

Unsteady Mixed Convection Heat and Mass Transfer Past an Infinite Porous Plate with Thermophoresis Effect

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Abstract: -

An analysis has been developed in order to study the unsteady mixed convection flow of an incompressible fluid past an infinite vertical porous plate with thermophoresis particle deposition effect. The governing equations are solved numerically using an implicit finite difference technique. The selected numerical method is validated by comparing the results with the analytical solutions. Numerical results for the details of the velocity profiles which are shown on graphs have been presented. It is found that the steady state values of thermophoretic deposition velocity reached faster as the thermophoresis constant decreased and the temperature ratio increased.

Key-Words: - Mixed convection, Heat and mass transfer, Unsteady, Thermophoresis, Plate

1 Introduction

The study of heat and mass transfer in the boundary layer induced by a vertical porous surface is important in several manufacturing processes in industry, which include the boundary layer along material handling, the extrusion of plastic sheets, the cooling of an infinite metallic plate. Many authors (see, for example, Elliot [1], Sakiadis [2], Kerboua and Lakis [3], Corcione [4]) have studied the problem of combine thermal convection from a semi-infinite vertical plate. Ching [5] uses the integral method to study the heat and mass transfer by mixed convection from vertical plates with constant wall temperature and concentration in porous media saturated with an electrically conducting fluid in the presence of a transverse magnetic field.

Thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and toward a cold one. Small particles, such as dust, when suspended in a gas temperature gradient, experience a force in the direction opposite to the temperature gradient.

This phenomenon has many practical applications in removing small particles from gas streams, in determining exhaust gas particles trajectories from combustion devices, and in studying the particulate material deposition on turbine blades. It has been also shown that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber performance.

Goren [6] studied the role of thermophoresis of a viscous and incompressible fluid, the classical problem of flow over a flat plate is used to calculate deposition rates and it is found that the substantial changes in surface deposition can be obtained by increasing the difference between the surface and free stream temperatures. Gokoglu and Rosner [7], Park and Rosner [8] obtained a set of similarity solutions for the two dimensional laminar boundary layers and stagnation point flows respectively. Chio [9] obtained the similarity solutions for the problem of a continuously moving surface in a stationary incompressible fluid, including the combined

effects of convection, diffusion, wall velocity and thermophoresis. Grag and Jayaraj [10] discussed the thermophoresis of small particles in forced convection laminar flow over inclined plates. Epstein et al. [11] have studied the thermophoresis transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid. Chiou [12] has considered the particle deposition from natural convection boundary layer flow on isothermal vertical cylinder.

Consideration in this work is given to the thermophoresis effects on unsteady mixed convection heat and mass transfer problems from infinite vertical porous surfaces. Numerical results for the velocity, temperature and concentration profiles as well as the thermophoresis velocity, under the effect of different dimensionless groups are presented.

2 Analysis

Consider an unsteady convection boundary layer flow of a fluid past an infinite isothermal vertical plate of constant temperature T_w and concentration C_w . The ambient temperature is T_∞ and concentration C_∞ . The plate temperature T_w and concentration C_w is higher than the ambient temperature T_∞ and concentration C_∞ . It is assumed that the fluid properties are constant except the influence of density variation with temperature is considered only in the body force term. The flow is assumed to be in the x -direction, which is along the vertical plate in the upward direction, and y -axis is taken to be normal to the plate, Fig. 1. Allowing for both Brownian motion of particles and thermophoretic transport, the governing equations are, Chiou, M. C. (1991),

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) + v \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} + \frac{\partial(Cv_t)}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

The physical problem assumes the following boundary conditions:

$$\begin{aligned} t \leq 0 \quad & u(y,t) = 0, T(y,t) = T_\infty, C(y,t) = C_\infty \\ t > 0 \quad & u(0,t) = 0, v(0,t) = v_w, T(0,t) = T_w, \\ & C(0,t) = C_w \\ t > 0 \quad & u(\infty,t) = u_o, T(\infty,t) = T_\infty, C(\infty,t) = C_\infty \end{aligned} \quad (5)$$

Here x and y are the dimensional distance along and normal to the plate, respectively. (u, v) are the averaged velocity components along the x and y , directions respectively, v_w is the suction velocity, T is the temperature, C is the concentration, β_T and β_C are the coefficient of thermal expansion of temperature and concentration respectively. ν is the kinematic viscosity, α is the thermal diffusivity and D is the Brownian diffusion coefficient.

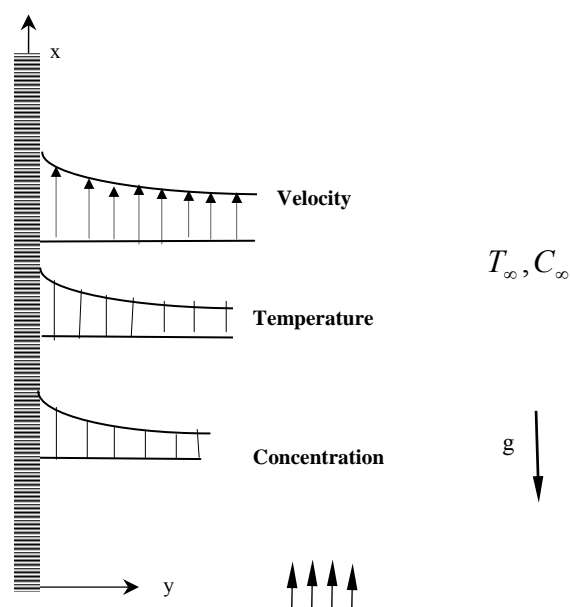


Fig. 1: Schematic diagram for flow mode coordinate system.

The thermophoretic velocity v_t can be expressed in the form,

$$v_t = -k \frac{\nu}{T} \frac{\partial T}{\partial Y} \tag{6}$$

Where k is the thermophoretic coefficient. In order to non-dimensionalize the governing equations, we introduce the following non-dimensional parameters:

$$\begin{aligned} U = \frac{u}{u_o}, \quad Y = Ra \frac{y}{l}, \quad \lambda = Ra \frac{\nu_w}{u_o}, \quad V_t = Ra \frac{\nu_t}{u_o}, \\ \tau = \frac{u_o}{l} t, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \tag{7}$$

Where $u_o = gK \beta_T (T_w - T_\infty) / \nu$ is the characteristic velocity, $Ra = gK \beta_T (T_w - T_\infty) / \nu \alpha$ is the Rayleigh number, l is the characteristic length of the plate and λ is the suction parameter. The dimensionless form of the governing equations and their boundary conditions are reduced to

$$\frac{\partial U}{\partial \tau} - \lambda \frac{\partial U}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 U}{\partial Y^2} + B(\theta + N\phi) \tag{8}$$

$$\frac{\partial \theta}{\partial \tau} - \lambda \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial Y^2} \tag{9}$$

$$\frac{\partial \phi}{\partial \tau} - \lambda \frac{\partial \phi}{\partial Y} + \frac{\partial(\phi + V_t)}{\partial Y} = \frac{1}{Le} \frac{\partial^2 \phi}{\partial Y^2} \tag{10}$$

$$V_t = -k \frac{Pr}{N_t + \theta} \frac{\partial \theta}{\partial Y} \tag{11}$$

$$\begin{aligned} \tau \leq 0 \quad U(Y, \tau) = 0, \quad \theta(Y, \tau) = 0, \quad \phi(Y, \tau) = 0 \\ \tau > 0 \quad U(0, \tau) = 0, \quad \theta(0, \tau) = 1, \quad \phi(0, \tau) = 1 \\ \tau > 0 \quad U(\infty, \tau) = 1, \quad \theta(\infty, \tau) = 0, \quad \phi(\infty, \tau) = 0 \end{aligned} \tag{12}$$

where $B = Gr Ra / Pr$ is the dimensionless mixed convection parameter and $Gr = \nu g \beta_T (T_w - T_\infty) / u_o^3$ is the thermal Grashof number. $Pr = \nu / \alpha$ and $Le = \alpha / D$ are the Prandtl and Lewis numbers, $N_t = (T_w - T_\infty) / T_\infty$ is the thermophoresis parameter and $N = \beta_C (C_w - C_\infty) /$

$\beta_T (T_w - T_\infty)$ is the buoyancy parameter.

The quantity of physical interest is the wall thermophoretic deposition velocity that can be express as:

$$V_{tw} = -k \frac{Pr}{N_t + 1} \frac{\partial \theta}{\partial Y} \Big|_{Y=0} \tag{13}$$

3 Solution Methodology

The mass conservation equation is a nonlinear equation. Furthermore, the mass and energy equations are coupled with the momentum equation, for such reasons, the system of equations with the corresponding boundary conditions (8-12) are solved numerically using an implicit finite-difference technique similar to Crank-Nicolson method, which is discussed by Anderson [13]. All the first-order derivatives with respect to τ are replaced by using formula of the form:

$$\frac{\partial C}{\partial \tau} = \frac{(C_j^{n+1} - C_j^n)}{\Delta \tau} \tag{14}$$

All the second-order derivatives with respect to Y are replaced by using formula of the form:

$$\begin{aligned} (1/2)(C_{j+1}^{n+1} - C_{j+1}^n) + \\ (1/2)(C_j^{n+1} - 2C_j^n + C_{j-1}^{n+1}) + \\ \frac{\partial^2 C}{\partial Y^2} = \frac{(1/2)(C_{j-1}^{n+1} + C_{j-1}^n)}{(\Delta Y)^2} \end{aligned} \tag{15}$$

The concentration equation is transformed to finite difference equations by applying the central difference approximations to the first and second derivatives. The finite difference equations form a tri-diagonal system can be solved by the tri-diagonal solution scheme. The effect of the grid size ΔY and $\Delta \tau$ on the numerical solution had been studied. The results drawn here are independent on the grid size. The grid spacing used here are those largest values of ΔY and $\Delta \tau$ which does not alter the solution, this process have important effects on computational time.

In order to verify the accuracy of the selected numerical method, the energy equation is a linear equation and can be solved analytically by Laplace transform technique; it is suitable to note that the nonlinearity in the concentration equation are due to the thermophoretic velocity. Furthermore, the code developed in this investigation was validated by comparing the results obtained by this numerical method with analytical solution of the energy equation using the Laplace transform technique. The solution of the energy equation is given by:

$$\theta(Y, \tau) = 1/2 \exp(-\lambda Y) \left[\exp(\lambda Y) \operatorname{erfc} \left(\frac{Y}{2\tau^{1/2}} + (\lambda \tau)^{1/2} \right) + \exp(-\lambda Y) \operatorname{erfc} \left(\frac{Y}{2\tau^{1/2}} - (\lambda \tau)^{1/2} \right) \right] \quad (16)$$

Fig 2 illustrates a comparison between the numerical and analytical solutions for the steady state temperature distribution. It is seen that the agreement between the results are excellent. This has established confidence in the numerical results to be reported in this paper.

4 Results And Discussion

In the present work, the numerical solutions were conducted to investigate the influence of the thermophoresis coefficient k , the buoyancy ratio N , the Lewis number Le and the temperature ratio N_t . For all numerical calculations the Pandtl number and the suction parameter are assigned a value of 1.0 ($Pr = 1.0$, $\lambda = 1.0$).

The effect of suction parameter λ on steady state temperature distribution is shown in Fig. 2. It can be concluded that the temperature profiles decrease as the values of the suction parameter λ increases. This leads the thermal boundary layer thickness to decrease.

Figs. 3 show the effect of buoyancy ratio N on steady state velocity profiles for $B = 1.0$, $N_t = 50.0$, $k = 0.5$, $Le = 10.0$, $\lambda = 1.0$ and $Pr = 1.0$. Positive values of N indicate aiding flow. The Figure shows that as the buoyancy parameter

increased the velocity increased due to favorable slip velocities near vertical surfaces and concentration contribution in immigration of fluid particles from the vertical surfaces.

Fig. 4 display the effect of Lewis number Le on steady state concentration profiles for $B = 1.0$, $N_t = 50.0$, $k = 0.5$, $N = 10.0$, $\lambda = 1.0$ and $Pr = 1.0$. Increases in the Lewis number tends to increases the buoyancy-induced flow along the surface at the expense of reduced concentration and its boundary layer thickness.

Fig. 5 represents the time dependent thermopheretic deposition velocity for $B = 1.0$, $N_t = 50.0$, $k = 0.5$, $N = 10.0$, $\lambda = 1.0$, $Pr = 1.0$ and at different thermophoresis coefficient. The Figure shows that as the thermophoresis coefficient is increased the wall thermophoresis velocity is also increased; this is due to favorable temperature gradients. Also the figure shows that as the time is increased the thermophoresis velocity is decreased up to steady state conditions which is reached more fast for low values of thermophoresis coefficient.

Fig. 6 shows the time dependent thermophoresis deposition velocity values for $B = 1.0$, $k = 0.5$, $N = 10.0$, $\lambda = 1.0$, $Pr = 1.0$ and at different values of temperature ratio $N_t = 10, 20, 50, 100$. It is clear that the thermophoresis values are decreased when temperature ratios are increased; this is due to small temperature differences between vertical surface and free stream conditions. Clearly, this is why the steady state thermophoresis velocity need less time for higher values of temperature ration.

5 Conclusions

The unsteady heat and mass transfer mixed convection problem of a Newtonian fluid over an infinite vertical porous plate in the presence of thermophoresis particle deposition effect were studied. Based on the obtained results, the following conclusions can be reported.

1- It was found that the thermophoretic deposition velocity increased as the

thermophoresis constant k increased and as temperature ration N_t decreased.

2- The increase of Le number concentration boundary layer at steady state conditions decreases due to an.

3- The steady state values of thermophoretic deposition velocity reached faster as the thermophoresis constant decreased and the temperature ratio increased.

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6 Nomenclature

C	Fluid concentration
c_p	Specific heat capacity
D	Brownian diffusion coefficient
g	Gravitational acceleration
Gr	Grashof number
k	Thermophoresis coefficient
Le	Lewis number, α_m/D
N	Buoyancy ratio, $[\beta_c(C_w - C_\infty)/\beta_T(T_w - T_\infty)]$
N_t	Dimensionless temperature ratio, $T_\infty/[T_w - T_\infty]$
Pr	Prandtl number, ν/α
Ra	Local Rayleigh number, $Kg\beta(T_w - T_\infty)x/\nu\alpha$
T	Temperature
u, v	Velocity components in x - and y -directions
v_t	Thermophoresis velocity
v_{tw}	Thermophoresis velocity at wall
V_t	Dimensionless thermophoresis velocity, $v_t x / \alpha_m$
V_{tw}	Dimensionless thermophoresis velocity at wall
x, y	Axial and normal coordinates

Greek symbols:

- α Effective thermal diffusivity of the porous medium
- β_T Coefficient of thermal expansion,
 $(-1/\rho)(\partial\rho/\partial T)_p$
- β_C Coefficient of concentration expansion,
 $(-1/\rho)(\partial\rho/\partial C)_p$
- λ Suction parameter
- θ Dimensionless temperature
- Φ Dimensionless concentration
- μ Dynamic viscosity
- ν Kinematic viscosity
- ρ Fluid density

Subscripts

- w Surface conditions
- ∞ Free stream condition
- T Thermophoresis effects

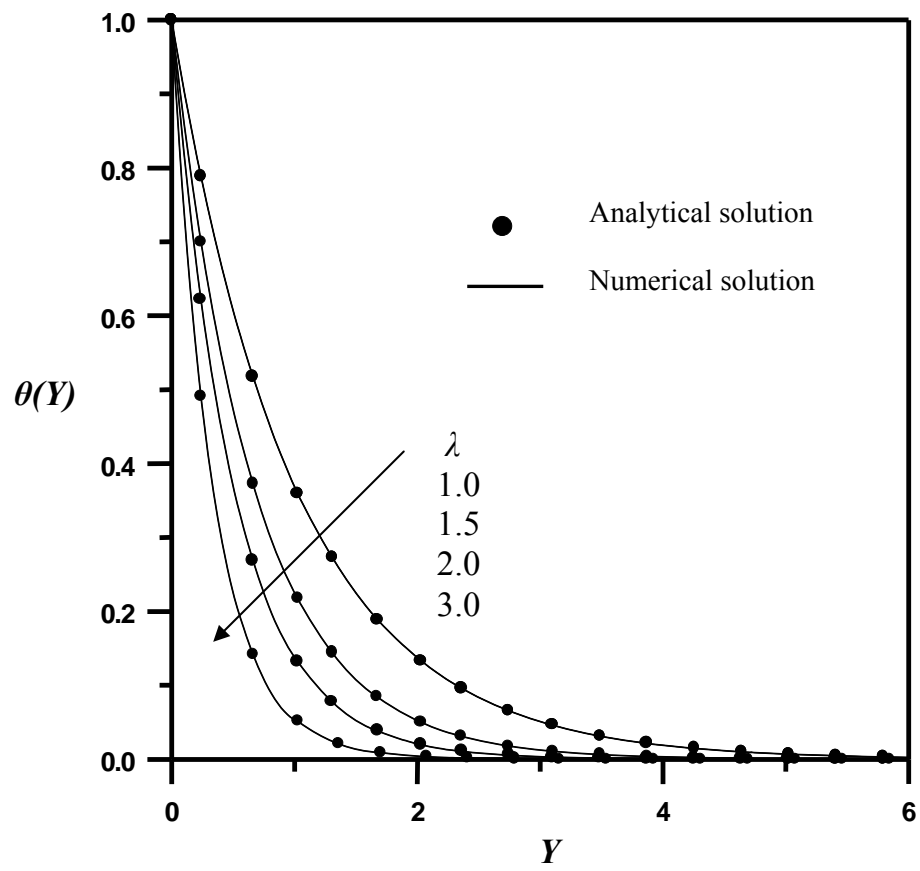


Fig. 2 Temperature distribution for different values of λ

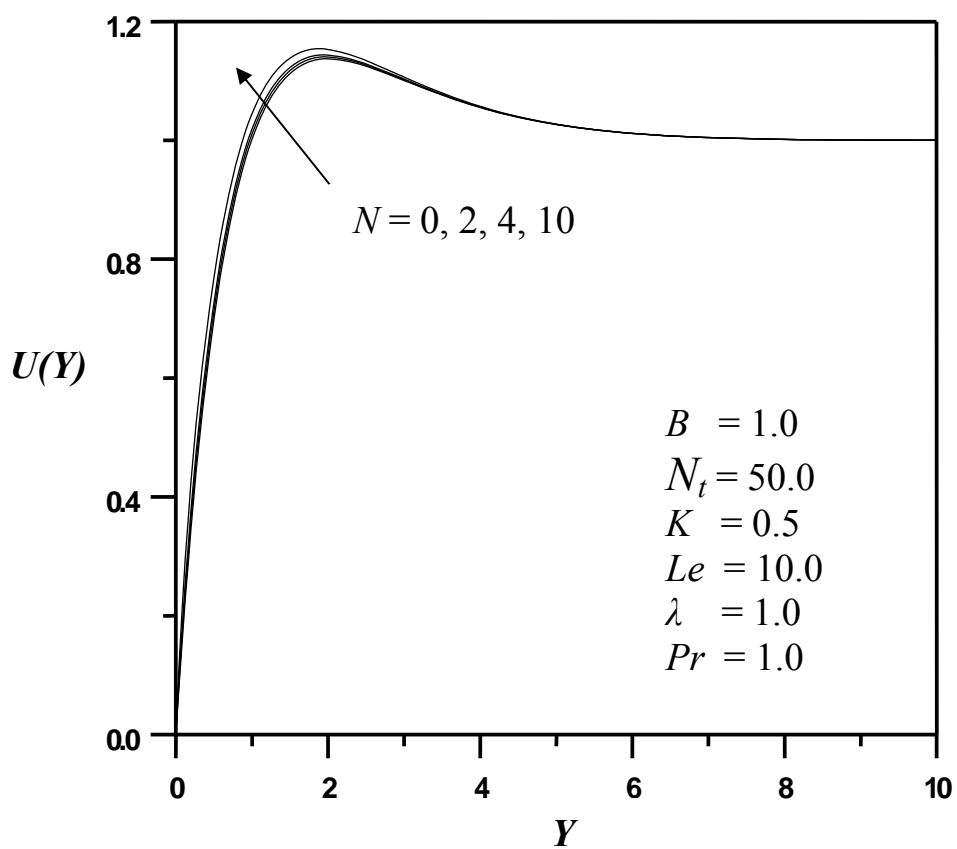


Fig. 3 Effect of buoyancy parameter on velocity distribution

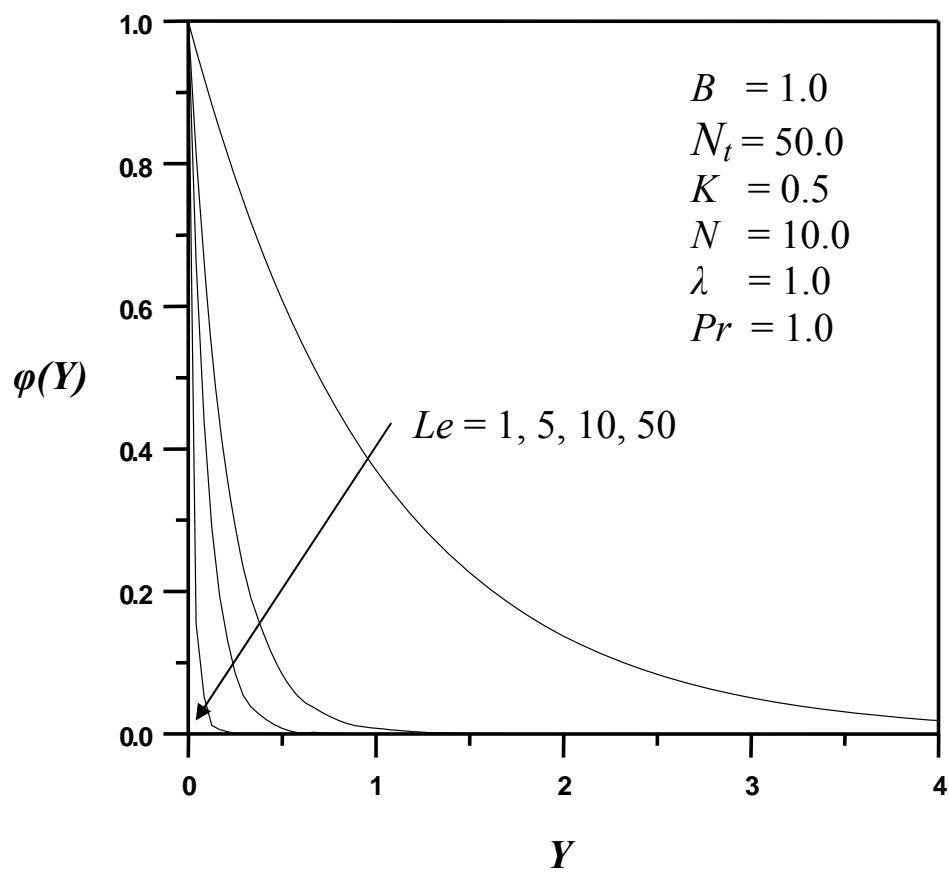


Fig. 4 Effect of Lewis number on concentration distribution

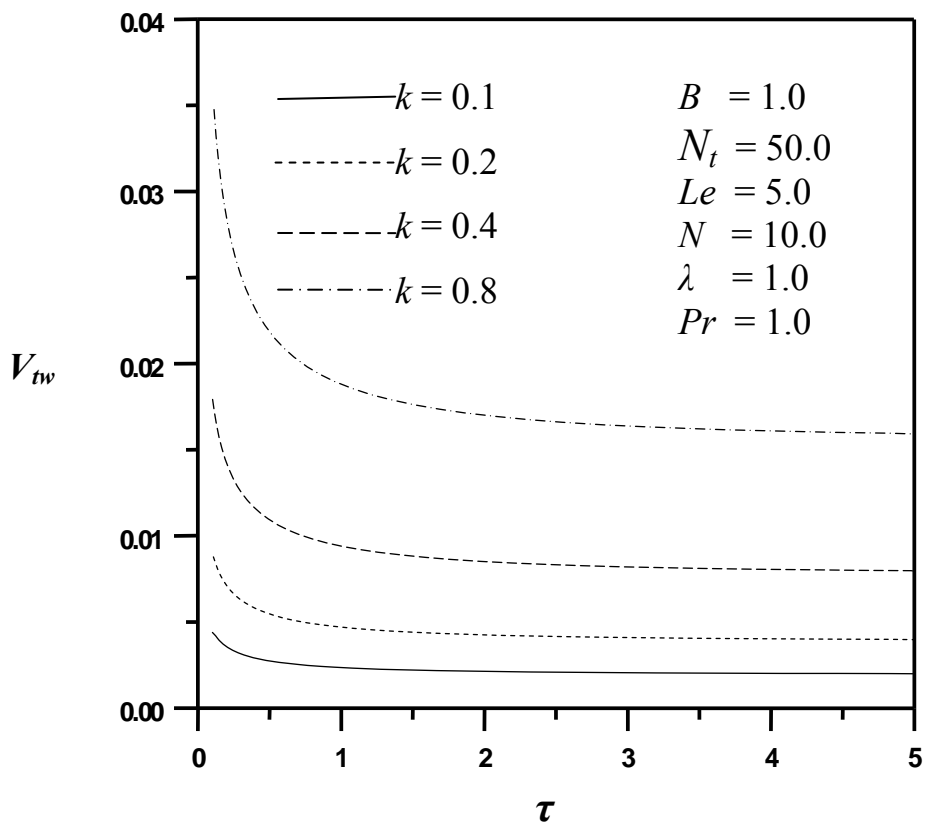


Fig. 5 Effect of thermophoresis coefficient on time dependent thermopheretic deposition velocity

