x_{2}))g_{L}(A_{3}) + (f(x_{4}) - f(x_{3}))g_{L}(A_{4})$ (15)

 $= 2 \times 1 + (5 - 2) \times 0.8030 + (7 - 5) \times 0.5667 + (9 - 7) \times 0.3$ = 8.2424

## 5 Experiment and Result

When an examinee finishes writing a specified Chinese character, the system will evaluate this character with an entire overview. The output of this assessment includes the grading of character, the name and assess of each stroke, and some suggestions to improve your character writing skill. The result of assessing writing Chinese character "  $\pm$ " is displayed in the Fig 3 and Fig 4 shows that the result for the same character with some error of this character writing. If a character is written with the wrong types of strokes, system will display these wrong strokes in red as a warning to the examinee.



Figure 3 Overall assessment of "生"



Figure 4 Overall assessment of "生" with wrong strokes

A real data set with 121 samples from a primary school in Taiwan and a Chinese school in Australia including 16 independent variables, examination scores of 16 Chinese characters<sup>1</sup>, and one dependent variable, the overall score of character writing tests at school is applied to evaluate the performances of two traditional additive scoring models and two Fuzzy Choquet integral scoring models based on  $\lambda$  -measure, L -measure ( L =5, L =10) to compute the mean square error (MSE) of the dependent variable. The formula of MSE is

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$
(16)

Responding the ratio of the credit hour for 16 Chinese characters, all of the fuzzy measures about the independent variables are assigned the same singleton measures as follows,

$$\begin{cases} g_{\mu}(\{x_1\}), g_{\mu}(\{x_2\}), ..., g_{\mu}(\{x_{16}\}) \\ = \{0.04, 0.08, 0.08, 0.04, 0.08, 0.04, 0.06, 0.06, \\ 0.08, 0.08, 0.09, 0.06, 0.05, 0.04, 0.09, 0.04 \}, \\ \mu = \lambda, L = 5, L = 10.$$
(17)

When the singleton measures of any fuzzy measure  $\mu$  are given, all the event measures of  $\mu$  can be found, and then, the Choquet integral based on  $\mu$  can be computed. The fuzzy measures L, has infinitely many solutions of fuzzy measure can be selecting and we choose the values as L=5,

L=10, and exploit proposed four forecasting models computing their MSE value. The experimental results of all forecasting models are listed in Table 3.

In Table 3, we found the following results,

(i) Choquet Fuzzy scoring model outperforms tradition additive scoring model.

(ii) Choquet Fuzzy scoring using L -measure outperforms model using  $\lambda$  -measure.

(iii) Choquet Fuzzy scoring model based on L -measure with L =5 and L =10 have similar performance.

(iv) Among traditional scoring the performance of polytomous model is better than that of dichotomous model.

scoring	approach	MSE
model		
traditional	dichotomous model	12.10408
additive	polytomous model	4.72045
Chaquat	$\lambda$ - measure	3.67360
integral	L - measure( $L$ =5)	3.31482
integral	L - measure( $L$ =10)	3.12901

Table3 MSE of four scoring models

## 6 Conclusion and Future Works

In this research, we developed a computer-based assessment system and proposed a new scoring model based on Choquet integral with fuzzy measure for stroke order of Chinese character writing. An educational data experiment was conducted to compare the performance of 4 approaches: 2 traditional additive approaches and 2 fuzzy scoring approaches based on Choquet integral with  $\lambda$  -measure and L -measure. Both fuzzy scoring models have better performance and traditional dichotomous model has the worst result.

We will continue to improve the performance of stroke order scoring method using other fuzzy measures. The assessment system can be adjusted to further educational applications such as a teaching tool to assist teachers with scoring and correcting students' mistakes or as an adaptive learning tool for students,

The system could also be integrated to an international level, where it could be adapted to other countries which have fixed strokes for each character, such as Japan (Kanji), Korean, and China (simplified Chinese).

<sup>&</sup>lt;sup>1</sup> 三竹同土交小生母回米找白木也送山

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