A Mathematical Approach to Estimate Air Rate in a Heating Furnace

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Abstract: - Heat transfer is extremely important in a wide range of materials processing techniques. Therefore, it is important to understand these flows and develop methods to minimize or control their effects. The transport in furnaces and ovens used for heat treatment strongly influence the quality of the product. However, quantitative information on the dependence of product quality, process control and optimization on the thermal transport is often unavailable.

Research in thermal materials processing is largely directed at the basic processes and underlying mechanisms, physical understanding, effects of different transport mechanisms and physical parameters, general behavior and characteristics, and the thermal process undergone by the material. It is usually a long-term effort, which leads to a better quantitative understanding of the process under consideration. However, it can also provide inputs, which can be used for design and development.

In order to develop an appropriate mathematical model for a given materials processing system, several idealizations and simplifications were made to make the problem amenable to an analytical solution. A general procedure, which includes considerations of transient versus steady-state transport, number of spatial dimensions needed, neglecting of relatively small effects, idealizations such as isothermal or uniform heat flux conditions, and characterization of material properties, may be adopted to obtain the usual simplifications in analysis. In addition, the simplifications and approximations that lead to a mathematical model also indicate the dominant variables in a problem. This helps in developing efficient physical or experimental models.

This paper focuses on the link between basic research on the underlying transport mechanisms and the engineering aspects associated with the process and the system. The results are focused on mathematical interpretation of the heat transfer processes that occur in industrial heating equipments. These are the bases for future researches regarding simulation and experimental models.

Key-words: - Air rate, Convection, Furnace, Heating, Mathematical model, Materials processing

NOMENCLATURE

a – the thermal diffusivity
c_p – specific heat at constant pressure
g - gravitational acceleration
g_n - gravitational acceleration towards the flowing direction
Gr - the Grashof number
Kc - consistency index for non-Newtonian fluid,
L – width of the boundary layer
Nu - the Nusselt number
p – the position pressure
Pr - the Prandtl number
Q_v - fluid volume debit
Re - the Reynolds number
t - temperature
t_0 – initial temperature
t_p – wall temperature
w - fluid flow velocity
w_x - fluid flow velocity on x direction
w_z - fluid flow velocity on z direction
w_{zmed} – fluid medium flow velocity on z direction
x, y, z - coordinate distances

GREEK SYMBOLS

α – convection coefficient
β - angle of flow
δ - thickness of the fluid film
λ- conductivity
τ – flow time
τ_{yx} – shear stress on plane y-x
ρ – fluid density
ν – kinematic viscosity
η – dynamic viscosity
$\Gamma\left(\frac{4}{3}\right)$ - Euler function of a second type.

1 Introduction

Materials processing is one of the most important and active areas of research in heat transfer today. It is also critical to use the fundamental understanding of materials processing in the design and optimization of the relevant systems. [1,2] Heat transfer is extremely important in a wide range of materials processing techniques. Therefore, it is important to understand these flows and develop methods to minimize or control their effects. The transport in furnaces and ovens used for heat treatment strongly influence the quality of the product. However, quantitative information on the dependence of product quality, process control and optimization on the thermal transport is often unavailable. [3]

Research in thermal materials processing is largely directed at the basic processes and underlying mechanisms, physical understanding, effects of different transport mechanisms and physical parameters, general behavior and characteristics, and the thermal process undergone by the material. It is usually a long-term effort, which leads to a better quantitative understanding of the process under consideration. [4, 5] However, it can also provide inputs, which can be used for design and development.

Some of the important considerations that arise when dealing with the thermal transport in the processing of materials make the mathematical and numerical modeling of the process and the associated system for materials processing very involved and challenging.

Special procedures and techniques are generally needed to satisfactorily simulate the relevant boundary conditions and material property variations, as a part of heat transfer application.

1.1 Thermal Processing of Materials

Thermal processing of materials refers to manufacturing and material fabrication techniques that are strongly dependent on the thermal transport mechanisms. With the substantial growth in new and advanced materials like composites, ceramics, different types of polymers and glass, coatings, specialized alloys and semiconductor materials, thermal processing has become particularly important since the properties and characteristics of the product, as well as the operation of the system, are largely determined by heat transfer mechanisms.

A few important materials processing techniques in which heat transfer plays a very important role are listed in Fig 1.

Fig. 1 Different types of thermal materials processing operations, along with examples of common techniques

1.	PROCESSES WITH PHASE CHANGE casting, continuous casting, crystal growing, drying
2.	HEAT TREATMENT annealing, hardening, tempering, surface treatment, curing, baking
3.	FORMING OPERATIONS hot rolling, wire drawing, metal forming, extrusion, forging
4.	CUTTING laser and gas cutting, fluid jet cutting, grinding, machining
5.	BONDING PROCESSES soldering, welding, explosive bonding, chemical bonding
6.	POLYMER PROCESSING extrusion, injection molding, thermoforming
7.	REACTIVE PROCESSING chemical vapor deposition, food processing
8.	POWDER PROCESSING powder metallurgy, sintering, sputtering, processing of nano-powders and ceramics
9.	GLASS PROCESSING optical fiber drawing, glass blowing, annealing
10.	COATING thermal spray coating, polymer coating
11.	OTHER PROCESSES composite materials processing, microgravity materials processing, rapid prototyping

This list contains both traditional processes and new or emerging methods. In the former category, it can include welding, metal forming, polymer extrusion, casting, heat treatment and drying.

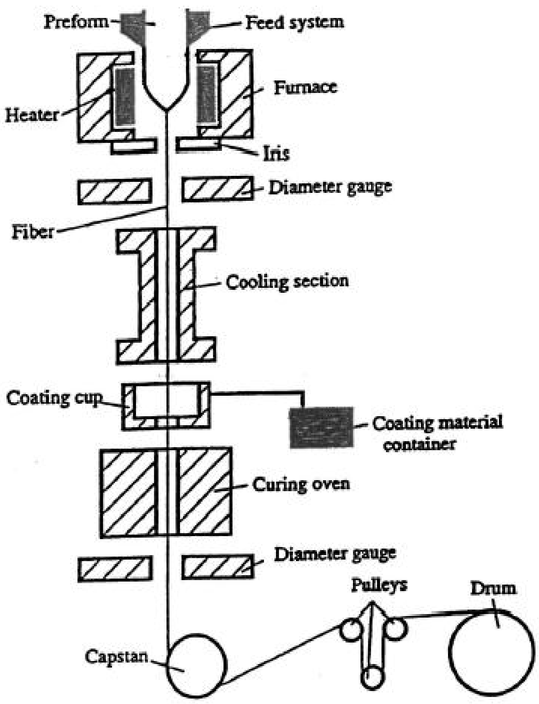
Similarly, in the latter category, it can include crystal growing, chemical vapor deposition and other thin film manufacturing techniques, thermal sprays, fabrication of composite materials, processing of nano-powders to fabricate system components, optical fiber drawing and coating, microgravity materials processing, laser machining and reactive extrusion. The choice of an appropriate material for a given application is an important consideration in the design and optimization of processes and systems [1].

A few thermal materials processing systems are also sketched in Fig. 2.

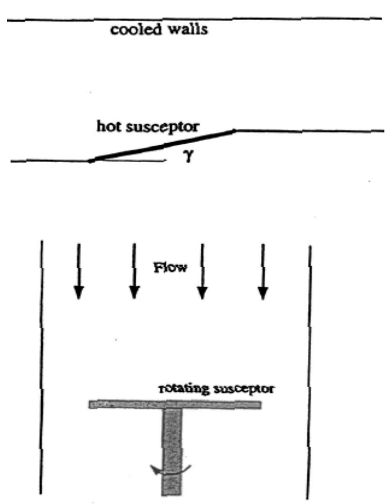
In all these processes, the quality and characteristics of the final product and the rate of fabrication are strong functions of the underlying thermal transport processes.

Many books and review articles have discussed important practical considerations and the fundamental transport mechanisms in the area of manufacturing and materials processing [2–24].

Fig. 2 Sketches of a few common manufacturing processes that involve thermal transport in the material being processed: a. optical fiber drawing; b. chemical vapor deposition.[24]



a.



b.

1.2 Basic Research Versus Engineering

Research in thermal materials processing is largely directed at the basic processes and underlying mechanisms, physical understanding, effects of different transport mechanisms and physical parameters, general behavior and characteristics, and the thermal process undergone by the material. It is usually a long-term effort, which leads to a better quantitative understanding of the process under consideration. However, it can also provide inputs, which can be used for design and development.

Engineering studies in materials processing, on the other hand, are concerned with the design of the process and the relevant thermal system, optimization, product development, system control, choice of operating conditions, improving product quality, reduction in costs, process feasibility, enhanced productivity, repeatability, and dependability.

Fig. 3 Various steps involved in the design and optimization of a thermal system and in the implementation of the design[24]

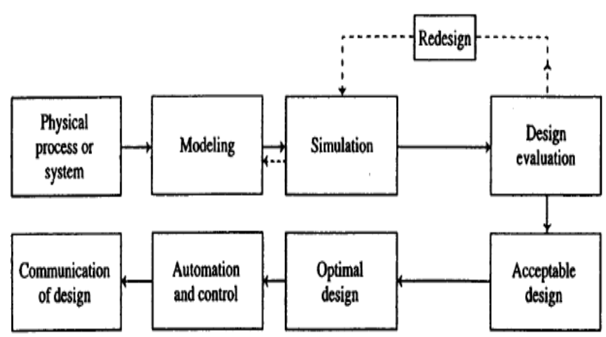


Figure 3 shows a schematic of the different steps that are typically involved in the design and optimization of a system. The iterative process to obtain an acceptable design by varying the design variables is indicated by the feedback loop connecting simulation, design evaluation and acceptable design. There is a feedback between simulation and modeling as well, in order to improve the model representation of the physical system on the basis of observed behavior and characteristics of the system, as obtained from simulation. Optimization of the system is undertaken after acceptable designs have been obtained.[22, 25, 26, 27]

Some of the important considerations that arise when dealing with the thermal transport in the processing of materials are given in Figure 4.

Figure 4. Important considerations in thermal materials processing[24]

1.	COUPLING OF TRANSPORT WITH MATERIAL CHARACTERISTICS	different materials, properties, behavior, material structure
2.	VARIABLE MATERIAL PROPERTIES	strong variation with temperature, pressure and concentration
3.	COMPLEX GEOMETRIES	complicated domains, multiple regions
4.	COMPLICATED BOUNDARY CONDITIONS	conjugate conditions, combined modes
5.	INTERACTION BETWEEN DIFFERENT MECHANISMS	surface tension, heat and mass transfer, chemical reactions, phase change
6.	MICRO-MACRO COUPLING	micro-structure changes, mechanisms operating at different length and time scales
7.	COMPLEX FLOWS	non-Newtonian flows, free surface flows, powder and particle transport
8.	INVERSE PROBLEMS	non-unique multiple solutions, iterative solution
9.	DIFFERENT ENERGY SOURCES	laser, chemical, electrical, gas, fluid jet, heat
10.	SYSTEM OPTIMIZATION AND CONTROL	link between heat transfer and manufacturing system

All these considerations make the mathematical modeling of the process and the associated system for materials processing very involved and challenging. Special procedures and techniques are generally needed to satisfactorily model the relevant boundary conditions and material property variations. The results from these provide inputs for the design and optimization of the relevant system, as well as for the choice of the appropriate operating conditions. Experimental techniques and results are also closely linked with the mathematical modeling in order to simplify the experiments and obtain useful results in terms of important dimensionless parameters.

It is necessary for heat transfer researchers to thoroughly understand the concerns, intricacies and basic considerations that characterize materials processing in order to make a significant impact on the field and to play a leadership role.[28, 29] The dependence of the characteristics of the final product on the heat transfer must be properly understood and characterized so that analysis or experimentation can be used to design processes to achieve desired product characteristics and production rates.[24, 30-32]

1.3 Mathematical Modeling

Modeling is one of the most crucial elements in the design and optimization of thermal materials processing systems. Practical processes and systems are generally very complicated and must be simplified through idealizations and approximations to make the problem solvable. This process of simplifying a given problem so that it may be represented in terms of a system of equations, for analysis, or a physical arrangement, for experimentation, is termed modeling. Once a model is obtained, it is subjected to a variety of operating conditions and design variations. If the model is a good representation of the actual system under consideration, the outputs obtained from the model characterize the behavior of the given system. This information is used in the design process as well as in obtaining and comparing alternative designs by predicting the performance of each design, ultimately leading to an optimal configuration of certain equipment.

A mathematical model is one that represents the performance and behavior of a given system in terms of mathematical equations.

Obtaining an accurate model is the extremely important in thermal systems design, since it provides flexibility in obtaining quantitative and qualitative results that are needed as inputs for design. A correct mathematical model stays at the base for simulation, which is a modern approach that optimizes the costs of designing a product. In addition, the simplifications and approximations that lead to a mathematical model also indicate the dominant variables in a problem. This helps in developing efficient physical models.

Numerical models are based on the mathematical model and allow one to obtain, using a computer, quantitative results on the system behavior for different operating conditions and design parameters.[33, 34]

1.4 Idealizations and Simplifications

Fluid technology plays a major role in industries, such as general mechanical engineering, aeronautical engineering, medical technology and process engineering. In many cases, fluid dynamics is a decisive factor in the optimization of a technology (such as in the reduction of flow resistance of a car, or improvement in the efficiency of a pump). During the last decade, computational fluid dynamics has steadily replaced the study of complex flow behavior by means of purely experimental methods.[33] There is a very important aspect that must be considered and that relates to the mathematical modeling of the overall thermal system, which usually consists of several components, since the process undergone by the material results from the energy exchange with the various components of the system.

Consider a typical electrical furnace, which consists of the heater, walls, insulation, enclosed gases and the material undergoing heat treatment. The transport mechanisms in all these components are coupled through the boundary conditions. Thus, the heater exchanges thermal energy with the walls, the gases and the material.[35, 36] Similarly, the material undergoing heat treatment is in energy exchange with the heater, the walls and the gases. The gas flow is driven by an externally imposed pressure difference, such as that due to a fan, by moving materials in continuous processing, and by buoyancy. Each individual component may first be mathematically modeled as uncoupled from the others, by employing prescribed boundary conditions. Then, these individual models can be combined, employing the appropriate coupling through the boundary conditions. This procedure provides a systematic approach to the mathematical modeling of the system, which may be a simple one or a complicated practical one. Once the simulation of the system is achieved the design and optimization of the process as well as of the system may be undertaken.[37-42]

An important aspect in the design of systems for the thermal processing of materials is the use of the available knowledge base on the process to guide the design and operation of the system. The knowledge base typically includes relevant information on existing systems and processes, current practice, knowledge of an expert in the particular area, material property data, and empirical data on equipment and transport, such as heat transfer correlations.[41, 43]

1.4.1 Boundary Conditions.

Many of the boundary and initial conditions used in materials processing are the usual no-slip conditions for velocity and the appropriate thermal or mass transfer conditions at the boundaries. Similarly, the normal gradients are taken as zero at an axis or plane of symmetry, temperature and heat flux continuity is maintained in going from one homogeneous region to another, and initial conditions are often taken as zero flow at the ambient temperature, representing the situation before the onset of the process. For periodic processes, the initial conditions are arbitrary.[15, 16-18, 29]

1.4.2 Other Simplifications

The basic nature of the underlying physical processes and the simplifications that may be obtained under various circumstances can be best understood in terms of dimensionless variables that arise when the governing equations and the boundary conditions are nondimensionalized. The commonly encountered governing dimensionless parameters are the Reynolds number Re, the Grashof number Gr, the Prandtl number Pr and the Nusselt number Nu. These are defined as:

$$Re = \frac{w_{zmed}\delta}{\nu}; \quad Nu = \frac{\alpha\delta}{\lambda}; \quad Pr = \frac{\eta c_p}{\lambda}; \eta = \nu\rho \qquad (1)$$

The dimensionless equations may be used to determine the various regimes over which certain simplifications can be made, such as creeping flow at small Re and boundary layer at large Re.[24, 25, 40, 42]

1.5 Material Considerations

The properties of the material undergoing thermal processing are very important in the modeling of the process, in the interpretation of experimental results and in the determination of the characteristics of the final product. The ranges of pressure, concentration and temperature are usually large enough to make it necessary to consider material property variations.

Usually, the dependence of the properties on temperature *T* is the most important effect. This leads to nonlinearity in the governing equations and couples the flow with the energy transport. Thus the solution of the equations and the interpretation of experimental results become more involved than for constant property circumstances. Average constant property values at different reference conditions are frequently employed to simplify the solution [25,26]. However, most manufacturing processes require the solution of the full variable-property problem for accurate predictions of the resulting transport.

Various models are employed to represent the viscous or rheological behavior of fluids of practical interest. Frequently, the fluid is treated as a Generalized Newtonian Fluid with the non-Newtonian viscosity function given in terms of the shear rate which is related to the second invariant of the rate of strain tensor.

For instance, time-independent viscoinelastic fluids without a yield stress are often represented by the power-law model, given by

$$\tau_{yx} = K_c \left|\frac{dw_x}{dy}\right|^{n-1} \frac{dw_x}{dy}$$

where K_c is the consistency index, and n the power law fluid index. Note that $n=1$ represents a Newtonian fluid. For $n<1$, the behavior is pseudoplastic (shear thinning) and for $n>1$, it is dilatant (shear thickening).

There are several other important considerations related to material properties, such as constraints on the temperature level in the material, as well as on the spatial and temporal gradients, for instance in the manufacturing of plastic-insulated wires [33].

Similarly, constraints arise due to thermal stresses in the material and are particularly critical for brittle materials such as glass and ceramics.

1.6 Analytical

Due to the complexity of the governing equations and the boundary conditions, analytical methods can be used in very few practical circumstances and numerical approaches are generally needed to obtain the solution. However, analytical solutions are very valuable since they provide results that can be used for validating the numerical model, physical insight into the basic mechanisms and expected trends, and results for limiting or asymptotic conditions.

2 Problem Formulation

The operation temperature for the heating processes at average temperatures is up to 600°C. In this context, the heat transfer prevails through radiation. In order to intensify the change of heat, one shall focus on the convection and on raising the air velocity inside the heating chamber.

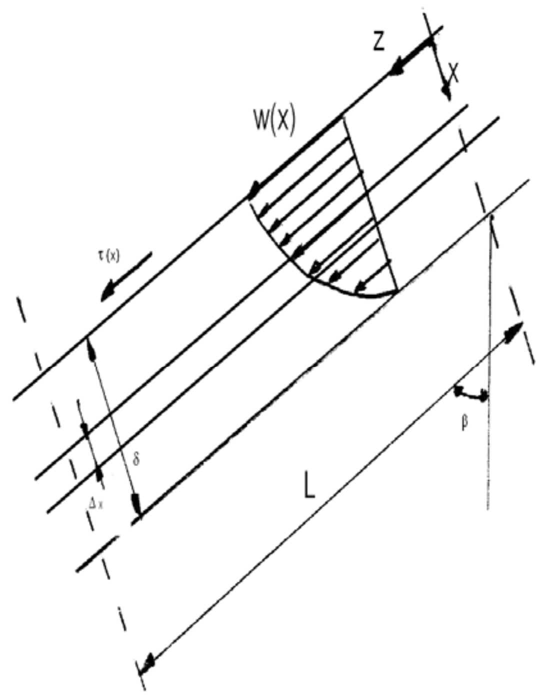
The heat losses that occur when changing the air circulation direction inside the working chamber shall not be taken into account in the first phase of modeling.

The basic conditions for the mathematical modeling are:

- the heating environment: air
- the thermal transfer type: radiation + free convection in laminar regime;
- the operation temperature: maximum 600°C.

The variation law of the average air velocity alongside the heated wall shall be obtained and the intervention paths over the heating processes intensification shall be established in the next phases, according to the results obtained following the preliminary experiments. For modeling it was considered a general case, illustrated in figure 5.

Figure 5. The movement of a fluid film alongside an inclined wall



The governing equations for convective heat transfer in materials processing are derived from the basic conservation principles for mass, momentum and energy. [9] For a pure viscous fluid, these equations may be written as

$$x\frac{\partial t}{\partial z} = B\frac{\partial^2 t}{\partial x^2}$$

with

$$B = \frac{a\nu}{g\delta} \qquad (1)$$

The boundary conditions for a short contact time shall be:

- $z = 0$ and $x > 0$ $\quad t = t_0$
- $x \to \infty$ $\quad$ z finite, $\quad t = t_0$ $\quad$ (17)
- $x = 0$ $\quad z > 0$ $\quad t = t_p$

In these conditions it can note:

$$T = \frac{t - t_0}{t_p - t_0};\quad \theta = \frac{x}{\sqrt[3]{9Bz}} \tag{2}$$

Equation becomes:

$$\frac{dT}{d\theta} = Ce^{-\theta^3},$$

with solution:

$$T = \frac{1}{\Gamma\left(\frac{4}{3}\right)} \int e^{-\theta^3} d\theta \tag{3}$$

where:

$$\Gamma\left(\frac{4}{3}\right) = \int_0^\infty t^{1/3} e^{-t} dt \tag{4}$$

is Euler function.

Solving this equation is rather hard, so Nusselt relations had been used:

$$Nu = 0.0942\, Re\, \Pr\left(\frac{\delta}{L}\right) + 1.88 \tag{4}$$

where L is furnace principal dimension.

3 Problem Solution

In order to develop an appropriate mathematical model for a given materials processing system, several idealizations and simplifications were made to make the problem amenable to an analytical solution. A general procedure, which includes considerations of transient versus steady-state transport, number of spatial dimensions needed, neglecting of relatively small effects, idealizations such as isothermal or uniform heat flux conditions, and characterization of material properties, may be adopted to obtain the usual simplifications in analysis. In addition, the simplifications and approximations that lead to a mathematical model also indicate the dominant variables in a problem. This helps in developing efficient physical or experimental models. So, the results are demonstrated below.

Using H. Brauer relations and eq. (4), it get:

$$w_{zmed} = \frac{1}{\delta}\int_0^\delta w_z dx = \frac{g\delta^2 \cos\beta}{3\nu}$$

However, taking into account the most used criteria's from equations (1) it get:

$$\frac{\alpha\delta}{\lambda} = 0{,}0942 \frac{w_{zmed}\delta}{\nu} \frac{\eta c_p}{\lambda} \frac{\delta}{L} + 1{,}88 \tag{6}$$

Considering:

$$a = \frac{\lambda}{\rho c_p} \tag{7}$$

it get:

$$\frac{\alpha\delta}{\lambda} = 0{,}0942 \frac{w_{zmed}\delta^2}{La} + 1{,}88 \tag{94}$$

The flow rate that passes through the boundary layer, having the width *L*, is (Eq. 8):

$$Q_v = L\delta w_{zmed} = \frac{gL\delta^3 \cos\beta}{3\nu} \tag{8}$$

And, making the replacement:

$$\frac{\alpha\delta}{\lambda} = 0{,}0942 \frac{Q_v\delta}{L^2 a} + 1{,}88$$
$$\Rightarrow \delta\left(\frac{\alpha}{\lambda} - 0{,}0942\frac{Q_v}{L^2 a}\right) = 1{,}88 \tag{7}$$

Finally for medium air speed:

$$w_{zmed} = \frac{1{,}88^2 g}{3\nu\left(\frac{\alpha}{\lambda} - 0{,}0942\frac{Q_v}{L^2 a}\right)^2} \tag{8}$$

Making elementary calculus:

$$w_{zmed} = \frac{11{,}55\lambda^2 L^4 a^2}{\nu\left(aL^2\alpha - 0{,}0942\lambda Q_v\right)^2} \tag{9}$$

This final equation represents a calculus formula for medium air rate inside the heated furnace.

This equation is very important for further development in designing heating equipments. It leads to a better understanding of air rate inside the working chamber and it can ease the simulation model.

At a quick view it can observe the influence of the working chamber geometry and of the heating temperature. At this point it has to underline the importance of the equipment design in heat transfer rate. There are many studies that concludes that oval furnaces are more energy efficient than the classic box furnaces. The problem with the oval inner space is the design of the curvature of the lateral walls.[25-29, 37, 44, 45]

Here it can be pointed that obtaining a mathematical model cannot be the end. It has to be the base of the research. Further studies regarding simulation of heating and experimental work are needed.

4 Conclusion

Several important considerations in the design and operation of practical thermal materials processing systems were discussed in the preceding sections. These included issues like rate of fabrication, quality of the product, and feasibility of the process. However, there are obviously many other aspects that need to be considered in the design and optimization of the system and for the selection of the operating conditions. Some of these are outlined here.

An important consideration in industrial heating is the air flow inside the working chamber since it determines the homogeneity of the heating process. The downstream motion of fluid particles may be considered for a better understanding of the heat transfer process.

Similarly, in other thermal materials processing systems, important aspects that are particularly relevant to the process under consideration arise and must be taken into account by the simulation and experimentation in order to provide the appropriate inputs for system design and optimization. These relate to engineering issues like durability, maintenance, availability of different materials and components, and the convenience and practical range of operating conditions.[24, 27-30, 46, 47]

A mathematical model is one that represents the performance and behavior of a given system in terms of mathematical equations. These models are the most important ones in the design of thermal systems, since they provide considerable flexibility and versatility in obtaining quantitative results that are needed as inputs for design. Mathematical models form the basis for simulation, so that the behavior and characteristics of the system may be investigated without actually fabricating a prototype. In addition, the simplifications and approximations that lead to a mathematical model also indicate the dominant variables in a problem. This helps in developing efficient physical or experimental models.

This paper focuses on the link between basic research on the underlying transport mechanisms and the engineering aspects associated with the process and the system. The understanding of thermal processing of materials has grown significantly over the last three decades. Many new and improved techniques have been developed, along with new materials, new processing systems and better control on product quality and production costs.

Also, experimental results are strongly needed for validation of models and for providing inputs and insight for future model development.

As a general conclusion it may say that further researches focused on different chamber geometries are needed. This will help design a more energy efficient heating equipment and a more process flexible one, starting with an oval geometry with different curvatures angles.

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