Heat capacity of vertical ground heat exchangers with single U-tube installation in the function of time

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Abstract: - One of the major problems of ensuring optimal working of ground source heat pump systems is a heat transfer around vertical ground heat exchanger. The working of vertical U-tube can be understood as a heat exchanger between the ground and the heat carrying medium. In our case this heat carrying medium is fluid, which transfers the extracted heat from the ground to the heat pump. In winter we extract heat from the ground and in summer we transfer heat to the ground. In our paper we propose a simple calculation model to calculate the temperature change and the thermal resistance in vertical ground heat exchangers with single U-tube installation. We made calculations to obtain the amount of extractable heat from the U-tube in the function of different mass flow. We did these calculations for several periods of time, 1 day, 1 year, 10 years. We found that the amount of extractable heat in winter and in summer is between 10 – 80 % in the function of mass flow, and with elapsing time its values decline.

Key-words: - heat pump, U-tube, heat transfer, heat flow, thermal resistance, system theory.

1 Introduction

In recent years, a large number of residential and commercial buildings have been installed with ground coupled heat pump systems for space cooling, heating and even hot water supply because of their higher efficient, low maintenance cost and environmental friendliness. Most of the ground coupled heat pumps use vertical ground heat exchangers which usually offer higher energy performance than the horizontal ground heat exchangers due to the less temperature fluctuation in the ground.

In the Carpathian basin, but mainly on the territory of Hungary the crust of the earth is thinner than the average; therefore its geothermal features are very good. Under the ground surface in the earth core levels from the decomposition of radioactive isotopes heat is produced. Its flow directed towards the surface is geothermal energy. The global average of the geothermal gradient is 33 m/°C, while in Hungary it is only 18-22 m/°C. The average value of the heat flow from the inner core of the ground is 80-100 mW/m^2 according to the heat flow map of Hungary, which is almost the double of the average value measured on the mainland [1].

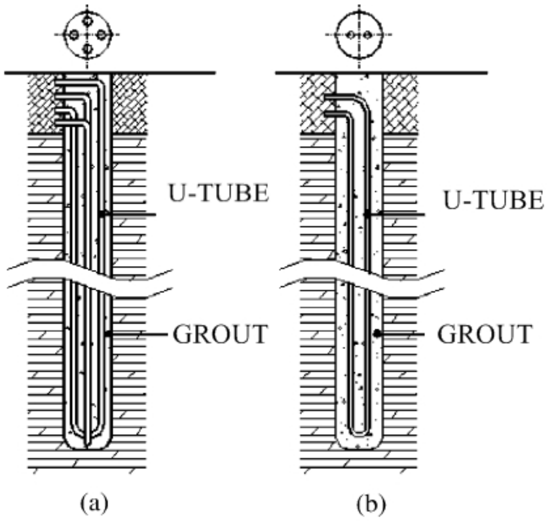


Fig. 1: Location of double (a) and single (b) U-tubes in boreholes

The primary side heat sources of heat pumps operating with water-water sources are the following: underground waters and heat of the earth (geothermal energy). The geothermal energy is extracted from the ground by ground heat exchangers with U-tubes. The installation of U-tubes can be vertical and horizontal. In our paper we deal with heat extraction of vertically installed single U-tubes. For the vertical U-tubes boreholes are made with a diameter of 75 – 150 mm, with a depth range 40 – 200 m. In these boreholes single and double U-tubes are installed. (Fig. 1) After the installation the ground heat exchanger is filled with bentonite grout. This grout ensures better heat transfer and it blocks the underground waters [2].

2 Review of heat transfer modelling in the case of U-tubes

In a descending and ascending branch of the U-tubes, the fluid gets warm and forwards the heat to the heat pump through a heat exchanger. The modelling of this heat transfer is a complex problem. The process of heat transfer is affected by many variables, such as ground temperature, ground humidity, the structure of the ground and the thermal features, furthermore the location of underwater. There are many authors, who deal with these problems, such as Zeng [2], Kalman [4], Kavanaugh [5], Yavusturk and Splitter [6]. During the modelling the heat transfer can be regarded as a steady or unsteady state. Theoretically steady state never occurs during the heat extraction process. Several months after steady operation , the heat transfer process is steady with good approximation. Among others, Zeng [2] describes short term unsteady processes.

If we take the processes of heat transfer and the working of U-tubes as steady, then for the description of heat transfer between the U-tube and the ground we can use the following very simple formula,

$$T_f(t) - T_b(t) = q_b(t) R_r, \tag{1}$$

where R_b is overall thermal resistance, which includes the resistance of heat transfer in the ground and grout furthermore the resistance of the heat transfer between U-tube and the fluid [3].

The process of warming of fluid can be described with the following formula

$$\frac{q_b(t) H}{\dot{m} \cdot c_p} = T_{fi}(t) - T_{fo}(t), \tag{2}$$

The main problem in the modelling is determining R_r the overall heat transfer thermal resistance.

The borehole thermal resistance is determined by a number of parameters, including the composition and flow rate of the circulating fluid, borehole diameter, grout and U-tube material as well as arrangement of flow channels. Models for practical engineering designs are often oversimplified in dealing with the complicated geometry inside the boreholes [2].

A one-dimensional model [13] has been recommended, conceiving the legs of the U-tubes as a single equivalent pipe inside the borehole, which leads to a simple expression

$$R_r = \frac{1}{2 \cdot \pi \cdot k_r} \cdot \ln\left(\frac{r_{pipe}}{\sqrt{N} \cdot r_{pipe}} \right) + R_{pipe}. \tag{3}$$

Another effort to describe the borehole resistance has used the concept of the shape factor of conduction and resulted in an expression [2]

$$R_r = \left[k_r \cdot \beta_0 \cdot \left(\frac{r_{borehole}}{r_{pipe}} \right)^{\beta_1} \right]^{-1}. \tag{4}$$

where parameters β_0 and β_1 were obtained by means of curve fitting of effective borehole resistance determined in laboratory measurements [14]. In this approach only a limited number of influencing factors were considered, and all the pipes were assumed to be of identical temperature as a precondition.

By a different approach Hellstrom [15] has derived two-dimensional analytical solution of the borehole thermal resistances in the cross-section perpendicular to the borehole with arbitrary numbers of pipes, which are superior to empirical expression. Also on assumptions of identical temperatures and heat fluxes of all the pipes in it the borehole resistance has been worked out of symmetrically disposed double U-tubes as [2]

$$R_r = \frac{1}{2 \cdot \pi \cdot k_r} \left[\ln\left(\frac{r_{borehole}}{r_{pipe}} \right) - \frac{3}{4} + \left(\frac{D}{r_{borehole}} \right)^2 - \frac{1}{4} \ln\left(1 - \frac{D^8}{r_{borehole}^8} \right) - \frac{1}{2} \ln\left(\frac{\sqrt{2} D}{r_{pipe}} \right) - \frac{1}{4} \left(\frac{2D}{r_{pipe}} \right) \right] + \frac{R_{pipe}}{4}. \tag{5}$$

On the other hand, Mei and Baxter [16] considered the two-dimensional model of the radial and longitudinal heat transfer, which was solved with a finite difference scheme. Recently, Yavuzturk et al. [6] employed the two-dimensional finite element method to analyze the heat conduction in the plane perpendicular to the borehole for short time step responses.

Requiring numerical solutions, these models are of limited practical value for use by designers of ground coupled heat pump systems although they result in more exact solutions for research and parametric analysis of ground heat exchangers.

In our paper, we give a simple calculation method for overall thermal resistance. We take into the account thermal resistance of the U-tube (R_{pipe}), thermal resistance of the backfill (R_{grout}) and the thermal resistance of the ground (R_{ground}).

3 Simple calculating method for the heat transfer in single U-tubes

Optimizing the heat pump systems is performed by system theory models [7]. The operation of geothermal heat pump systems is affected by ground temperature and heat transfer processes in the ground, because the ground temperature determines the maximum extractable heat capacity. It basically determines the coefficient of performance (COP). Therefore we lay a big emphasis on modelling this process, i.e. on obtaining exact numerical values of the temperature change in the ascending branch of the U-tube. By knowing the rate of this warming, we can make an exact calculation for the borehole depth in function of required capacity of the unit.

In our paper we use a simple calculating model to determine the temperature change and the extractable maximum heat capacity. In our calculations we use steady and unsteady models. The heat flux through the top and the end of the borehole is neglected because the size of the borehole diameter is much smaller than its depth.

By setting up our model, we use the following hypotheses:

1. In 10 m depth, the ground temperature is not affected by the outdoor temperature changes, so the season changes are not influencing parameters. In 10 m depth the ground temperature is 10 °C.
2. The ground temperature change is linear; in 100 m depth we assume 16 °C.

3.1 Bases of the calculation model

The temperature change of the fluid is described by the following differential equations:

For the descending branch of the U-tube

$$\dot{m} \cdot c_v \cdot \frac{dT_1(H)}{dH} = s \pm \frac{\left(T_{ground} - T_1(H)\right)}{R_1} \pm q', \qquad (6)$$

for the ascending branch of the U-tube

$$\dot{m} \cdot c_v \cdot \frac{dT_2(H)}{dH} = s \pm \frac{\left(T_{ground} - T_2(H)\right)}{R_2} \pm q', \qquad (7)$$

where H is the borehole depth, T_1 and T_2 describes the temperature of the fluid in the function of depth (H). q' in equations (3), (4) shows the mutual influence of the U-tube (Fig. 2). R_1 and R_2 are overall thermal resistances around the U-tube. The mutual influence can be calculated by the following equation [8]:

$$\frac{q'}{\lambda \cdot (T_1 - T_2)} = \frac{2\pi}{\cosh^{-1}\left[\frac{4l^2 - D^2 - d^2}{2 \cdot D \cdot d}\right]}. \qquad (8)$$

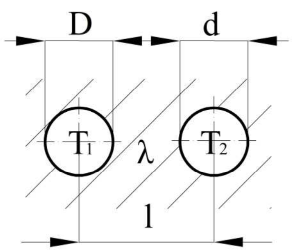


Fig. 2: Mutual influence of U-tube in endless space

In equation (5) T_1 and T_2 describe the fluid temperature in each part of the U-tube, D and d represent the diameter of the U-tube (in our case D=d), l the distance between the parts of the U-tube and λ represents the heat conductivity of the grout. We study the descending and ascending temperature change in a separate coordinate system (Fig. 3).

The previously shown equations add up to a system of linked differential equations. The linked differential equations contain two unknown functions $T_1(H)$ and $T_2(H)$. These equations are solved by applying the method of serial approach as follows. In the 0^{th} approach we neglect the mutual interaction of the branches of the U-tube and we solve the equations (3) and (4) separately. The solutions are as follows:

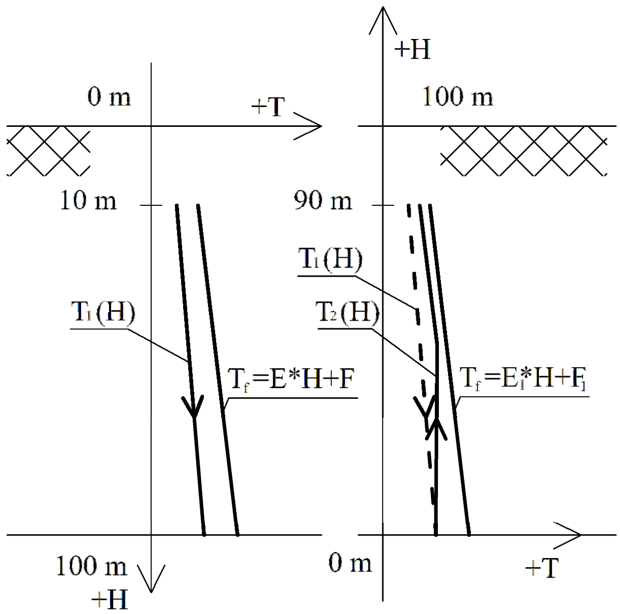
$$T_1(H) = s \cdot R_1 - E \cdot m \cdot c_v \cdot R_1 + F + E \cdot H + e^{-\frac{H}{R_1 \cdot m \cdot c_v}} \cdot C \qquad (9)$$

$$T_2(H) = s \cdot R_2 - E_1 \cdot m \cdot c_v \cdot R_2 + F_1 + E_1 \cdot H + e^{-\frac{H}{R_2 \cdot m \cdot c_v}} \cdot C_1 \qquad (10)$$

These two solutions are shown in coordinate systems (Fig. 3).

In the following phase we correct the obtained functions for $T_1(H)$ and $T_2(H)$ so that we take into account the interactions of the U-tube parts according to the (8) equation. In the equation we substitute the functions $T_1(H)$ and $T_2(H)$ with the obtained results in the 0^{th} approach and we solve again the equations (6) and (7). We proceed numerically, by ΔH steps from 10 m to 100 m and vice versa from 100 m to 10 m. We continue this method and the function correction recursively.

In the previously shown calculation method the appropriate solution calculated for values R_1 and R_2 is problematic. In the following chapter we give an exact method to obtain solution for these thermal resistances.



E = 0.06 E_1 = -0.067
F = 10 F_1 = 16
Fig. 3: Coordinate systems for solution

4 Determining R_1 and R_2 thermal resistances considering the unsteady operation of the U-tubes

We determine the thermal resistance with equation (11) for steady states, where we calculate the sum of thermal resistance of particular system elements. For unsteady state the method is the same, because the process is very slow, and we can model it by the method of serial approach.

Therefore the value of R_1 and R_2 can be obtained by the following simple formula for steady and unsteady process (Fig. 4):

$$R_r = R_1 = R_2 = R_{ground} + R_{grout} + R_{pipe} \tag{11}$$

We apply the method of Carslaw-Jaeger to determine the thermal resistance R_{ground} which is represented in Fig 5. Carslaw-Jaeger [9] introduced in the scientific literature how the distribution of temperature and density of heat flux is changing on the surface of cylinder in the function of time around a circular cylinder in the infinite space.

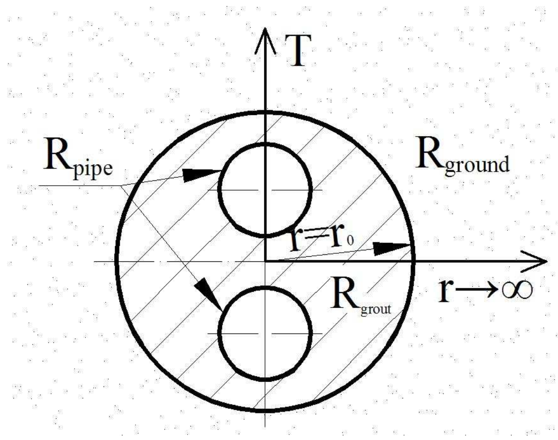


Fig. 4: Parts of the overall thermal resistance

With the help of Carslaw-Jaeger method we present the solution of the problem. Carslaw-Jaeger defined the problem as follows: The region bounded internally by the circular cylinder.

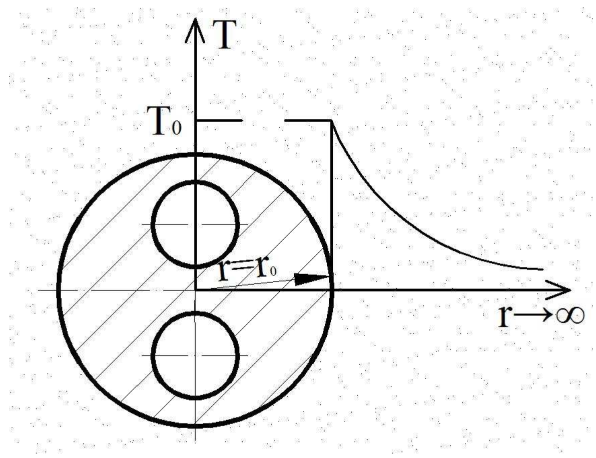


Fig 5: Distribution of temperature around a circular cylinder in infinite space

$$\frac{d^2\overline{\Theta}}{dr^2} + \frac{1}{r}\cdot\frac{d\overline{\Theta}}{dr} - q^2\overline{\Theta} = 0, \qquad r > r_0, \tag{12}$$

where $q^2 = \dfrac{s}{\kappa}$.

If $r \to \infty$ and $\overline{\Theta} = T_0/s$, $r = r_0$, the solution is:

$$\overline{\Theta} = \frac{T_0 K_0(qr)}{sK_0(qr_0)}. \tag{13}$$

By using the inversion thesis according to Carslaw and Jaeger [9]:

$$\vartheta = \frac{T_0}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\lambda\cdot\tau}\frac{K_0(\mu r)d\lambda}{K_0(\mu r_0)\lambda}, \tag{14}$$

where $\mu = \sqrt{\lambda/\kappa}$, and K_0 is a modified Bessel function of the second kind, zero order.

If $\lambda = \kappa u^2 e^{i\pi}$, then:

$$2\int_0^\infty e^{-\kappa u^2\tau}\frac{K_0(rue^{\frac{1}{2}\pi i})}{K_0(r_0 ue^{\frac{1}{2}\pi i})}\frac{du}{u} = 2\int_0^\infty e^{-\kappa u^2\tau}\frac{J_0(ur)-iY_0(ur)}{J_0(ur_0)-iY_0(ur_0)}\frac{du}{u}, \tag{15}$$

since

$$K_0(ze^{\frac{1}{2}\pi i}) = -\frac{1}{2}\pi\cdot i\cdot H_0^2(z) = -\frac{1}{2}\pi\cdot i\left[J_0(z)-iY_0(z)\right].$$

Combining these correlations:

$$\vartheta = T_0 + \frac{2T_0}{\pi}\int_0^\infty e^{-\kappa u^2\tau}\frac{J_0(ur)Y_0(ur_0)-Y_0(ur)J_0(ur_0)}{J_0^2(r_0u)+Y_0^2(r_0u)}\frac{du}{u}, \tag{16}$$

The asymptotic analysis of the Bessel functions (13) is used for small time units in the Laplace transformed form of the solution:

$$\overline{\Theta} = \frac{T_0}{s}\left(\frac{r_0}{r}\right)^{\frac{1}{2}} e^{-q(r-r_0)}\left\{1+\frac{(r-r_0)}{8r_0rq}+\frac{(9r_0^2-2r_0r-7r^2)}{128r_0^2r^2q^2}\dots\right\}.$$

The re-transformed form of which is:

$$\vartheta = \frac{T_0r_0^{1/2}}{r^{1/2}}\,erfc\frac{r-r_0}{2\sqrt{(\kappa\cdot\tau)}} + \frac{T_0(r-r_0)(\kappa\cdot\tau)^{\frac{1}{2}}}{4r_0^{\frac{1}{2}}r^{\frac{1}{2}}}\,ierfc\frac{r-r_0}{2\sqrt{(\kappa\cdot\tau)}} + \frac{T_0(9r_0^2-2r_0r-7r^2)\kappa\cdot\tau}{32r_0^{\frac{3}{2}}r^{\frac{5}{2}}}\,i^2erfc\frac{r-r_0}{2\sqrt{(\kappa\cdot\tau)}}+\dots, \tag{17}$$

Since the U-tubes extract heat from the ground while working, the temperature of the ground around the U-tube declines simultaneously and the quantity of extractable heat gradually declines, too. This phenomenon can be modelled with the method shown by Carslaw-Jaeger [9].

According to the outer radius of the U-tube the heat flux in the function of time is:

$$\dot{q} = -\lambda_{ground}\left[\frac{\partial\vartheta}{\partial r}\right]_{r=r_0} = \frac{4T_0\lambda_{ground}}{r_0\pi^2}\int_0^\infty e^{-\kappa u^2\tau}\frac{du}{u[J_0^2(r_0u)+Y_0^2(r_0u)]}. \tag{18}$$

Integral (12) for lower values of the Fourier number approximately is:

$$\dot{q} = \frac{\lambda_{ground}T_0}{r_0}\left\{(\pi\cdot Fo)^{-\frac{1}{2}}+\frac{1}{2}-\frac{1}{4}\left(\frac{Fo}{\pi}\right)^{\frac{1}{2}}+\frac{1}{8}Fo\dots\right\}, \tag{19}$$

for larger values of Fo numbers is:

$$\dot{q} = \frac{2T_0\lambda_{ground}}{r_0}\left\{\frac{1}{\ln(4Fo)-2\gamma}-\frac{\gamma}{[\ln(4Fo)-2\gamma]^2}-\dots\right\}, \tag{20}$$

($\gamma = 0.57$, Euler number)

As T_0 is a beyond temperature (the difference between the temperatures of the borehole's wall and the distant ground) the unsteady heat transfer thermal resistance can be defined by the following:

$$R_{ground} = \frac{T_0}{\dot{q}} = \frac{r_0}{2\cdot\lambda_{ground}\cdot\left\{\frac{1}{\ln(4Fo)-2\gamma}-\frac{\gamma}{[\ln(4Fo)-2\gamma]^2}-\dots\right\}}. \tag{21}$$

It is demonstrable that for the larger values of Fo the value of R_{ground} changes very slowly, with a good approximation it can be considered as constant in a fixed period of time.

With the above stated equations we can calculate the value of the thermal resistance between the ground and all of the U-tube in different depths and the amount of the heat flux arriving to the walls of the U-tube in the function of time. It is demonstrable that the process of ground temperature decreasing is very slow. After 1 year of operation the heat transfer can be defined as a steady state. The change of the Fo number in the function of time is shown in Table 1.

Table 1: Fo number change in the function of time

	10 s	**1 hour**	**1 day**
τ [s]:	10	3600	86400
Fo	0,040192	14,46907	347,2577
	1 month	**1 year**	**10 year**
τ [s]:	2592000	946080000	9460800000
Fo	10417,73	3802471	38024713

Table 2 shows the values of unsteady thermal resistance R_{ground}, which are calculated by equation (21) and with the average value of heat conduction $\lambda_{ground} = 2.42$ W/mK.

Table 2: Unsteady thermal resistance R_{ground} change in the function of time

	10 s	1 hour	1 day
R_g [mK/W]	0.008	0.012	0.022
	1 month	**1 year**	**10 year**
R_g [mK/W]	0.033	0.053	0.060

The inner space between the U-tube and the borehole is filled up with bentonite, in order to stop porosity and inner air. With the grout we increase the heat flux between the heat carrier fluid and the ground. Thermal resistance of the grout can be calculated by the following equation [8]

$$\frac{q'}{\lambda_{grout} \cdot (T_1 - T_w)} = \frac{2 \cdot \pi}{\cosh^{-1}\left[\frac{\left(D_{borehole}^2 + D^2 - 4 \cdot l^2\right)}{2 \cdot D_{borehole} \cdot D}\right]}, \qquad (22)$$

where D and d are outer diameters of U-tube, $D_{borehole}$ is a diameter of the borehole, l is a distance between U-tube and midpoint of the borehole. λ_{grout} = 2,09 W/mK is thermal conductivity of the bentonite (Fig. 5).

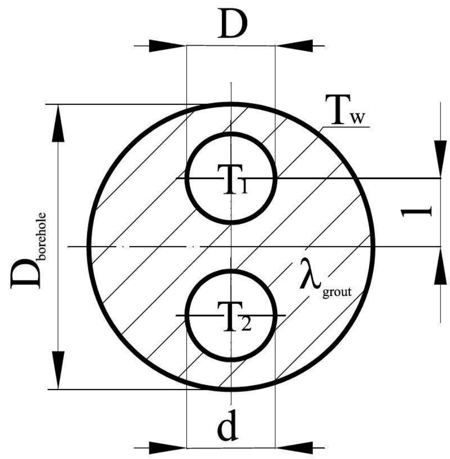


Fig. 5

Solving equation (22) we obtain solution for the thermal resistance of the grout, which is the following:

$$R_{grout} = \frac{\cosh^{-1}\left[\frac{\left(D_{borehole}^2 + D^2 - 4 \cdot l^2\right)}{2 \cdot D_{borehole} \cdot D}\right]}{2 \cdot \pi \cdot \lambda_{grout}} \qquad (23)$$

In our situation D=d.

With the help of the above described formula for calculating the thermal resistance of the grout is R_{grout} = 0,089 mK/W. It is taken into account that the U-tube is located eccentrically in the borehole.

The overall unsteady thermal resistance can be obtained if to the results shown in Table 2 are added to the thermal resistance of the plastic U-tube pipe, which value is 0,085 mK/W and to the value of the grout's thermal resistance. The values of the overall unsteady thermal resistances are shown is Table 3.

Table 3: Overall thermal resistance R_r change in the function of time

	10 s	1 hour	1 day
R_r [mK/W]	0.165	0.185	0.195
	1 month	**1 year**	**10 year**
R_r [mK/W]	0.207	0.226	0.234

5 Monthly calculated results for an operating single U-tube

Hereby I propose a computation sample. This calculation is made for the following months: February, May, August and November. For each month the method is the same, the only changes are in the ground temperature, because in the first 10 m its value is affected by the ambient temperature.

Basic data are as follows:

- The outer diameter of U-tube pipes is 32 mm;
- The absolute roughness of inner walls of U-tube is 0.00015 m;
- The outer diameter of boreholes is 140 mm;
- After placing the U-tube in the borehole, the inner space is filled by bentonite to stop the porosity;
- The fluid flow in the U-tube is turbulent;
- The distance between the descending and ascending branches of the U-tube is 3.3 cm;
- Entering water temperature is 3 °C in each month.

In the examples (6), (7) and (8) we calculated the outgoing temperature change from the U-tube and the extracted heat from the ground with the help of equations and following the method of serial approach for the periods τ = 1 day, 1 year and 10 year. Values of overall thermal resistances R_1 and R_2 are taken from Table 3. The following tables (Table 6 – 17) in the following chapters show iterations and improvements of the results step by step from the 0^{th} approach to the 2^{nd} approach. The iterations are finished at the 2^{nd} approach. Calculated values in the tables (Table 6 – 17) show fluid warming, from 0 m to 100 m in the descending part, and from 100 m to 0 m, in the ascending part of the

U-tube. $\dot{Q}$ [kW] represents the extractable heat from the U-tube under the given conditions.

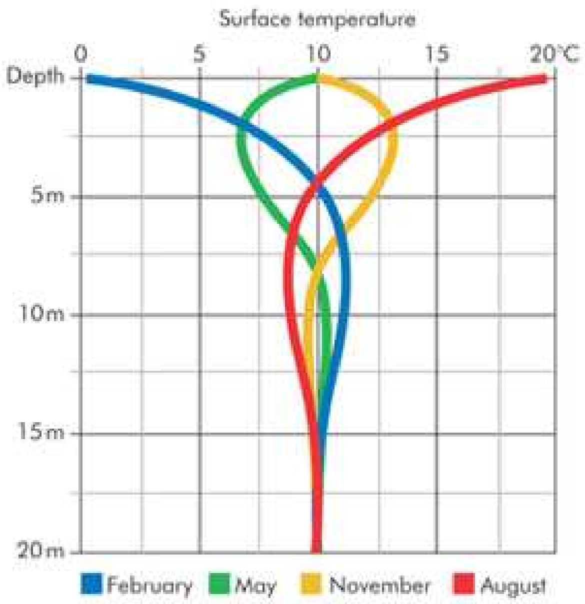


Fig. 6: Ground's temperature change during months: February, May, August and November [12]

5.1 Obtained results for February

Table 6: τ = 1 day

H [m] \ m [kg/s]	0.95	0.53	0.32
10	3.03	3.05	3.09
50	3.76	4.52	5.49
100	4.45	5.73	7.40
50	4.79	5.99	7.43
10	4.53	4.90	4.90
0	4.44	4.69	4.56
Q [kW]	5.73	3.77	2.10

Table 7: τ = 1 year

H [m] \ m [kg/s]	0.95	0.53	0.32
10	3.02	3.05	3.08
50	3.67	4.33	5.21
100	4.27	5.41	6.92
50	4.57	5.65	6.98
10	4.34	4.69	4.70
0	4.26	4.51	4.41
Q [kW]	5.04	3.35	1.89

Table 8: τ = 10 year

H [m] \ m [kg/s]	0.95	0.53	0.32
10	3.02	3.05	3.08
50	3.67	4.33	5.21
100	4.27	5.41	6.92
50	4.57	5.65	6.98
10	4.34	4.69	4.70
0	4.26	4.51	4.41
Q [kW]	5.04	3.35	1.89

5.2 Obtained results for May

Table 9: τ = 1 day

H [m] \ m [kg/s]	0.95	0.53	0.32
10	3.07	3.12	3.21
50	3.80	4.58	5.58
100	4.49	5.78	7.46
50	4.83	6.04	7.50
10	4.56	4.96	4.99
0	4.51	4.81	4.75
Q [kW]	6.00	4.03	2.35

Table 10: τ = 1 year

H [m] \ m [kg/s]	0.95	0.53	0.32
10	3.06	3.11	3.18
50	3.70	4.39	5.29
100	4.30	5.45	6.98
50	4.60	5.69	7.04
10	4.37	4.74	4.78
0	4.32	4.61	4.57
Q [kW]	5.28	3.58	2.10

Table 11: τ = 10 year

H [m] \ m [kg/s]	0.95	0.53	0.32
10	3.06	3.10	3.18
50	3.67	4.34	5.23
100	4.25	5.38	6.88
50	4.55	5.62	6.94
10	4.33	4.69	4.73
0	4.28	4.56	4.52
Q [kW]	5.12	3.48	2.05

5.3 Obtained results for August

Table 12: τ = 1 day

m [kg/s] / H [m]	0.95	0.53	0.32
10	3.15	3.26	3.45
50	3.87	4.69	5.75
100	4.55	5.89	7.60
50	4.89	6.14	7.63
10	4.63	5.07	5.18
0	4.64	5.04	5.11
Q [kW]	6.55	4.54	2.83

Table 13: τ = 1 year

m [kg/s] / H [m]	0.95	0.53	0.32
10	3.13	3.23	3.39
50	3.76	4.49	5.45
100	4.36	5.55	7.11
50	4.66	5.78	7.17
10	4.43	4.84	4.95
0	4.44	4.81	4.88
Q [kW]	5.75	4.02	2.53

Table 14: τ = 10 year

m [kg/s] / H [m]	0.95	0.53	0.32
10	3.12	3.22	3.38
50	3.73	4.45	5.38
100	4.31	5.47	7.00
50	4.60	5.70	7.06
10	4.39	4.79	4.90
0	4.40	4.76	4.83
Q [kW]	5.57	3.91	2.46

5.4 Obtained results for November

Table 15: τ = 1 day

m [kg/s] / H [m]	0.95	0.53	0.32
10	3.11	3.21	3.35
50	3.84	4.65	5.68
100	4.53	5.85	7.54
50	4.86	6.10	7.58
10	4.60	5.03	5.11
0	4.59	4.95	4.96
Q [kW]	6.33	4.33	2.63

Table 16: τ = 1 year

m [kg/s] / H [m]	0.95	0.53	0.32
10	3.10	3.18	3.31
50	3.73	4.45	5.38
100	4.33	5.51	7.06
50	4.63	5.75	7.12
10	4.41	4.80	4.88
0	4.39	4.73	4.75
Q [kW]	5.56	3.84	2.35

Table 17: τ = 10 year

m [kg/s] / H [m]	0.95	0.53	0.32
10	3.10	3.17	3.30
50	3.71	4.40	5.32
100	4.29	5.44	6.95
50	4.58	5.67	7.01
10	4.36	4.75	4.83
0	4.35	4.68	4.71
Q [kW]	5.39	3.73	2.29

6 Conclusions

The results shown in the tables (Table 6 – 17) are presented in Fig. 6 – 9. From the calculated results the following conclusion can be made. The out-going temperature of the fluid T_2 (H = 0 m) at every period of time in the function of mass flow has a maximum, which can be found in the interval 0.4 – 0.5 kg/s. However, the extractable heat does not have a maximum.

In the case of each mass flow value, the warming of the temperature stops at around 50 m depth in the ascending branch of the U-tube, after which the temperature of the fluid is decreasing while moving toward the surface. From the calculation we can see that with the increase in mass flow the quantity of extractable heat is increasing as well. We managed to obtain equation of the time dependent transient thermal resistance of the heat conduction of the bore. We suggest using thermal resistance calculation with equation in practice (21) following by Carslaw-Jaeger's [9] model. Using equation (21) is theoretically proven. The accuracy of calculation is however affected by how precise information we have of the heat conductivity of the ground in the surroundings of the U-tube. Our results presented hereby correspond by size with the results calculated by GLD 3.0 [10] software R_r = 0.124 mK/W and with the results calculated by researchers Zeng, Diao and Fang [2].

We can draw interesting conclusions from the results of the calculations. The output of the examined

U-tube is rapidly changing in the function of mass flow. There is a smaller difference in the amount of extractable heat in winter and in summer months. For example, for February after the first day of working Q=2.1 kW and for August Q=2.83 kW if the mass flow is m=0.32 kg/s. If the mass flow is m=0.95 kg/s than these values change to the following Q=5.73 kW and Q=6.55 kW. Fig 10 provides detailed information of the above described case.

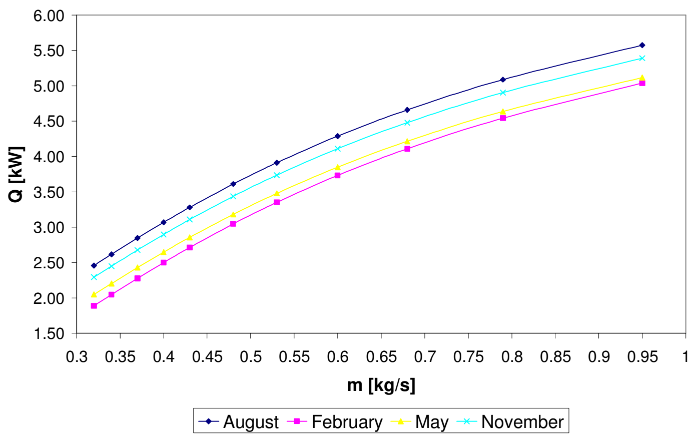


Fig 6: Change of the extractable heat in the function of mass flow after 10 year

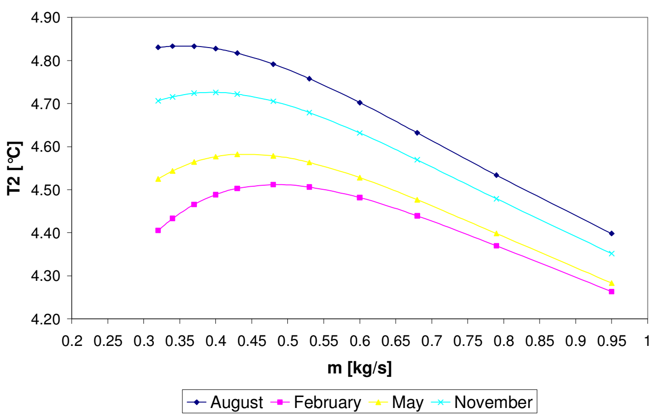


Fig 7: Change of the out-going temperature in the function of mass flow after 10 year

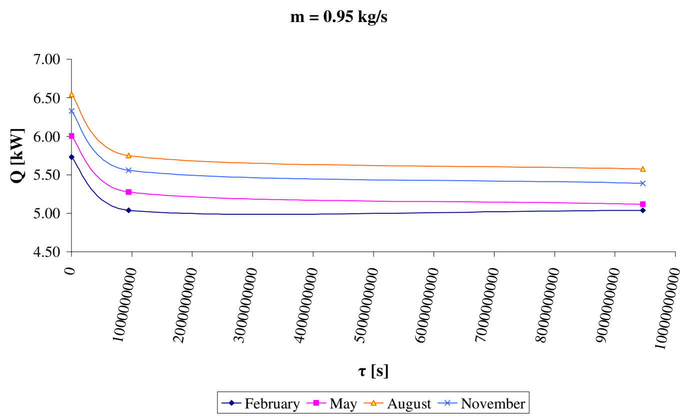


Fig 8: Change of the extractable for each month in the function of time for mass flow $\dot{m} = 0.95$ kg/s

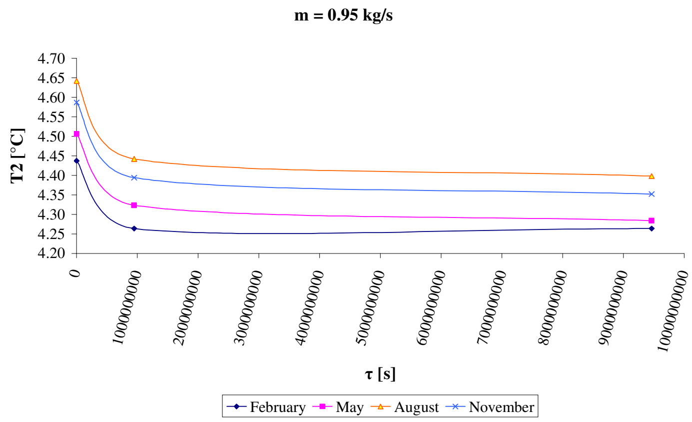


Fig 9: Change of the outgoing temperature for each month in the function of time for mass flow $\dot{m} = 0.95$ kg/s

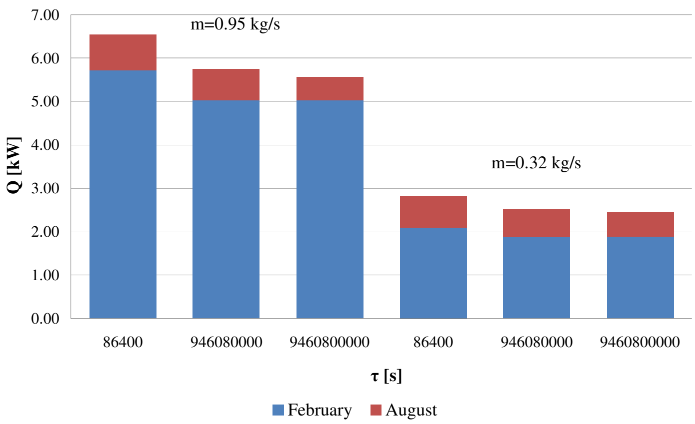


Fig. 10: Change of the extractable heat in the function of time with different mass flow for months February and August

Symbols:
T_1 – Descending fluid's temperature;
T_2 – Ascending fluid's temperature;
ϑ, T_0 – Beyond temperature;
m – Mass flow;
H – Depth;
c_v – Specific heat,
A – Surface;
λ – Coefficient of Heat Conductivity;
s – Friction's heat capacity;
q – Heat flow;
Q – Heat Capacity;
Fo – Fourier number;
τ – Time;
r – Radius;
D, d – diameter of the U-tube;
$D_{borhole}$ – Outer diameter of the borehole;
R_1 – Overall thermal resistance of the descending pipe;
R_2 – Overall thermal resistance of the ascending pipe;
R_r – Overall unsteady Thermal Resistance;
γ – Euler's number;
E, F, E_1, F_1 – Integral constants;
$K_{0,}$ J_0, Y_0 – Bessel functions;

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