Thermal Parameters of Heavy-current Lines in the Process of Formulation of Optimal Design of These Devices

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Abstract: - The paper presents a mathematical model of calculation of electrodynamic parameters in the constructions of three-phase heavy-current lines (power busducts). Particular attention has been paid to determining temperatures of the system. For purpose of electrodynamic analysis the method of integral equations has been used. The presented calculation model was used in the process of optimization of the designs of heavy-current busducts, aimed at minimization of the investment and operational costs of these devices. Genetic algorithms were used for the optimization purpose. Thermal calculation was carried out for a definite physical system. The effect of predefined electrodynamic parameters on the optimization results has been analyzed.

Key-Words: - thermal calculation, electrodynamical parameters, heavy current lines, optimization, genetic algorithms, power busducts

1 Introduction

While designing and constructing modern technical equipment the optimization process is very significant. Among the most important optimization criteria there are minimization of raw material (construction material) consumption, minimization of energy loss during operation of the devices, and ecological aspects (interaction of the device with the environment). Possibilities of the construction optimization are delimited by some functional parameters of the considered objects. Among the most important technological aspects of most of the devices there are thermal parameters of their particular parts. The thermal calculation is usually of highly complex nature, as it depends (often randomly) on many varying factors, like the condition and colour of the surface, varying convection, random ventilation phenomena, etc.

The work deals with the equipment designed for electric power transmission, i.e. three-phase heavy-current lines (power busducts). A mathematical model is proposed for electrodynamic calculation of the busducts with particular consideration of the thermal aspect. Transmission capacity of these devices is affected by thermal phenomena which belong to the most important parameters determining operational properties of the busducts. The electrodynamic calculation presented here is used in practice in the optimization process of heavy-current busducts.

The present paper provides the results of thermal calculation carried out for a concrete physical system. Dependence of obtained results of the optimization process on variation of some electrodynamic parameters has been analyzed. It has been considered which of the factors affects the optimization to the greatest degree. Such an analysis enables improving the optimization computation model. The optimization performed correctly allows for significant savings of raw material necessary for busduct manufacturing and for reduction of power loss during their operation. Such an undertaking provides definite financial savings in manufacturing and operational costs of the considered heavy-current busducts.

2 Description of the Analyzed System

The consideration presented in the paper pertains to three-phase heavy-current busducts being one of the elements of electric power supply systems. They are used in almost all locations of a power system, in the range of medium voltage, for purposes of power transmission up to 200 MVA. Examples of their use may be as follows:

- connection of a generator to the unit transformers in power plants, and transformers to the medium-voltage switchgears;
- connections between the switchgear sections,
- heavy duty power outlets operating under particularly difficult conditions, frequently occurring e.g. in underground hydro-electric power stations;
- power transmission to the mines, steel works, large industrial plants, city centres;
- as the supplying elements of production bays provided with welders, industrial automats, electro-thermal (e.g. arc) furnaces;
- in large buildings – connections of the supply networks to the building transformers, etc.

The paper presents analysis of a three-phase shielded heavy-current busduct in which air is an insulation medium (Fig. 1). The phase conductors in the form of oval-section pipes of the dimensions: a – the large diagonal, b – the small diagonal, and g – thickness of oval wall are distributed each 120º inside the cylindrical shield of inside R_s and outside R_o radii. The height of conductor suspensions amounts to h.

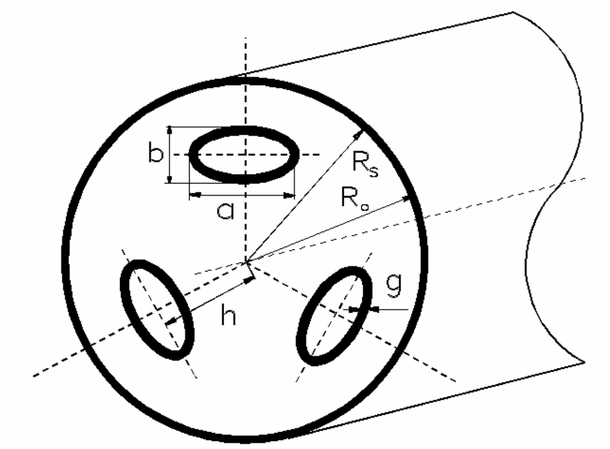

Fig. 1. Geometry of the considered system

For the considered devices the electrodynamical parameters should be calculated (as, for example, temperatures of the conductors and the shield, power losses of the system, voltage gradients, and electrodynamical forces) that, in consequence, are to be used for purposes of the optimization process of the considered heavy-current busducts.

3 Description of Mathematical Model of Electrodynamic Calculation

Alternating current flowing through electric power devices gives rise to many effects of electromagnetic, thermal, and electrodynamic nature as, e.g. real power loss, heating of the system structure and the environment, the forces arising between the system parts [1]. Among such devices the heavy-current busducts may be mentioned as well. The dynamical and thermal phenomena occurring in them may be analyzed based on the information on electrodynamic field distribution and real power loss. For example, knowledge of the power loss caused by eddy currents induced therein is necessary, particularly if the losses make significant part of total real power loss of the considered structure. Total power loss transformed into heat in the systems and ability of carrying it away determine the working temperature of the device, being one of basic structural parameters.

In order to describe the electromagnetic field the method of integral equations has been used in the paper. The integral equation of current density distribution of a single working conductor has been formulated with consideration of geometry of the system. In the kernel of the equation such phenomena have been considered as the skin effect of phase conductors, their approach, and the effect of eddy current induced in the shield. Such a procedure enabled finding the solution that allowed for analyzing the electromagnetic phenomena arising in the current busducts and their surrounding, considering only a definite part of the analyzed area (in this case the surface area of one of the working conductors).

Knowledge of approximate distribution of the current density vector enables determining the values of real power loss in the conductors and the shield. Information on the power loss emitted from the system allows to define the thermal conditions of the conductors and the shield. Moreover, the electrodynamic forces acting between the conductors and the electrical strength of the system are determined, that is more comprehensively described in [1,2].

3.1 Determining the distribution of current density and power loss of the system

In the considered system the magnetic vectorial potential $\mathbf{A}(r,\varphi,z)$ has only one component in the direction of z-axis and depends only on the (r,φ), coordinates, i.e. $\mathbf{A}(r,\varphi,z) = \mathbf{1}_z\ A(r,\varphi)$, meeting the following relationships in particular domains (Fig. 2) [1]:

- in the I domain (inside the shield), i.e. $0 \le r \le R_S$:

$$\mathbf{A}_I(r,\varphi) = \mathbf{A}_1(r,\varphi) + \mathbf{A}_2(r,\varphi) \qquad (1)$$

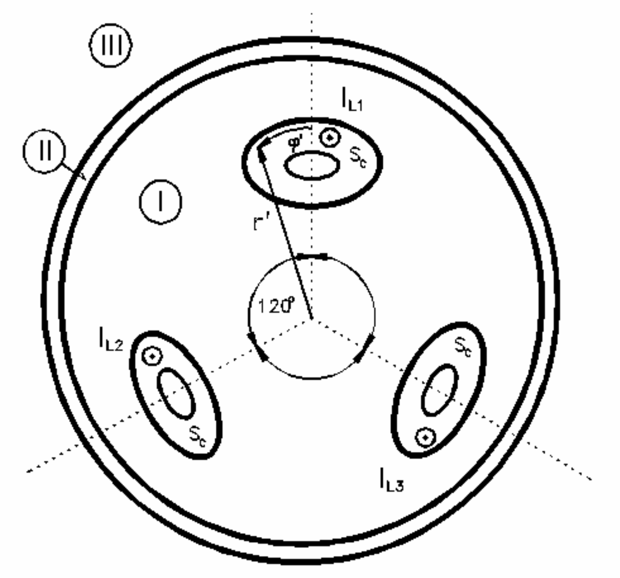

Fig. 2. Analyzed system with marked particular areas for calculations

In the above relationship the potential $\mathbf{A}_1(r,\varphi)$ is due to the currents flowing in the phase conductors and may be expressed by the formula:

$$\mathbf{A}_1(r,\varphi) = = \frac{3}{4\pi}\mu_O \int_{S_C} J(r',\varphi')\sum_{i=1}^{\infty}\left[a_i \sin i(\varphi-\varphi')+b_i \cos i(\varphi-\varphi')\right]\frac{x^i}{i}r'dr'd\varphi' \tag{2}$$

The potential $\mathbf{A}_2(r,\varphi)$ is generated by the currents induced in the shield and, in consequence, satisfies the Laplace equation [1,3,4]:

$$\nabla^2 \mathbf{A}_2(r,\varphi)=0 \tag{3}$$

- in the II domain (the shield material), i.e. for $R_S \le r \le R_O$:

$$\nabla^2 \mathbf{A}_{II}(r,\varphi) = j\omega\,\mu_O\,\mu_S\gamma_S\mathbf{A}_{II}(r,\varphi) \tag{4}$$

- in the III domain (outside the shield), i.e. for $r \ge R_O$:

$$\nabla^2 \mathbf{A}_{III}(r,\varphi) = 0 \tag{5}$$

In the above relationships: S_c – is the cross-section area of a single phase conductor, J – current density in the L1 phase, ω – pulsation ($\omega=2\pi f$), μ_O – magnetic permeability of vacuum; μ_S – relative magnetic permeability of the shield material, γ_S – electrical conductivity of the shield material. The coefficients of the formulae are presented in the papers [1,2,4].

Moreover, the following boundary conditions should be met at the boundaries of particular areas:

- for $r = R_S$:

$$A_I(r,\varphi) = A_{II}(r,\varphi)$$
$$H_{I\varphi}(r,\varphi) = H_{II\varphi}(r,\varphi) \tag{6}$$

-for $r = R_O$:

$$A_{II}(r,\varphi) = A_{III}(r,\varphi)$$
$$H_{II\varphi}(r,\varphi) = H_{III\varphi}(r,\varphi) \tag{7}$$

Proper transformations lead to the integral Fredholm equation of current density $J(r,\varphi)$ in the phase currents and the shield [1-7]:

$$J(r,\varphi) - J(r_O,\varphi_O) + + \frac{3}{4\pi}j\omega\mu_O\gamma_C\int_S J(r',\varphi')\left[K(r',\varphi',r,\varphi)-K(r',\varphi',r_O,\varphi_O)\right]dr'\,d\varphi' = 0 \tag{8}$$

$$\int_S J(r',\varphi')r'\,dr'\,d\varphi'=I \tag{9}$$

where: (r_O,φ_O) is a reference point, γ_c electrical conductivity of the conductor; $K(r',\varphi',r,\varphi)$ – a kernel of the integral equation determined by the relationship [1,4,6]:

$$K(r',\varphi',r,\varphi) = \sum_{i=1}^{\infty}\left[a_i \sin i(\varphi-\varphi') + b_i \cos i(\varphi-\varphi')\right]\frac{x^i}{i}r' + +\sum_{i=1}^{\infty}\left[F_{1i} \sin i(\varphi-\varphi') + F_{2i} \cos i(\varphi-\varphi')\right](r\,r')^i\,r \tag{10}$$

The method of moments being a variant of the Ritz method, allows for converting the integral equations into a system of algebraic equations [1,5]. Solution of the system defines distribution of current density in the conductors and the shield.

Knowledge of approximate distribution of the current density vector enables determining the amount of real power loss P_C and P_S of the conductors and their shield, respectively, that is presented by the following relationships (in a descretized form) [1,4,6]:

$$P_C = \frac{3}{\gamma_C}\sum_{m=1}^{N}\left|J_m(r_m,\varphi_m)\right|^2\,\Delta S_m \tag{11}$$

and

$$P_S = \frac{1}{\gamma_S}\sum_{m=1}^{N}\left|J_{S_m}(r_m,\varphi_m)\right|^2\,\Delta S_m \tag{12}$$

3.2 Determination of temperature distribution of the shield

Taking into account that busduct length many times exceeds its cross dimensions the entire thermal power emitted in the phase conductors propagates in radial direction. Based on temperature measurements it may be assumed that it flows uniformly. Such a thermal flux is additionally complemented by the thermal power emitted in the shield. The power density however, remains non uniform, taking into account inhomogeneous character of the current density induced in the shield. This in turn gives evidence that the temperature distribution may also be inhomogeneous.

Total thermal power emitted from the considered system is carried to the environment by convection and radiation. Mathematical description of the exchange is made, among others, in terms of empirical coefficients, taking into account the physical and geometrical parameters of the considered system. Values of the coefficients may be accurately determined only on empirical way. Values of the coefficients assumed for the calculation allow for estimating the temperature of the working conductor and the shield.

Distribution of the power density released from the shield is expressed by the relationship [1,2,4,6]:

$$\rho(\mathrm{r},\varphi) = \frac{|\,\mathrm{J_S}(\mathrm{r},\varphi)\,|^2}{\gamma_\mathrm{S}} \tag{13}$$

Results of many computations carried out for aluminum shields of 3 mm thickness have shown that $\rho(\mathrm{r},\varphi)$ is a function which depends strongly on the variable φ, of symmetrical properties each 120^o. On the other hand, ρ depends on the r variable only to small degree. The total power emitted from the phase conductors propagates radially and only approximately uniformly. At the inner shield surface the following boundary condition is met [2,4,6]:

$$\frac{\mathrm{P_C}}{2\pi\,\mathrm{R_S}} = -\,\lambda_\mathrm{S}\,\frac{\partial \mathrm{T}}{\partial \mathrm{r}} \qquad \text{for} \qquad \mathrm{r} = \mathrm{R_S} \tag{14}$$

Temperature inside the shield meets the Poisson equation:

$$\nabla^2\,\mathrm{T}(\mathrm{r},\varphi) = -\,\frac{\rho(\mathrm{r},\varphi)}{\lambda_\mathrm{S}} \tag{15}$$

The thermal power is transferred from the outer shield surface to the environment by convection and radiation [1,2,6]:

$$\lambda\,\frac{\partial\,\mathrm{T}(\mathrm{r},\varphi)}{\partial\,\mathrm{r}} = -\,\alpha_\mathrm{CR}\,\left[\mathrm{T}(\mathrm{r},\varphi) - \mathrm{T_O}\right] \qquad \text{for} \qquad \mathrm{r} = \mathrm{R_O} \tag{16}$$

with: $$\alpha_\mathrm{CR} = \alpha_\mathrm{C} + \alpha_\mathrm{R} \tag{17}$$

α_C - surface film conductance for convection:

$$\alpha_\mathrm{C} = \frac{\lambda_\mathrm{S}\cdot 0.235\cdot\left(\mathrm{G_R}\ \mathrm{P_R}\right)^{0.25}}{\mathrm{R_O}} \tag{18}$$

α_R - surface film conductance for radiation:

$$\alpha_\mathrm{R} = 5.67\cdot 10^{-8}\cdot\varepsilon\ \frac{\left[\mathrm{T}^4(\mathrm{r}=\mathrm{R_O}) - \mathrm{T_O^4}\right]}{\mathrm{T}(\mathrm{r}=\mathrm{R_O}) - \mathrm{T_O}} \tag{19}$$

$\mathrm{G_R}$ - Grashof number:

$$\mathrm{G_R} = 156.96\cdot\frac{\mathrm{T}(\mathrm{r}=\mathrm{R_O}) - \mathrm{T_O}}{\mathrm{T}(\mathrm{r}=\mathrm{R_O}) + \mathrm{T_O}}\cdot\frac{\mathrm{R_O^3}}{\nu^2} \tag{20}$$

In the above formulas [1,6,8,9,10]: $\mathrm{P_C}$ – is the active power loss (per unit length) of a phase conductor; $\mathrm{P_R}$ – is the Prandtl number (read from tables); $\mathrm{T_O}$ – temperature of the environment; ε – emissivity factor of the shield material (the factor, similarly as the other material data, should be each time experimentally determined for a given material), λ_S – thermal conductivity of the shield material, ν – air kinematic viscosity (read from tables).

The shield temperature may be considered as a sum of two excitations [1,6]:

$$\mathrm{T}(\mathrm{r},\varphi) = \mathrm{T_O} + \Delta\mathrm{T_{CS}}(\mathrm{r}) + \Delta\mathrm{T_S}(\mathrm{r},\varphi) \tag{21}$$

where $\Delta\mathrm{T_{CS}}(\mathrm{r})$ – the increase of temperature caused by real power released in the phase conductors (being a function of only r variable, due to radial propagation of thermal power); $\Delta\mathrm{T_S}(\mathrm{r},\varphi)$ – the increase of temperature caused by real power released in the shield (being a function of both r and φ variables since the distribution of the real power released in the shield depends on both these variables).

The increase of temperature $\Delta\mathrm{T_{CS}}(\mathrm{r})$ meets the Laplace equation [1,2,4,8,9]:

$$\frac{\partial}{\partial r}\left[r \frac{\partial T_{CS}(r)}{\partial r} \right] = 0 \quad \text{for} \quad R_3 \le r \le R_4 \tag{22}$$

Solution of the equation (22) with consideration of (14) and (16) boundary conditions gives [1,2,6]:

$$T_{CS}(r) = \frac{P_C}{2\pi \lambda_S}\left(\ln \frac{R_O}{r} + \frac{\lambda_S}{\alpha_{CR} R_O} \right) \tag{23}$$

As it was mentioned above, the density of the power released from the shield is a symmetrical function repeated each 120°. The assumption that the whole power ΔP_S released from a shield sector is concentrated to circular sectors of $r = R_x$ and the angle 2β, enables stating that power surface density amounts to:

$$\sigma_O = \frac{\Delta P_S}{2\beta R_X} = \rho(r_K, \varphi_K)\, dR_X \int dl \tag{24}$$

where: $R_X = (R_S + R_O)/2$; $\rho(r_K, \varphi_K) = \rho_K$ – volumetric density of the power emitted from the k-th shield sector.

The surface density may be presented as a series:

$$\sigma(\varphi) = \sigma_O \sum_{n=1}^{\infty} B_n \cos n\varphi \tag{25}$$

taking into account [1]:

$$B_n = \begin{cases} \dfrac{3\beta}{\pi} & \text{dla} \quad n = 0 \\ \dfrac{6 \sin n\varphi}{n\pi} & \text{dla} \quad n = 3k \\ 0 & \text{dla} \quad n \ne 3k \end{cases} \tag{26}$$

In such approach the stationary temperature field is described by a system of Laplace equations [1,4,5,6]:

$$\begin{aligned} \nabla^2 T_1 = 0 \qquad & R_S \le r \le R_X \\ \nabla^2 T_2 = 0 \qquad & R_X \le r \le R_O \end{aligned} \tag{27}$$

where T_1, T_2 – temperatures of inner and outer shield surface, respectively.

In result of the transformations finally, for $r = R_O$ one obtains [1,6]:

$$\Delta T_S(r = R_4, \varphi) = \sum_{k=1}^{N} \rho_K \sum_{n=1}^{\infty} \left[D_{3n} R_4^{n} \cos n(\varphi - \varphi_k) + D_{4n} R_4^{-n} \cos n(\varphi - \varphi_k) \right] + D_5 \ln R_4 + D_6 \tag{28}$$

The coefficients occurring in the equations are determined in [1,2,4,6]. Temperature of the inner shield wall (for $r = R_S$) may be similarly determined.

3.3 Determination of conductor temperature distribution

Solution of the system of integral equations provides distribution of current density J_j and, at the same time, real power values are attributed to particular j-th points of the conductor cross-section [4,6]:

$$P_j = \frac{1}{\gamma_C} \left| J_j \right|^2 \Delta S_j \tag{29}$$

Thickness of the conductor wall is relatively small, while the thermal conductivity of the material is rather high. Hence, it might be assumed that under stationary condition the temperature along the cross-section wall thickness is constant, varying only along its length. Taking into account such an assumption the thermal model of the conductor cross-section may be presented in the form of a set of $l_1, l_2, \ldots, l_m, \ldots l_M$ lines corresponding to definite layers of the conductor cross-section. The beginning and the end of the branch l_m is defined by the points l_{mp} and l_{mk}, respectively. Then, to particular points of the $l_{m1}, l_{m2}, \ldots, l_{mn}$ line may be assigned the values P_j of real power emitted in the discretized elements of the phase conductor cross-section (Fig. 3 and Fig. 4).

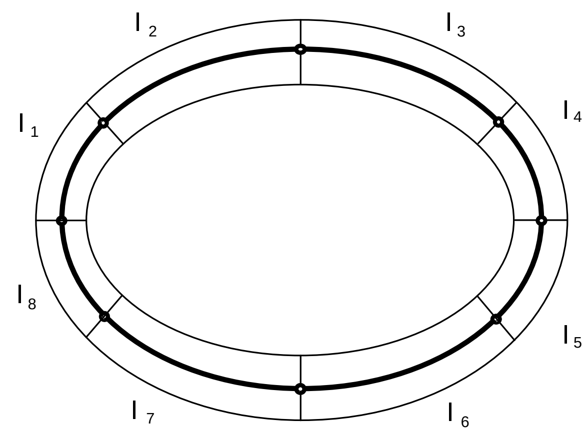

Fig. 3. Cross-section of the phase conductor and its division into subdomains

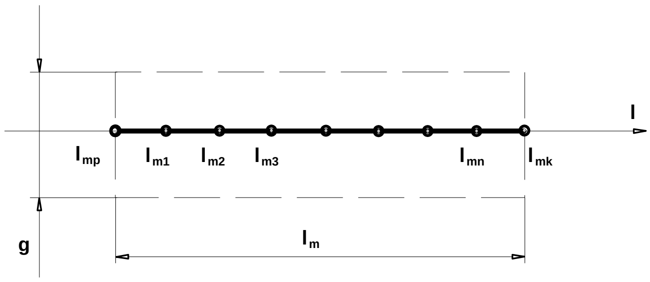

Fig. 4. Discretization of the conductor cross-section subdomain

In case of such a formulation of the system the temperature growth above the environment temperature in the branch l_m meets the Poisson equation [1,4,6]:

$$\lambda_C \frac{\partial^2 T(l)}{\partial l^2} = - p(l) \qquad (30)$$

where:

$$p(l) = \frac{1}{g} \sum_{j=1}^{n} P_j \, \delta(l - l_{mj}) - \frac{2\alpha}{g} T(l) \qquad (31)$$

is a difference between the branch power emitted from n inside sources and the power carried to the environment with a constant α coefficient. On the other hand, the function $\delta(l-l_{mj})$ is the Dirac delta.

Based on the Poisson equation a differential equation is obtained that describes stationary distribution of temperature increment along the l_m branch:

$$\frac{\partial^2 T(l)}{\partial l^2} - \frac{2\alpha}{\lambda g} T(l) = - \frac{1}{\lambda g} \sum_{j=1}^{n} P_j \, \delta(l - l_{mj}) \qquad (32)$$

The transformations give finally [1,6]:

$$T_m(l) = C_{1m} \exp(a\,l) + C_{2m} \exp(-a\,l) + \frac{1}{2\lambda g a} \sum_{j=1}^{n} P_j \exp\left(-a\left|l - l_{mj}\right|\right) \qquad (33)$$

The constants C_{1m} and C_{2m} are determined based on the boundary conditions of elementary conductor sectors. The continuity condition of heat flux for the elementary sectors may be presented in the form:

$$\lambda \frac{\partial T(l)}{\partial l}\bigg|_{l=l_{jk}} = \lambda \frac{\partial T(l)}{\partial l}\bigg|_{l=l_{ip}} \qquad (34)$$

for the l_i and l_j values corresponding to the points of conductor sector contact. Moreover, in all the contact nodes the temperature continuity condition must be met:

$$T(l)\Big|_{l=l_{jk}} = T(l)\Big|_{l=l_{ip}} \qquad (35)$$

The above relationships formulated for all the nodes provide a system of algebraic equations in terms of C_{1m} and C_{2m} unknowns. Solution of the system gives constant values C_1 and C_2 for all the elements of the considered system [1,2,6]. Distribution of temperature increments of particular elements is obtained from the equation (33). To the temperatures obtained this way the shield temperature should be added (obtained on the grounds of previous calculation) that is here considered as the environment temperature of the phase conductors.

3.4 Consideration of coupling between electromagnetic and temperature fields

The calculation model used here makes allowance for coupling between the electromagnetic and thermal fields. The coupling occurs on the conductance of the conductors and shield. The temperature changes cause the changes in conductivity of the shield and conductors and in result power losses, and vice versa. These dependencies may be expressed as [4]:

$$T = f(\rho,\gamma,\lambda) \quad \text{and} \quad \rho = f(T,\gamma) \qquad (36)$$

The calculations must be carried out with iterative methods. Before the calculations an assumption must be made about temperature of the conductors and the shield. The assumption must be checked at the end of the calculations. Once the error exceeds 0.5 K a new assumption must be made and the calculation repeated.

3.5 Analysis of electric interactions of the system

Analysis of electric field distribution of three-phase shielded heavy-current busducts and, on the basis of the above, determination of the locations of critical stress occurrence, is very important from the point of view of the use of these devices, being, at the same time, one of optimization elements of their designing.

Determination of electric strength against breakdown is a highly difficult task, as the strength depends on many factors, like pressure, humidity, temperature, possible dust and dirt occurrence inside the system, etc. On the other hand, possible changes in electric critical stress may be related to micro-spears in the system that may arise during the technological process.

For purposes of accurate analysis of the electric interactions the method of integral equations may be used.

Taking into account the linear charge τ(α,β) the potential may be expressed by the relationship [11]:

$$v(x,y) = \frac{1}{2\pi\varepsilon} \int_L \tau(\alpha,\beta)\ \ln\frac{1}{r}\ dL \qquad (37)$$

where: r = |AB|, A(x,y) – the point in which the potential is calculated; B(α,β) – any point located at the L line, τ – linear density of the charge, ε – permittivity.

A system of three very long phase conductors located in an earthed shield may be replaced with linearly distributed charges. It is assumed that the integration line L is approximated by a broken curve, the segments of which are described by the equations of the type $y = cx + d$. The curvilinear integral (37) may be then replaced with an iterated integral:

$$v(x,y) = \frac{\sqrt{1+c^2}}{2\pi\varepsilon} \int_{\alpha_1}^{\alpha_2} \tau(\alpha,\beta)\ \ln\frac{1}{r}\ d\alpha \qquad (38)$$

where: α_1, α_2 – are x-coordinates of the beginning and end of the line segment, respectively.

In order to determine linear density of the charge τ the line L is divided into N elementary segments, with the assumption that linear charge density of an elementary segment remains constant and equal to τ_i. The equation (38) takes then a form:

$$v(x,y) = \sum_{i=1}^{N} \frac{\sqrt{1+c^2}}{2\pi\varepsilon}\ \tau_i\ (\alpha_i', \beta_i') \int_{\alpha_i}^{\alpha_{i+1}} \ln\frac{1}{r}\ d\alpha_i \qquad (39)$$

where:

$$r = \sqrt{(x-\alpha_i)^2 + (y-\beta_i)^2} = \sqrt{(x-\alpha_i)^2 + (y - c\alpha_i - d)^2}$$

with: α_i, α_{i+1} – being x-coordinates of the beginning and end of the elementary segment, respectively; (α_i', β_i') – the coordinates of middle point of the i-th segment.

Every segment of the current busducts has a potential $V(x,y) = V_j$ The presented relationships allow, upon appropriate transformations, to obtain a system of algebraic equations [1,2,11]:

$$\begin{bmatrix} C_{11} & C_{12} & \dots & C_{1N} \\ C_{21} & C_{22} & \dots & C_{2N} \\ \dots & \dots & \dots & \dots \\ C_{N1} & C_{N2} & \dots & C_{NN} \end{bmatrix} \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \qquad (40)$$

where:

$$C_{ij} = \frac{\sqrt{1+c^2}}{2\pi\varepsilon} \int_{\alpha_i}^{\alpha_{i+1}} \ln \frac{1}{\sqrt{(x_j-\alpha_i)^2 + (y_j - c\,\alpha_i - d)^2}}\ d\alpha_i \qquad (41)$$

while for the element describing its own interaction with itself (i = j):

$$C_{ii} = -\frac{1}{2}\sqrt{1+c^2}\ \ln(1+c^2)(\alpha_{i+1} - \alpha_i) + \sqrt{1+c^2}\ (\alpha_i - \alpha_{i+1})\ (\ln\left|\frac{\alpha_i - \alpha_{i+1}}{2}\right| - 1) \qquad (42)$$

Solution of the system of equations (40) provides the linear density of the charge τ_i in the elementary busduct segments. The potential in any point of the considered region is determined from the equation (39) for a known linear charge distribution at the conductors surface. On the other hand, the E_x and E_y components of electric field intensity are determined from the relationship [11]:

$$E_x = -\frac{\partial u}{\partial x} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\sqrt{1+c^2}\ \tau_i(\alpha_i', \beta_i') \int_{\alpha_i}^{\alpha_{i+1}} \frac{2(x-\alpha_i)}{(x-\alpha_i)^2 + (y - c\alpha_i - d)^2}\ d\alpha_i \qquad (43)$$

$$E_y = -\frac{\partial u}{\partial y} = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\sqrt{1+c^2}\ \tau_i(\alpha_i', \beta_i') \int_{\alpha_i}^{\alpha_{i+1}} \frac{2(y - c\alpha_i - d)}{(x-\alpha_i)^2 + (y - c\alpha_i - d)^2}\ d\alpha_i \qquad (44)$$

Full analysis of the electric field was necessary in case of former designs of the heavy-current busducts in which sharp edges of the phase conductor cross-section occurred (in such a case geometry of the busduct has been imposed by the electric strength conditions). In case of oval cross-section phase conductors the cross-section geometry of the heavy-current busduct is determined chiefly by thermal conditions of the system, while the electric strength is obtained by analysis of electric stresses in the system weak points [1,2,4,11]. Under such a condition the analytical expressions for electric field intensity may be used that are related to geometrically definite parts of the system. Many research and computation trials carried out for the considered system have clearly shown that critical points of the analyzed geometry are (Fig. 5 and Fig. 6):

a) E1 – maximal electric field intensity between the phase conductors:

$$\left| E1 \right| = \frac{U \cos \psi}{2 d\, \varepsilon_r \sin^2 \psi \, \ln \mathrm{ctg} \frac{\psi}{2}} \quad (45)$$

b) E2 – maximal electric field intensity between the conductor and the shield at the shorter oval axis:

$$\left| E2 \right| = \frac{U}{(R_3 - h_{iz})\, \varepsilon_r \, \ln \frac{R_3}{R_3 - h_{iz}}} \quad (46)$$

c) E3 – maximal electric field intensity between the conductor and the shield at the longer oval axis:

$$\left| E3 \right| = \frac{U}{c\, \varepsilon_r \sqrt{\left(\frac{x}{c}\right)^2 - 1}\, \left(\mathrm{arcosh} \frac{a}{2c} - \mathrm{arsinh} \frac{x}{c} \right)} \quad (47)$$

with: $$c = \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{b}{2}\right)^2}$$

In the above formulas: ψ – half of the apex angle of the phase conductor cross-section; d – distance between the phase conductors; R_3 – internal radius of the shield; h_{iz} – insulator height; a – large diameter of the conductor; b – small diameter of the conductor; x – the distance between the centre of symmetry of the shield along the large diameter of the conductor; ε_r – specific inductive capacity; U – peak value of withstood proof surge voltage in kV (determined based on standards).

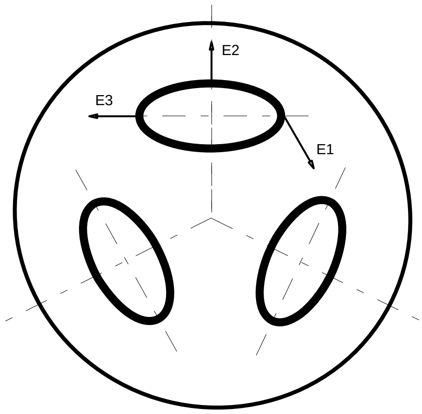

Fig. 5. Locations of system critical stresses

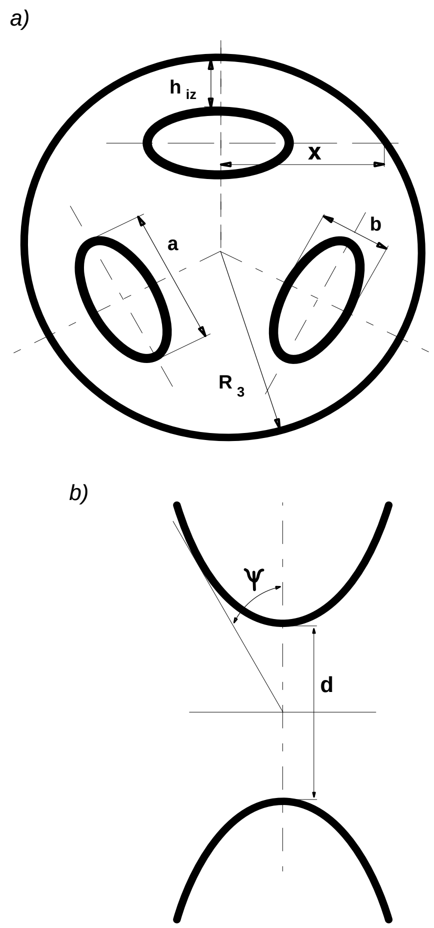

Fig. 6. Geometric dimensions characterizing the system from the point of view of critical stress calculation

4 Optimization problems

The optimization is aimed at saving the raw materials and the power at the stages of manufacturing and operating of the three-phase heavy-current busducts.

As an optimization criterion of heavy-current busducts the minimal cost of the consumed construction material (i.e. the investment cost), and unit cost of real power loss within the duration of the use of the device (i.e. the operational cost k_e) have been adopted. Therefore, the objective function S_K is a sum of variable construction and operational costs of the considered busducts, depending on geometrical dimensions of the system [1,2,4]:

$$S_K = k_i + k_e = f(a, b, g, h, R_s) \quad (48)$$

The constraints Z_i of the optimization process include allowable electrodynamic parameters of the system (temperatures of the conductors and shield, electrodynamic forces, and electric strength) and real (technologically justified) ranges of their geometrical dimensions. In order to check the optimization problem with these constraints the criterial function S_z has been used for unconditional optimization purposes, with the penalty factor P_i, resulting in precluding the solutions for which the constraints are not met [1,2]:

$$S_Z = f(a, b, g, h, R_s) + \sum_{i=1}^{m} P_i (Z_i)^2 N[Z_i] \quad (49)$$

where:

$$N[Z_i] = \begin{cases} 1 & \text{for} \quad Z_i > 0 \quad \text{infringement of the constraints} \\ 0 & \text{for} \quad Z_i \le 0 \quad \text{fulfilment of the constraints} \end{cases} \quad (50)$$

Consideration of greater number of the factors in the analysis of the system and the use of more sophisticated mathematical apparatus for computation of the object parameters (the determination of which is required in order to cope with all the optimization conditions) provide the criterial functions in an inexplicit form, having many local optimal points. In such cases an appropriate choice of optimization method (in accordance with the considered technological problem) becomes an important element.

In the optimization of a simple (unimodal) function the deterministic methods (among them the Gauss-Seidel method presenting a classical deterministc approach, the method of maximum slope, i.e. a gradient method with minimization of the direction, and the method of coupled directions – with varying metric) are very effective. On the other hand, in case of multimodal functions the deterministic methods usually stall in one of many local optimal points (reaching no real optimal point) [1,2,12,13,14].

The optimum in the global sense may be found only with the help of indeterministic methods (as Monte Carlo or the genetic algorithms – the evolutionary method). Another worth considering solution may consist in combination of the indeterministic methods (in order to find the starting point) with the deterministic ones (for finding the accurate optimal point).

In the present paper in order to reach the global optimal point of multimodal criterial function the indeterministic method of the genetic algorithms is used [1,2,12,14].

Simplicity of the genetic algorithms consists in the use of several simple operations performed on code series. It consists in converting the objective function parameters (i.e the decisive variables) into a binary form and processing the information in a random manner. Operation of the method is based on a general “Darwin Principle” saying that a stronger (better adapted) individual survives while the weaker one “dies”. In case of the genetic algorithms the better adaptation concerns the code series having higher values of the objective function called the adaptation function. Better adapted individuals have better chance for introduction of their representatives to the next population. This is conducive to the fact that the next population includes the code series distinguished by the best adaptation, in result of a simultaneously random and deterministic procedure [14]. Mathematical formulation of the above mentioned rule has a form:

$$m(H, p+1) = m(H, p) \cdot (\bar{f} + c\,\bar{f})/\bar{f} = (1 + c) \cdot m(H, p) \quad (51)$$

where:
m(H,p) – expected number of the representatives of the H scheme in the p population,
m(H,p+1) – expected number of the representatives of the H scheme in the p+1 population,
c – an experimental constant,
$\bar{f} = \sum_{j=1}^{n} f_j / n$ – average adaptation of the whole population,
f_j – adaptation of the j-th individual,
n – the number of the individuals belonging to the population.

Since in the genetic algorithms the best individuals are selected, the criterial function (consisting in the functional minimization) should be transformed into the adaptation function (requiring maximization). The methods of these transformations are presented in [14].

The foundations of every genetic algorithm are based on several elementary operations, i.e. reproduction, crossing, and mutation. The reproduction operation consists in copying the code series into the next population based on the value of the adaptation function. On the other hand, the crossing operation consists in random mating of the code series of one population into pairs that represent a single decisive variable and, afterwards, random exchanging of a definite number of bits between them. The random exchange of the definite number (defined based on the assumed probability level) of bits is called mutation (in particular cases such a process may disturb the algorithm thus leading to the lack of convergence).

The genetic algorithm in its elementary version includes the following operations: selection according to the roulette rule, simple crossing with random mating, and simple mutation. The selection according to the roulette rule is a process distinguished by large variation, hence, the eventual number of copies is significantly dispersed around its expected value (thus resulting in poor convergence of the computation). The paper presents a modification of the selection operation based on the De Jong Expected Value Model and Brindle's random selection without repetitions. In order to prevent losing the best individual such an individual, representing the best among the current solutions, is "always" transferred into a randomly selected place. Another modification consists in scaling of the adaptation. In the present algorithm the linear scaling is used [1,2].

The crossing and mutation probabilities were constant, amounting to 0.8 and 0.005, respectively. A constant population size of 16 individuals was kept. The optimization process was carried out with 80 generations.

5 Results of the calculation performed

The calculation has been performed for a three-phase shielded heavy-current busduct. Aluminum is used as structural material of the conductors and the shield. The system is located in still air, screened from solar radiation. For purposes of the calculation the following data are assumed: rated voltage U = 12 kV, rated current I = 4 kA, the frequency f = 50 Hz, the environment temperature $T_O = 308$ K, allowable shield temperature $T_S = 338$ K, allowable conductor temperature $T_C = 378$ K, conductivity of conductor and shield material $\gamma = 34.6 \cdot 10^6$ S/m, emissivity coefficient of the conductor and shield materials 0.5.

Conductor and shield temperature distribution has been calculated for a busduct of the parameters: diagonal of the conductor oval – the large one a = 148 mm and the small one b = 125 mm, conductor wall thickness g = 14 mm, height of conductor suspension h = 178 mm, internal shield radius $R_S = 547$ mm, external shield radius $R_O = 550$ mm, surface film conductance $\alpha = 4.5$ W/(m$^2 \cdot$ K), thermal conductivity of conductor and shield heat $\lambda = 220$ W/(m $\cdot$ K). Results of the calculation are presented in Fig. 7. Calculated temperature distributions are presented with the accuracy to two decimal places, only in order to depict the scale of temperature differences of particular points of the system. Under real temperature computation such accuracy is useless, since the temperatures determined this way depend on many factors, as condition of the conductor and shield surface (and, in consequence the surface film conductance), arrangement of the busduct (even slight deviation from horizontal position may affect the convection conditions) or environmental conditions (random air motion around the busduct or consideration of the insolation in case of location in an open area).

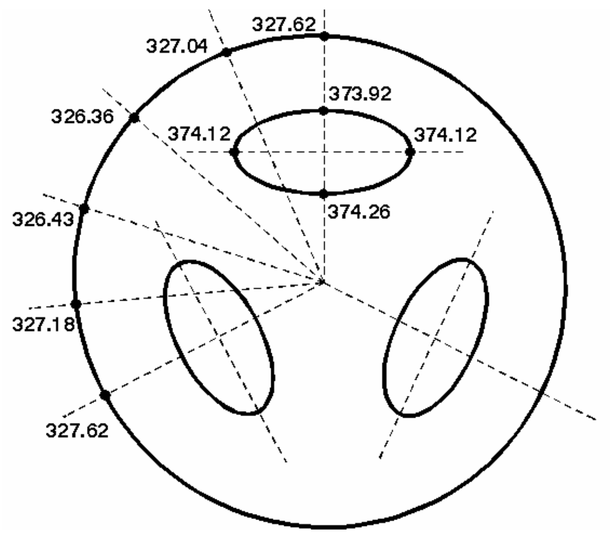

Fig. 7. Results of temperature distribution T[K] calculation of the conductors and shield

For the same system the temperatures of the conductors and the shield have been calculated with respect to:

a) the current in the range 1÷4 kA. Changes in current load capacity of the busducts of definite geometry are conducive to intensive temperature changes, both of the conductors and the shield (Fig. 8). Temperature difference between the conductors corresponding to the current variation range specified above amounts to 34 K, while in case of the shield – to 8.5 K.

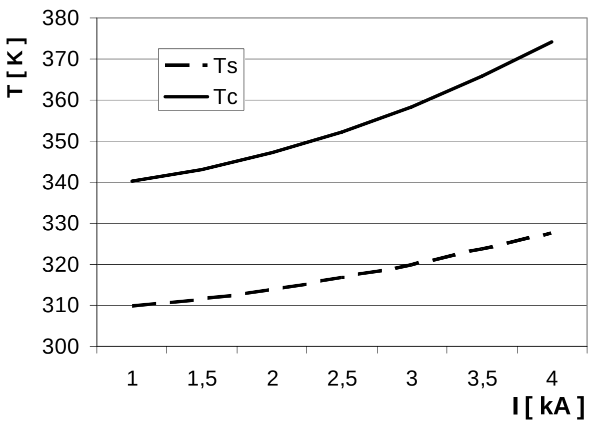

Fig.8. Changes in conductor and shield temperature with regard to current variations

b) electrical conductivity of the conductor material in the range $28 \div 36 \cdot 10^6$ S/m. Temperature difference between the conductors corresponding to the range of conductivity of the conductor material specified above amounts to 8.5K, while in case of the shield – to 4K (Fig. 9).

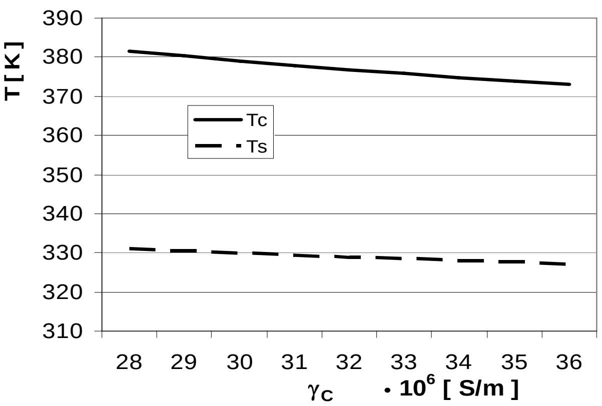

Fig.9. Changes in conductor and shield temperature with regard to varying conductivity of the conductor material

c) electrical conductivity of the shield material in the range $28 \div 36 \cdot 10^6$ S/m. The difference in calculation results of the shield temperature for the range of conductivity of the conductor material specified above was lower than 0.5 K. Temperature of the conductors remained unchanged (the difference below 0.1 K). Due to insignificant temperature changes no plots of the results are drawn up.

Moreover, optimization calculation has been carried out for the heavy-current busducts. Dependence of optimal total cross-section area of the S profile and the power loss P versus varying electrical conductivity of the conductor material γ_C is shown in Fig. 10.

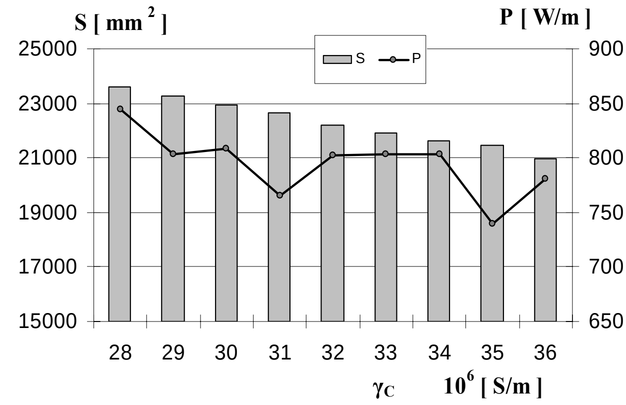

Fig. 10. Dependence of optimization results of heavy-current busducts versus varying conductivity of the conductor material

Fig. 11 and Fig. 12 show the relationships between optimal total cross section area S and power loss P versus the current intensity and voltage of the busduct, respectively.

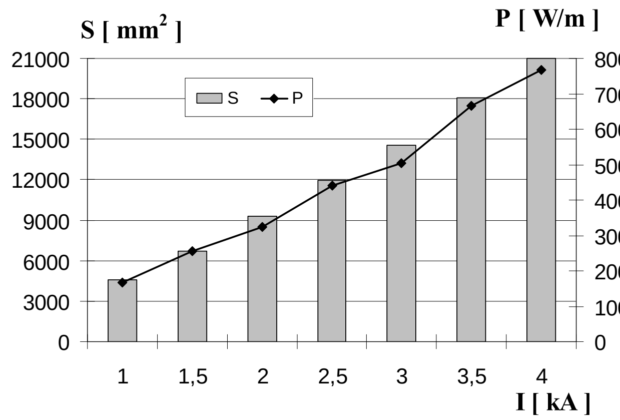

Fig. 11. Relationship between the results of heavy-current busduct optimization and current variation

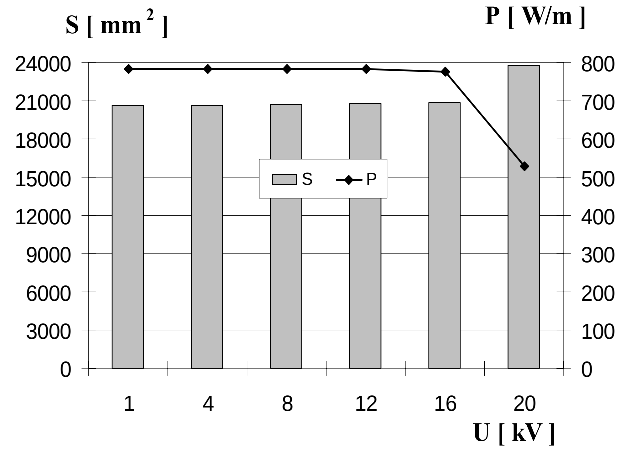

Fig. 12. Relationship between the results of heavy-current busduct optimization and voltage variation

6 Conclusion

On the grounds of the multi-variant computation carried out for the heavy-current busducts and its comparison to measurements made in physical systems [1,2,4,6] it was found that the calculation error is less than 1 percent. Accuracy of the thermal calculation depends on many factors, among which the following might be mentioned: correctness of the assumed mathematical model, proper consideration of the environment conditions (e.g. random air motion, consideration of solar radiation or the effect of other heat sources existing in the vicinity), and exactness in assuming the coefficients defining the kind of the material and condition of the surface of particular elements (e.g. changes in colour or roughness caused by external factors).

Conductor and shield temperature is significantly affected by the variations of electrical conductivity of the conductors and the shield. It means, in consequence, that it is worth making the phase conductors of a material of high conductivity, while the use of high conductivity aluminum in case of the shield is unprofitable. Similar conclusions may be drawn based on the optimization results of the present paper and former publications of the author.

Multi-variant calculations (carried out in other author's papers too) give evidence that the results of optimization of the considered busducts are chiefly determined by thermal factors (the effect of electrodynamic forces and electrical strength is much smaller). Therefore, appropriate thermal calculation of the system is of high importance for the optimization process.

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