Research on static and dynamic heat balance of the human body

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Abstract: This paper attempts to give a brief summary of the concept, objectives and research methodology of comfort theory and, in particular, thermal comfort for international building service engineers. It also provides an overview of the findings of international research and the goals of ongoing Hungarian projects (mainly carried out at the Department of Building Service Engineering and Process Engineering at the Budapest University of Technology and Economics). Moreover, the importance of comfort theory and its application by engineers will be highlighted. We intend to present the basic equations of the classical comfort theory, the equations of the human body's static heat balance, and the so called comfort equation as well as its sensitivity depending on the thermic parameters of the microclimate. The PMV expression describing the thermal comfort of the human body and its sensitivity, derivatives and cross-derivatives is emphasized, too. The concept of dynamic thermal sensation and the possibilities of its research is going to be introduced, which is possible only through the presentation of system theory, system dynamics and regulation theory.

Keywords: Thermal Comfort, Static and Dynamic Heat Balance, Thermal Sensation, Human Body.

1 INTRODUCTION

The architectural and building engineering design of spaces suitable for human occupation and work aims to meet the so-called comfort requirements optimally.
Human comfort requirements cover the air quality of the occupied space and its thermal attributes in addition to other ergonomic, sociological and psychological criteria, they we naming parameters of microclimate.
According to the research approach used in the last two decades (comfort theory, thermal comfort – research) the analysis of the joint impact of microclimate parameters can be based on the energy balance of an active or sedentary human who has a mechanical or thermal connection with his/her environment.

2 CLASSIFYING AND MEASURING THERMAL COMFORT

Human response to thermal environment and the satisfaction with thermal comfort are expressed by subjective thermal sensation. Thermal comfort is linked to the heat balance of the human body: the time and the adaptation reactions required to achieve this balance and whether it is agreeable for the given person and what skin temperature and perspiration are measured.
ASHRAE expresses subjective thermal sensation using the thermal sensation scale:

+3 (hot), +2 (warm), +1 (slightly warm), 0 (neutral), -1 (slightly cool), -2 (cool), -3 (cold)

The agreeable thermal sensation falls between –1 and +1 on the scale. In an experiment conducted with several thousand live subjects the test persons had to declare (after a relaxation interval) how they felt in the examined microclimate and what value they voted for on the scale.
P. O. Fanger based the investigation of thermal sensation on writing up and calculating the energy balance of the human body and established a mathematical connection between the solutions of the energy balance and the thermal sensation scale. [1]

If the thermal impacts of the environment change, a human conducting a certain activity in a given thermal environment will try to set off the balance of heat production and heat loss by increasing or reducing heat loss in a transient process. Fanger examined only the static heat balance of the human body, after the heat equilibrium caused by the transient process set in.
In our papers we demonstrate the research the heat balance of human body.

3 STATIC THERMAL COMFORT EQUATION

Fangers experiments have shown that the thermal sensation (thermal comfort) of an active person is deemed agreeable (optimal) if the following criteria are met:

-The static energy balance describing the thermal and mechanical connection between the human and his environment is zero, and the resultant of the heat produced and lost as well as the performed work is zero,

-Human skin temperature remains within a narrow range,

-Perspiration remains within a given range.

This was expressed by Fanger by the following inequalities:

$$a < t_s < b\,,\ c < E_{sw} < d\,. \tag{1}$$

According to the experiments skin temperature in the state of agreeable thermal comfort only depends on metabolism and heat loss through perspiration is similarly linked to metabolic heat in a defined statistical connection.
Fanger found that skin temperature in the state of agreeable thermal comfort has the following regressive relationship with metabolic heat.

$$\bar{t}_s = 35{,}7 - 0{,}032\frac{M}{A_{Du}} \quad [{}^oC]\,. \tag{2}$$

Fanger also discovered that the regressive relationship between heat loss by perspiration and metabolic heat in the state of agreeable thermal comfort is as follows:

$$Q_{sw} = 0{,}42\left[\frac{M}{A_{Du}}(1-\eta) - 58{,}15\right] \quad [\mathrm{W}]. \tag{3}$$

The comfort equation describing the heat balance of the human body in the state of thermal comfort is the following:

$$\begin{aligned}
&\frac{M}{A_{Du}}(1-\eta) - \\
&-3{,}05.10^{-3}\left[5733 - 6{,}99\cdot\frac{M}{A_{Du}}(1-\eta) - p_a\right] \\
&-0{,}42\left[\frac{M}{A_{Du}}(1-\eta) - 58{,}15\right] - \\
&-1{,}7.10^{-5}\frac{M}{A_{Du}}(5867 - p_a) - 0{,}0014\frac{M}{A_{Du}}(34 - t_a) = \\
&= 3{,}96.10^{-8} f_{cl}\left[(t_{cl}+273)^4 - (t_{mrt}+273)^4\right] + \\
&+ f_{cl}h_c(t_{cl} - t_a).
\end{aligned} \tag{4}$$

This equation is discussed in detail by book [1] and [2] listed in the References.

4 CO-DEPENDENCY OF COMFORT PARAMETERS

Fanger processed his findings about the dependence of the solution of comfort equation and the factors influencing the thermal comfort (with using the pertaining derivatives) in diagrams [1]. Using the comfort equation derivatives describing the co-dependency of the activity level, the clothing and the climate parameters are obtained. Derivatives express the change that is produced by the unit change on the variable in the numerator while the value of comfort equation remains zero, witch we are present in follows:

Change in t_a according to t_{mrt} (if t_{cl} is constant):

$$\frac{\partial t_a}{\partial t_{mrt}} = -\frac{3.168\times10^{-7} f_{cl}\left(t_{mrt}+273\right)^3}{2 f_{cl}h_c + 0{,}0014\frac{M}{A_{Du}}}\,. \tag{5}$$

Change in t_a according to p_a (if t_{cl} is constant):

$$\frac{\partial t_a}{\partial p_a} = \frac{-0{,}00305 - 0{,}000017\frac{M}{A_{Du}}}{2 f_{cl}h_c + 0{,}0014\frac{M}{A_{Du}}}\,. \tag{6}$$

Change in t_a according to t_{mrt} (if t_{cl} is not constant):

$$\frac{\partial t_a}{\partial t_{mrt}} = \frac{4C_4 t_{mrt}^{\ 3}}{1 - A_4\frac{M}{A_{Du}} I_{cl}\left(C_5 - 3C_4 t_{cl}^{\ 3}\right)}\,. \tag{7}$$

Change in t_a according to p_a (if t_{cl} is not constant):

$$\frac{\partial t}{\partial p_a} = \frac{C_1 + C_2 \frac{M}{A_{Du}} - C_5 + A_3 I_{cl} \frac{M}{A_{Du}}}{1 - C_3 \frac{M}{A_{Du}} + 4C_4 t^3 + C_5}. \tag{8}$$

Change in t_a according to v (if t_{cl} is not constant):

$$\frac{\partial t}{\partial v} = \frac{t - t_{cl}}{1 + C_3 \frac{M}{A_{Du}} + 4C_4 t^3 + C_5}. \tag{9}$$

Change in t_a according to M / A_{Du} (if t_{cl} is not constant):

$$\frac{\partial t}{\partial \frac{M}{A_{Du}}} = \frac{C_2 p_a + C_3 t - C_0}{1 + 4C_4 t_{cl}{}^3 X + 4C_4 t^3 - C_5 X + C_5}, \tag{10}$$

where:

$$A_1 = \frac{-35.7 - 41.9 I_{cl} + 3.05 \cdot 10^{-3} I_{cl} p_a}{-0.6 I_{cl}(1-\eta) - 0.028(1-\eta) + 0.15 I_{cl} - 1.7 \cdot 10^{-5} p_a I_{cl} - 0.0014 t_a I_{cl}},$$

$$A_2 = \frac{-35.7 + 0.028 \frac{M}{A_{Du}}(1-\eta) - 17.5 I_{cl} + 3.05 \cdot 10^{-3} I_{cl} p_a + 24.4 I_{cl}}{-0.6(1-\eta) + 0.15 - 1.7 \cdot 10^{-5} p_a - 0.0014 t_a},$$

$$A_3 = -58823.5\Big(-35.7 + 6.92 I_{cl} - 0.15 I_{cl} \frac{M}{A_{Du}} + 3.05 \cdot 10^{-3} I_{cl} p_a + $$
$$+0.0014 I_{cl} \frac{M}{A_{Du}} t_a + 0.028 \frac{M}{A_{Du}}(1-\eta) + 0.6 I_{cl} \frac{M}{A_{Du}}(1-\eta)\Big),$$

$$A_4 = -714.286\Big(-35.7 + 6.92 I_{cl} - 0.15 I_{cl} \frac{M}{A_{Du}} + 3.033 \cdot 10^{-3} I_{cl} p_a + $$
$$+0.028 \frac{M}{A_{Du}}(1-\eta) + 0.6 I_{cl} \frac{M}{A_{Du}}(1-\eta)\Big),$$

$$C_0 = \frac{-6.94 - 0.305 \cdot 10^{-3} p_a - 2 f_{cl} h_c t_a + 6.5 f_{cl} t_{cl} + 2 f_{cl} h_c t_{cl}}{0.45 + 1.7 \cdot 10^{-5} p_a + 0.0014 t_a - 0.6\eta} +$$
$$+ \frac{0.035 f_{cl} t_{cl}{}^2 + 8.65 \cdot 10^{-5} f_{cl} t_{cl}{}^3 + 7.92 \cdot 10^{-8} f_{cl} t_{cl}{}^4}{0.45 + 1.7 \cdot 10^{-5} p_a + 0.0014 t_a - 0.6\eta} -$$
$$- \frac{6.45 f_{cl} t_{mrt} - 0.035 f_{cl} t_{mrt}{}^2 - 8.65 \cdot 10^{-5} f_{cl} t_{mrt}{}^3 - 7.92 \cdot 10^{-8} f_{cl} t_{mrt}{}^4}{0.45 + 1.7 \cdot 10^{-5} p_a + 0.0014 t_a - 0.6\eta},$$

$$C_1 = \frac{-6.94 - 0.45 \frac{M}{A_{Du}} - 2 f_{cl} h_c t_a - 0.0014 \frac{M}{A_{Du}} ta + 6.5 f_{cl} t_{cl} + 2 f_{cl} h_c t_{cl}}{3.05 \cdot 10^{-3} + 1.7 \cdot 10^{-5} \frac{M}{A_{Du}}} +$$
$$+ \frac{0.035 f_{cl} t_{cl}{}^2 + 8.6 \cdot 10^{-5} f_{cl} t_{cl}{}^3 + 7.92 \cdot 10^{-8} f_{cl} t_{cl}{}^4}{3.05 \cdot 10^{-3} + 1.7 \cdot 10^{-5} \frac{M}{A_{Du}}} -$$
$$- \frac{6.45 f_{cl} t_{mrt} - 0.035 f_{cl} t_{mrt}{}^2 - 8.6 f_{cl} t_{mrt}{}^3 - 7.92 \cdot 10^{-8} f_{cl} t_{mrt}{}^4 + 0.6 \frac{M}{A_{Du}} \eta}{3.05 \cdot 10^{-3} + 1.7 \cdot 10^{-5} \frac{M}{A_{Du}}},$$

$$C_2 = \frac{-6.94 - 0.305 \cdot 10^{-3} p_a - 2 f_{cl} h_c t_a + 6.5 f_{cl} t_{cl} + 2 f_{cl} h_c t_{cl}}{1.7 \cdot 10^{-5}} +$$
$$+ \frac{0.035 f_{cl} t_{cl}{}^2 + 8.6 \cdot 10^{-5} f_{cl} t_{cl}{}^3 + 7.92 \cdot 10^{-8} f_{cl} t_{cl}{}^4}{1.7 \cdot 10^{-5}} -$$
$$- \frac{6.45 f_{cl} t_{mrt} - 0.035 f_{cl} t_{mrt}{}^2 - 8.6 \cdot 10^{-5} f_{cl} t_{mrt}{}^3 - 7.92 \cdot 10^{-8} f_{cl} t_{mrt}{}^4 -}{1.7 \cdot 10^{-5}} -$$
$$- \frac{0.45 \frac{M}{A_{Du}} - 0.0014 \frac{M}{A_{Du}} t_a + 0.6 \frac{M}{A_{Du}} \eta}{1.7 \cdot 10^{-5}},$$

$$C_3 = \frac{-6.94 - 0.305 \cdot 10^{-3} p_a - 2 f_{cl} h_c t_a + 6.5 f_{cl} t_{cl} + 2 f_{cl} h_c t_{cl}}{0.0014} +$$
$$+ \frac{0.035 f_{cl} t_{cl}{}^2 + 8.6 \cdot 10^{-5} f_{cl} t_{cl}{}^3 + 7.92 \cdot 10^{-8} f_{cl} t_{cl}{}^4}{0.0014} -$$
$$- \frac{6.45 f_{cl} t_{mrt} - 0.035 f_{cl} t_{mrt}{}^2 - 8.6 \cdot 10^{-5} f_{cl} t_{mrt}{}^3 - 7.92 \cdot 10^{-8} f_{cl} t_{mrt}{}^4 -}{0.0014} -$$
$$- \frac{0.45 \frac{M}{A_{Du}} - 1.7 \cdot 10^{-5} \frac{M}{A_{Du}} p_a + 0.6 \frac{M}{A_{Du}} \eta}{0.0014},$$

$$C_4 = \frac{\frac{M}{A_{Du}}(1-\eta) - 3.05 * 10^{-3}\left[5733 - 6.99 \frac{M}{A_{Du}}(1-\eta) - p_a\right]}{3.96 \cdot 10^{-8} f_{cl}} -$$
$$- \frac{0.42\left[\frac{M}{A_{Du}}(1-\eta) - 58.18\right] - 1.7 \cdot 10^{-5} \frac{M}{A_{Du}}(5867 - p_a)}{3.96 \cdot 10^{-8} f_{cl}} -$$
$$- \frac{0.0014 \frac{M}{A_{Du}}(34 - t_a) - 3.96 \cdot 10^{-8} f_{cl} \cdot 273^4 + 3.96 \cdot 10^{-8} f_{cl} t_{mrt}{}^4}{3.96 \cdot 10^{-8} f_{cl}} +$$
$$+ \frac{3.96 \cdot 10^{-8} f_{cl} 273^4 f_{cl} h_c t_{cl} + f_{cl} h_c t_a}{3.96 \cdot 10^{-8} f_{cl}},$$

$$C_5 = \frac{-6.94 - 0.45 \frac{M}{A_{Du}} - 0.305 \cdot 10^{-3} p_a - 1.7 \cdot 10^{-5} \frac{M}{A_{Du}} p_a + 6.5 f_{cl} t_{cl} + 2 f_{cl} h_c t_{cl}}{2 f_{cl} hc + 0.0014 \frac{M}{A_{Du}}} +$$
$$+ \frac{0.035 f_{cl} t_{cl}{}^2 + 8.6 \cdot 10^{-5} f_{cl} t_{cl}{}^3 + 7.92 \cdot 10^{-8} f_{cl} t_{cl}{}^4}{2 f_{cl} hc + 0.0014 \frac{M}{A_{Du}}} -$$
$$- \frac{6.45 f_{cl} t_{mrt} - 0.035 f_{cl} t_{mrt}{}^2 - 8.6 \cdot 10^{-5} f_{cl} t_{mrt}{}^3 - 7.92 \cdot 10^{-8} f_{cl} t_{mrt}{}^4 + 0.6 \frac{M}{A_{Du}} \eta}{2 f_{cl} hc + 0.0014 \frac{M}{A_{Du}}},$$

$$X = -A_1 - A_2 I_{cl} - A_3 I_{cl} p_a + A_4 I_{cl} t + A_4 \frac{M}{A_{Du}} I_{cl}.$$

5 PMV THE COMPLEX MEASUREMENT VALUE OF THERMAL COMFORT

5.1 Theoretical and empirical fundamentals of PMV

Fanger introduced the term thermal load of the human body which is calculated by differing from 0 in the comfort equation as follows [1], [2]:

$$L = \frac{M}{A_{Du}}(1-\eta) - 3{,}05.10^{-3}\left[5733 - 6{,}99\frac{M}{A_{Du}}(1-\eta) - p_a\right] - 0{,}42\left[\frac{M}{A_{Du}}(1-\eta) - 58{,}15\right] - 1{,}7.10^{-5}\frac{M}{A_{Du}}(5867 - p_a) - 0{,}0014\frac{M}{A_{Du}}(34 - t_a) - 3{,}96.10^{-8} f_{cl}\left[(t_{cl}+273)^4 - (t_{mrt}+273)^4\right] + f_{cl} h_c (t_{cl} - t_a). \tag{11}$$

Conducting masses of experiments and using the live subjects' votes Fanger looked for a mathematically defined function between the ASHRAE scale [1] and thermal load L on an empirical basis.

Fanger found the following regressive function between the ASHRAE scale and the L values of thermal load:

$$Y = PMV = \left(0{,}303.e^{-0{,}036\frac{M}{A_{Du}}} + 0{,}028\right).\left\{\frac{M}{A_{Du}}(1-\eta) - 3{,}05.10^{-3} * \left[5733 - 6{,}99\frac{M}{A_{Du}}(1-\eta) - p_a\right] - 0{,}42\left[\frac{M}{A_{Du}}(1-\eta) - 58{,}15\right] - 1{,}7.10^{-5}\frac{M}{A_{Du}}(5867 - p_a) - 0{,}0014\frac{M}{A_{Du}}(34 - t_a) - 3{,}96.10^{-8} f_{cl}\left[(t_{cl}+273)^4 - (t_{mrt}+273)^4\right] + f_{cl} h_c (t_{cl} - t_a)\right\}. \tag{12}$$

Thermal comfort indicator Y calculated with the above equation was called PMV, the predicted mean vote by Fanger [1], [8].

The physically reasonable solutions of the thermal comfort indicator fall between -3 and +3. PMV can be used in two ways:

- PMV is calculated for different activity levels, various environmental and clothing parameters to classify our environment and thermal sensation: between -1 and +1 thermal comfort is described as slightly cool, neutral and slightly warm.
- the inverse of the above: environmental microclimate parameters are defined for the activity level and clothing linked to a specified PMV or PMV range (practically -1, +1 range) to meet the criteria of the given PMV.

The above tasks can be carried out using mathematical operations on equation (9).

To perform the tasks the derivatives of the PMV function has been produced in function of the different microclimate parameters and I_{cl} characterizing the activity level and clothing. Derivatives also showed the sensitivity of PMV in function of the various factors and in case of slight changes provided us with answers about how to compensate for the impact of a given factor by modifying the value of another factor.

5.2 Sensitivity and derivatives of PMV

The (PMV) thermal sensation indicator is expressed by its derivatives. Fanger produced these derivatives numerically and introduced them with help of diagrams. The following presented the explicit mathematical functions of the derivatives; of witch we are present in follows:

Change in PMV according to the outside temperature (if t_{cl} is constant):

$$\frac{\partial Y}{\partial t_a} = \left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}} + 0{,}028\right)\left(0{,}0014\frac{M}{A_{Du}} + f_{cl}.h_c\right). \tag{13}$$

Change in PMV according to the partial water vapour pressure (if t_{cl} is constant):

$$\frac{\partial Y}{\partial p_a} = \left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}} + 0{,}028\right)\left(0{,}00305 + 1{,}7.10^{-5}\frac{M}{A_{Du}}\right). \tag{14}$$

Change in PMV according to M / A_{Du} (if t_{cl} is constant):

$$\frac{\partial Y}{\partial \frac{M}{A_{Du}}} =$$

$$\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right)\cdot\left(1-1{,}7\cdot10^{5}\left(5867-p_a\right)-\right.$$
$$-0{,}0014\left(34-t_a\right)-0{,}39865\left(1-\eta\right)-\eta\big)-0{,}010908e^{-0{,}036\frac{M}{A_{Du}}}\cdot$$
$$\left(-1{,}7\cdot10^{-5}\frac{M}{A_{Du}}\left(5867-p_a\right)-0{,}0014-\frac{M}{A_{Du}}\left(34-t_a\right)-\right.$$
$$-f_{cl}h_c\left(t_{cl}-t_a\right)-3{,}9\cdot10^{-8}f_{cl}\left(\left(273+t_{cl}\right)^4-\left(273+t_{mrt}\right)^4\right)-$$
$$-0{,}00305\left(5733-p_a-6{,}99\frac{M}{A_{Du}}\left(1-\eta\right)\right)-$$
$$\left.-0{,}42\left(\frac{M}{A_{Du}}\left(1-\eta\right)-58{,}15\right)+\frac{M}{A_{Du}}\left(1-\eta\right)\right). \tag{15}$$

Change in PMV according to the mean radiant temperature (if t_{cl} is constant):

$$\frac{\partial Y}{\partial \bar{t}_{mrt}}=1{,}584\cdot10^{-7}\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right)f_{cl}\left(273+t_{mrt}\right)^3. \tag{16}$$

Change in PMV according to h_c (if t_{cl} is constant):

$$\frac{\partial Y}{\partial h_c}=-\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right).f_{cl}\left(t_{cl}-t_a\right). \tag{17}$$

Change in PMV according to air velocity (if t_{cl} is constant):

$$\frac{\partial Y}{\partial v}=-\frac{6{,}05\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right)f_{cl}\left(t_c-t_a\right)}{\sqrt{v_a}}. \tag{18}$$

Change in PMV according to h_c (if t_{cl} is not constant):

$$\frac{\partial Y}{\partial h_c}=\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right)*$$
$$\left(\frac{1{,}584\cdot10^{-7}f_{cl}^{\,2}\cdot\left(t_{cl}+273\right)^3I_{cl}\left(t_{cl}-t_a\right)}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}-f_{cl}\left(t_{cl}+t_a\right)\right)-$$
$$f_{cl}h_c\left(-\frac{I_{cl}f_{cl}\left(t_{cl}-ta\right)}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}\right). \tag{19}$$

Change in PMV according to the ambient temperature (if t_{cl} is not constant):

$$\frac{\partial Y}{\partial t_a}=$$
$$\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right)\cdot$$
$$\left(\frac{1{,}584\cdot10^{-7}f_{cl}^{\,2}\cdot\left(t_{cl}+273\right)^3I_{cl}t_{cl}h_c}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}-f_{cl}h_c+\right.$$
$$\left.f_{cl}h_c\left(-\frac{I_{cl}f_{cl}\left(t_{cl}-ta\right)}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}\right)\right). \tag{20}$$

Change in PMV according to the mean radiant temperature (if t_{cl} is not constant):

$$\frac{\partial Y}{\partial t_{mrt}}=$$
$$\left(0{,}303e^{-0{,}036\frac{M}{A_{Du}}}+0{,}028\right)*$$
$$\left(\frac{2{,}509056\cdot10^{-14}f_{cl}^{\,2}\cdot\left(t_{cl}+273\right)^3I_{cl}\left(t_{mrt}+273\right)^3}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}-1{,}584\cdot10^{-7}f_{cl}\left(t_{mrt}+273\right)^3\right.$$
$$\left.f_{cl}h_c\left(\frac{1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{mrt}+273\right)^3}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}\right)\right). \tag{21}$$

Change in PMV according to M/A_{Du} (if t_{cl} is not constant):

$$\frac{\partial Y}{\partial\frac{M}{A_{Du}}}=$$
$$\left(-0{,}010908e^{-0{,}036\frac{M}{A_{Du}}}\right)\cdot\left(0{,}603195\frac{M}{A_{Du}}\left(1-\eta\right)+6{,}93735+\right.$$
$$+0{,}00305p_a-1{,}7\cdot10^{-5}\frac{M}{A_{Du}}\left(5867-p_a\right)-0{,}014\frac{M}{A_{Du}}\left(34-t_a\right)-$$
$$-3{,}96\cdot10^{-8}f_{cl}\left(\left(t_{cl}+273\right)^4-\left(t_{mrt}+273\right)^4-f_{cl}h_c\left(t_{cl}-t_a\right)\right)-$$
$$-\frac{1{,}584\cdot10^{-7}f_{cl}\left(t_{cl}+273\right)^3\left(0{,}028-0{,}028\eta\right)}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}+$$
$$\left.+\frac{3{,}96\cdot10^{-8}f_{cl}h_c\left(0{,}028-,028\eta\right)}{-1{,}584\cdot10^{-7}I_{cl}f_{cl}\left(t_{cl}+273\right)^3+I_{cl}f_{cl}h_c+1}\right). \tag{22}$$

6 DYNAMIC HEAT BALANCE OF HUMAN BODY

Many people examined the dynamic heat balance in the past years some of them: B. W. Olesen, Muhsin Kilic, Omer Kaynakli, Mihaela Baritz, Luciana Cristea, Diana Cotoros and Ion Balcu.

Mihaela Baritz, Luciana Cristea, Diana Cotoros and Ion Balcu are presented the differential equation describe the dynamic heat balance of human body, can by calculated as fallows [12]:

$$k\left(\frac{\partial^2 T}{\partial r^2}+\frac{\omega}{r}\frac{\partial T}{\partial r}\right)+q_m+\rho_{bl}w_{bl}c_{bl}\left(T_{artbl}-T\right)=\rho_c\frac{\partial T}{\partial t} \tag{23}$$

TSENS and DISC values can be calculated by the following equations [10]:

$$TSENS=\begin{cases}0{,}4685(T_b-T_{b,c}) & T_b<T_{b,c}\\ 4{,}7\eta_e(T_b-T_{b,c})/(T_{b,h}-T_{b,c}) & T_{b,c}\le T_b\le T_{b,h}\\ 4{,}7\eta_e+0{,}685(T_b-T_{b,h}) & T_{b,h}<T_b\end{cases} \tag{24}$$

$$DISC=\begin{cases}0{,}4685(T_b-T_{b,c}) & T_b<T_{b,c}\\ \dfrac{4{,}7(Q_{e,rsw}-Q_{e,rsw,req})}{Q_{e,\max}-Q_{e,rsw,req}-Q_{e,dif}} & T_{b,c}\le T_b.\end{cases} \tag{25}$$

Scales of TSENS, +-5 intolerable hot/ cold, +-4 very hot/cold, +-3 hot/cold, +-2 warm/cool, +-1 slightly warm/cool, 0 neutral

Fort he DISC, 0 comfortable, +-1 slightly uncomfortable but acceptable, +-2 uncomfortable and unpleasant, +-3 very uncomfortable, +-4 limited tolerance, +-5 intolerable.

B. W. Olesen, Muhsin Kilic and Omer Kaynakli developed a mathematical model to describe the dynamic heat balance of the human body, taking into account the heat storage capacity of the human body and clothing as seen on Fig. 3.

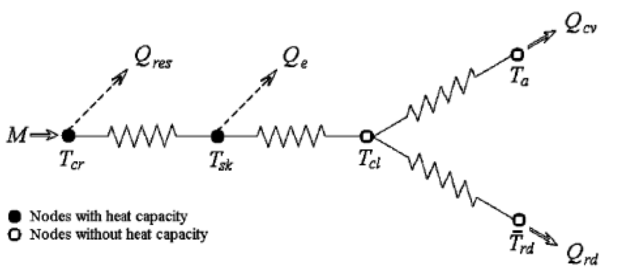


Fig. 3

Heat flows originating from the core of the human body are calculated with help of heat resistances defined for certain body parts. Heat leaving the body surface is conducted via clothes, then with radiation and convection. Theoretically, the dry and hidden heat generated by exhalation should be taken into account as well. The considered heat resistances are calculated with the following formulas:

The total thermal resistance (R_t) and the total evaporative resistance ($R_{e,t}$) for each segment can by calculated as follows [13]:

$$R_t(i)=R_a(i)\frac{r(i,0)}{r(i,nl)}+\sum_{j=1}^{nl}\left[R_{al}(i,j)\frac{r(i,0)}{r(i,j-1)}+R_f(i,j)\frac{r(i,0)}{r(i,j)}\right], \tag{26}$$

$$R_{e,t}(i)=R_{e,a}(i)\frac{r(i,0)}{r(i,nl)}+\sum_{j=1}^{nl}\left[R_{e,al}(i,j)\frac{r(i,0)}{r(i,j-1)}+R_{e,f}(i,j)\frac{r(i,0)}{r(i,j)}\right], \tag{27}$$

where R_a and $R_{e,a}$ are he thermal and evaporative resistances of the outer air layer, R_{al} and $R_{e,al}$ are the thermal and evaporative resistances of the air layer between the clothing layers. Detailed information about these resistances may be found in McCullough et al. [13] and Kaynakli et al. [14].

The sensible heat losses (convective and radiative) for each segment are calculated as follows:

$$Q_{s,sk}(i)=\frac{T_{sk}(i)-T_0(i)}{R_t(i)} \tag{28}$$

where T_{sk} and T_o are the skin and operative temperatures. Operative temperature:

$$T_0(i)=\frac{h_{cv}(i)T_a+h_{rd}\overline{T}_{rd}}{h_{cv}(i)+h_{rd}} \tag{29}$$

where h_{cv} and h_{rd} are convective and radiative heat transfer coefficients, h_{rd} is assumed 4.7 W/m^{2}K [10]. The convective heat transfer coefficients for entire body and of the body are given in de Dear et al. [11].

Evaporative heat loss from skin ($Q_{e,sk}$) depends on the difference between the water vapour pressure at the skin (p_{sk}) and in the ambient environment (p_a), and the amount of moisture on the skin (w)

$$Q_{e,sk}(i)=\frac{w(i)(p_{sk}(i)-p_a)}{R_{e,t}(i)} \tag{30}$$

Total skin wettedness (w) includes wettedness due to regulatory sweating (w_{rsw}) and to diffusion through the skin (w_{dif}). w_{rsw} and w_{dif} are given by:

$$w_{rsw} = \frac{\dot{m}_{rsw} h_{fg}}{Q_{e,\max}} \quad (31)$$

$$w_{dif} = 0{,}06(1 - w_{rsw}) \quad (32)$$

where the $Q_{e,max}$ is the maximum evaporative potential, m_{rsw} is the rate of sweat production, h_{fg} the heat of vaporization of water.

The blood flow between the core and skin per unit of skin area can be expressed mathematically as:

$$\dot{m}_{b1} = \frac{\left[\dfrac{(6{,}3 + 200WSIG_{cr})}{1 + 0{,}5CSIG_{sk}}\right]}{3600} \quad (33)$$

where WSIG and CSIG are warm and cold signal from the body thermoregulatory control mechanism, respectively.

The heat exchange between the core and skin can be written as:

$$Q_{cr,sk} = \left(K + c_{p,bl}\dot{m}_{bl}\right)\left(T_{cr} - T_{sk}\right) \quad (34)$$

where K is average thermal conductance, $c_{p,bl}$ is specific heat of blood.

The rate of sweat production per unit of skin area is estimated by:

$$\dot{m}_{rsw} = 4{,}7 \times 10^{-5} WSIG_b \exp\left(\frac{WSIG_{sk}}{10{,}7}\right). \quad (35)$$

In his live subject experiments Fanger placed his subjects under various activities and thermal loads where the produced and lost heat was not in balance at the initial phase of the examined interval. This was expressed by the deviation from 0 in thermal load L of the human body. Naturally if the human body was able to undergo an adaptation process the heat balance was achieved and the L thermal load of the human body turned 0. Through their votes live subjects were describing this adaptation process. Their votes obviously expressed what new core temperature was required to achieve the new L=0 heat balance and whether it was agreeable or disagreeable for them. This shows the weak point of Fanger's theory: the regressive straight lines of skin temperatures and perspiration values considered agreeable are deemed valid thus also covering the discomfort state of heat balance.

This contradiction can be mathematically presented in the following figure:

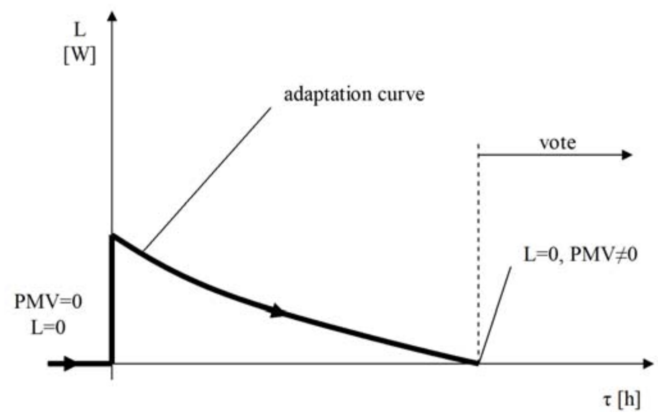


Fig. 1

To develop the so-called static PMV Fanger took his starting point as the extrapolation of the dynamic energy balance of the human body and called a hypothetical condition static PMV. In a stochastic sense there exists a dynamic PMV that classified the ongoing changes of the actual thermal load L of the human body in time and describes the changes of the human response in time as well as the response of the human body to the changes in the environment in time. This theory can grasp static conditions by recording the environmental parameters, eliminating changes in time and revealing the asymptotic conditions. To create such a simplified (and stochastically valid) theory we must start with the fact that the response of the human body to environmental changes can be described by the type of differential equation known from the regulation theory [3]:

$$A_2 v'' + A_1 v' + A_0 v = u \quad \text{or}$$
$$A_1 v' + A_0 v = u \quad (36)$$

The variables are in theory vector variables but may be component variables as well. Variable components of **u** are environmental impacts affecting humans i.e. t_a, t_{mrt}, v_{air}, p_a, while the components of **v** variable are the physiological responses of humans i.e. t_{core}, t_{skin}, t_{sw}, Q_L, Q_{SW}, Q_D, S, C, clothing, body mass, work intensity, other characteristics, constant parameters.

In a stochastic sense equation (36) can be written up for thermal load L in humans and for the relationships of responses given to thermal load intensity.

If the functions in time of the physiological responses given to the unit jump function of the various environmental parameters were or are available we could define the so-called temporary functions of the human body. The solution of equation (36) for a unit jump input is shown graphically on Fig. 2:

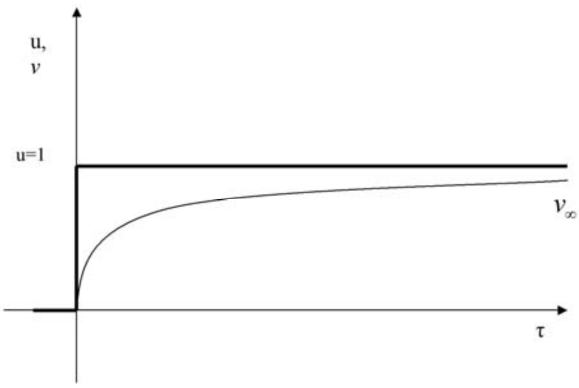


Fig. 2

If there are no jump unit inputs and physiological responses given to them available in terms of environmental variable and impacts we can still define the temporary function for the physiological response of the human body in a stochastic sense, using Duhamel's principle.

Duhamel's principle describes the following: in the event the time function of a disturbance in the system is known then the time function for the system's response can be expressed by the convolution integral:

$$v(t) = \int_0^t h'(t-\tau)u(\tau)d\tau \,. \tag{37}$$

According to equation (37) if we knew the temporary function for a human physiological response to an environmental impact affecting a person then the time function for the examined physiological response can be determined for the time function of the examined environmental impact.

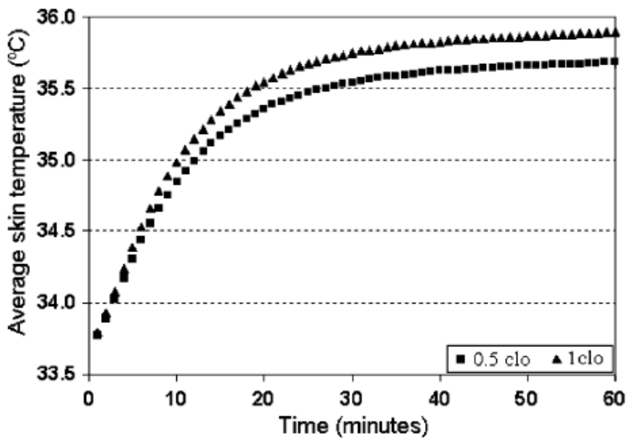
We can also do the inverse task: if we know the time function for a human physiological response to the time function of an environmental impact then using convolution integral (37) we can in theory determine the temporary function for the examined response. Our task is then to resolve an integral equation which is in fact far from being as difficult as it looks because by solving equation (36) the temporary function is

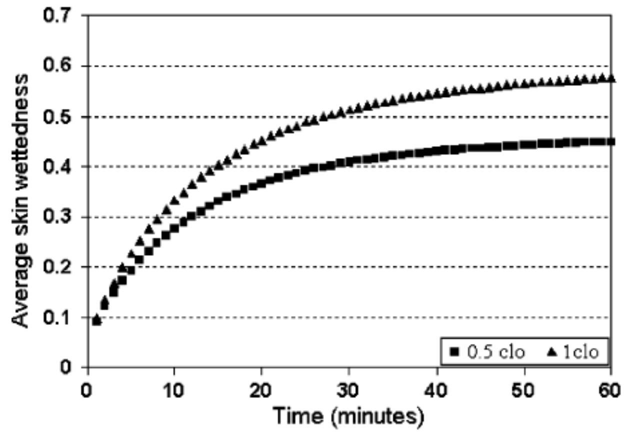
$$h(t) = \frac{1}{A_0}\left(1 - e^{-\frac{A_0}{A_1}t}\right) \tag{38}$$

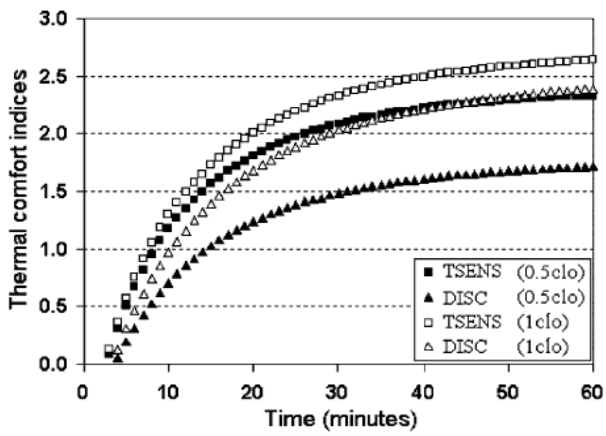
Expression (38) is substituted into convolution integral equation (37). If value pairs **u v** are available among the measurement results we can integrate expression (38) to get algebraic equations to determine A_0 and A_1 coefficients.

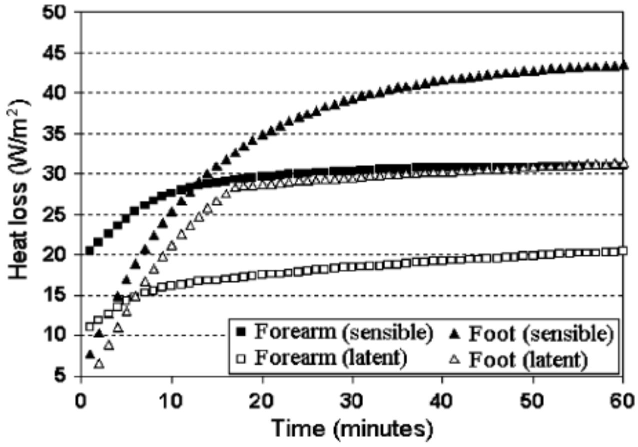
The above discussed examinations enable us to write up dynamic thermal sensation if thermal load L is considered the input of equation (37) and the output is Y and PMV parameters.

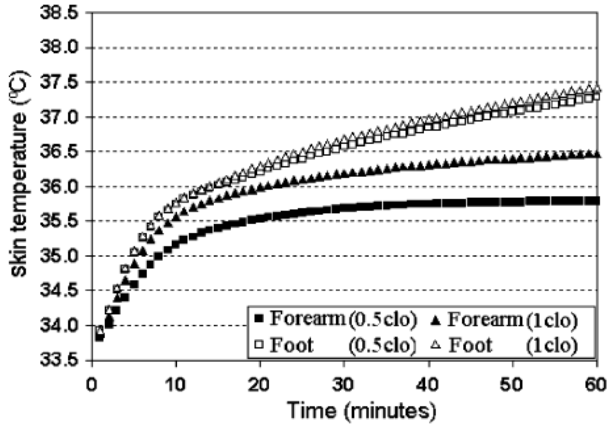
We would like to draw your attention to the work carried out by Muhsin Kilic and Omer Kaynakli whose publication processed the actual adaptations of the human body in time with regard to the response functions of the human body to the various runnings of the different environmental impacts in time. Some of these are presented below [4]. The diagrams graphically show the temporary functions of heat loss, TSENS (thermal sensation index), average skin temperature, average skin wettedness under the given environmental parameters (disturbances).

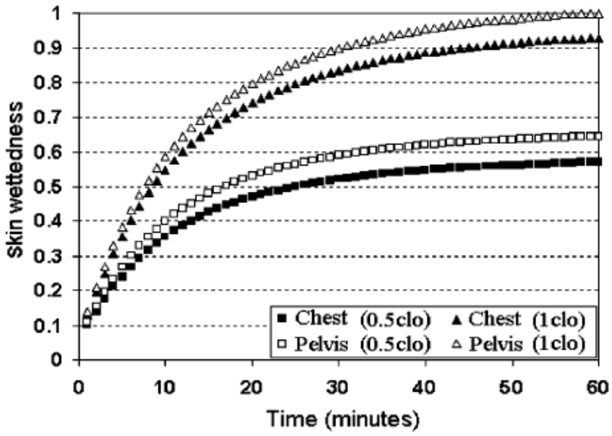












We must note that unlike Fanger the above mentioned authors used the term perceived temperature recommended by ASHRAE to describe PMV:

Based on the diagrams presented by the authors we defined for the temporary functions the A_0 and A_1 parameters of expression $h(t)=\frac{1}{A_0}\left(1-e^{-\frac{A_0}{A_1}t}\right)$ belonging to the $A_1 v' + A_0 v = u$ differential equation [Table 1 and Table 2].

Table 1

		clo	Constants A_0	Constants A_1	Environment
Heat loss	Sensible	0,5	0,1	1,55	M=80W/m², $t_a=t_{mrt}$=30°C
		1	0,1	1,64	R=50%, V=0.1m/s
	Latent	0,5	0,025	0,388	
		1	0,022	0,367	
Average skin temperature		0,5	0,5	8,25	
		1	0,44	7,1	
Average skin wettedness		0,5	2,67	45,3	
		1	2	37	
Thermal comfort indices	TSENS	0,5	0,4	5,8	
		1	0,36	4,54	
	DISC	0,5	0,57	6,57	
		1	0,4	6,6	
Heat loss	Forearm	sensible	0,08	1,04	M=80W/m², $t_a=t_{mrt}$=30°C
		latent	0,08	1	R=50%, V=0.1m/s
	Foot	sensible	0,025	0,45	
		latent	0,036	0,53	
Skin temperature	Forearm	0,5	0,44	5,78	
		1	0,364	4,54	
	Foot	0,5	0,3	3,33	
		1	0,3	3,5	
Skin wettedness	Forearm	0,5	2,857	50	
		1	1,67	27,63	
	Foot	0,5	1,11	12,78	
		1	1,11	10,56	
Heat loss	Chest	sensible	0,087	2,83	M=80W/m², $t_a=t_{mrt}$=30°C
		latent	0,025	0,475	R=50%, V=0.1m/s
	Pelvis	sensible	0,1	3,15	Rcl=0.5clo
		latent	0,027	0,51	
Skin temperature	Chest	0,5	0,42	10,83	M=80W/m², $t_a=t_{mrt}$=30°C
		1	0,377	9,245	R=50%, V=0.1m/s
	Pelvice	0,5	0,42	10	
		1	0,377	8,33	
Skin wettedness	Chest	0,5	2	35	
		1	1,18	19,41	
	Pelvice	0,5	1,667	29,167	
		1	1,11	16,66	

Table 2

		m/s	Constants A_0	Constants A_1	Environment
Heat loss	Sensible	0,05	0,1	1,5	M=80W/m², $t_a=t_{mrt}$=30°C
		0,3	0,08	1,76	R=50%, Rcl=0.5clo,
	Latent	0,05	0,024	0,412	
		0,3	0,031	0,49	
Average skin temperature		0,05	0,44	7,556	
		0,3	0,57	10,29	
Average skin wettedness		0,05	1,82	32,72	
		0,3	5	80	
Thermal comfort indices	TSENS	0,05	0,36	4,54	
		0,3	0,5	6,75	
	DISC	0,05	0,4	5,8	
		0,3	1	17,5	
Heat loss	Forearm	sensible	0,067	0,9	M=80W/m², $t_a=t_{mrt}$=30°C
		latent	0,033	0,567	R=50%, Rcl=0.5clo,
	Foot	sensible	0,057	0,89	
		latent	0,036	0,618	
Skin temperature	Forearm	0,05	0,44	4,89	
		0,3	0,5	6,75	
	Foot	0,05	0,25	3,125	
		0,3	0,29	3,714	
Skin wettedness	Forearm	0,05	2	36	
		0,3	5	90	
	Foot j	0,05	1,11	8,33	
		0,3	1,11	13,89	
Heat loss	Chest	sensible	0,087	2,74	M=80W/m², $t_a=t_{mrt}$=30°C
		latent	0,031	0,523	R=50%, Rcl=0.5clo
	Pelvis	sensible	0,095	3,14	
		latent	0,031	0,554	
Skin temperature	Chest	0,05	0,44	11,33	
		0,3	0,44	12,44	
	Pelvice	0,05	0,44	10,67	
		0,3	0,5	13,75	
Skin wettedness	Chest	0,05	1,43	24,29	
		0,3	3,33	46,67	
	Pelvice	0,05	1,25	20,625	
		0,3	2,5	40	

7 Conclusion

In our research paper we analyzed the comfort equation describing the static and the so called static comfort equation of the human body's thermal balance. The latter describes the human body's thermal balance in such a state, when, given the microclimatic parameters, the human's general condition and thermal sensation are pleasurable, i.e. when the skin temperature and wettedness remain within a narrow, comfortable range. We developed the derivatives and cross derivatives of the comfort equation, which describe the sensitivity of this equation depending on the microclimatic parameters. In other words, how big of an alteration in one of the microclimatic parameters can compensate the alteration of a different microclimatic parameter. Our paper presents the fundamentals of PMV, which is used to satisfy human comfort requirements optimally in premises serving for human residence and work. We described the sensitivity of PMV, produced by deriving PMV according to the influencing parameters. Additionally, we introduced the basics of creating temporary functions from dynamic thermal balance.
Based on the calculations of Muhsin Kilic and Omer Kaynakli, we conducted concrete calculations on our examples complied to define the temporary functions.

Terms:

H internal heat production in the human body
$\dot{Q}_L$ dry respiration heat loss
$\dot{Q}_{re}$ latent respiration heat loss
$\dot{Q}_d$ heat loss by skin diffusion
$\dot{Q}_{SW}$ heat loss by evaporation of sweat
$\dot{Q}_K$ heat transfer from the skin to the outer surface of the human clothing
$\dot{Q}_S$ heat loss by radiation
$\dot{Q}_C$ heat loss by convention
Icl thermal resistance of clothing
L thermal load
$\frac{M}{A_{Du}}$ metabolic rate
t_b mean skin temperature
t_c surface temperature of clothing
t_a ambient temperature
t_m internal (core) temperature of the human body
t_s surface temperature
$\dot{q}_{konv}$ heat loss by convention
$\dot{q}_{sug}$ heat loss by radiation
W external mechanical work
t_l air temperature
t_{mrt} mean radiant temperature
v_{air} relative air velocity
p_a partial water vapour pressure
$\frac{Q_{SW}}{A_{Du}}$ heat loss by skin diffusion and evaporation of sweat
k tissue conductivity
T tissue temperature
r radius
w geometry parameter
q_m metabolism
ρ_{bl} density of the blood
w_{bl} blood perfusion rate
T_{artbl} arterial blood temperature
c_{bl} heat capacity of blood
t time
ρ tissue density

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