Optimal Run Time for EMQ Model with Backordering, Failure-In-Rework and Breakdown Happening in Stock-Piling Time

YUAN-SHYI PETER CHIU, SHUN-SHENG WANG,
CHIA-KUAN TING*, HSIEN-JU CHUANG,
YU-LUNG LIEN
Department of Industrial Engineering and Management, Chaoyang University of Technology
168 Jifong East Road, Wufong, Taichung 413,
TAIWAN
*E-mail: djk@cyut.edu.tw (CHIA-KUAN TING) http://www.cyut.edu.tw/~iem/teacher.htm

Abstract: - This study examines the optimal run time for the economic manufacturing quantity (EMQ) model with failure-in-work, backlogging, and random breakdown happening in stock-piling time. A recent article by Chiu and Chiu [Mathematical modeling for production system with backlogging and failure in repair, Journal of Scientific & Industrial Research, 65 (2006) 499-506] investigated optimal lot-size for EMQ model with backordering and failure-in-rework. By incorporating random machine breakdown-another inevitable reliability factor into their model, this research examines its effects on the optimal run time and on the long-run average costs. Mathematical modeling and cost analysis are employed and the renewal reward theorem is used to cope with variable cycle length. Convexity of the expected production- inventory cost function is proved. An optimal replenishment policy that minimizes overall costs is derived for such an unreliable system. Numerical example is provided to show its practical usage. Managers in the field can adopt this run time decision to establish their own robust production plan accordingly.

Key-Words: - Manufacturing, Run time, Breakdown, Failure-in-rework, Backorder

1 Introduction

The economic order quantity (EOQ) model [1] uses mathematical techniques to balance the setup cost and holding cost, and derives an optimal ordering quantity that minimizes overall purchase costs. The economic manufacturing quantity (EMQ) model, an extension to the EOQ model, is often used in manufacturing sector for determining the optimal production lot-size that minimizes the long-run average production-inventory costs [2-3]. Despite the simplicity of EOQ/EMQ models, they are still applied nowadays and are still the basis for the analyses of more complex systems [4-7].

One implicit assumption of conventional EMQ model is that the manufactured items are of perfect quality. However, in real world production systems, owing to many unpredictable factors, generation of random defective items is inevitable. Many research papers have been carried out to address imperfect quality issues in EMQ model [8-15]. Boone et al. [9] investigated the impact of imperfect processes on the production run time. They built a model in an attempt to provide managers with guidelines to choose the appropriate production run times to cope with both the defective items and stoppages occurring due to machine breakdowns. Cheung and Hausman [11] formulated an analytical model of preventive maintenance (PM) and safety stock (SS) strategies in a production environment subject to random machine breakdowns. They illustrated the trade-off between investing in the two options (PM and SS) and also provided optimality conditions under which either one or both strategies should be implemented to minimize the associated cost function. Both the deterministic and exponential repair time distributions are analyzed in detail in their study. Lee and Rosenblatt [13] studied an EPQ model with joint determination of production cycle time and inspection schedules, and they derived a relationship that can be used to determine the effectiveness of maintenance by inspection. Zhang and Gerchak [15] considered joint lot sizing and inspection policy in an EOQ model with random yield.

The defective items, in some circumstances such as the plastic goods in plastic injection molding process, the printed circuit board assembly (PCBA) in PCBA manufacturing, etc., can be reworked and repaired. Therefore, overall production-inventory costs can be reduced significantly [16-22]. Hayek and Salameh [21] assumed that all of the defective items produced are repairable and derived an optimal operating policy for EPQ model under the effect of reworking of imperfect quality items. Jamal et al. [22] studied optimal manufacturing batch size with rework process at a single-stage production system.

Due to the excess demands, stock-out situations may arise occasionally. Sometimes, shortages are permitted and they are backordered and satisfied in the very next replenishment. Hence, production inventory costs can be decreased substantially [16-17,20]. Another common reliability factor that troubles the production practitioners most is the random breakdown of production equipment. Therefore, to effectively control and manage the disruption caused by random breakdown in order to minimize overall production costs becomes the critical task to most production planners. Therefore, determination of the optimal run time for manufacturing systems subject to machine failures has received extensive attention from researchers in the past decades [23-32].

Examples of research are surveyed as follows. Groenevelt et al. [23] proposed two inventory control policies to deal with machine failures. One assumes that the production of the interrupted lot is not resumed (called no resumption-NR policy) after a breakdown. The other policy considers that the production of the interrupted lot will be immediately resumed (called abort/resume-AR policy) after the breakdown is fixed and if the current on-hand inventory falls below a certain threshold level. Repair time is assumed to be negligible, and effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions were investigated. Makis and Fung [29] studied effects of machine failures on the optimal lot size as well as on optimal number of inspections. Formulas for the long-run expected average cost per unit time was obtained. Then the optimal production/inspection policy that minimizes the expected average costs was derived. Chiu et al. [26] considered the optimal run time for EPQ model with scrap, rework and random breakdown. They have proved theorems on conditional convexity of the integrated cost function and on bounds of the production run time. Then, an optimal run time was located by the use of the bisection method based on the intermediate value theorem. Chung [25] investigated bounds for production lot sizing with machine breakdowns. He obtained the upper and lower bounds of the optimal lot sizes for both of the aforementioned extensions (i.e. NR and AR policies) to the model introduced by Groenevelt et al. [23]. By incorporating a random breakdown taking place during stock-piling time into model studied by Chiu and Chiu [16], this paper examines its effect on the optimal run time and on the long-run average production-inventory costs. Since little attention was paid to aforementioned area, this paper intends to fill the gap.

2 Modeling and Analysis

The description of proposed model is as follow. Suppose a manufactured item's annual demand rate is λ and its production rate is P per year, where P is much larger than λ. Production system may randomly produce x portion of defective items at a rate d, where $d=Px$. All items produced are screened and the inspection cost per item is included in the unit production cost C. Assuming that the production rate of perfect quality items must always be greater than or equal to the sum of the demand rate λ and the defective rate d. Hence, the following condition must hold: $(P-d-\lambda)>=0$ or $(1-x-\lambda/P)>=0$. All of the defective items are reworked right after the regular production process ends, at a reworking rate of P_1. A θ_1 portion of the reworked items fails the repairing and becomes scrap.

Shortages are allowed and backordered, they will be satisfied in the next replenishment. Further, according to the mean time between failures (MTBF) data, a machine breakdown may happen randomly in stock-piling time (see Figure 1), an abort/resume inventory control policy is adopted in this study. Under such a policy, when a breakdown takes place the machine is under corrective maintenance immediately, a constant repair time is assumed, and the interrupted lot will be resumed right after the restoration of machine. Figure 1 depicts the level of on-hand inventory of perfect quality items in proposed EMQ model.

The following cost parameters are considered: the cost for repairing and restoring machine M, unit manufacturing cost C, unit repair cost for each defective item reworked C_R, disposal cost for each scrap item C_S, setup cost K, unit holding cost h, unit shortage/backordered cost b, and unit holding cost per reworked item h_1. Additional variables used are listed as follows.

H_1 = level of on-hand inventory when machine breakdown occurs,

H_2 = level of on-hand inventory when machine is repaired and restored,

H_3 = level of on-hand inventory when regular production process ends,

H_4 = the maximum level of perfect quality inventory when rework finishes,
T = the production cycle length,
T_1 = production run time (i.e. uptime) to be determined by the proposed study,
Q = production lot size for each cycle,
B = the maximum backorder level allowed for each cycle,
t = production time before a random breakdown occurs,
t_r = time required for repairing and restoring the machine,
t_5 = time required for filling the backorder quantity B,
t_2 = time needed to rework the defective items,
t_3 = time required for depleting all available perfect quality on-hand items,
t_4 = shortage permitted time,
$TC(T_1,B)$ = total production-inventory costs per cycle,
$TCU(T_1,B)$ = total production-inventory costs per unit time (e.g. annual),
$E[TCU(T_1,B)]$ = the expected total production-inventory costs per unit time.

The following derivation procedure is similar to what was used by past studies [16,21]. From Figure 1, one can obtain the following: the cycle length T; the levels of on-hand inventory H_1, H_2, H_3 and H_4; production run time T_1; time for reworking defective items t_2; time required for depleting all available on-hand items t_3; shortage allowed time t_4 and time for refilling B (maximum backorder quantity) t_5.

$$T = T_1 + t_2 + t_3 + t_4 + t_r \tag{1}$$

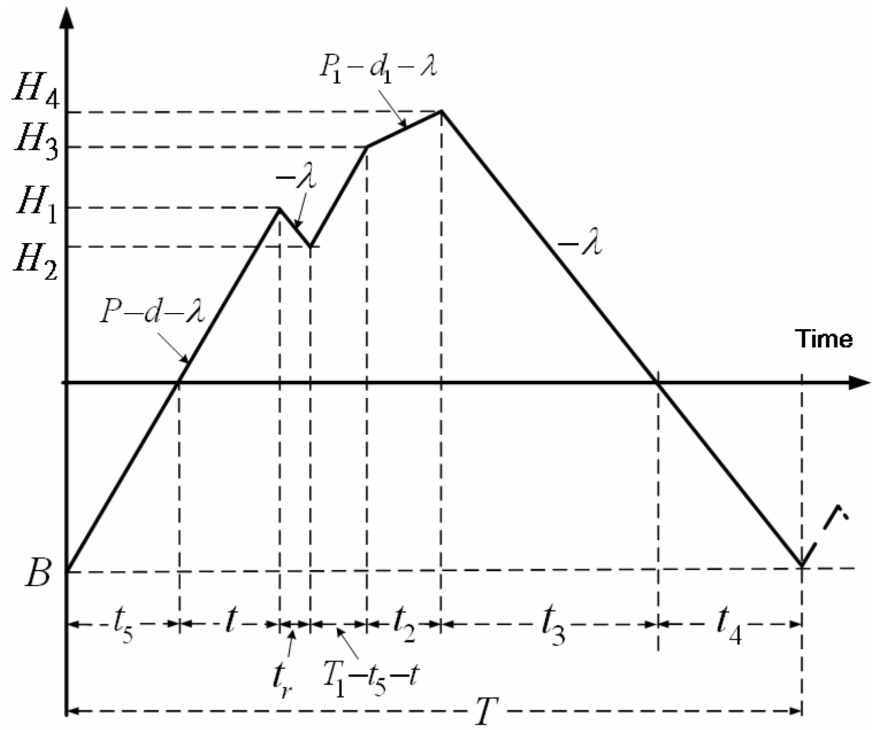

Fig.1: On-hand inventory of perfect quality items in EMQ model with failure-in-rework and breakdown happening in stock-piling time

$$H_1 = (P - d - \lambda)t \tag{2}$$

$$H_2 = H_1 - t_r\lambda = H_1 - g\lambda \tag{3}$$

$$H_3 = H_2 + (P - d - \lambda)\cdot(T_1 - t_5 - t) \tag{4}$$

$$H_4 = H_3 + (P_1 - d_1 - \lambda)t_2 \tag{5}$$

$$T_1 = \frac{Q}{P} \tag{6}$$

$$t_2 = \frac{(d \cdot T_1)}{P_1} \tag{7}$$

$$t_3 = \frac{H_4}{\lambda} \tag{8}$$

$$t_4 = \frac{B}{\lambda} \tag{9}$$

$$t_5 = \frac{B}{P-d-\lambda} \tag{10}$$

where $d=Px$ and let g be the constant machine repair time, i.e. $t_r = g$.

The level of on-hand defective items for the proposed EMQ model is shown in Figure 2.

Total defective items produced during the production run time T_1 are:

$$d \cdot T_1 = x \cdot Q \tag{11}$$

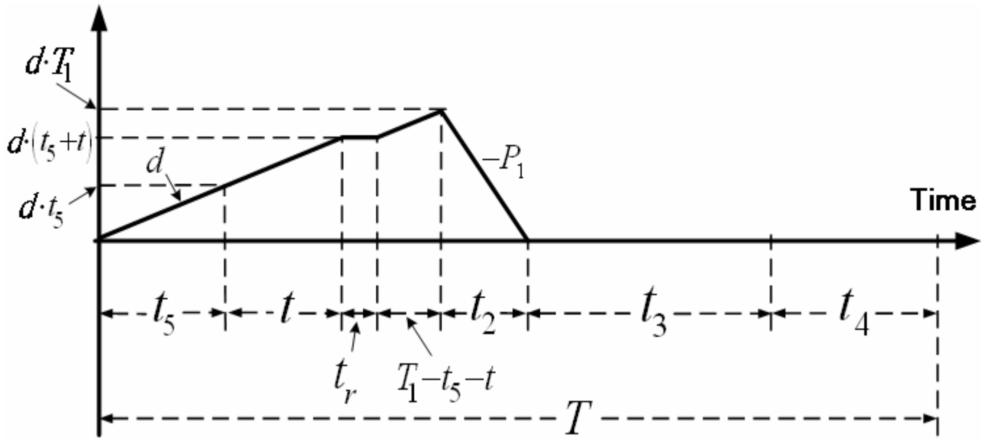

Fig.2: On-hand inventory of defective items in EMQ model with failure-in-rework and breakdown occurring in stock-piling time

Figure 3 illustrates the on-hand inventory level of scrap items. During the rework process, production rate of scrap items and total scrap items produced are as follows:

$$d_1 = P_1 \cdot \theta_1 \text{ ; where } 0 \le \theta_1 \le 1 \tag{12}$$

$$d_1 \cdot t_2 = (P_1 \cdot \theta_1) \cdot t_2 = (d \cdot T_1) \cdot \theta_1 = (x \cdot Q) \cdot \theta_1 \tag{13}$$

Total production-inventory cost per cycle $TC(T_1,B)$ is:

$$\begin{aligned} TC(T_1,B) &= K + M + C\cdot(PT_1) + C_R\cdot(PT_1\cdot x) + C_S\cdot(PT_1\cdot x)(1-\theta_1) \\ &+ h\left[\frac{H_1(t)}{2} + \frac{H_1+H_2}{2}(t_r) + \frac{H_2+H_3}{2}(T_1-t_5-t) + \frac{H_3+H_4}{2}(t_2) + \frac{H_4(t_3)}{2}\right] \\ &+ h\left[\frac{d(t_5+t)}{2}(t_5+t) + (t_5+t)t_r + \frac{(t_5+t)+dT_1}{2}(T_1-t_5-t)\right] \\ &+ h_1\left[\frac{dT_1}{2}(t_2)\right] + b\left[\frac{B}{2}(t_5) + \frac{B}{2}(t_4)\right] \end{aligned} \tag{14}$$

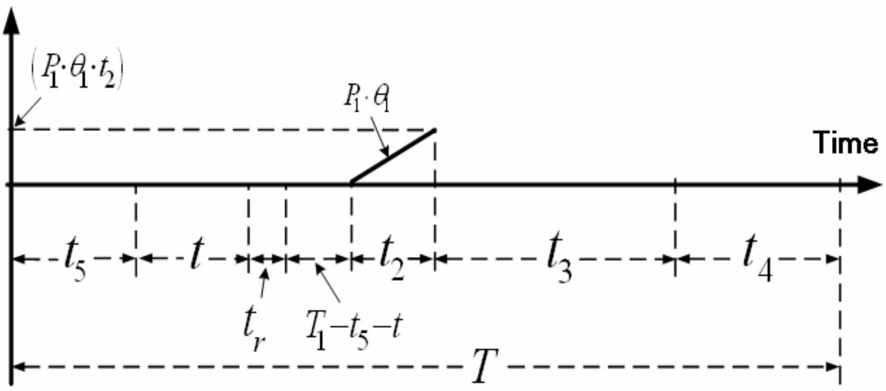

Fig.3: On-hand inventory level of scrap items

Substituting all related parameters from equations (1) to (13) in equation (14), $TC(T_1,B)$ becomes:

$$TC(T_1,B)=C\cdot P\cdot T_1+K+M+C_R\cdot T_1\cdot P\cdot x+C_S\cdot T_1\cdot P\cdot x\cdot\theta_1$$
$$+\frac{h}{2}\left\{\frac{P^2}{\lambda}\left(1-2x\theta_1+x^2\theta_1^2\right)T_1^2+\frac{(1-x)}{\lambda\left(1-x-\frac{\lambda}{P}\right)}B^2-PT_1^2+2Px\theta_1T_1^2\right\}$$
$$+\frac{b(1-x)}{2\lambda\left(1-x-\frac{\lambda}{P}\right)}B^2+\frac{P^2x^2}{2P_1}\left[h_1-h(1-\theta_1)\right]T_1^2-h\frac{P}{\lambda}T_1B(1-x\theta_1)$$
$$+\frac{hg}{\left(1-x-\frac{\lambda}{P}\right)}(B+g\lambda)-hPgT_1(1-x\theta_1)+hPgt \quad (15)$$

Because of the random defective/scrap rates and the uniformly distributed breakdown is assumed to occur in stock-piling time, the production cycle length is not constant.

Thus, to take the randomness of scrap rate and breakdown into account, one can employ the renewal reward theorem in production- inventory cost analysis to cope with the variable cycle length and use the integration of $TC(T_1,B)$ to deal with random breakdown happening in stock- piling time.

The long-run expected costs per unit time can be computed as follows.

$$E\left[TCU(T_1,B)\right]=\frac{E\left[\int_0^{T_1-t_5}TC(T_1,B)\cdot f(t)\,dt\right]}{E[T]}=\frac{E\left[\int_0^{T_1-t_5}TC(T_1,B)\cdot(1/t_5)\,dt\right]}{T_1P\left(1-\theta_1E[x]\right)/\lambda} \quad (16)$$

Substituting all related parameters from Equations (1) to (13) in equation (16), E[$TCU(T_1,B)$] becomes:

$$E\left[TCU(T_1,B)\right]=\lambda\left[C\frac{1}{1-\theta_1E[x]}+C_R\frac{E[x]}{1-\theta_1E[x]}+C_S\theta_1\frac{E[x]}{1-\theta_1E[x]}\right]$$
$$+\frac{\lambda(K+M)}{PT_1}\frac{1}{1-\theta_1E[x]}+\frac{h}{2}\left[\left(1-\frac{\lambda}{P}\right)PT_1-2B\right]\frac{1}{1-\theta_1E[x]}$$
$$+\frac{\lambda PT_1}{2P_1}\left[h_1-h(1-\theta_1)\right]\frac{E\left[x^2\right]}{1-\theta_1E[x]}+\frac{hPT_1\theta_1^2}{2}\frac{E\left[x^2\right]}{1-\theta_1E[x]}-\frac{h\lambda g}{2}\frac{1}{1-\theta_1E[x]}$$
$$+\frac{B^2}{2PT_1}(b+h)E\left(\frac{1-x}{1-x-\frac{\lambda}{P}}\right)\frac{1}{1-\theta_1E[x]}+h\theta_1\left[B-\left(1-\frac{\lambda}{P}\right)PT_1\right]\frac{E[x]}{1-\theta_1E[x]}$$
$$+\frac{h\lambda g}{2PT_1}(B+\lambda g)E\left(\frac{1}{1-x-\frac{\lambda}{P}}\right)\frac{1}{1-\theta_1E[x]}+h\lambda g\theta_1\frac{E[x]}{1-\theta_1E[x]} \quad (17)$$

Let $E_0=\frac{1}{1-\theta_1E[x]}$; $E_1=\frac{E[x]}{1-E[x]\theta_1}$; $E_2=\frac{E\left[x^2\right]}{1-\theta_1E[x]}$;

$$E_3=\frac{1}{1-\theta_1E[x]}E\left[\frac{1-x}{1-x-\frac{\lambda}{P}}\right];E_4=\frac{1}{1-\theta_1E[x]}E\left[\frac{1}{1-x-\frac{\lambda}{P}}\right]$$

Then Eq. (17) becomes:

$$E\left[TCU(T_1,B)\right]=\lambda\left[CE_0+C_RE_1+C_S\theta_1E_1\right]+\frac{\lambda(K+M)}{PT_1}E_0$$
$$+\frac{h}{2}\left[\left(1-\frac{\lambda}{P}\right)PT_1-2B\right]E_0+\frac{\lambda PT_1}{2P_1}\left[h_1-h(1-\theta_1)\right]E_2$$
$$+\frac{hPT_1\theta_1^2}{2}E_2-\frac{h\lambda g}{2}E_0+\frac{B^2}{2PT_1}(b+h)E_3$$
$$+h\theta_1\left[B-\left(1-\frac{\lambda}{P}\right)PT_1\right]E_1+\frac{h\lambda g}{2PT_1}(B+\lambda g)E_4+h\lambda g\theta_1E_1 \quad (18)$$

3 Optimal Run Time and Lot Size

By minimizing the long-run average cost function *E[TCU(T_1,B)]*, one obtains the optimal production run time. For proof of convexity of *E[TCU(T_1,B)]*, one can utilize the Hessian matrix equations [33] and verify the existence of the following:

$$\begin{bmatrix}T_1 & B\end{bmatrix}\cdot\begin{pmatrix}\frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial T_1^2} & \frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial T_1\partial B}\\ \frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial T_1\partial B} & \frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix}>0 \quad (19)$$

E[TCU(T_1,B)] is strictly convex only if equation (19) is satisfied, for all T_1 and B different from zero:

$$\begin{bmatrix}T_1 & B\end{bmatrix}\cdot\begin{pmatrix}\frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial T_1^2} & \frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial T_1\partial B}\\ \frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial T_1\partial B} & \frac{\partial^2E\left[TCU(T_1,B)\right]}{\partial B^2}\end{pmatrix}\cdot\begin{bmatrix}T_1\\ B\end{bmatrix} \quad (20)$$
$$=\frac{2\lambda(K+M)}{PT_1}E_0+\frac{hg^2\lambda^2}{PT_1}E_4>0$$

Equation (20) is resulting positive because all parameters are positive. Hence, *E[TCU(T_1,B)]* is a strictly convex function. It follows that for the optimal uptime T_1 and maximal backorder level B, one can differentiate *E[TCU(T_1,B)]* with respect to T_1 and with respect to B, and solve linear systems of equations (21) and (22) by setting these partial derivatives equal to zero.

$$\frac{\partial E[TCU(T_1,B)]}{\partial T_1} = -\frac{\lambda(K+M)}{PT_1^2}E_0 + \frac{h}{2}P\left(1-\frac{\lambda}{P}\right)E_0 - \frac{h\lambda gB}{2PT_1^2}E_4 + \frac{\lambda P}{2P_1}\left[h_1 - h(1-\theta_1)\right]E_2 - \frac{hg^2\lambda^2}{2PT_1^2}E_4 - h\theta_1 P\left(1-\frac{\lambda}{P}\right)E_1 + \frac{hP\theta_1^2}{2}E_2 - \frac{B^2}{2PT_1^2}(b+h)E_3 \quad (21)$$

$$\frac{\partial E[TCU(T_1,B)]}{\partial B} = -hE_0 + \frac{B}{PT_1}(b+h)E_3 + h\theta_1 E_1 + \frac{h\lambda g}{2PT_1}E_4 \quad (22)$$

$$\therefore T_1^* = \frac{1}{P}\sqrt{\frac{2\lambda(K+M)E_0 + h\lambda^2 g^2 E_4\left(1-\frac{hE_4}{4(b+h)E_3}\right)}{\left(h\left(1-\frac{\lambda}{P}\right)E_0 + \frac{\lambda}{P_1}[h_1 - h(1-\theta_1)]E_2 - \frac{h^2}{(b+h)E_3} - 2h\theta_1\left(1-\frac{\lambda}{P}\right)E_1 + h\theta_1^2 E_2\right)}} \quad (23)$$

$$\therefore B^* = \frac{h}{(b+h)E_3}\left(PT_1^* - \frac{\lambda g}{2}E_4\right) \quad (24)$$

From Equations (6), (23), and (24), one can obtain the optimal lot-size Q^* and optimal backorder level B^* as follows:

$$Q^* = \sqrt{\frac{2\lambda(K+M)E_0 + h\lambda^2 g^2 E_4\left(1-\frac{hE_4}{4(b+h)E_3}\right)}{\left(h\left(1-\frac{\lambda}{P}\right)E_0 + \frac{\lambda}{P_1}[h_1 - h(1-\theta_1)]E_2 - \frac{h^2}{(b+h)E_3} - 2h\theta_1\left(1-\frac{\lambda}{P}\right)E_1 + h\theta_1^2 E_2\right)}} \quad (25)$$

$$B^* = \frac{h}{(b+h)E_3}\left(Q^* - \frac{\lambda g}{2}E_4\right) \quad (26)$$

4 Solution Verification

Suppose machine breakdown is not a factor to be considered, then the repairing cost and time for the failure machine, M=0 and g=0, Equations (25)-(26) become the same equations as were given by [16]:

$$Q^* = \sqrt{\frac{2K\lambda}{\left(h\left(1-\frac{\lambda}{P}\right) + \frac{\lambda}{P_1}[h_1 - h(1-\theta_1)]E\left[x^2\right] + h\theta_1^2 E\left[x^2\right] - \frac{h^2\cdot\left[1-\theta_1 E[x]\right]^2}{(b+h)E\left[\frac{1-x}{1-x-\frac{\lambda}{P}}\right]} - 2h\theta_1\left(1-\frac{\lambda}{P}\right)E[x]\right)}} \quad (27)$$

$$\text{and} \quad B^* = \left(\frac{h}{b+h}\right)\left(\frac{1}{E_3}\right)\cdot Q^* = \left(\frac{h}{b+h}\right)\cdot\frac{1-\theta_1\cdot E[x]}{E\left(\frac{1-x}{1-x-\frac{\lambda}{P}}\right)}\cdot Q^* \quad (28)$$

Further, suppose that the regular production process produces no defective items, i.e. x = 0, then Equations (27)-(28) become the same equations as were presented by the classic EMQ model with shortages permitted and backordered [3,34]:

$$\text{If } x=0, \text{ then } Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}}\cdot\sqrt{\frac{b+h}{b}} \quad (29)$$

$$\text{and} \quad B^* = \left[\frac{h}{(b+h)}\left(1-\frac{\lambda}{P}\right)\right]\cdot Q^* \quad (30)$$

5 Examples and Discussion

5.1 Numerical Example 1

Consider a manufactured item can be produced at a rate of 18,000 units and demand of this item is 3,000 units per year. A random percentage of defective items produced x, follows a uniform distribution over the interval [0, 0.15]. All imperfect quality items are reworked at a rate of P_1= 6,000 units per year, and a θ_1=0.4 portion of the reworked items fails the repairing and becomes scrap. The following are additional parameters used:

- M = \$500 repair cost for each breakdown,
- g = 0.018 years, time needed to repair and restore the machine,
- K = \$240 for each production run,
- C = \$2.00 per item,
- C_R = \$1.2 for each item reworked,
- C_S = \$1.00 disposal cost for each scrap item,
- h = \$0.6 per item per unit time,
- h_1 = \$0.8 per item per unit time,
- b = \$0.8 per item backordered per unit time.

By using equations (23)-(24) one can obtain the optimal T_1*=0.2189 years and backorder level B*=1,371. From equation (18), the long-run average costs $E[TCU(T_1^*,B^*)]$= \$7,460.73. Figure 4 shows variation of defective rate x effects on optimal run time T_1*. It indicates that as x increases, the value of run time T_1* decreases.

Figure 5 illustrates the behavior of the optimal production run time with respect to defective rate x and scrap rate θ_1.

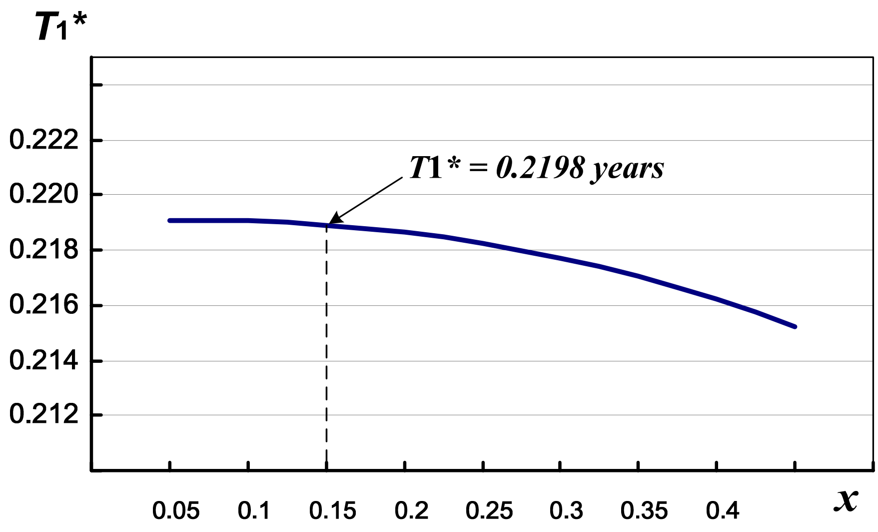

Fig.4: Variation of defective rate *x* effects on optimal run time T_1*

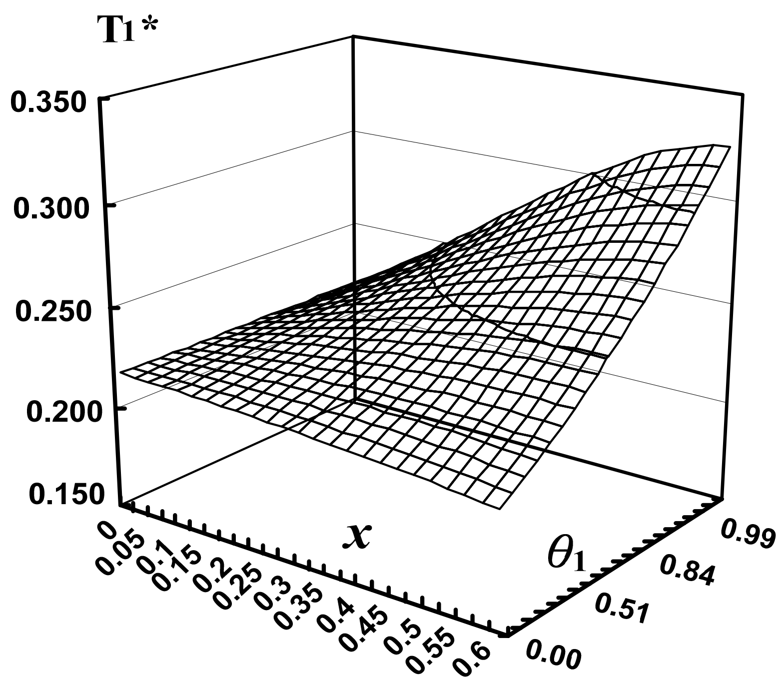

Fig.5: Behavior of the optimal production run time with respect to defective rate *x* and scrap rate θ_1

Variation of defective rate *x* and scrap rate θ_1 effects on optimal backorder quantity *B** is displayed in Figure 6.

Figure 7 depicts the behavior of $E[TCU(T_1*,B*)]$ with respect to defective rate *x* and scrap rate θ_1. It points out that as θ_1 increases, the value of $E[TCU(T_1*,B*)]$ increases slightly and for different *x* values, as *x* increases, $E[TCU(T_1*,B*)]$ increases significantly.

When dealing with an unreliable manufacturing system with breakdown occurring in stock-piling time, suppose the result of the present study is not available, one can only use a closely related lot-size solutions given by [16] and obtain run time T_1 = 0.1247 years and *B* = 782 units. Plugging T_1 and *B* into equation (18) of this paper, one obtains $E[TCU(T_1,B)]$=\$7,645.<u>32</u>.

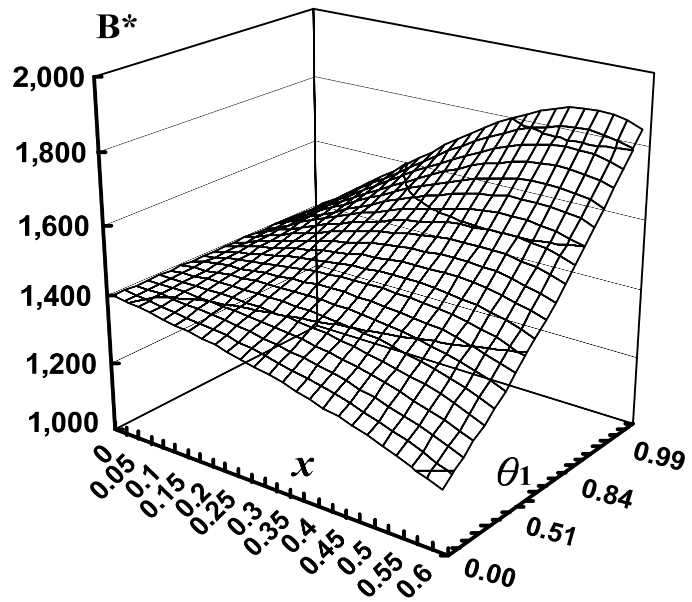

Fig.6: Variation of defective rate x and scrap rate θ_1 effects on optimal backordering quantity B^*

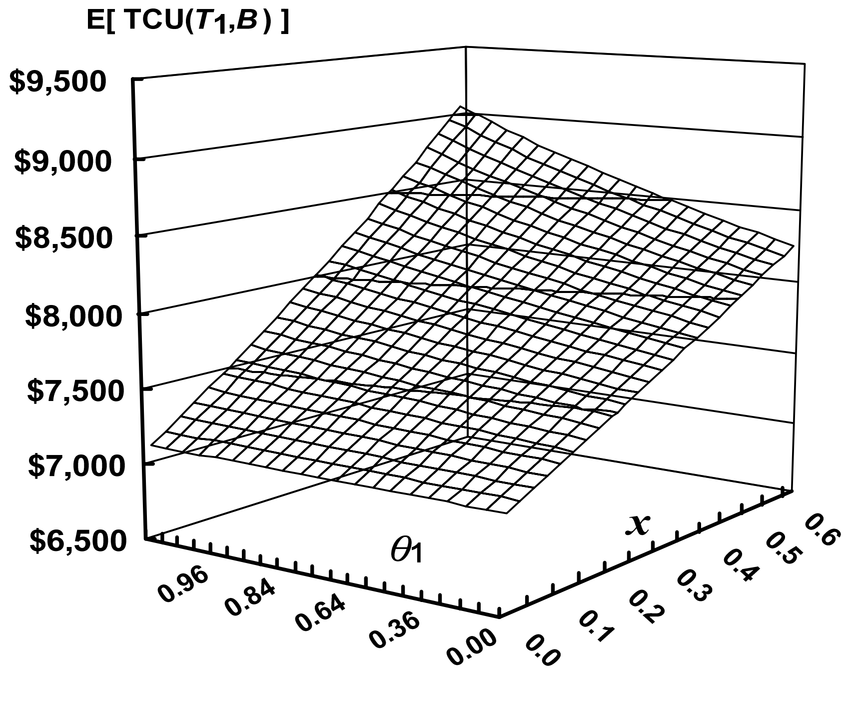

Fig.7: Behavior of $E[TCU(T_1^*,B^*)]$ with respect to θ_1 and x

As a result, it pays extra 12.64% on total setup and holding costs than the optimal costs obtained by using the optimal run time decisions (i.e. equation (18)) from the present study.

5.2 Numerical Example 2

Suppose another manufactured item can be produced at a rate of 44,000 units and its annual demand is 11,000 units. A random percentage of defective items produced *x*, follows a uniform distribution over the interval [0, 0.20]. All imperfect quality items are reworked at a rate of P_1= 2,640 units per year, and a θ_1=0.3 of the reworked items fails the repairing and becomes scrap. Other parameters used are:

- M = \$600 repair cost for each breakdown,
- g = 0.036 years, time needed to repair and restore the machine,
- K = \$350 for each production run,
- C = \$2.40 per item,
- C_R = \$1.8 for each item reworked,
- C_S = \$0.80 disposal cost for each scrap item,
- h = \$1.00 per item per unit time,
- h_1 = \$1.20 per item per unit time,
- b = \$1.40 per item backordered per unit time.

Applying equations (23)-(24) one obtains the optimal T_1*=0.1549 years or 8.05 weeks, and backorder level B*=1,959. From equation (18), the long-run average costs $E[TCU(T_1*,B*)]$= \$32,523. By using equation (6), one obtains its optimal lot-size Q*= 6815, it can then be used for consequent materials requirement planning based on product structure diagram [35-36].

Figure 8 shows variation of defective rate *x* effects on optimal production lot size Q_1*. It shows that as *x* increases, the value of Q* decreases significantly.

Variation of defective rate effects on optimal expected inventory cost function E[TCU(Q*,B*)] is depicted in Figure 9.

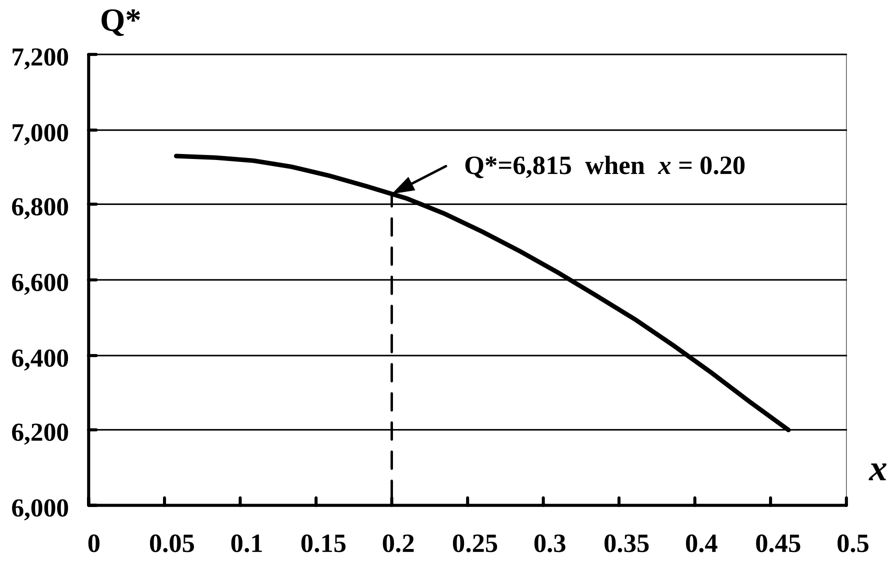

Fig. 8: Variation of defective rate *x* effects on optimal lot size Q*

6 Conclusion

This study provides a complete solution procedure for determining optimal run time for EMQ model with backordering of excess demand, failure-in-rework, and breakdown happening in stock-piling time. This procedure includes the mathematical modeling, the use of renewal reward theorem to cope with variable cycle length, derivation of the long-run average production-inventory cost function, proof of convexity of the cost function, and determining the optimal replenishment run time that minimizes expected the overall costs. Numerical example with discussion is also presented to confirm its practical usage and contribution of this research.

With an in-depth investigation on such an unreliable manufacturing system, optimal operating discipline and related facts of the system can now be revealed and managers in the field can adopt the result of this run time decision to establish their own robust production plan accordingly.

For future research, to investigate the effect of random breakdown with abort/resume inventory control policy and occurring in the backorder filling time on same model will be one of the interesting topics.

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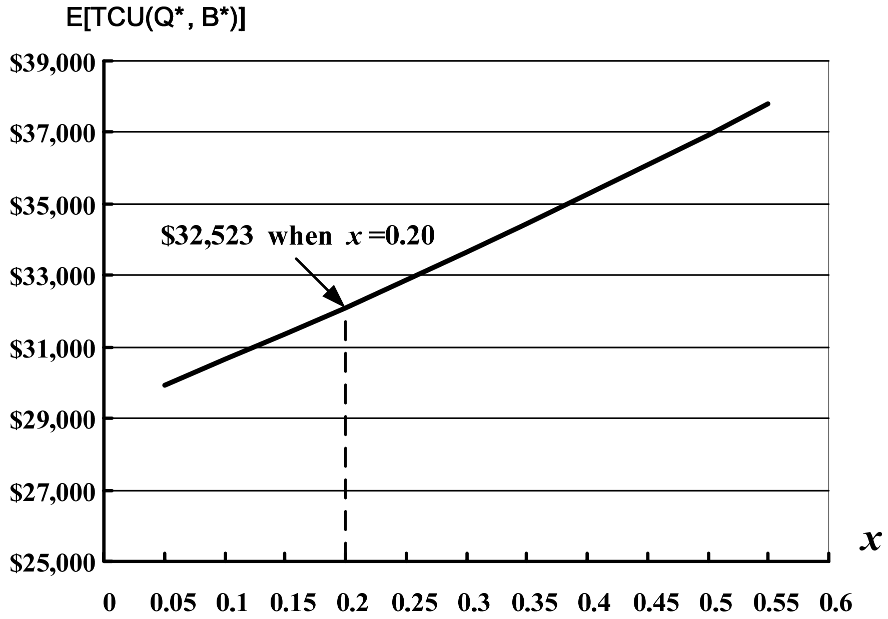

Fig. 9: Variation of defective rate effects on optimal expected inventory cost function E[TCU(Q^*,B^*)]

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