

# Effects of Thermal Radiation and Free Convection Currents on the Unsteady Couette Flow Between Two Vertical Parallel Plates with Constant Heat Flux at one Boundary

M. NARAHARI

Fundamental and Applied Sciences Department  
Universiti Teknologi PETRONAS  
31750 Tronoh, Perak  
MALAYSIA  
[marneni@petronas.com.my](mailto:marneni@petronas.com.my)

*Abstract:* - An exact analysis of the natural convection in unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation is performed. The flow is induced by means of Couette motion and free convection currents occurring as a result of application of constant heat flux on the wall with a uniform vertical motion in its own plane while constant temperature on the stationary wall. The fluid considered here is a gray, absorbing-emitting but non-scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the analysis. The dimensionless governing partial differential equations are solved using Laplace transform technique. Numerical results for the velocity, the temperature, the skin-friction, the Nusselt number, the volume flow rate and the vertical heat flux are shown graphically. The effect of different parameters like thermal radiation parameter, Grashof number, Prandtl number and time are discussed. It is observed that the momentum and thermal boundary layer thickness decreases owing to an increase in the value of the radiation parameter. An increase in the Grashof number is found to increase the velocity of air and water and to decrease the skin-friction at the moving plate.

*Key-Words:* - Vertical channel, Natural convection, Couette flow, Constant heat flux, Radiation.

## 1 Introduction

The fluid flow between parallel plates by means of Couette motion is a classical fluid mechanics problem that has applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, and also in many material processing applications such as extrusion, metal forming, continuous casting, wire and glass fiber drawing, etc. This problem has received considerable attention in the case of horizontal parallel plates [1]-[15] than vertical parallel plates. An analysis of flow formation in Couette motion between vertical parallel plates was presented by Schlichting and Gersten [16]. This problem is of fundamental importance as it provides the exact solution and reveals how the velocity profiles varies with time, approaching a linear distribution asymptotically, and how the boundary layer spreads throughout the flow field.

Free convection in vertical channels has been studied widely in the last few decades under different physical effects [17]-[27] due to its importance in many engineering applications such

as cooling of electronic equipments, design of passive solar systems for energy conversion, cooling of nuclear reactors, design of heat exchangers, chemical devices and process equipment, geothermal systems, and others. However, very few papers deal with free convection in Couette motion between vertical parallel plates. Singh [28] studied the effect of free convection in Couette motion. He has considered the unsteady free-convective flow of a viscous incompressible fluid between two vertical parallel plates at constant but different temperatures and one of which is impulsively started in its own plane and the other is kept stationary. This problem was further extended for magnetohydrodynamic case by Jha [29]. Fully-developed laminar free convection Couette flow between two vertical parallel plates with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion has been analyzed by Jain and Gupta [30]. The physical effect of external shear in the form of Couette flow of a Bingham fluid in a vertical parallel plane channel with constant temperature differential across the walls was investigated analytically by Barletta and Magyari

[31]. Steady fully-developed combined forced and free convection Couette flow with viscous dissipation in a vertical channel has been investigated analytically by Barletta et al. [32]. In their study, the moving wall is thermally insulated and the wall at rest is kept at a uniform temperature.

The aim of the present paper is to provide an exact analysis of unsteady free convection in Couette motion between two vertical parallel plates in the presence of thermal radiation, where the moving plate is subject to constant heat flux and the plate at rest is isothermal. Exact solutions are derived for the velocity and temperature fields using Laplace transform technique. These solutions are useful to gain a deeper knowledge of the underlying physical processes and it provides the possibility to get a benchmark for numerical solvers with reference to basic flow configurations. The mathematical analysis and the solution of the velocity field for two different cases – one valid for fluids with Prandtl numbers different from unity and the other for which the Prandtl number is unity – have been presented in Section 2, the results are discussed in Section 3 and the conclusions are set out in Section 4.

## 2 Mathematical Analysis

Consider the unsteady free-convective Couette flow of an incompressible viscous radiating fluid between two infinite vertical parallel plates separated by a distance  $h$ . The  $x'$ -axis is taken along one of the plates in the vertically upward direction and the  $y'$ -axis is taken normal to the plate. Initially, at time  $t' \leq 0$ , the two plates and the fluid are assumed to be at the same temperature  $T'_h$  and stationary. At time  $t' > 0$ , the plate at  $y' = 0$  starts moving in its own plane with an impulsive velocity  $U$  and is heated by supplying heat at constant rate whereas the plate at  $y' = h$  is stationary and maintained at a constant temperature  $T'_h$ . It is also assumed that the radiative heat flux in the  $x'$ -direction is negligible as compared to that in the  $y'$ -direction. As the plates are infinite in length, the velocity and temperature fields are functions of  $y'$  and  $t'$  only. Then under the usual Boussinesq's approximation, the flow of a radiating fluid is shown to be governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_h) + \nu \frac{\partial^2 u'}{\partial y'^2} \quad (1)$$

and

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \quad (2)$$

The initial and boundary conditions are as follows:

$$\left. \begin{aligned} t' \leq 0 : u' = 0, T' = T'_h & \quad \text{for } 0 \leq y' \leq h, \\ t' > 0 : u' = U, \frac{\partial T'}{\partial y'} = -\frac{q}{k} & \quad \text{at } y' = 0, \\ u' = 0, T' = T'_h & \quad \text{at } y' = h. \end{aligned} \right\} \quad (3)$$

where  $g$  is the acceleration due to gravity,  $\beta$  the volumetric co-efficient of thermal expansion,  $\nu$  the kinematic viscosity,  $\rho$  the density,  $k$  the thermal conductivity,  $C_p$  the specific heat at constant pressure,  $q$  the constant heat flux,  $q_r$  the radiative heat flux in  $y'$ -direction,  $T'$  the fluid temperature, and  $u'$  is the fluid velocity.

The radiative heat flux term is simplified by making use of the Rosseland approximation [33] as

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T'^4}{\partial y'} \quad (4)$$

where  $\sigma$  is the Stefan-Boltzmann constant and  $k^*$  is the mean absorption coefficient. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If temperature differences within the flow are sufficiently small such that  $T'^4$  may be expressed as a linear function of the temperature, Then the Taylor series for  $T'^4$  about  $T'_h$ , after neglecting higher order terms, is given by

$$T'^4 \cong 4T_h'^3 T' - 3T_h'^4 \quad (5)$$

It is emphasized here that equation (5) is widely used in computational fluid dynamics involving radiation absorption problems [34] in expressing the term  $T'^4$  as a linear function.

In view of Eqs. (4) and (5), Eq. (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_h'^3}{3k^*} \frac{\partial^2 T'}{\partial y'^2} \quad (6)$$

In order to solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities:

$$\left. \begin{aligned} y = \frac{y'}{h}, t = \frac{t'v}{h^2}, u = \frac{u'}{U}, \theta = \frac{T' - T'_h}{(hq/k)}, \\ \Pr = \frac{\mu C_p}{k}, R = \frac{kk^*}{4\sigma T_h^3}, Gr = \frac{g\beta h^3 q}{Uk\nu} \end{aligned} \right\} \quad (7)$$

where  $Gr$  is the thermal Grashof number,  $\Pr$  the Prandtl number,  $R$  the radiation parameter,  $t$  the dimensionless time,  $u$  the dimensionless velocity,  $y$  the dimensionless coordinate axis normal to the plate,  $\mu$  the coefficient of viscosity and  $\theta$  is the dimensionless temperature.

Then in view of Eqs. (7), Eqs. (1), (6) and (3) reduces to the following non-dimensional form of equations:

$$\frac{\partial u}{\partial t} = Gr \theta + \frac{\partial^2 u}{\partial y^2} \quad (8)$$

$$3R \Pr \frac{\partial \theta}{\partial t} = (3R + 4) \frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

The initial and boundary conditions are

$$\left. \begin{aligned} t \leq 0: u = 0, \theta = 0 \quad \text{for } 0 \leq y \leq 1, \\ t > 0: u = 1, \frac{\partial \theta}{\partial y} = -1 \quad \text{at } y = 0, \\ u = 0, \theta = 0 \quad \text{at } y = 1. \end{aligned} \right\} \quad (10)$$

The solutions of Eqs. (8) and (9) under the initial and boundary conditions (10) by Laplace transform technique is given by

$$\begin{aligned} u(y,t) = & \sum_{n=0}^{\infty} [f_1(a,t) - f_1(b,t)] \\ & + \frac{Gr}{(N-1)\sqrt{N}} \sum_{n=0}^{\infty} [f_3(b,t) - f_3(a,t)] \\ & + 2 \sum_{m=0}^n (-1)^m \{f_3(c,t) - f_3(d,t)\} \\ & + (-1)^n \{f_3(b\sqrt{N},t) - f_3(a\sqrt{N},t)\} \end{aligned} \quad (11)$$

$$\theta(y,t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{\infty} (-1)^n [f_2(a\sqrt{N},t) - f_2(b\sqrt{N},t)] \quad (12)$$

where

$$a = 2n + y, b = 2n + 2 - y, N = \frac{3R \Pr}{3R + 4},$$

$$c = 2m\sqrt{N} - 2m + a, d = 2m\sqrt{N} - 2m + b,$$

$$f_1(z,t) = \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right),$$

$$f_2(z,t) = 2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{z^2}{4t}\right) - z \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right),$$

$$\begin{aligned} f_3(z,t) = & \frac{1}{3}(4t + z^2) \sqrt{\frac{t}{\pi}} \exp\left(-\frac{z^2}{4t}\right) \\ & - \left(zt + \frac{z^3}{6}\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right), \end{aligned}$$

$z$  is a dummy variable and  $f_1, f_2, f_3$  are functions of dummy variable.

Using the expression (11), the skin-friction at the moving hot plate  $y = 0$  in non-dimensional form is given by

$$\begin{aligned} \tau_0 = & \frac{\tau'_0 h}{\mu U} = - \frac{\partial u}{\partial y} \Big|_{y=0} \\ & = \sum_{n=0}^{\infty} [f_4(n,t) + f_4(n+1,t)] \\ & - \frac{Gr}{(N-1)\sqrt{N}} \sum_{n=0}^{\infty} \{f_5(n,t) + f_5(n+1,t)\} \\ & - 2 \sum_{m=0}^n (-1)^m \{f_5(w,t) + f_5(w+1,t)\} \\ & + (-1)^n \{f_6(n\sqrt{N},t) + f_6((n+1)\sqrt{N},t)\} \end{aligned} \quad (13)$$

and the skin-friction at the stationary plate  $y = 1$  is given by

$$\begin{aligned} \tau_1 = & - \frac{\partial u}{\partial y} \Big|_{y=1} \\ & = 2 \sum_{n=0}^{\infty} f_4\left(\frac{2n+1}{2}, t\right) \\ & - \frac{2Gr}{(N-1)\sqrt{N}} \sum_{n=0}^{\infty} \left[ f_5\left(\frac{2n+1}{2}, t\right) \right. \\ & \left. - 2 \sum_{m=0}^n (-1)^m f_5\left(\frac{2w+1}{2}, t\right) \right. \\ & \left. + (-1)^n f_6\left(\frac{(2n+1)\sqrt{N}}{2}, t\right) \right] \end{aligned} \quad (14)$$

Using the expression (12), it is also interesting to study the rate of heat transfer at the moving hot plate  $y = 0$  which is expressed as Nusselt number by

$$Nu_0 = \frac{qh}{k(T' - T'_h)} = - \frac{1}{\theta(0,t)} \left. \frac{\partial \theta}{\partial y} \right|_{y=0} = \frac{1}{\theta(0,t)} \quad (15)$$

and the Nusselt number at the stationary plate  $y = 1$  is given by

$$Nu_1 = - \left. \frac{\partial \theta}{\partial y} \right|_{y=1} = 2 \sum_{n=0}^{\infty} (-1)^n f_1((2n+1)\sqrt{N}, t) \quad (16)$$

where

$$w = m(\sqrt{N} - 1) + n,$$

$$f_4(z, t) = \frac{1}{\sqrt{\pi t}} \exp\left(-\frac{z^2}{t}\right),$$

$$f_5(z, t) = (2z^2 + t) \operatorname{erfc}\left(\frac{z}{\sqrt{t}}\right) - 2z\sqrt{\frac{t}{\pi}} \exp\left(-\frac{z^2}{t}\right),$$

$$f_6(z, t) = (2z^2 + t\sqrt{N}) \operatorname{erfc}\left(\frac{z}{\sqrt{t}}\right) - 2z\sqrt{\frac{Nt}{\pi}} \exp\left(-\frac{z^2}{t}\right),$$

$$\theta(0, t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{\infty} (-1)^n \left[ f_2(2n\sqrt{N}, t) - f_2(2(n+1)\sqrt{N}, t) \right],$$

$\tau'_0$  is the dimensional skin-friction at the plate  $y = 0$  and  $f_4, f_5, f_6$  are functions of dummy variable  $z$ .

Another two important quantities for this problem are the non-dimensional volume flow rate between the plates and the non-dimensional vertical heat flux defined, respectively, by the following equations [35]-[37]:

$$M = \int_0^1 u \, dy \quad (17)$$

and

$$Q = \int_0^1 u \theta \, dy \quad (18)$$

### 2.1 Solution in the absence of radiation

In the absence of thermal radiation, i.e. in the pure convection case which numerically corresponds to  $R \rightarrow \infty$ , the energy equation in non-dimensional form becomes

$$\operatorname{Pr} \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \quad (19)$$

Since  $N = \operatorname{Pr}$  as  $R \rightarrow \infty$ , therefore the solution of the problem in the absence of radiation can be obtained from the equations of (11) and (12) simply by replacing  $N$  by  $\operatorname{Pr}$ . Thus the velocity and temperature expressions in the absence of thermal radiation are given by

$$u(y, t) = \sum_{n=0}^{\infty} [f_1(a, t) - f_1(b, t)] + \frac{Gr}{(\operatorname{Pr}-1)\sqrt{\operatorname{Pr}}} \sum_{n=0}^{\infty} \left[ [f_3(b, t) - f_3(a, t)] + 2 \sum_{m=0}^n (-1)^m [f_3(c_1, t) - f_3(d_1, t)] + (-1)^n [f_3(b\sqrt{\operatorname{Pr}}, t) - f_3(a\sqrt{\operatorname{Pr}}, t)] \right] \quad (20)$$

$$\theta(y, t) = \frac{1}{\sqrt{\operatorname{Pr}}} \sum_{n=0}^{\infty} (-1)^n \left[ f_2(a\sqrt{\operatorname{Pr}}, t) - f_2(b\sqrt{\operatorname{Pr}}, t) \right] \quad (21)$$

where

$$c_1 = 2m\sqrt{\operatorname{Pr}} - 2m + a, d_1 = 2m\sqrt{\operatorname{Pr}} - 2m + b.$$

It is clear that the solution for temperature field given by Eq. (21) is valid for all values of the Prandtl number whereas the solution for velocity field given by Eq. (20) is not valid for fluids of Prandtl number unity. As the Prandtl number is a measure of the relative importance of the viscosity and thermal conductivity of the fluid, the case  $\operatorname{Pr} = 1$  corresponds to those fluids whose momentum and thermal boundary layer thicknesses are of the same order of magnitude. Thus the solution for the velocity field has to be re-derived from Eqs. (19) and (8) when  $\operatorname{Pr} = 1$ . It can be shown that

$$\begin{aligned}
 u(y,t) = & f_7(y,t) + \sum_{n=0}^{\infty} [f_1(a,t) - f_1(b-2,t)] \\
 & + Gr \sum_{n=0}^{\infty} [f_8(c_2,t) - f_8(d_2,t)] \quad (22) \\
 & + \frac{yGr}{2} \sum_{n=0}^{\infty} (-1)^n [f_8(a,t) + f_8(b,t)]
 \end{aligned}$$

where

$$c_2 = 2(n+1) + a, d_2 = 2n + b$$

$$f_7(z,t) = 1 + \operatorname{erf}\left(\frac{z}{2\sqrt{t}}\right)$$

$$f_8(z,t) = \left(\frac{z^2}{2} + t\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right) - z\sqrt{t/\pi} \exp\left(-\frac{z^2}{4t}\right)$$

### 3 Results and Discussion

An exact solution to the problem of natural convection in unsteady Couette flow between two long vertical parallel plates in the presence of constant heat flux and thermal radiation have been presented in the preceding section. In order to get the physical insight into the problem, the numerical values of the temperature field, the velocity field, the skin-friction, the Nusselt number, the volume flow rate and the vertical heat flux are computed for different values of the system parameters such as Radiation parameter ( $R$ ), Grashof number ( $Gr$ ), Prandtl number ( $Pr$ ) and time ( $t$ ). Figure 1 presents the temperature profiles of air ( $Pr = 0.71$ ) for different values of  $t$  and  $R$ . It is seen that the temperature increases with increasing time in the presence of radiation and in the case of pure convection (which numerically corresponds to  $R \rightarrow \infty$ ) i.e. in the absence of radiation. Moreover, the temperature is found to decrease due to an increase in the radiation parameter. When radiation is present, the thermal boundary layer was always found to thicken, which may be explained by the fact that radiation provides an additional means to diffuse energy. For  $R = 10$  the temperature profile is found to increase 4.26 % of the pure convection case at the moving plate when  $t = 0.4$ . The thickening of the thermal boundary layer is more significant for small values of  $t$  and  $R$ . Furthermore, the temperature profiles attain their maximum value near the moving hot wall ( $y = 0$ ) and decreases

smoothly to zero at the stationary wall ( $y = 1$ ) of the vertical channel.

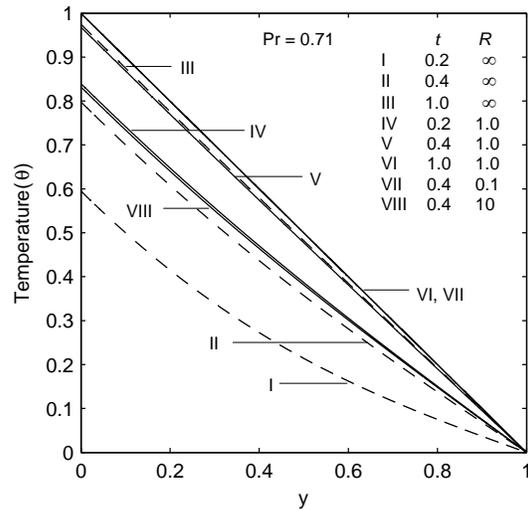


Fig. 1 Temperature profiles

Figure 2 presents the velocity profiles for both air and water ( $Pr = 7.0$ ) in the case of pure convection ( $R \rightarrow \infty$ ) for different values of  $Gr$  and  $t$ . It is seen that the velocity of air and water increases with increasing  $Gr$  and  $t$ . At a smaller  $t$ , the velocity distribution is monotonic, but at a higher time it passes through a maximum near the moving plate when the buoyancy effect partly suppresses the inertial effects of the plate velocity. Moreover, the velocity of air is greater than the velocity of water. Physically this is possible because fluids with high Prandtl number have greater viscosity, which makes the fluid thick and hence move slowly.

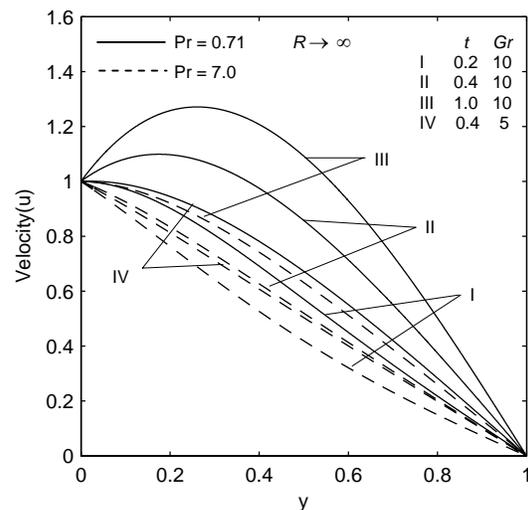


Fig. 2 Velocity profiles for different  $t$ ,  $Pr$  and  $Gr$  (Pure convection case)

Figure 3 presents the velocity profiles of air for different values of  $t$ ,  $Gr$  and  $R$ . It is observed that the velocity increases with increasing  $t$  and  $Gr$ . Physically this is possible because as the Grashof number or time increases, the contribution from the buoyancy force near the moving hot plate become more significant and hence a small rise in the fluid velocity near the plate is observed. Further, it is observed that the fluid velocity decreases with increasing value of  $R$ . This result may be explained by the fact that an increase in the radiation parameter  $R (=kk^*/4\sigma T_h')$  for fixed  $k$  and  $T_h'$  means an increase in the Rosseland mean absorption coefficient  $k^*$ . When radiation is present, the momentum boundary layer was found to be thickened, which is in agreement with the observation made earlier with regard to the temperature variations of air. For  $R=10$  the velocity profile is found to increase 2.52 % of the pure convection case near the moving plate at  $y = 0.3$  when  $t = 0.4$ .

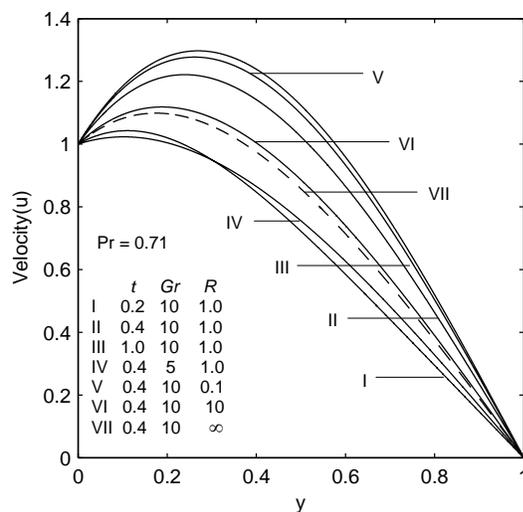


Fig. 3 Velocity profiles of air for different  $t$ ,  $Gr$  and  $R$

Figure 4 presents the skin-friction variation with  $t$  in the pure convection case for different values of  $Pr$  and  $Gr$  at the moving plate. It is observed that the skin-friction increases with increasing  $Pr$  whereas it decreases with increasing  $Gr$  and  $t$ . A large Prandtl number implies more prominent viscous effects causing an enhanced frictional force. Figure 5 presents the skin-friction variation of air flow with time in the presence of radiation effects for different values of  $R$  and  $Gr$  at the moving plate. It is observed that the skin-friction increases with increasing  $R$  and it decreases with increasing  $Gr$  and  $t$ . Furthermore, from figures 4 and 5, it is interesting

to note that for small values of time, skin-friction is more affected by  $Pr$ ,  $Gr$  and  $R$  while less affected for large values of time. The value of the skin-friction becomes negative after some time, indicating that there occurs a reverse type of flow near the moving plate. Physically this is possible as the motion of the fluid is due to the plate motion in the upward direction against the gravitational field. Thus it can be expected that large values of  $Gr$  may cause flow separation even at small values of  $t$ .

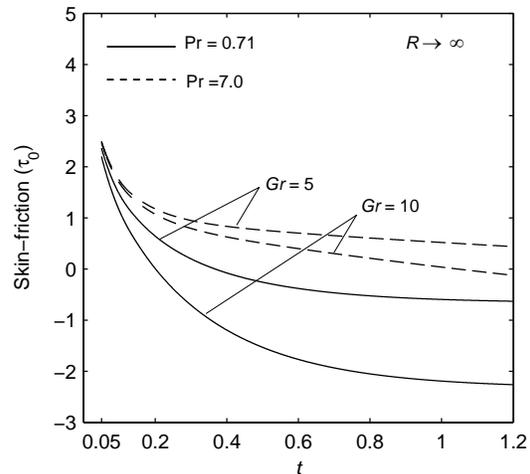


Fig. 4 Skin-friction ( $\tau_0$ ) variation with  $t$  (Pure convection case)

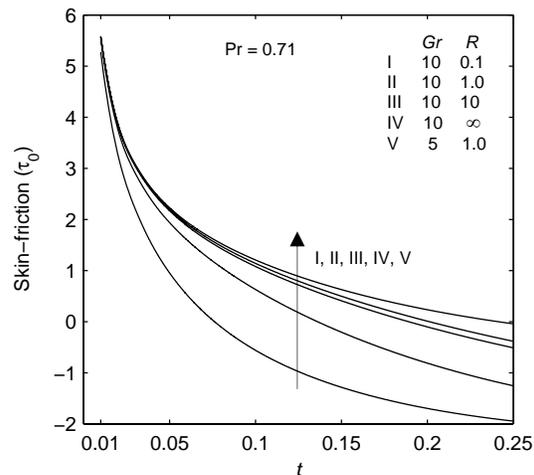


Fig. 5 Skin-friction ( $\tau_0$ ) variation with  $t$

In the case of pure convection, figures 6 and 7 presents the variation of the skin-friction with time for different values of  $Gr$  and  $Pr$  at the stationary plate ( $y = 1$ ). It is observed that the skin-friction is increasing with increasing  $Gr$  to a certain value of  $t$  and it decreases gradually to the steady state value as time increases. That is the friction curves assume

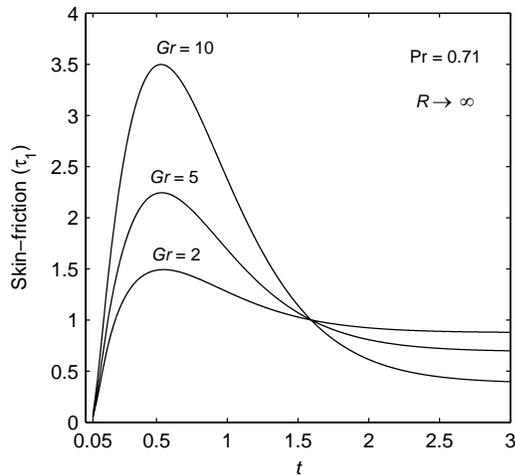


Fig. 6 Skin-friction ( $\tau_1$ ) variation with  $t$

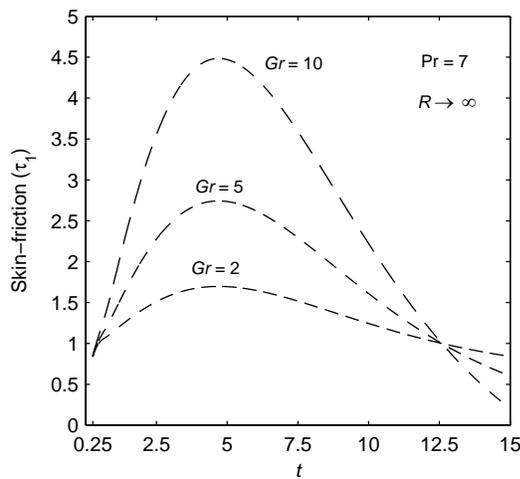


Fig. 7 Skin-friction ( $\tau_1$ ) variation with  $t$

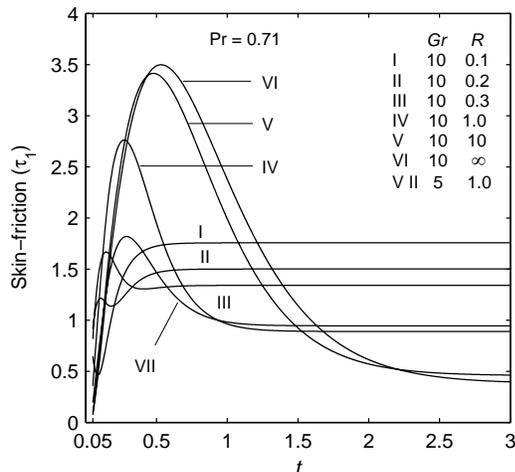


Fig. 8 Skin-friction ( $\tau_1$ ) variation with  $t$

parabolic shapes with an increase in  $t$ . It is also clear that the skin-friction is higher for large values of  $Pr$ . Figure 8 presents the skin-friction variation with time for different values of  $Gr$  and  $R$  at the stationary plate. It is noted that the peak values of the skin-friction is attained at low time, and the behavior of the skin-friction distribution is completely oscillatory.

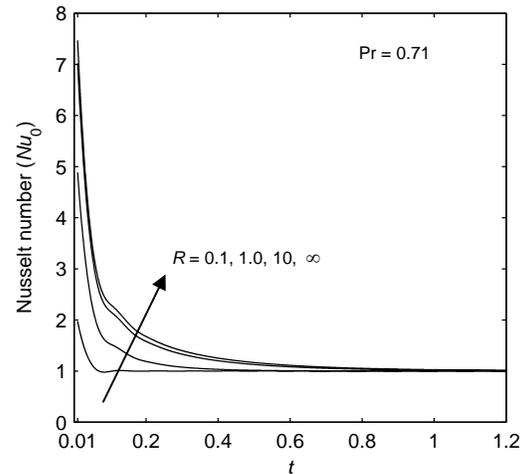


Fig. 9 Nusselt number ( $Nu_0$ ) variation with  $t$

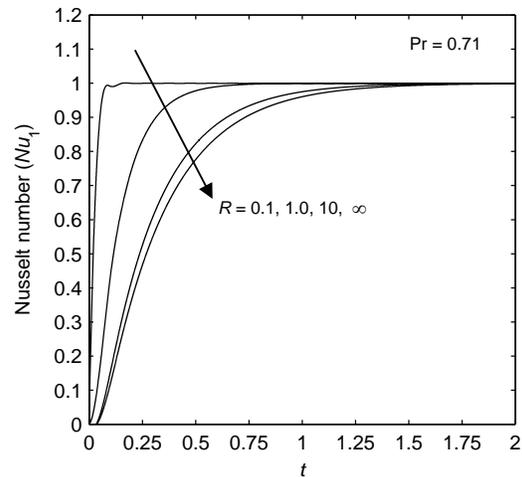


Fig. 10 Nusselt number ( $Nu_1$ ) variation with  $t$

Figures 9 and 10 shows the effect of  $R$  on the variation of the Nusselt number with  $t$  for  $Pr = 0.71$  at the moving and stationary plates respectively, we see that the Nusselt number is increasing with increasing values of  $R$  at small values of  $t$  and it is decreasing with an increase in time at the moving plate. But, the trend is completely opposite at the stationary plate. As time increases, the effect of  $R$  on the Nusselt number is not significant.

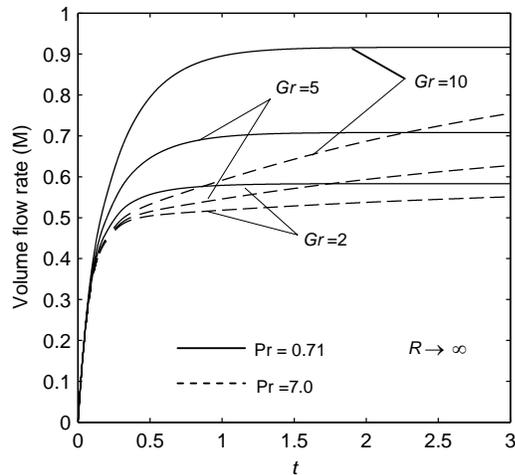


Fig. 11 Volume flow rate variation with  $t$  (Pure convection case)

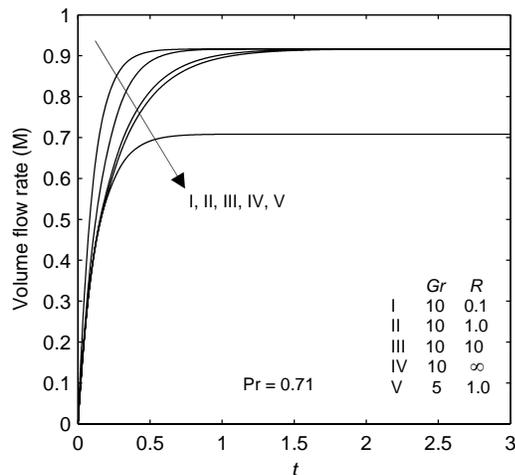


Fig. 12 Volume flow rate variation with  $t$

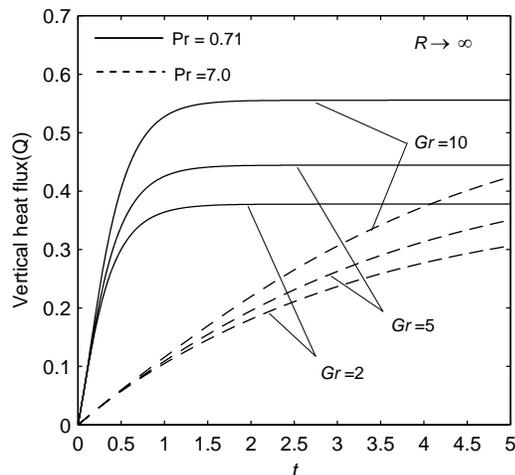


Fig. 13 Vertical heat flux variation with  $t$  (Pure convection case)

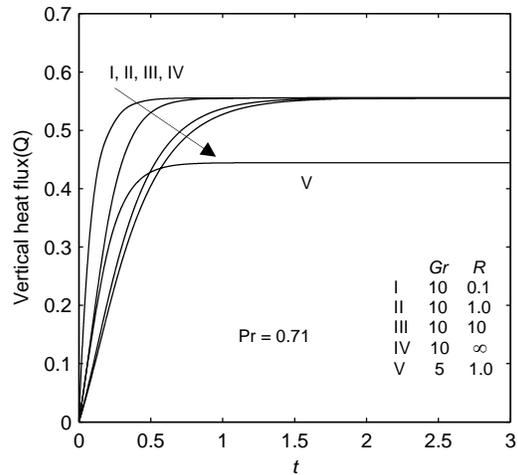


Fig. 14 Vertical heat flux variation with  $t$

Figure 11 presents the variation of volume flow rate with  $t$  for different values of  $Gr$  and  $Pr$  in the case of pure convection. It is observed that the volume flow rate increases with increasing  $Gr$  and it decreases with increasing  $Pr$ , which are in agreement with the observations made earlier with regard to the velocity field. Figure 12 presents the variation of Volume flow rate with  $t$  for different values of  $R$  and  $Gr$ . It is observed that the volume flow rate is decreasing with increasing values of  $R$  and these variations are significant only for small values of time. The volume flow rate increase with increasing values of  $Gr$  in the presence of radiation.

Figure 13 shows the effects of Grashof number and Prandtl number on vertical heat flux at different time in the case of pure convection. We see that at a given time, vertical heat flux increases significantly with increasing Grashof number and it decreases with increasing Prandtl number. The effect of Grashof number is relatively small at lower time and becomes quite pronounced at higher time. Figure 14 shows the effects of radiation parameter and Grashof number on vertical heat flux at different time when  $Pr = 0.71$ . We see that the vertical heat flux decreases with increasing radiation parameter and these variations are significant at lower values of time. The vertical heat flux is also increasing with increasing values of Grashof number in the presence of radiation.

### 4 Conclusion

An exact solution to the problem of unsteady natural convective Couette flow of a viscous incompressible fluid in a parallel plane vertical channel in the presence of constant heat flux and thermal radiation have been derived. The dimensionless governing

partial differential equations are solved by the usual Laplace transform technique. The effect of different parameters such as radiation parameter, Grashof number, Prandtl number and time are studied. Conclusions of the study are as follows:

1. The temperature of the fluid increases with increasing time whereas it decreases due to an increase in the value of radiation parameter.
2. In the case of pure convection (i.e. in the absence of radiation), the velocity of the fluid increases with increasing Grashof number and time, but falls owing to an increase in the Prandtl number.
3. The velocity of the fluid increases with increasing Grashof number and time but it decreases owing to an increase in the value of the radiation parameter.
4. The momentum and thermal boundary layers are found to thicken when the radiation is present.
5. In the pure convection case the skin-friction increases with increasing Prandtl number whereas it decreases with increasing Grashof number and time at the moving plate.
6. The skin-friction at the moving plate increases with increasing values of the radiation parameter but it decreases with increasing values of Grashof number and time for air flows.
7. The Nusselt number increases owing to an increase in the value of radiation parameter but it decreases with an increase in time at the moving plate.
8. The Nusselt number decreases owing to an increase in the value of radiation parameter but it increases with an increase in time at the stationary plate.
9. In the case of pure convection, the volume flow rate and the vertical heat flux are increasing with increasing values of Grashof number and time. But, these are decreasing with increasing values of Prandtl number.
10. The volume flow rate and the vertical heat flux are decreasing with increasing values of radiation parameter.

We may conclude therefore, that the interaction between the radiation, buoyancy forces and the applied shear induced by a uniform vertical motion of the hot wall can affect the configuration of the flow field significantly.

#### References:

- [1] R. N. Jana and N. Datta, Couette Flow and Heat Transfer in a Rotating System, *Acta Mechanica*, Vol. 26, 1977, pp. 301-306.
- [2] V. M. Soundalgekar, Hall Effects in MHD Couette Flow with Heat Transfer, *IEEE*

*Transactions on Plasma Science*, Vol. PS-14, No. 5, 1986, pp. 579-583.

- [3] A. K. Singh, N. C. Sacheti and P. Chandran, Transient Effects in Magneto-hydrodynamic Couette Flow with Rotation: Accelerated Motion, *International Journal of Engineering Sciences*, Vol. 32, 1994, pp. 133-139.
- [4] A. J. Kearsley, A steady State Model of Couette Flow with Viscous Heating, *International Journal of Engineering Sciences*, Vol. 32, 1994, pp. 179-186.
- [5] S. K. Ghosh, Effect of Hall Currents on MHD Couette Flow in a Rotating System with Arbitrary Magnetic Field, *Czech Journal of Physics*, Vol. 52, 2002, pp. 51-63.
- [6] J. Kumar, C. Lakshmana Rao and M. Massoudi, Couette Flow of Granular Materials, *International Journal of Non-Linear Mechanics*, Vol. 38, 2003, pp. 11-20.
- [7] C. K. Choi, T. J. Chung and M. C. Kim, Buoyancy Effects in Plane Couette Flow Heated Uniformly From Below with Constant Heat Flux, *International Journal of Heat and Mass Transfer*, Vol. 47, 2004, pp. 2629-2636.
- [8] H. A. Attia and M. E. Sayed-Ahmed, Hall Effect on Unsteady MHD Couette Flow and Heat Transfer of a Bingham Fluid with Suction and Injection, *Applied Mathematical Modelling*, Vol. 28, 2004, pp. 1027-1045.
- [9] S. H. Hashemabadi, S. Gh. Etemad and J. Thibault, Forced Convection Heat Transfer of Couette-Poiseuille Flow of Nonlinear Viscoelastic Fluids Between Parallel Plates, *International Journal of Heat and Mass Transfer*, Vol. 47, 2004, pp. 3985-3991.
- [10] P. D. S. Verma and M. M. Sehgal, Couette Flow of Micropolar Fluids, *International Journal of Engineering Sciences*, Vol. 42, 2004, pp. 65-78.
- [11] O. Aydin and M. Avci, Laminar Forced Convection with Viscous Dissipation in a Couette-Poiseuille Flow Between Parallel Plates, *Applied Energy*, Vol. 83, 2006, pp. 856-867.
- [12] P. Mebine, Radiation Effects on MHD Couette Flow with Heat Transfer Between Two Parallel Plates, *Global Journal of Pure and Applied Mathematics*, Vol. 3, No. 2, 2007, pp. 191-202.
- [13] B. K. Das, M. Guria and R. N. Jana, Unsteady Couette Flow in a Rotating System, *Meccanica*, Vol. 43, 2008, pp. 517-521.
- [14] H. A. Attia, The Effect of Variable Properties on the Unsteady Couette Flow with Heat Transfer Considering the Hall Effect, *Communications in Nonlinear Science and*

- Numerical Simulation*, Vol. 13, 2008, pp. 1596-1604.
- [15] O. A. Bég, H. S. Takhar, J. Zueco, A. Sajid and R. Bhargava, Transient Couette Flow in a Rotating non-Darcian Porous Medium Parallel Plate Configuration: Network Simulation Method Solutions, *Acta Mechanica*, Vol. 200, 2008, pp. 129-144.
- [16] H. Schlichting and K. Gersten, *Boundary-Layer Theory*, 8<sup>th</sup> Revised and Enlarged Edition, Springer, 2001.
- [17] W. Aung, Fully Developed Laminar Free Convection Between Vertical Plates Heated Asymmetrically, *International Journal of Heat and Mass Transfer*, Vol. 15, No. 8, 1972, pp. 1577-1580.
- [18] O. Miyatake, T. Fujii and H. Tanaka, Natural Convection Heat Transfer Between Vertical Parallel Plates – One Plate with a Uniform Heat Flux and the Other Thermally Insulated, *Heat Transfer – Japanese Research*, Vol. 2, No. 1, 1973, pp. 25-33.
- [19] H. M. Joshi, Transient Effects in Natural Convection Cooling of Vertical Parallel Plates, *International Communications in Heat and Mass Transfer*, Vol. 15, 1988, pp. 227-238.
- [20] K. T. Lee and W. M. Yan, Laminar Natural Convection Between Partially Heated Vertical Parallel Plates, *Heat and Mass Transfer*, Vol. 29, 1994, pp. 145-151.
- [21] A. K. Singh, H. R. Gholami and V. M. Soundalgekar, Transient Free Convection Flow Between Two Vertical Parallel Plates, *Heat and Mass Transfer*, Vol. 31, 1996, pp. 329-331.
- [22] A. Barletta, Heat Transfer by Fully Developed Flow and Viscous Heating in a Vertical Channel with Prescribed Wall Heat Fluxes, *International Journal of Heat and Mass Transfer*, Vol. 42, 1999, pp. 3873-3885.
- [23] M. Narahari, S. Sreenadh and V. M. Soundalgekar, Transient Free Convection Flow Between Long Vertical Parallel Plates with Constant Heat Flux at One Boundary, *Journal of Thermophysics and Aeromechanics*, Vol. 9, No. 2, 2002, pp. 287-293.
- [24] B. K. Jha, A. K. Singh and H. S. Takhar, Transient Free Convection Flow in a Vertical Channel Due to Symmetric Heating, *International Journal of Applied Mechanics and Engineering*, Vol. 8, No. 3, 2003, pp. 497-502.
- [25] A. Campo, O. Manca and B. Morrone, Numerical Investigation of the Natural Convection Flows for Low-Prandtl Fluids in Vertical Parallel-Plates Channels, *ASME Journal of Applied Mechanics*, Vol. 73, 2006, pp. 96-107.
- [26] A. K. Singh and T. Paul, Transient Natural Convection Between Two Vertical Walls Heated/Cooled Asymmetrically, *International Journal of Applied Mechanics and Engineering*, Vol. 11, No. 1, 2006, pp. 143-154.
- [27] M. Narahari, Oscillatory Plate Temperature Effects of Free Convection Flow of Dissipative Fluid Between Long Vertical Parallel Plates, *International Journal of Applied Mathematics and Mechanics*, Vol. 5, No. 3, 2009, pp. 30-46.
- [28] A. K. Singh, Natural Convection in Unsteady Couette Motion, *Defense Science Journal*, Vol. 38, No. 1, 1988, pp. 35-41.
- [29] B. K. Jha, Natural Convection in Unsteady MHD Couette Flow, *Heat and Mass Transfer*, Vol. 37, 2001, pp. 329-331.
- [30] N. C. Jain and P. Gupta, Three Dimensional Free Convection Couette Flow with Transpiration Cooling, *Journal of Zhejiang University SCIENCE A*, Vol. 7, No. 3, 2006, pp. 340-346.
- [31] A. Barletta and E. Magyari, Buoyant Couette-Bingham Flow Between Vertical Parallel Plates, *International Journal of Thermal sciences*, Vol. 47, 2008, pp. 811-819.
- [32] A. Barletta, S. Lazzari and E. Magyari, Buoyant Poiseuille-Couette Flow with Viscous Dissipation in a Vertical Channel, *Zeitschrift für angewandte Mathematik und Physik ZAMP*, Vol. 59, 2008, pp. 1039-1056.
- [33] R. Siegel and J. R. Howell, *Thermal Radiation Heat Transfer*, 4<sup>th</sup> Edition, Taylor & Francis, 2002.
- [34] T. J. Chung, *Computational Fluid Dynamics*, Cambridge University Press, 2002.
- [35] W. Aung, Fully Developed Laminar Free Convection Between Vertical Plates Heated Asymmetrically, *International Journal of Heat Mass Transfer*, Vol. 15, 1972, pp. 1577-1580.
- [36] A. Pantokratoras, Fully Developed Laminar Free Convection with Variable Thermophysical Properties Between Two Open-Ended Vertical Parallel Plates Heated Asymmetrically with Large Temperature Differences, *ASME Journal of Heat Transfer*, Vol. 128, 2006, pp. 405-408.
- [37] A. Pantokratoras, The Classical Plane Couette-Poiseuille Flow with Variable Fluid Properties, *ASME Journal of Fluids Engineering*, Vol. 128, 2006, pp. 1115-1121.