The stability of fire extinguishing rocket motor

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Abstract: - The aim of this paper consists in developing a model for realistic calculation, but at the same time not a very complicated one, in order to determine the operating parameters of a rocket motor with solid propellant (RMSP). The model results will be compared with experimental data and the quality of the model will be evaluated. The study of stability RMSP will be made accordingly to Liapunov theory, considering the system of parametric equations perturbed around the balance parameters. The methodology dealing with the stability problem consists in obtaining the linear equations and the verification of the eigenvalues of the stability matrix. The results are analyzed for a functional rocket motor at low pressure, which has the combustion chamber made of cardboard, motor used for fire extinguishing rocket. The novelty of the work lies in the technique to tackle the stability problem for the operation of rocket motors at low pressure, representing specific applications for civil destination.

Key-Words: - Rocket, Motor, Solid propellant, Stability, Liapunov theory, Low pressure, Fire-extinguishing

NOMENCLATURE

λ - Ratio between velocity in exit plane and velocity in throat area;
ρ - Gas density in burning chamber;
ρ_p - Propellant density
ψ - Ratio between propellant mass consumed and total propellant mass;
φ - Erosion factor;
σ - Ratio between instantaneous burning surface and initial burning surface;
σ_T - Ratio between instantaneous propellant cross surface and initial propellant cross surface;
k - Gas specific heats ratio;
A_t - Throat area;
A_e - Exit area;
F - Motor thrust;
I_{sp} - Specific impulse;
I_Σ - Total impulse;
Q_C -Heat quantity educts by burning reaction of 1 kilogram propellant;
q - Amount of heat transferred to the combustion chamber in time unit (heat flow);
D - Coefficient of variation of burning rate with initial propellant temperature;
l -Length of the propellant grain;
$\dot{m}_p$ -Propellant consuming mass in time unit;

p - Gas pressure in burning chamber;
p_e - Gas pressure in exit area;
p_H - Atmospheric pressure;
R - Gas constant in burning chamber;
T - Gas temperature in burning chamber;
T_{0N} - Normal propellant temperature for burning rate;
T_{in} - Initial propellant temperature;
u - Burning rate;
u_N - Linear burning rate in normal conditions:
U - Energy;
V - Volume of the burning chamber;
V_0 - Initial volume of the burning chamber;
V_p - Propellant volume;
w_e -Gas velocity in exit plane;
w_t -Gas velocity in throat plane;
S - Instantaneous burning surface;
S_T - Instantaneous propellant cross surface;

1 Introduction

Using missiles into civilian area involve a series of specific measures for compliance with environmental restrictions like a greater degree of safety in operation, and persons' protection.
An example of such an application is the fire-extinguishing rocket, which has a motor made of cardboard, ecological, non-hazardous but with low operating pressure. This type of technical problem causes the need for a scientific approach to support the technological effort of achieving such a missile motor capable of stable operating at low pressure, which is the subject to approach in this work.
One of the main challenges in designing rocket motor with solid propellant - RMSP are determination of the functional its parameters and analyzing their stability.
The problems of stability of combustion can be addressed by different ways both experimental and theoretical, a series of methods and models being shown in the works [4], [5], [6], [7]. Note that the paper [4] proposes a different approach of stability for linear and non-linear phenomena. Unlike this, in our work the approach will be unitary, being focused on a particular and difficult case, that of the combustion at low pressure.
In our study we will develop a non linear model for calculus of the functional parameters of RMSP, followed by the analysis of the evolution of balance stability regarded as the basic movement. Stability analysis for the perturbed equations of the RMSP will be made according to Liapunov theory, by placing them in the linear form.
Remember that Liapunov theory said "If we can prove that linear form of the equations system is stable then its initial non-linear form is also stable".
Resuming, our work has two purposes:
- Scientific one – to check the possibility of applying Liapunov theory [9] to analyze the stability of the balance parameters of RMSP at low pressure;
- Technical one – to design the rocket motor for the fire-extinguishing rocket.

2 RMSP internal ballistic model

An important parameter in an internal ballistic model for a rocket engine is the burning rate of propellant. In the case of a **R**ocket **M**otor with **S**olid **P**ropellant (**RMSP)**, the burning rate is called regression rate and is given by a relation indicated in paper [1]

$$u = \varphi(x) a p^{\nu}, \tag{1}$$

where the erosion factor has been denoted with $\varphi(x)$ and the coefficient a can be expressed by:

$$a = u_N p_H^{-\nu} e^{D(T_{in}-T_N)} \tag{2}$$

where $e^{D(T_{in}-T_N)}$ shows the influence of the variation of the initial propellant temperature and p_H means atmospheric pressure. Exponent D, parameter ν and regression rate u_N are determined experimentally under the normal propellant temperature (T_N).
To assess the erosive phenomenon we use the parameter named in [1] "Pobedonosetov" parameter:

$$x = (S - S_T)/(S_{cam} - S_T), \tag{3}$$

which allows us to determine the erosion factor:

$$\varphi(x) = \begin{cases} 1 + 3{,}2\times10^{-3}(x-100) & for\ x > 100; \\ 1 & for\ x \le 100 \end{cases}. \tag{4}$$

In order to obtain the surface burning area, we define the parameter:

$$\psi = (V - V_0)/V_p \tag{5}$$

In this case, the quadratic fitting can be used:

$$\sigma(\psi) = \begin{cases} a_2\psi^2 + a_1\psi + 1 & for\ \psi < 1; \\ 0 & for\ \psi \ge 1 \end{cases}; \tag{6}$$

$$\sigma_T(\psi)=\begin{cases} b_2\psi^2+b_1\psi+1 & for\ \psi<1; \\ 0 \quad for\ \psi\geq 1 \end{cases}; \tag{7}$$

and burning area and propellant cross-section become:

$$S(\psi)=S_0\sigma(\psi); \tag{8}$$

$$S_T(\psi)=S_{T0}\sigma_T(\psi). \tag{9}$$

Altogether, by simple geometrical reasoning, the volume variation in time is given by:

$$\dot{V}=S(\psi)\varphi(x)ap^m, \tag{10}$$

relation which represents volume equation.

Using the continuity equation, the variation of the mass in the burning chamber is the difference between the mass produced in time unit by the propellant burning and the exits mass from motor through the nozzle in time unit:

$$\frac{\partial(\rho V)}{\partial t}=\dot{m}_{in}-\dot{m}_{out}, \tag{11}$$

where V is the volume of the burning chamber, and ρ is gas density inside the burning chamber, $\dot{m}_{in}$ is the input mass generated from the propellant combustion inside the motor chamber and $\dot{m}_{out}$ is the output mass ejected through the nozzle of the rocket motor. The input mass per time unit is given by the propellant input:

$$\dot{m}_{in}=\dot{m}_p, \tag{12}$$

and the output mass in time unit is the mass flow through the nozzle:

$$\dot{m}_{out}=\Lambda A_t\sqrt{p\rho}, \tag{13}$$

where A_t is the throat area, p is the pressure in chamber, ρ is gas density, and:

$$\Lambda=\sqrt{k\left(2/(k+1)\right)^{(k+1)/(k-1)}} \tag{14}$$

Taking into account that the propellant consuming mass in time unit is:

$$\dot{m}_p=\rho_p\dot{V}, \tag{15}$$

developing relation (11) we obtain the density equation:

$$\dot{\rho}=\left(\rho_p-\rho\right)\frac{\dot{V}}{V}-\frac{\Lambda A_t}{V}\sqrt{p\rho}. \tag{16}$$

Taking into account the equation (10), the density equation becames:

$$\dot{\rho}=\left(\rho_p-\rho\right)S\varphi aV^{-1}p^{\nu}-\Lambda A_tV^{-1}p^{1/2}\rho^{1/2}. \tag{17}$$

Beside the volume equation (10) and density equation (17), we need the third equation expressing the change in temperature or pressure of the combustion products.

We consider that the input energy for the system is the heat quantity Q_C educts by burning reaction of m_p solid propellant.

Also, we take into account that the specific heat at constant volume C_V can be obtain from the relation:

$$C_V=R/(k-1). \tag{18}$$

where k is the ratio of specific heats and R is the gas constant in burning chamber.

To build the temperature equation, we start from the following relationship of energy balance:

$$\mathrm{d}U=\mathrm{d}U_1+\mathrm{d}U_2+\mathrm{d}U_3+\mathrm{d}U_4, \tag{19}$$

where the reaction energy of the propellant, given by:

$$\mathrm{d}U=Q_C\,\mathrm{d}m_p, \tag{20}$$

is converted into:

- internal energy growth due to additional gas from the combustion chamber:

$$\mathrm{d}U_1=C_VTV\,\mathrm{d}\rho=R(k-1)^{-1}TV\,\mathrm{d}\rho; \tag{21}$$

- energy in gas from the combustion chamber increased due to temperature variation:

$$\mathrm{d}U_2=\rho VC_V\,\mathrm{d}T=R(k-1)^{-1}\rho V\,\mathrm{d}T; \tag{22}$$

- kinetic energy due to gas flow:

$$\mathrm{d}U_3=k(k-1)^{-1}RT\,\mathrm{d}m_{out}. \tag{23}$$

- loss of energy due to the disposal of heat through the chamber walls:

$$\mathrm{d}U_4=q\,\mathrm{d}t, \tag{24}$$

where q is the amount of heat transferred to the combustion chamber in time unit (heat flow) $[J/s]$.

If we take the derivative of (19) with respect to time and then simplify it, we obtain:

$$Q_C\dot{m}_p/C_V=TV\dot{\rho}+\rho V\dot{T}+kT\dot{m}_{out}+q/C_V, \tag{25}$$

hence we obtain the temperature equation:

$$(k-1)Q_C\frac{\rho_p}{p}\frac{\dot{V}}{V}=\frac{\dot{\rho}}{\rho}+\frac{\dot{T}}{T}+\frac{k\Lambda A_t}{V}\sqrt{\frac{p}{\rho}}+\frac{(k-1)q}{pV}. \tag{26}$$

Taking into account that the state equation can be written in form:

$$\dot{p}/p=\dot{\rho}/\rho+\dot{T}/T, \tag{27}$$

we transform the temperature equation (26) into the pressure equation:

$$\begin{aligned}\dot{p}=&(k-1)\rho_pQ_CS\varphi aV^{-1}p^{\nu}-\\&-k\Lambda A_tV^{-1}\rho^{-1/2}p^{3/2}-(k-1)qV^{-1}\end{aligned}. \tag{28}$$

Having differential equations (17) and (28) solved, for temperature we can use the state relation:

$$T = p/(R\rho). \tag{29}$$

Regular paper [6], for the rate between throat area A_t and exit area of the nozzle A_e propose the relation:

$$\frac{A_t}{A_e} = \sqrt{\frac{2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{1-k}} \tilde{p}_e^{\frac{2}{k}} \left(1-\tilde{p}_e^{\frac{k-1}{k}}\right)}, \tag{30}$$

with the relative pressure given by:

$$\tilde{p}_e = p_e/p \ , \tag{31}$$

where p_e is the gas pressure in exit area.

If we take into account that the gas velocity in the exit plane is:

$$w_e = \sqrt{2\frac{k}{k-1}RT\left(1-\tilde{p}_e^{\frac{k-1}{k}}\right)} \ , \tag{32}$$

and the gas velocity in throat plane is:

$$w_t = \sqrt{2\frac{k}{k+1}RT} \ , \tag{33}$$

the velocity report becomes:

$$\lambda = w_e/w_t = \sqrt{\frac{k+1}{k-1}\left(1-\tilde{p}_e^{\frac{k-1}{k}}\right)} \ . \tag{34}$$

From (30) and (34) we can obtain the rate surfaces formula indicated in paper [5]:

$$\frac{A_t}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \lambda \left[1-\frac{k-1}{k+1}\lambda^2\right]^{\frac{1}{k-1}}, \tag{35}$$

The relation (35) leads to transcendental equation,

$$\lambda = \frac{A_t}{A_e}\left(\frac{k+1}{2}\right)^{\frac{1}{1-k}} \left[1-\frac{k-1}{k+1}\lambda^2\right]^{\frac{1}{1-k}}. \tag{36}$$

Figure 1 shows the left member λ and the right member denoted $f(\lambda)$ in the previous relation. The diagram can be used to estimate a graphic solution for equation (36) when k=1.4. Similar representations can be easily obtained for other values of k.

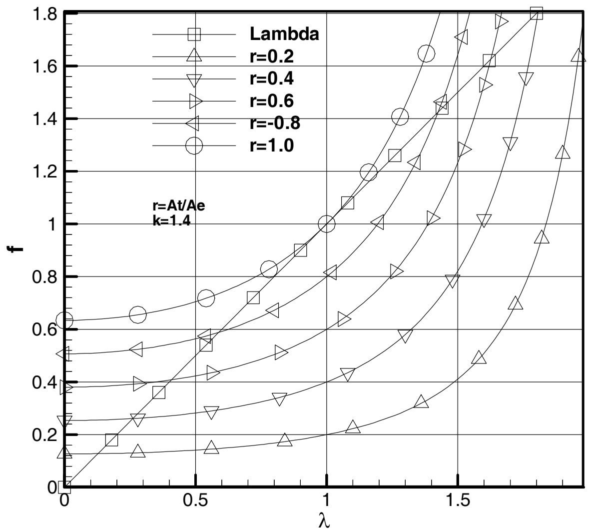

Fig. 1 Graphic representation of the transcendental equation

We can observe that the right member of the relation (36), satisfies the inequality:

$$\frac{\mathrm{d}\, f(\lambda)}{\mathrm{d}\,\lambda} > 1, \tag{37}$$

which means that relation (36) considerate like iterative, does not converge. In this case, we put this relation in Newton-Raphson form:

$$\lambda_{i+1} = \lambda_i - \frac{\lambda_i - f(\lambda_i)}{1-\left.\frac{\mathrm{d}\, f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda i}}. \tag{38}$$

If we denote:

$$a_1 = \frac{1}{1-k}\,; b_1 = \frac{1-k}{1+k}\,;\ c_1 = \frac{A_t}{A_e}\left(\frac{k+1}{2}\right)^{a1}, \tag{39}$$

we can write:

$$\begin{aligned} f(\lambda) &= c_1(1+b_1\lambda^2)^{a_1}\,; \\ f'(\lambda) &= 2a_1b_1c_1\lambda(1+b_1\lambda^2)^{a_1-1}, \end{aligned} \tag{40}$$

and an iterative relation can be obtained:

$$\lambda_{i+1} = \lambda_i - \frac{\lambda_i - c_1(1+b_1\lambda_i^2)^{a_1}}{1-2a_1b_1c_1\lambda_i(1+b_1\lambda_i^2)^{a_1-1}}. \tag{41}$$

Because this relation is independent from equations system, it can be applied separately, before system being solved. The iterative procedure is convergent for the initial value close to the final solution. We recommend starting from $\lambda = 2$.

Assuming constant ratio of specific heats throughout the expansion process, one finds out the thrust force relation indicated in paper [1]:

$$F = A_e p_H \left[\sigma_c \frac{p}{p_H} \frac{A_t}{A_e} k \left(\frac{2}{k+1} \right)^{\frac{1}{k-1}} \lambda - 1 \right] \quad (42)$$

where p_H is atmospheric pressure, and σ_c is overall loss of thrust by nozzle. The simplest nozzle is the conical one with a divergence cone half angle of 10-18 degrees. For such nozzles, part of the force of exhaust gases is orientated transversally and thus does not produce any thrust at all. In order to correct this phenomena one can use a correction factor related to the divergence cone half angle. Also other loses can appear, all of these can be taken into account using coefficient σ_c.

3 Balance parameters

The study of stability in operating a RMSP will be made accordingly to Liapunov theory, considering the system of parametric equations perturbed.

This means that one has to consider the system of parametric equations perturbed around the balance parameters. This involves a disturbance applied shortly on the evolution of balance, which will produce a deviation of the state variables. Developing in series the perturbed parametric equations in relation to status variables and taking into account the first order terms of the detention, we will get linear equations which can be used to analyze the stability in first approximation, as we proceed in most dynamic non linear problems.

Thus, for defining the evolution of balance, we consider:

$$\dot{p} = 0 \quad \dot{\rho} = 0 \, ; \dot{V} = Sap^{\nu} = ct \, . \quad (43)$$

Using these, from relations (16) and (28) we obtain:

$$(\rho_p - \rho)\dot{V} - \Lambda A_t p^{1/2} \rho^{1/2} = 0 \, ; \quad (44)$$

$$(k-1)\rho_p Q_C \dot{V} - k \Lambda A_t \rho^{-1/2} p^{3/2} - (k-1) q = 0 \, , \quad (45)$$

moreover:

$$\rho = \rho_p - \frac{\Lambda A_t}{\dot{V}} \sqrt{p\rho} \, ; \quad (46)$$

$$p = \frac{k-1}{k \Lambda A_t} \left(\rho_p Q_C \dot{V} - q \right) \sqrt{\frac{\rho}{p}} \, , \quad (47)$$

from which we obtain:

$$p = k_1 \sqrt{\frac{\rho}{p}} \, ; \quad \rho = k_2 - k_3 \sqrt{p\rho} \, , \quad (48)$$

where we denote:

$$k_1 = \frac{k-1}{k \Lambda A_t} \left(\rho_f Q_C \dot{V} - q \right); \; k_2 = \rho_p \, ; k_3 = \frac{\Lambda A_t}{\dot{V}} \, . \quad (49)$$

Finally the balance equations become:

$$\rho = p^3 / k_1^2 \, ; \quad p^3 + k_3 k_1 p^2 - k_1^2 k_2 = 0 \, . \quad (50)$$

The pressure equation can be arranged in transcendental form:

$$p = b p^{-2} - a \, , \quad (51)$$

where

$$a = k_3 k_1 \, ; \; b = k_1^2 k_2 \, . \quad (52)$$

This can be solved using iterative Newton-Raphson method:

$$p_{i+1} = p_i - \frac{p_i - b p_i^{-2} + a}{1 + 2 b p_i^{-3}} \, . \quad (53)$$

In order to help our analysis we will use dimensionless parameter ψ defined by relation (5).

4 Linear equations

In the context of the balance parameters established above, the operating equations (10), (17) and (28) can be put in linear form:

$$\Delta \dot{V} = a_V^V \Delta V + a_V^p \Delta p \, ;$$
$$\Delta \dot{\rho} = a_\rho^V \Delta V + a_\rho^\rho \Delta \rho + a_\rho^p \Delta p \, ;$$
$$\Delta \dot{p} = a_p^V \Delta V + a_p^\rho \Delta \rho + a_p^p \Delta p \, , \quad (54)$$

where, neglecting the erosion factor, the coefficients of the equations are:

$$a_V^V = E \dot{V} V^{-1} ; \; a_V^p = \nu \dot{V} p^{-1} ;$$
$$a_\rho^V = \left[(\rho_p - \rho) \dot{V} (E-1) + \Lambda A_t p^{1/2} \rho^{1/2} \right] V^{-2}$$
$$a_\rho^\rho = -\left[\dot{V} + 0.5 \Lambda A_t p^{1/2} \rho^{-1/2} \right] V^{-1} ;$$
$$a_\rho^p = \left[\nu (\rho_p - \rho) p^{-1} \dot{V} - 0.5 \Lambda A_t p^{-1/2} \rho^{1/2} \right] V^{-1} ;$$
$$a_p^V = \begin{bmatrix} (k-1)\rho_p Q_C \dot{V} (E-1) + \\ + k \Lambda A_t \rho^{-1/2} p^{3/2} + (k-1) q \end{bmatrix} V^{-2} ;$$
$$a_p^\rho = 0.5 k \Lambda A_t \rho^{-3/2} p^{3/2} V^{-1} ;$$
$$a_p^p = \begin{bmatrix} (k-1)\rho_p Q_C \nu \dot{V} p^{-1} - \\ -1.5 k \Lambda A_t \rho^{-1/2} p^{1/2} \end{bmatrix} V^{-1} \quad (55)$$

where:

$$E = \frac{\sigma'(\psi)}{\sigma(\psi)} \frac{V}{V_p} = \frac{2 a_2 \psi + a_1}{a_2 \psi^2 + a_1 \psi + 1} \frac{V}{V_p}$$

Finally we can put the linear system in regular form:

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} \, , \quad (56)$$

where the state vector is:

$$\mathbf{x} = \begin{bmatrix} V & \rho & p \end{bmatrix}^T , \tag{57}$$

and the stability matrix becomes:

$$\mathbf{A} = \begin{bmatrix} a_V^V & 0 & a_V^p \\ a_\rho^V & a_\rho^\rho & a_\rho^p \\ a_p^V & a_p^\rho & a_p^p \end{bmatrix} , \tag{58}$$

From the previous relation one can observe that all the stability coefficients a_i^j are dependent by volume.

5 Input data

For exemplifying the method, we will build a study model out of the motor test.

The rocket engine for the fire-extinguishing rocket is presented in figure 2.

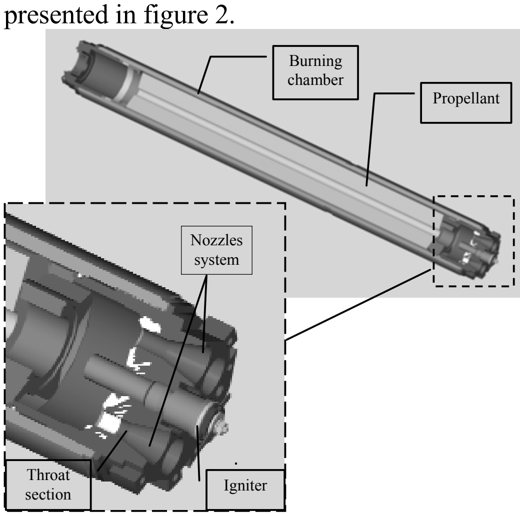

Fig.2 Rocket motor with solid propellant- RMSP for fire-extinguishing rocket

5.1 Propellant geometry

First we describe the geometry of propellant which is a cylinder, with a cylindrical hole inside, no insulated, so burning simultaneously on all surfaces (figure 3).

Denote instantaneous sizes:

R - Outside radius of the cylinder; r - inside radius of the cylinder; l - cylinder length, the burning area, terminal area and propellant volume are given by:

$$S = 2\pi(R+r)(R-r+l) \, ; S_T = \pi(R+r)(R-r) \, ;$$

$$V = S_T l = \pi(R+r)(R-r) \, . \tag{59}$$

If we denote x the linear burning distance, which at the time t is given by integrating the burning rate:

$$x = \int_0^t u \, \mathrm{d}t \, , \tag{60}$$

the main geometric quantities are rewritten as it follows:

$$R = R_0 - x \, ; \; r = r_0 + x \, ; \; l = l_0 - 2x \, , \tag{61}$$

from which the combustion areas and volume become:

$$S = S_0 - 4\pi(R_0 + r_0)x \tag{62}$$

$$S_T = S_{T0} - 2\pi(R_0 + r_0)x \tag{63}$$

$$V = V_0 - 2[S_{T0} + \pi(R_0 + r_0)l_0]x + \\ + 4\pi(R_0 + r_0)x^2 \tag{64}$$

where we denoted with index "0" the initial values for length, surfaces and volumes.

After processing we obtain:

$$\sigma = \frac{S}{S_0} = 1 - \frac{2x}{R_0 - r_0 + l_0} \, ; \; \sigma_T = \frac{S_T}{S_{T0}} = 1 - \frac{2x}{R_0 - r_0} \, ;$$

$$\psi = 1 - \frac{V}{V_0} = \frac{2x}{l_0} + \frac{2x}{R_0 - r_0} - \frac{4x^2}{(R_0 - r_0)l_0} \tag{65}$$

For the application the main geometrical quantities are:

$$R_0 = 33\,mm \, ; r_0 = 7\,mm \, ; \; l_0 = 319\,mm \, .$$

In this case, the initial areas are:

$$S_0 = 86708 \; mm^2 \, ; \quad S_{T0} = 3267 mm^2 \, .$$

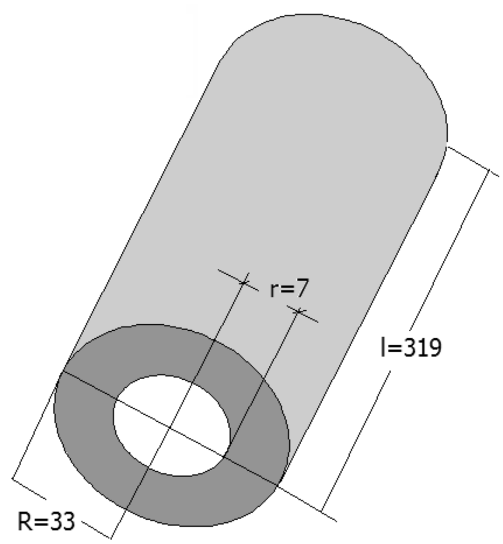

Fig. 3 Propellant geometry

Developing the relations (65) in a numerical form related to the parameter x results the dependence between the no dimensional areas σ, σ_T and the burn parameter ψ . By quadratic fitting we obtain:

$$\sigma(\psi) \cong 0.999951 - 0.069120\psi - 0.00614672\psi^2 \ ;$$
$$\sigma_T(\psi) \cong 0.999352 - 0.917169\psi - 0.0815622\psi^2 \ , \tag{66}$$

functions which are represented in figure 4:

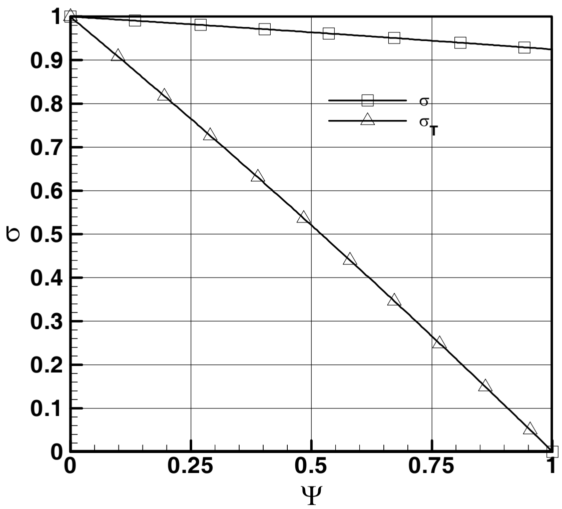

Fig. 4 Areas burning diagrams

5.2 Motor geometry

The motor geometry elements used for the test considered are:

-Combustion chamber cross surface:
$A_{cam} = 3739\, mm^2$;

- Throat area: $A_t = 490 mm^2$;
- Exit plane area: $A_e = 1206\ mm^2$.
- Burning chamber volume:
$V_{cam} = 1924555\ \ mm^3$

5.3 Propellant and process features

The features for the used propellant are:

- Propellant mass: $m_p = 1.834\, kg$;
- Propellant density: $\rho_p = 1790\, Kg / m^3$;
- Adiabatic gas coefficient of the combustion products $k = 1.4$;
- Gas constant: $R = 336.7\ \ J/Kg/K$;
- Linear burning rate in normal conditions: $u_N = 4.6\ mm / s$;
- Pressure exponent of burning law: $\nu = 0.18$;
- Coefficient of variation of burning rate with temperature: $D = 0.0038\, K^{-1}$;
- Heat quantity educts by burning reaction of 1 kilogram propellant: $Q_C = 4.9 \times 10^6\ J / Kg$;
- The quantity of heat transferred to the combustion chamber in time unit (heat flow) $q = 1000\, J / s$;
- Coefficient overall rate of loss of thrust by nozzle: $\sigma_c = 0.71$;

6 Results

From the computing model considered, we obtain numerical values for the rocket engine operating parameters. The rocket engine was tested on the "fire stand" and the pressure and thrust force versus time were measured and will be used as comparative data in this study. Figures 5 and 6 present the comparison between pressure obtained by relationship (38), respective thrust force calculated using (42) and the experimental pressure and thrust force of the test motor. Figures 7, 8, 9 and 10 show the influence of initial propellant temperature for the pressure, thrust force, gas density respective for the gas temperature in the burning chamber.

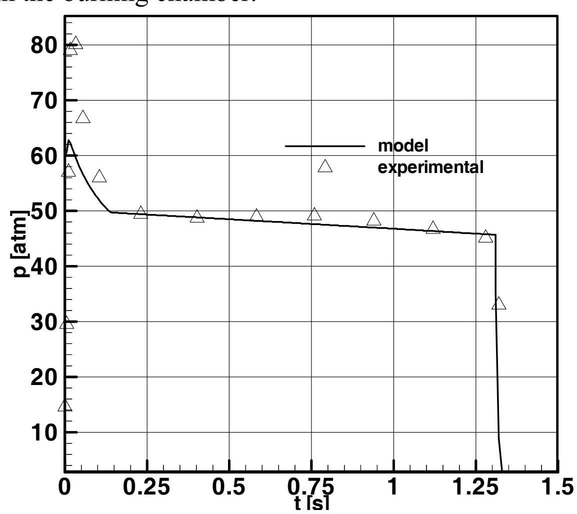

Fig. 5 Comparative pressure diagram

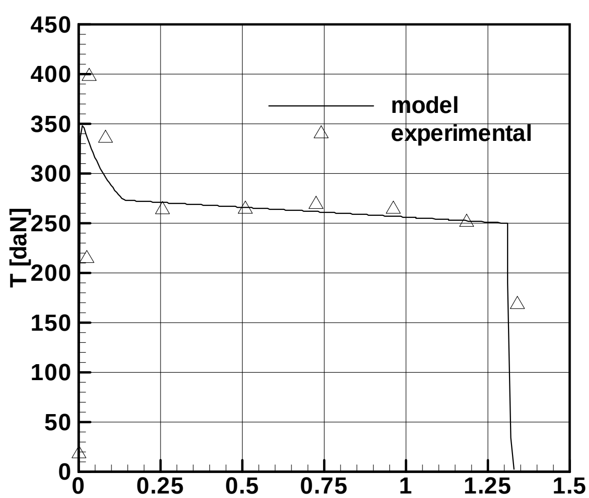

Fig. 6 Comparative thrust force diagram

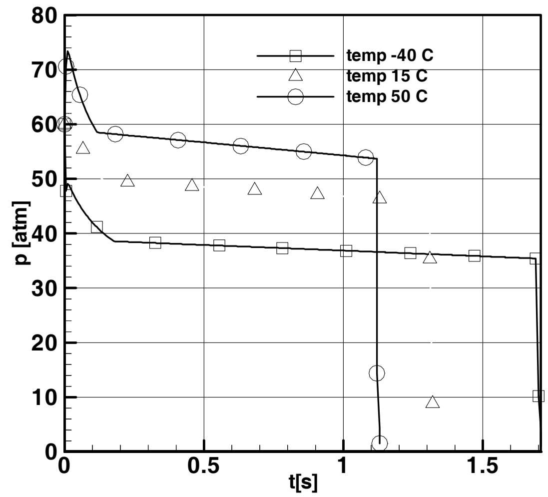

Fig. 7 Influence of initial propellant temperature for the pressure

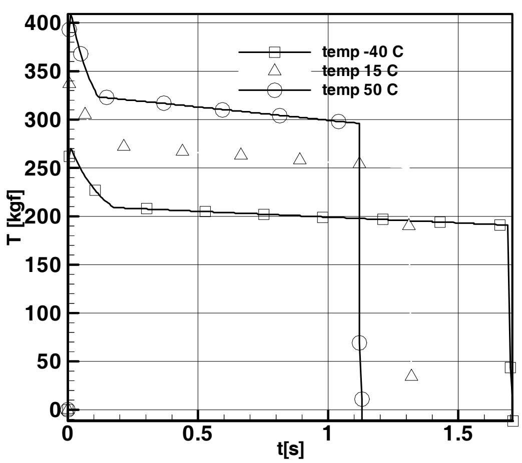

Fig. 8 Influence of initial propellant temperature for the thrust force

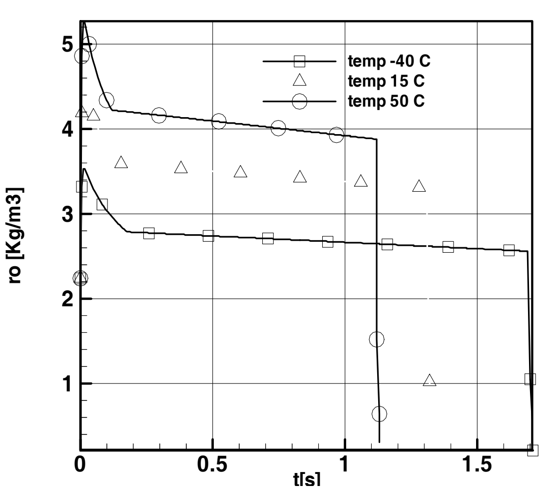

Fig. 9 Influence of initial propellant temperature for the gas density

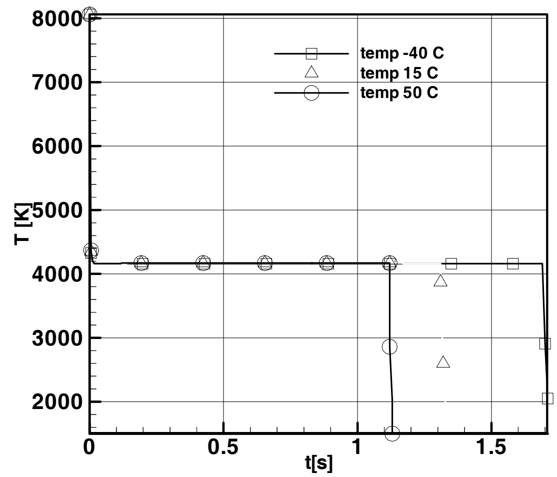

Fig. 10 Influence of initial propellant temperature for the gas temperature in burning chamber

Further on we will analyze the balance parameters and the dynamic stability of the operating RMSP. Thus, in figure 11 and 12 we are showed, for the considered application, the balance pressure ratio and the balance density ratio as functions of propellant mass ratio.

Henceforth, setting the basic trend, we can evaluate, using the matrix (58), the parametric stability of the operating motor. To do this in figure 13 there are given the real part of eigenvalues for the matrix corresponding to the stable balance parameters.

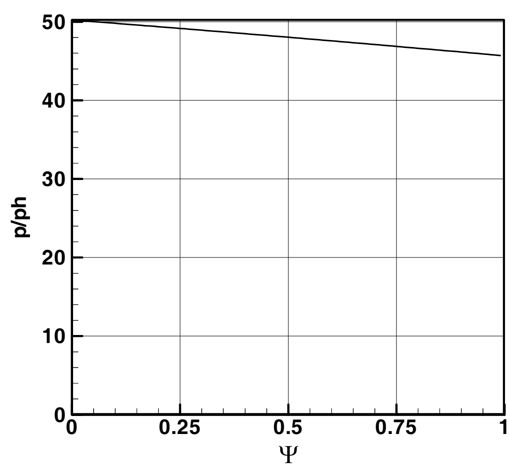

Fig. 11 Balance pressure ratio dependency on propellant mass ratio

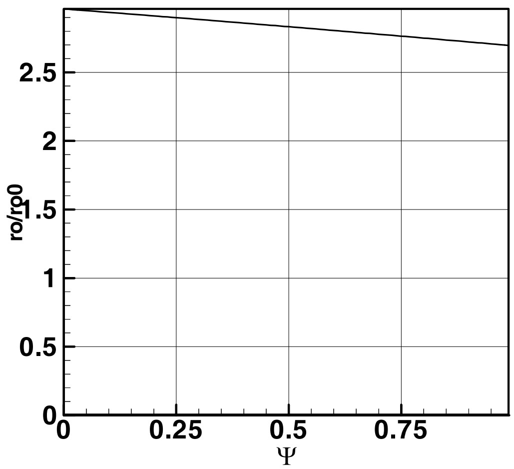

Fig. 12 Balance density ratio dependency on propellant mass ratio

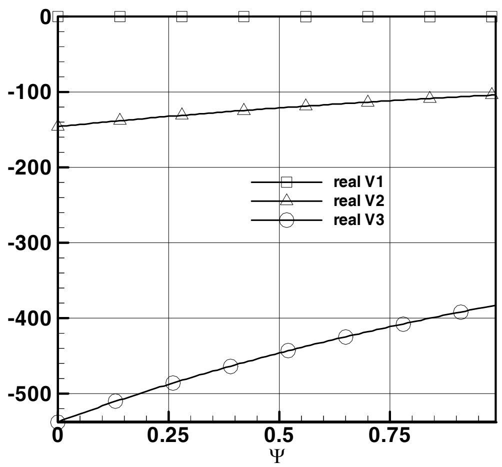

Fig. 13 Real part of eigenvalues for stable RMSP

7 Conclusions

As we resumed in the introductive part, our work followed two purposes:

Scientific one – to check the possibility of applying Liapunov theory to analyze the stability of the balance parameters of RMSP at low pressure. With this reason we obtained:

- A flexible parametric expression of the propellant surface which allows to use different propellant geometry without major modification of the input data structure;
- A good concordance between parametric non- linear equations of the RMSP and the experimental results as we can see in figures 5 and 6 where the comparative pressure diagram and the comparative thrust force diagram are shown;
- An algorithm to define the balance parameters and stability matrix;
- A comfortable method to evaluate the motor stability operating a low pressure evaluating eigenvalue of the stability matrix, as we show in figure 13.

Technical one – to design the rocket motor for the fire-extinguishing rocket that was successfully accomplished, as we can see in figure 14.

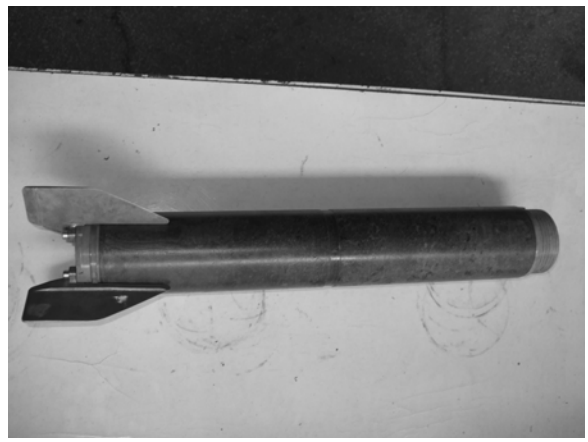

Fig. 14 RMSP for fire-extinguishing rocket

The rocket engine functionality was tested also in a shooting-range polygon, as you can see in figure 15.

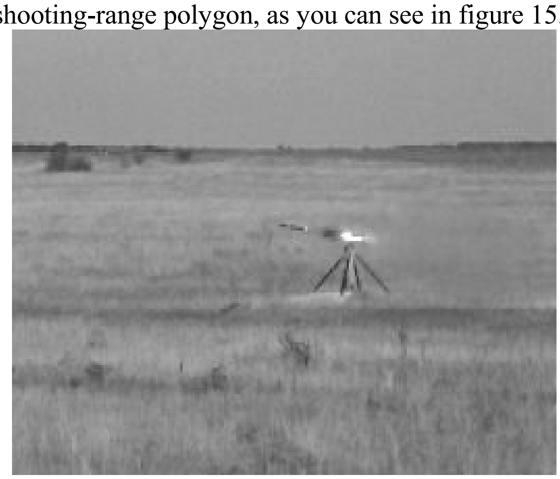

Fig 15 The rocket releases the launching system

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