# Application of Multifractal Analysis on Medical Images

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*Abstract:* - This paper shows results of computer analysis of images in the purpose of finding differences between medical images in order of their classifications in terms of separation malign tissue from normal and benign tissue. The diagnostics of malign tissue is of the crucial importance in medicine. Therefore, ascertainment of the correlation between multifractals parameters and “chaotic” cells could be of the great appliance. This paper shows the application of multifractal analysis for additional help in cancer diagnosis, as well as diminishing. of the subjective factor and error probability.

*Key-Words:* - Fractals**,** Multifractals, Hõlder exponent, Medical images, Carcinomas, FracLac, FracLab

## 1 Introduction

The purpose of this research was applying special method of image analysis in medical diagnostics. It consists of applying multifractal analysis on certain digital medical images, with purpose of diagnostics malign tissue. Multifractals main parameter is Hõlder’s *exponent* ([1], [2]).

$$\alpha = \frac{\log \mu(box)}{\log \varepsilon} \qquad (1)$$

μ (box) represents dimension of box and $\varepsilon$ dimension of longitude of box.

## 2 Applying multifractal analysis in image processing

Local and global information from the spectrum are used for segmentation, noise deletion and edge detection at picture points.

Image analisys is the fundamental component of computer visual problem, which can be applied in robotics, medicine and satellite images. Segmentation is an important step for description of the basic individual process. Signal gradients are gotten by filtering, whereby extreme values roughly correspond to the contours. After that, multi-resolution techniques can be used for "cleaning" of results. The main dissadvantage of this method is precision lost because of preliminary filtering.

Alternative aproach is observing of the image as a measure of the fix resolution. Iregularities of this measure can be examined with help of multifractal analisys. The general principe is the following one:

- at first, different measures and capacities are defined from image with a grey level.
- after that, according multifractal spectrum has to be calculated, enabling local (using $\alpha$ ) and global (using $f(\alpha)$) information. No hypotesys about the signal regularity are used.

Multifractal analisys (MF) can be successfuly used in image processing.

Importance and advantage of fractal and multifractal analisys (MFA), compared to "classical" signal analisys is in the way of handling of irregularities. MFA tries to extract the information directly from the singularity, whereby "classical" approach often obserevs LF filtered versions, with different filtering depths for irregularities observing and noise repressing. Actual, based on specific values of $\alpha$ and $f(\alpha)$, inhommogenity point can be isolated in original signal. By image pixel extraction, which satisfy chosen value of the parameter $\alpha$ or spectrum $f(\alpha)$, it is possible to extract some regions from the image using inverse multifractal analisys (IMFA), which cannot be extracted by any of known methods. Additional advantage is that such segmentation causes no degradation of the starting image: all pixels interrelations stay unchanged and therefore image details are completely kept. This characteristic is specialy important in medical diagnostics and therefore, the potential of application of IMFA in that field is enorm.

It was shown that a great number of fast changing signals of different nature (electrical signals, modern telecommunication traffic, meteorogical and biomedical signals) can be similary described. In order to express variabilites, it is needed to exam fractal characteristic. Usage of classic statistical methods in such case (mean values) can cause valuation errors. Significant singularities are indicated by multifractality of the process.

It is also concluded that the fractal dimension is a descriptor of early changes of patological neuron changes, which is an introduction into further understunding of mechanisms of complex changes, not only on the cellular level, but also in all cases of systems in which the principle of agreement and interaction between the elements can be described using basic pattern of relations between specific cells and belonging structures.

Computer specification of fractal dimension can become very important component of the image analisys in patology, thanks to the speed, automatisation and resetability of computer methods. Fractals are very important in two fields of medicine and patology:

- in quantification of complete structures (not only morfological aspect, but a functional one also, i.e. with the time component) and
- modelling of biological, development and patological processes.

## 3 Research methods and sample

Many researches have shown existing correlation between fractals and multifractals and on the other side tumours, and generally biological systems and pathology forms ([3]-[23]).

Multifractals analysis of images is based on definition of measures from images that are grey level. Then multifractal spectrum f (α) is calculated ([24]). There is no filtering, like in many others classical approaches.

Given patterns, which are got thanks to collaboration with Center for patology and judicial medicine of Military Medical Academy in Belgrade, are observed and photographed with a "coolscope" device, which is a combination of a microscope and a computer. Digital photos which are obtained are analized using a special software used for multifractal analisys. Positive results which are obtained open new possibilities of examining and potential application on other adenocancer, or other types of cancer and generaly, other types of medical images with the same purpose.

This method would help doctors in cases which are not clear defined, for additional diagnosis check, which is a very delicate problem in this field of science.

This research was directed to find differences in parameters of multifractals analysis between three groups of tissues:

1. Normal tissue of large intestine (colon), Fig.1

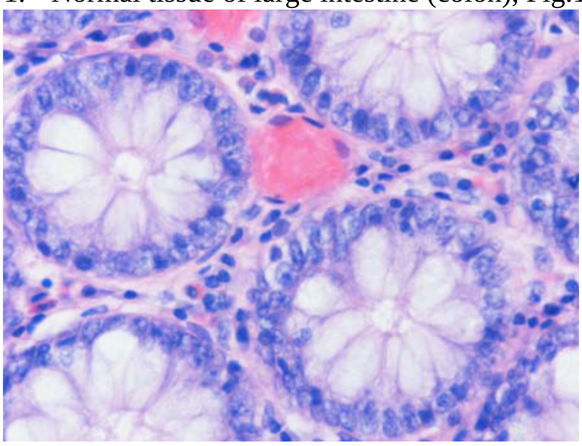


Fig. 1 Graphical representation of normal tissue of intestine

2. Tissue of large intestine (colon) with diagnose of malign tumours – cancer, Fig.2

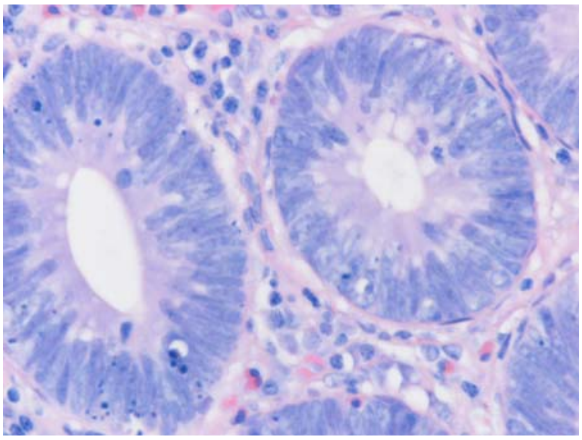


Fig. 2 Graphical representation of large intestine with a cancer diagnose

3. Tissue of large intestine (colon) with diagnose of benign tumours – adenomas, Fig.3

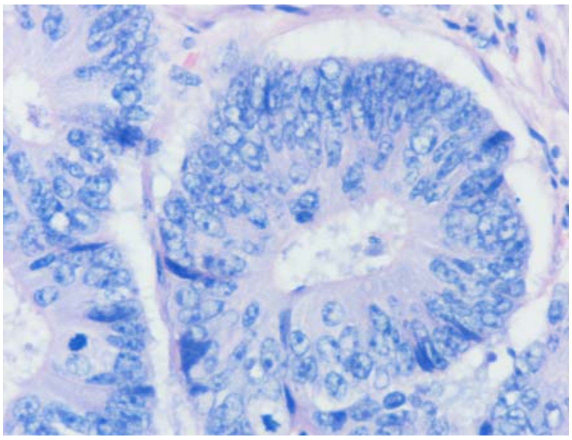


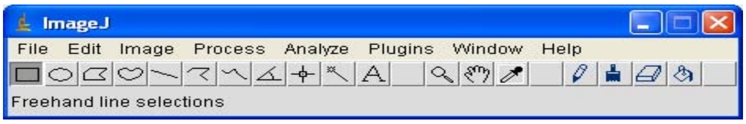
Fig. 3 Graphical representation of large intestine with adenomas diagnose

In order to get results, we applied next steps in research:

1. Calculating parameters of multifractal analysis for all three group of images
2. Exploring if there are differences of statistical significant between multifractal parameters for previous mentioned groups of tissues.

For multifractals analysis of digital medical images and gaining of parameters of multifractals analysis, we have used following programs:

1. Program "FracLac" for multifractals analysis of images ([25]), Fig.4
2. Program "FracLab" for multifractals image analysis ([26]), Fig.5
3. Program for statistical analysis of data SPSS (Statistical Package for Social Sciences), standard for statistical analysis results of clinical researches ([27]).



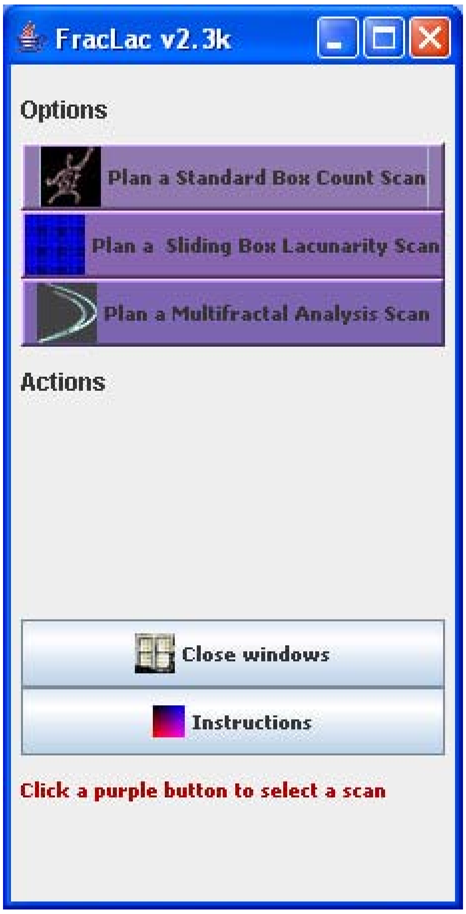




Fig.4 FracLac- dialog window

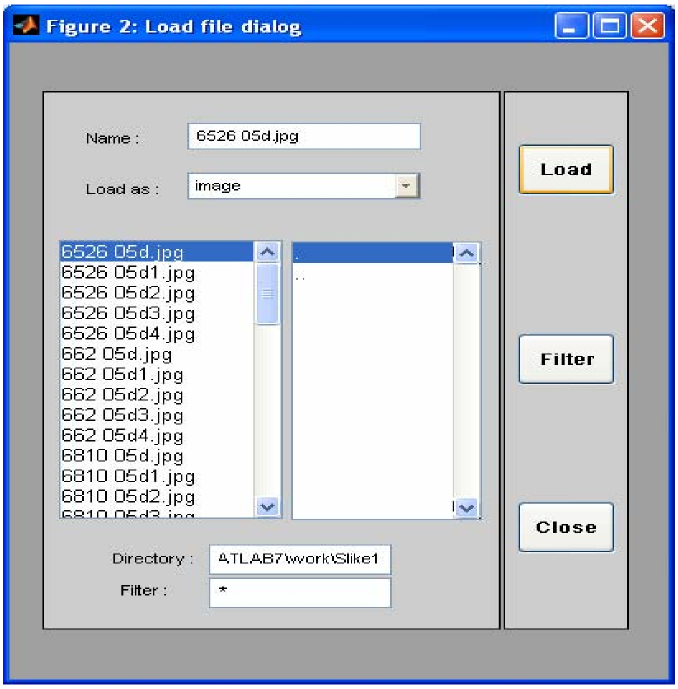



Fig.5 FracLab – dialog window

Sample was consisted of 750 images from 150 different preparations (from each preparations five different images), which were derived by biopsy from large intestine, *Adenocarcinoma tubulare coli*.

Multifractal analysis was applied on all images with programs FracLab and FracLac. Then average values have been calculated of each of multifractal parameter for five photographs from the same preparations, which were, then, analyzed with program for statistical analysis SPSS.

## 4 Multifractal Image Analysis Results

Multifractal analisys has been applied to all photos using FracLab i FracLac programs. After that, mean values of every multifractal paarameter for all of 5 photos of the same pattern are calculated and analised using SPSS programm from statistical analisys.

### 4.1. FracLac

At first, results of digital medical images will be examined using multifractal analisys and a FracLac programm.

Fig. 6 shows the normal tissue pattern from Fig.1, after transformation into the binary form using FracLac program.

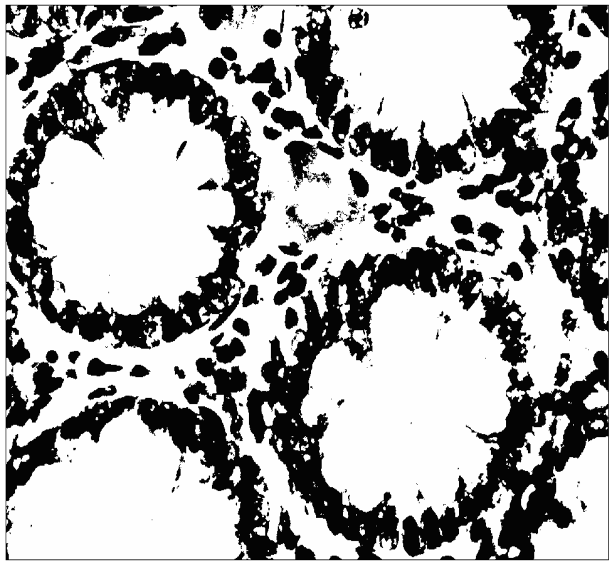


Fig. 6 Normal tissue pattern from Fig. 1 after transformation into binary form (FracLac)

Fig.7 shows $f(\alpha)$ obtained using programm FracLac for Fig. 6.

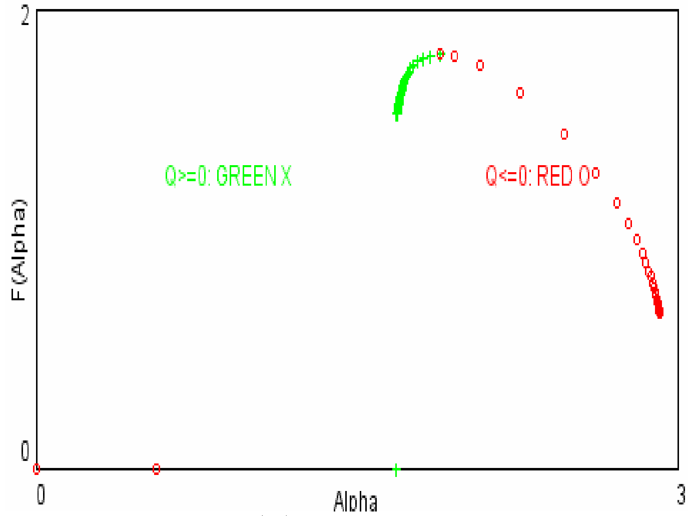



Fig. 7 $f(\alpha)$ from Fig. 6 (FracLac)

Fig.8 shows graphic of $D(Q)$ obtained using programm FracLac for Fig. 6.

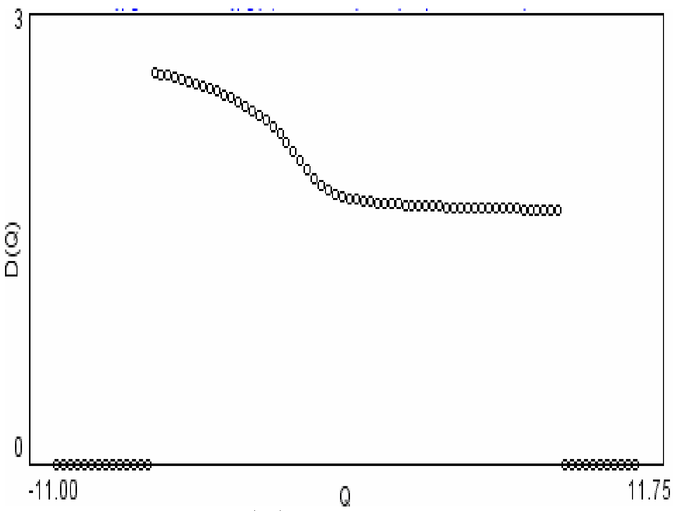



Fig. 8 $D(Q)$ from Fig. 6 (FracLac)

Fig. 9 shows adenom pattern picture from Fig. 2, after transformation into a binary form using FracLac.

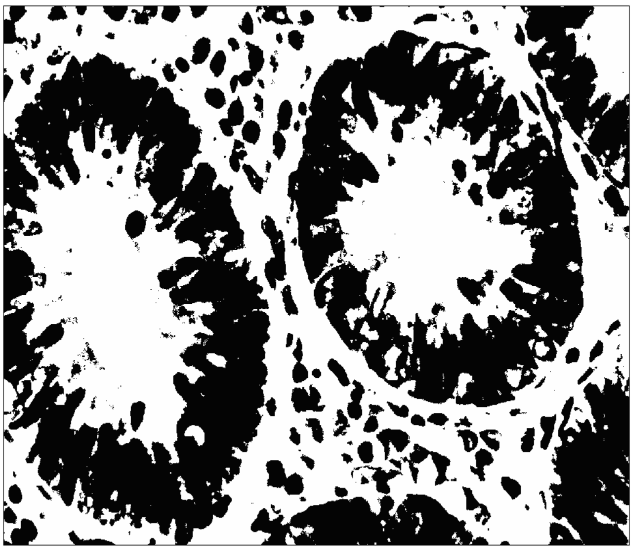


Fig. 9 Normal tissue pattern from Fig. 2 after transformation into binary form (FracLac)

Fig.10 shows graphic $f(\alpha)$ obtained using program FracLac for Fig. 9.

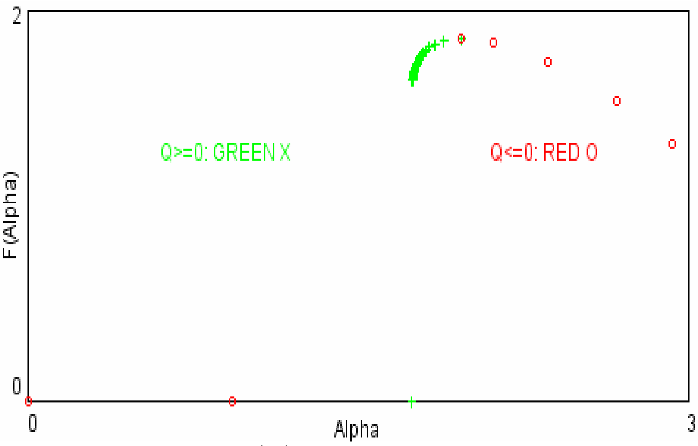



Fig. 10 $f(\alpha)$ from Fig. 9 (FracLac)

Fig.11 shows graphic $D(Q)$ obtained using program FracLac for Fig.9.

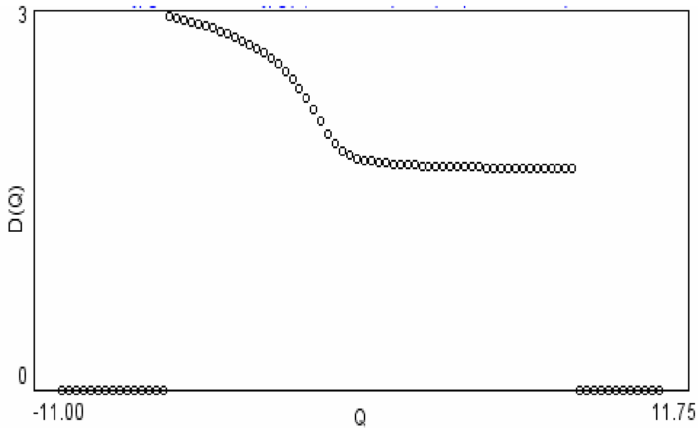



Fig.11 $D(Q)$ from Fig. 9 (FracLac)

Fig.12 shows picture of cancer tissue from Fig.5, after transformation into the binary form using programm FracLac.

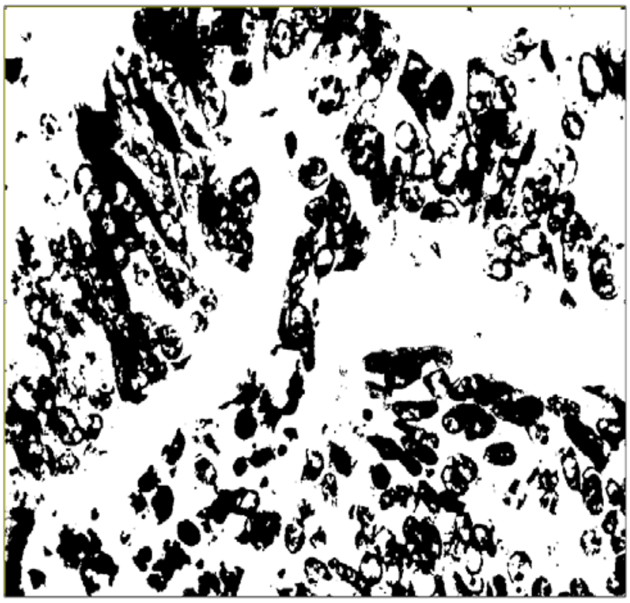


Fig. 12 Normal tissue pattern from Fig. 3 after transformation into binary form (FracLac)

Fig.13 shows graphic $f(\alpha)$ obtained using program FracLac for Fig.12.

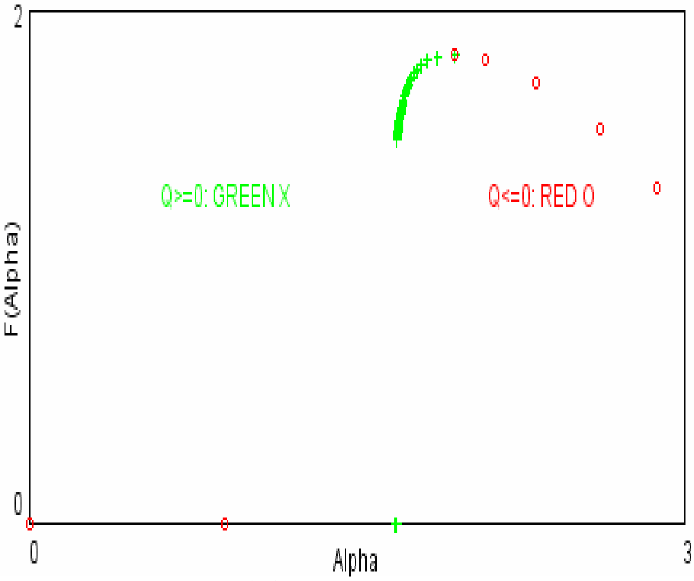



Fig.13 $f(\alpha)$ from Fig.12 (FracLac)

Fig.14 shows graphic $D(Q)$ obtained using programm FracLac for Fig. 12.

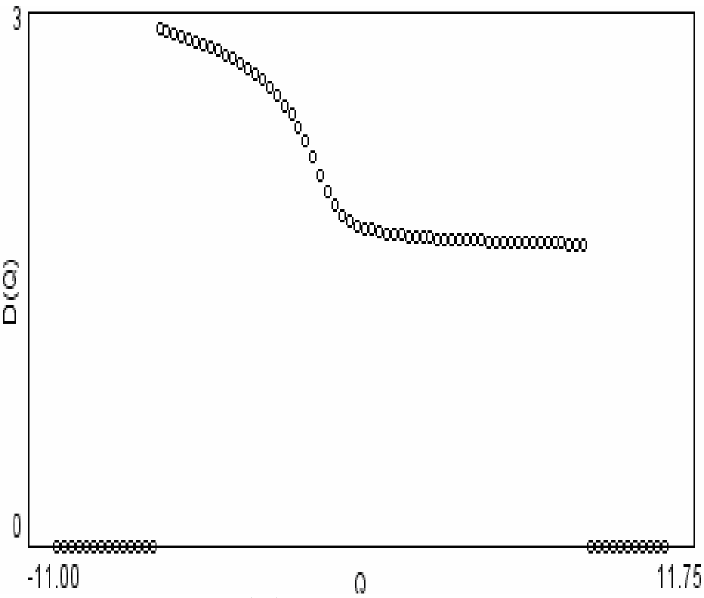



Fig. 14 $D(Q)$ from Fig. 12 (FracLac)

## 5 Influence of Luminity Changing on Results

Table 1, Table 2 and Table 3 show the influence of luminity change (-10% for 10% brighter images, +10% for 10% darker images) on results obtained using program FracLac to patterns of every of three observed groups.Table 1 shows results of obtained parameters of normal tissue from Fig. 1. It can be concluded that results are completely identical by standard luminity and by 10% lower luminity (darker picture). By 10% higher luminity lower calues are obtained, than by standard luminity.

| **Parameters** | **+10%** | **Standard luminity** | **-10%** |
|---|---|---|---|
| $D_{max}$ | 2,568 | 2,617 | 2,617 |
| $\overline{Q}$ | -6,75 | -6,5 | -6,5 |
| $\underline{\alpha}$ | 2,862 | 2,915 | 2,915 |
| $f(\alpha)_{min}$ | 0,66 | 0,684 | 0,684 |
| $\overline{\alpha}$ | 1,877 | 1,891 | 1,891 |
| $f(\alpha)_{max}$ | 1,804 | 1,814 | 1,814 |

Table 1 Influence of luminity changing on normal tissue pattern image

Table 2 show results of obtained parameters of adenom pattern in Fig.3. It can be concluded that here (as in case of a normal tissue) results are completely identical by standard luminity and by 10% lower luminity (darker picture). By 10% higher luminity lower values are obtained than by standard luminity, exept in case of $f(\alpha)_{min}$ (higher value for brighter picture).

| **Parameters** | **+10%** | **Standard luminity** | **-10%** |
|---|---|---|---|
| $D_{max}$ | 2,957 | 2,965 | 2,965 |
| $\overline{Q}$ | -6,25 | -6,25 | -6,25 |
| $\underline{\alpha}$ | 3,314 | 3,335 | 3,335 |
| $f(\alpha)_{min}$ | 0,721 | 0,654 | 0,654 |
| $\overline{\alpha}$ | 1,958 | 1,967 | 1,967 |
| $f(\alpha)_{max}$ | 1,85 | 1,861 | 1,861 |

Table 2 Influence of luminity changing on adenom tissue pattern image

Table 3 shows results of cancer pattern from Fig.5. It can be concluded that here ( as in case of normal tissue ) and results are totaly identical by standard luminity and 10% lower luminity (darker picture). By 10% higher luminity lower values are obtained than by standard luminity, exept in case of $\overline{Q}$ (identical).

| Parameters | +10% | Standard luminity | -10% |
|---|---|---|---|
| $D_{\max}$ | 2,877 | 2,911 | 2,911 |
| $\overline{Q}$ | -6,5 | -6,5 | -6,5 |
| $\underline{\alpha}$ | 3,219 | 3,24 | 3,24 |
| $f(\alpha)_{\min}$ | 0,658 | 0,772 | 0,772 |
| $\overline{\alpha}$ | 1,947 | 1,95 | 1,95 |
| $f(\alpha)_{\max}$ | 1,831 | 1,837 | 1,837 |

Table 3 Influence of luminity changing on cancer tissue pattern image

Generaly, it can be concluded that decrease of image luminity in program FracLac has no influence on results of multifractal analysis, but increase of image luminity causes decrease of calculated parameters of multifractal analisys.

## 6 Results of Medical Images Classifications

Classification results obtained using program FracLac compared to results obtained by program FracLab have shown higher reliability by differing of groups of observed tissue images. The reliability of classification between cancer and adenom groups obtained using programm FracLac were little bit lower (73%) compared to obtained result obtained using programm FracLab.

Statistic processing of obtained results of multifractal analisys of analized medical images groups, answers to the hypotesys set in this paper were given.

General conclusion is that the base of the general hypotesys is correct, i.e. that the parameters of multifractal analisys significantly differ for all three observed groups of normal tissue, cancer and adenom. In this way, the starting hypotesys about non differing of these groups is redrawn. This conclusion is valid for usage of both programms for multifractal picture analisys: FracLac and FracLab.

Further on, by specific analisys of results obtained by each of these two programms, even more precise conclusions can be obtained.

Using FracLac software, observed parameters are $D_{\max}$, $f(\alpha)_{\max}$, $\underline{\alpha}$, $\overline{Q}$, $\overline{\alpha}$ and $f(\alpha)_{\min}$. In case of FracLac programm, based on variance analisys (Tab. 4) it can be concluded that pictures of tissue groups with cancer, adenom and without patological changes statistic quite differ in parameters $D_{\max}$, $f(\alpha)_{\max}$, $\underline{\alpha}$, $\overline{Q}$ and $\overline{\alpha}$ (scheduled per discriminatory power). This can be concluded observing importance levels of *F statistics* on the level 0,01, which redrows the starting hypotesys about non existance of differnces. Onlyfor parameter $f(\alpha)_{\min}$ is the importance level of *F statistics* above level 0,05, which shows that observed groups of tissue pictures do not statistical significantly differ per this parameter, and only in this point the general hypotesys cannot be confirmed.

In case of specific hypotesys in this paper, results have to be discussed for each observed parameter of mulifractal analisys obtained using both programms.

In case of parameter $f(\alpha)_{\min}$, there is no sense checking if the specific hypotesys is true or not, as it was shown that there is no statistic importancy for that parameter.

Using programm FracLab, parameters $\alpha_{stdev}$, $f(\alpha)_{sr}$, $f(\alpha)_{stdev}$, $\overline{\alpha}$ and $\alpha_{sr}$ were observed. In case of FracLab programm, based on variance analisys (Tab. 21) it can be concluded that pictures of tissue groups with cancer, adenom and without patologic changes significantly differ in parameters $\alpha_{stdev}$, $f(\alpha)_{sr}$, $f(\alpha)_{stdev}$ and $\overline{\alpha}$. This can be concluded observing levels of importancy of *F statistics* that are under the level 0,01, which redraws the starting hypotesys about non existing differences. Only for parameter $\alpha_{sr}$ is the importance level of *F statistics* above levels 0,01 and 0,05, which shows that observed groups of tissue pictures significantly not differ in this parameter – in this case the general hypotesys is not true.

After all answers which were obtained using "hard" methods, "fine" data analisys using methods of canonic discriminatory analisys has been applied. All dimensions (subscales) were observed all together (their linear combination in discriminatory function). Two discriminatory functions were obtained, which are statistically important and which successfully discriminate these three groups.

At first, the software FracLac was applied and following multifractals parameters were observed: $D_{\max}$, $f(\alpha)_{\max}$, $\underline{\alpha}$, $\overline{Q}$, $\overline{\alpha}$ and $f(\alpha)_{\min}$. It was concluded, based on analyze of variance, that parameters which are of statistical significant for classifications groups of images of tissues with carcinomas, with adenomas and without pathological changes, are the following: $D_{\max}$, $f(\alpha)_{\max}$, $\underline{\alpha}$, $\overline{Q}$ and $\overline{\alpha}$ (ordered by power of discriminate).

Figures 15-19 show average values of all analyzed parameters, depending on the group, in case of the program FracLac.

The order of analyzed parameters of observed groups, starting with highest values of average values is next: group of images of normal tissue, group of images with carcinomas and group of images with adenomas.

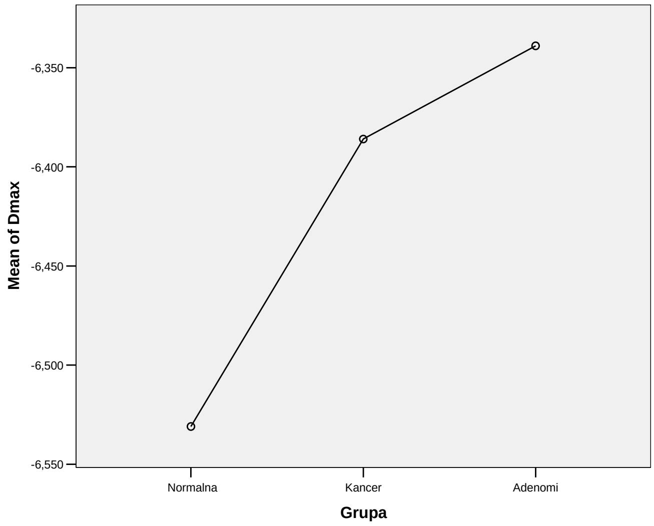



Fig.15 Graphical representation of average values for parameter $D_{\max}$ (gained with program FracLac) for all three investigated groups of tissue images

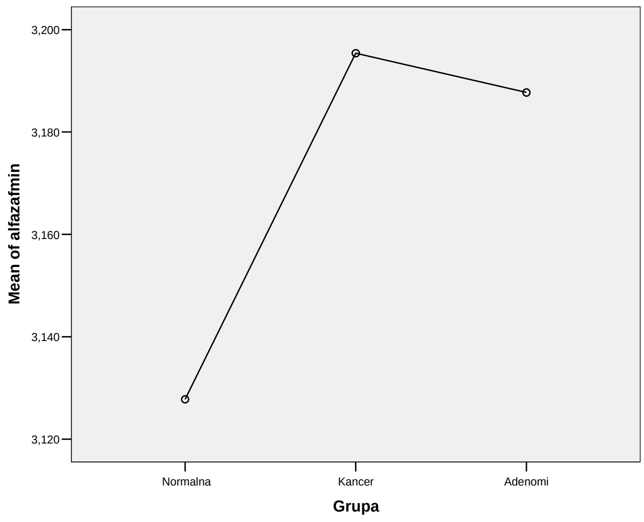



Fig. 16 Graphical representation of average values for parameter $\underline{\alpha}$ (gained with program FracLac) for all three investigated groups of tissue images

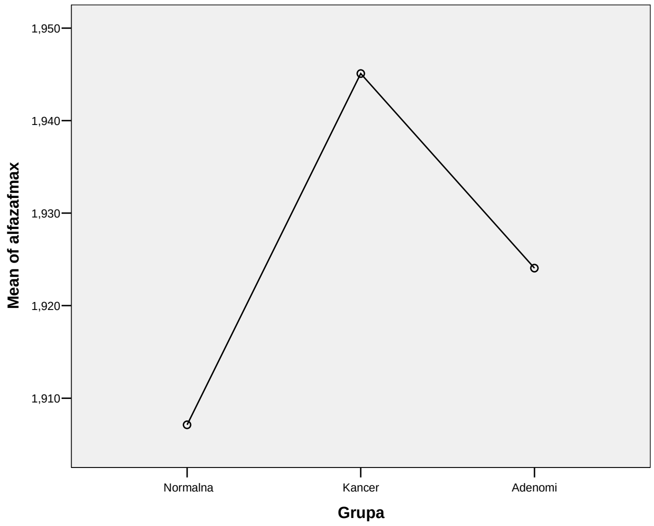



Fig.17 Graphical representation of average values for parameter $\overline{\alpha}$ (gained with program FracLac) for all three investigated groups of tissue images

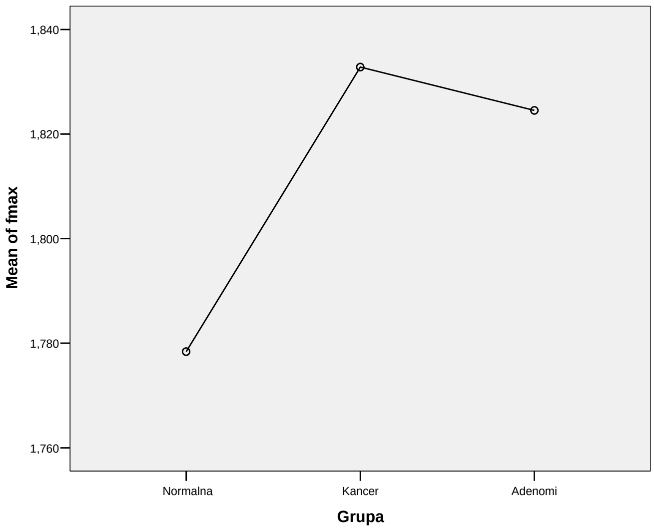



Fig. 18 Graphical representation of average values for parameter $f(\alpha)_{\max}$ (gained with program FracLac) for all three investigated groups of tissue images

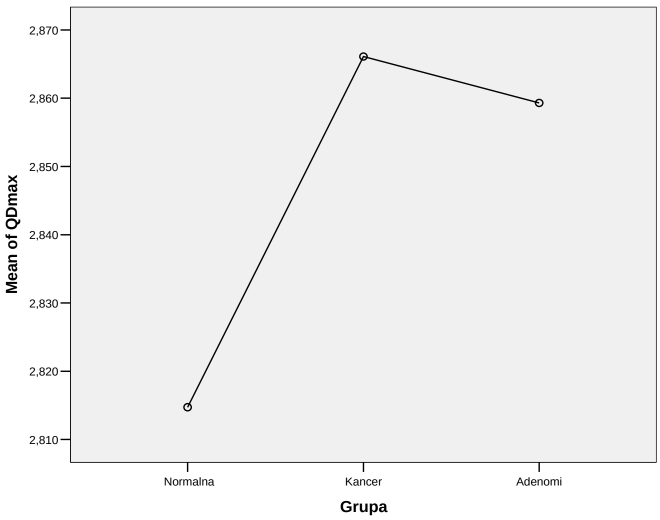



Fig. 19 Graphical representation of average values for parameter $\overline{Q}$ (gained with program FracLac) for all three investigated groups of tissue images

After FracLac, the software FracLab was applied for multifractal analysis of medical images and following parameters were observed: $\alpha_{stdev}$, $f(\alpha)_{sr}$, $f(\alpha)_{stdev}$, $\overline{\alpha}$ and $\alpha_{sr}$. It was concluded, based on analisys of variance, that parameters which are of statistical significant for classifications groups of images of tissues with carcinomas, with adenomas and without pathological changes, are following:: $\alpha_{stdev}$, $\alpha_{stdev}$, $f(\alpha)_{stdev}$, and $\overline{\alpha}$. Figures 19-23 show average values of all analyzed parameters, depending of group, in case of the program FracLab.

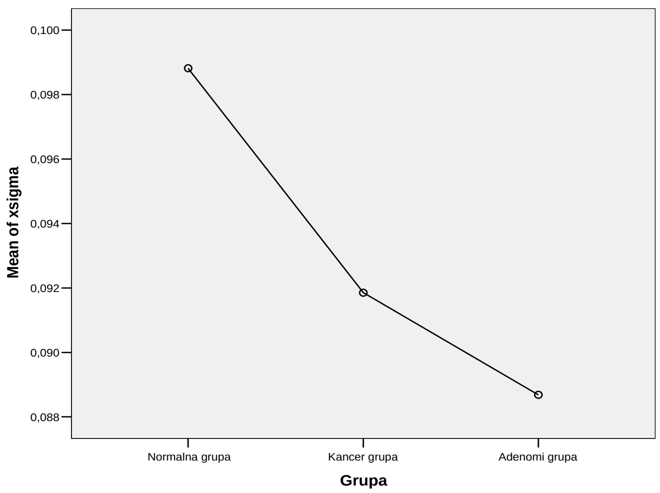



Fig. 20 Graphical representation of average values for parameter $\alpha_{stdev}$ (gained with program FracLab) for all three investigated groups of tissue images

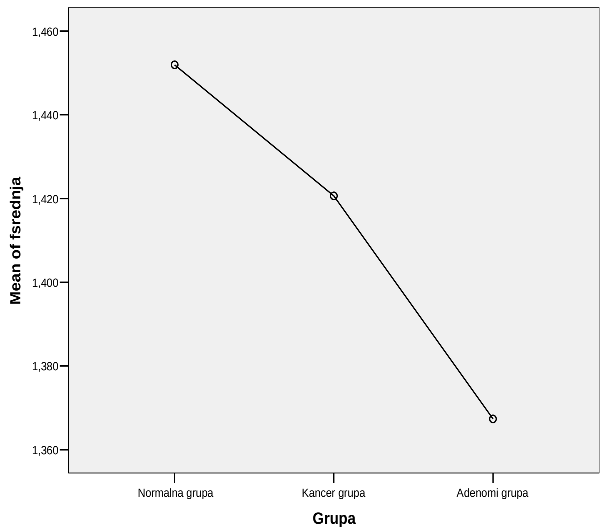



Fig. 21 Graphical representation of average values for parameter $f(\alpha)_{sr}$ (gained with program FracLab) for all three investigated groups of tissue images

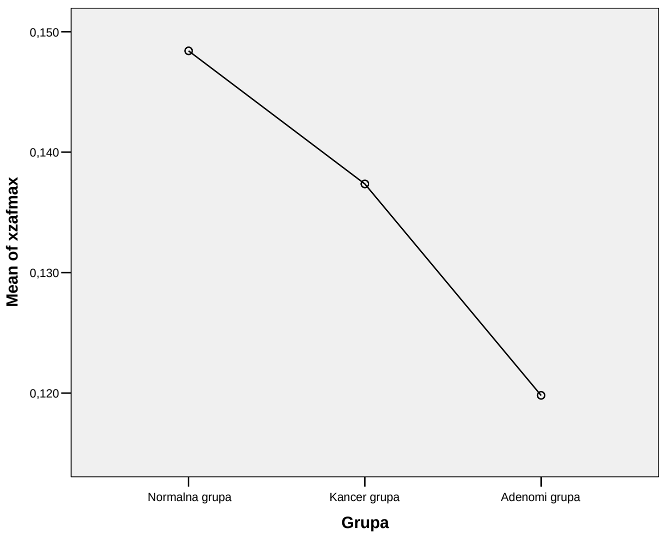



Fig.22 Graphical representation of average values for parameter $\overline{\alpha}$ (gained with program FracLab) for all three investigated groups of tissue images

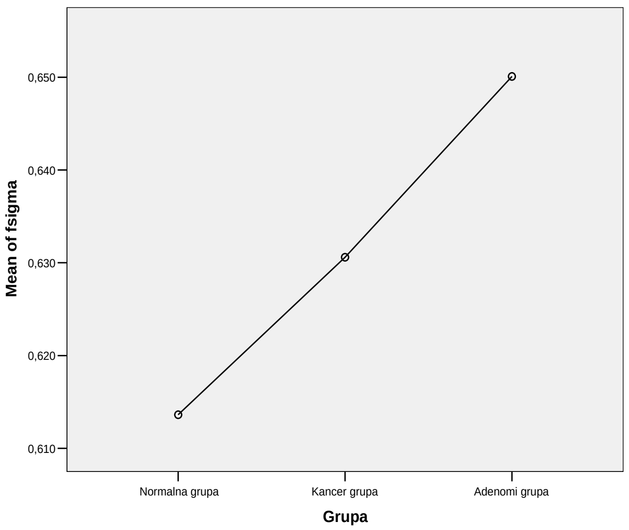



Fig. 23 Graphical representation of average values for parameter $f(\alpha)_{stdev}$ (gained with program FracLab) for all three investigated groups of tissue images

In case of program FracLac (shown on Table 4 ) reliability of classification of all three groups of images, based on multifractals parameters, is 65,3%, what is more compared to 60,7% in case of program FracLab. In case of program FracLac following results of successfully classificated images were gained: 80,0%, 73,0% and 85,0%, for next groups of images respectively: normal tissue and carcinomas, carcinomas and adenomas, normal tissue and adenomas.

| | **Normal Vs. Benign Vs. Malign** | **Normal Vs. Malign** | **Normal Vs. Benign** | **Benign Vs. Malign** |
|---|---|---|---|---|
| **FracLac parameters** | **65%** | **80%** | **85%** | **73%** |
| **FracLab parameters** | **61%** | **64%** | **80%** | **74%** |

Table 4 Classifications results

In case of program FracLab (shown on Table 4) for the same relations of groups of tissue images, respectively, there were gained following results of reliability: 64,0% (what is significant worst in comparison to the same case with program FracLac), 74,0% (what is better in comparison to the same case with program FracLac) and 80,0% (what is worse in comparison to the same case with program FracLac.

We can, based on previous results, conclude that classifications of medical images by multifractal analysis are very effective and that generally more in case of program FracLac.

# 6 Conclusion and the future work

In this paper, it has been shown that the basis of the general hypothesis is the correct one, i.e. that parameters of multifractal analysis significantly differ for all three investigated groups of tissue images. In this way, the zero (starting) hypotheses about non-differing of these groups have been neglected. This conclusion is valid for both programs of multifractal image analysis: FracLac and FracLab.

Multifractal analysis is even more sophisticated method than the fractal analysis and the research, which is the subject of this paper, shows the application of multifractal analysis for additional help in cancer diagnosis, as well as diminishing of the subjective factor and error probability.

Further method improvements are possible in a sense of speed of analysis, reliability, usage easiness etc. Ideas for further investigations ([40], [41] and [42]) are checks of multifractal analysis application on other cancer types and other organs.

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