

# Numerical analysis of coupled radiation and laminar forced convection in the entrance region of a circular duct for non-grey media: entropy generation

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*Abstract:* The current study analyses entropy generation induced by coupled forced convection and thermal radiation heat transfer, in circular tubes and for non-grey media. A laminar flow of over heated water vapour in the entrance region of the duct was investigated. A specially developed numerical model, based on the finite-volume technique, is used for the solutions of the governing differential equations. The radiative transfer coefficients are deduced by use of the "Ray Tracing" method through  $S_4$  directions, in combination with the "statistical narrow band correlated-k" (SNBCK) model. The local entropy generation distributions as well as the overall entropy generation in the whole flow fields are analyzed. The developing of velocity and temperature fields is also presented in this study.

*Keywords:* Entropy generation, Coupled convection and radiation, SNBCK model, non-grey gas.

## 1 Introduction

The analysis of the energy utilization and the entropy generation has become one of the primary objectives in designing a thermal system. In fact, the study of entropy production, or thermodynamics' second law analysis, is the gateway for optimisation studies in thermal equipments and systems. Bejan [1] and San et al. [2] proposed different analytical solutions for the entropy generation equation in several simple flow situations. They introduced the concept of entropy production number, irreversibility distribution ratio, and presented spatial distribution profiles of entropy generation. Bejan presented an analytical solution to the partial differential equation of the entropy production encountered in convective heat transfer for different configurations, where San et al. investigated the irreversible entropy generation for combined heat and mass transfer. Based on the concept of efficient exergy use and the minimal entropy generation principle, optimal designs of thermodynamic systems were widely proposed by the thermodynamic second law [3]. Khan and EL-Ghalban [4] showed that second law optimization proved to be a better method to optimize a shell and tube heat exchanger. Through entropy minimization techniques, it is possible to increase the efficiency and overall performance of all kinds of thermal systems.

Several studies thoroughly dealt with fluid flow irreversibility due to viscous effect and heat transfer

by conduction. However, the evaluation of entropy production due to radiative heat transfer in participating media was often overlooked. Arpaci [5,6] and Arpaci and Selamet [7-8] evaluated the radiative entropy production by analogy to that due to conduction, which is true in the case of an optically thick medium. In their study of entropy generation due to interaction of the radiative field with solid boundaries, Wright et al [9] extended and generalized this formalism for grey and non participating media. Recently, Caldas and Semiao [10,11] adapted this approach for non grey and participating media. The procedure adopted is completely compatible with standard radiative heat transfer calculation in engineering systems like the discrete ordinates method. They concluded that entropy generated through emission and absorption is much larger than that generated through scattering. Their studies are extended by Liu and Chu [12] to analyse the radiative entropy generation in the enclosures filled with semitransparent media. They declared that the numerical simulation method presented by Caldas and Semiao can be used in the entropy generation analysis of high temperature systems such as boilers and furnaces, in which radiation is the dominant mode of heat transfer. Zhang and Basu [13] lately investigated entropy flow and generation in radiative transfer between nonideal surfaces when multiple reflections are included. They showed that several approximate expressions found in the literature can result in

significant errors in entropy analysis even for diffuse-grey surfaces. Recently, Ben Nejma et al [14] established a numerical computation of combined gas radiation and forced convection. They gave special attention to entropy generation and its dependency on geometrical and thermodynamic parameters for a laminar flow of over heated water vapour, through two parallel plates.

The present paper is aimed at inquiring about interaction of thermal radiation and forced convection for non grey gas inside cylindrical pipes. The analysis of heat transfers and channel flows inside cylindrical ducts have been studied by many investigators [15-23] and are important issues for applications such as heat exchangers and cooling processes in nuclear reactors. A considerable number of reports were accumulated since radiation heat transfer became more and more important in engineering applications where higher temperatures and higher designing accuracies are required to improve the performance of power systems. In such systems, heat transfer results from coupled processes, in general, cannot be calculated separately. During the past, a number of experiments and numerical computations were studied by many researchers for describing the radiation effects on heat transfer by simultaneous convection, conduction and radiation. One of the classical studies of these investigations is conducted by Chen [24] to report the study of laminar heat transfer in a tube with nonlinear radiant heat-flux boundary conditions. Under the assumptions of fully developed laminar velocity distribution and constant thermophysical properties, Desoto [25] considered the interaction of two-dimensional radiation with conduction and convection in a non-isothermal, non-grey gas flowing in the entrance region of a tube with isothermal black walls by solving the integro-differential energy equation numerically. Pearce and Emery [15] worked on heat transfer by thermal radiation and laminar forced convection to an absorbing fluid in the entry region of a pipe. Using the finite difference technique, Echigo et al [26] examined the combined transfer with two-dimensional radiative analysis in a grey gas allowing an upstream propagation from the entrance of the hot tube wall. They concluded that the local Nusselt numbers and the mixed mean temperatures obtained by the 1-D analysis are lower than those obtained by 2-D one near the entrance of the heating section in the region down-stream the wall temperature jump. Wassel and Edwards [27] analyzed the influence of the molecular radiation in a laminar or turbulent pipe flow. Azad and Modest [28] presented a numerical study of combined radiation and convection in absorbing emitting and anisotropically scattering gas-particulate tube flow. Campo and Schuler [29] investigated the

study of thermal radiation and laminar forced convection in a gas pipe flow. Using an element-to-node approach for a hydrodynamically fully developed flow, Kassemi and Chung [30] studied the two-dimensional combined convection and radiation transfer from a grey isotropically scattering fluid in a reflecting channel. Kim and Lee [31] analysed the two-dimensional relation in a thermally developing Poiseuille flow of a grey and anisotropically scattering fluid between infinite plane parallel plates. The radiative part of the problem was solved by using the  $S_n$  discrete ordinates method. It was found that the two-dimensional effects are more pronounced when the channel optical thickness increases and when the conduction-radiation parameter and the scattering albedo decrease. Yang and Ebdian [18] studied thermal radiation and laminar forced convection in the entrance region of a pipe with axial conduction and radiation. They treated the 2-D thermal radiation for established laminar flow of a grey gas. Seo et al [32] extended their researches for combined convection and radiation in simultaneously developing flow and heat transfer with non-grey gas mixtures. Kamiuoto [33] applied carbon dioxide as non grey gas to analyze the combined laminar forced convection and thermal radiation heat transfer in a non black plane-parallel duct.

In this paper, special attention is devoted to the treatment of the spectral nature of radiation for over heated water vapor. The non-grey behavior of such gases, the multidimensional nature of radiative transfer and the resulting coupling between different volume elements in the medium are the main fundamental difficulties introduced in the problem of combined heat transfer for these media. The radiative transfer coefficients of the temperature-dependent medium are given by solving the "Radiative Transfer Equation" (RTE). Different kinds of numerical methods have actually been developed in order to solve this equation. Modest [34] cited most of these methods such as the spherical harmonic method, the zonal method, Monte Carlo method and the "discrete ordinate method (DOM)". The latter, which received great attention during the last few years, was proposed by Chandrasekhar [35] and later developed by Truelove [36] and Carlson and Lathrop [37]. Like the DOM, the "Ray Tracing" method [38], applied in our study through the  $S_n$  quadratures, is based on the selection of transfer directions and their associated weights. When the flowing medium is a radiating molecular gas, its complex absorption and emission spectra introduce an important difficulty in the simulation of these flows. In this case, the radiative properties of the medium vary generally so strongly and rapidly

across the spectrum and the grey gas approximation is today believed to be inaccurate [34]. Many approximate radiative property models were developed to represent molecular spectra of gases. While "line by line (LBL)" models are useful for providing benchmark solutions, they are not practical in multidimensional applications particularly when the radiative transfer is combined with other transfer modes, since it requires great computational times and storage volumes. Several other models were introduced to calculate radiative properties of gases like the "statistical narrow band (SNB)" [39,40], "correlated-k (CK)" [41], "Correlated-k with fictitious gases (CKFG)" [42,43], "exponential wide-band (EWB)" [44], "spectral line-based weighted-sum-of-grey-gases (WSGG)" model [45,46] and "absorption distribution function (ADF)" [47]. In the absence of LBL results, the SNB model is frequently considered as the most accurate non-grey gas radiation model. However, the implementation of this model into the RTE [48,49] has some shortcomings as it provides transmissivity instead of absorption coefficients [50] and contains a correlation term which requires a long computing time [51]. Initially developed for atmospheric modeling applications, the CK model has the advantage over the SNB model in that scattering effects can be accounted for. The implementation of the SNBCK model for radiation computation previously described by Liu et al [52] represents an effective approach to avoid the expensive on-line inversion of the cumulative distribution function [41,53,54]. In this case, the distribution function is obtained by inverse Laplace transformation of the SNB gas transmissivity in order to deduce gas absorption coefficients.

For any radiative quantity  $\bar{\phi}_v$  which depends on absorption coefficient, the base of this method is that the integration over wave number can be replaced by integration over the absorption coefficient.

$$\bar{\phi}_v = \frac{1}{\Delta\nu} \int_{\Delta\nu} \phi(\kappa_v) d\nu = \int_0^\infty f(k) \phi(k) dk \quad (1)$$

$$\text{Where } f(k) = \frac{1}{\Delta\nu} \frac{d\nu}{dk} \quad (2)$$

is defined as the normalized distribution function of the gas absorption coefficient inside  $\Delta\nu$  and  $f(k)dk$  represents the fraction of wavenumber inside  $\Delta\nu$ , where the gas absorption coefficient lies between  $k$  and  $k + dk$ . The distribution function  $f(k)$  in this

technique is first obtained by inverse Laplace transformation of the SNB gas transmissivity.

$$\bar{\tau}_v(L) = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-\kappa_v L) d\nu = \int_0^\infty f(k) \exp(-\kappa L) dk \quad (3)$$

In the SNB model, the narrow-band gas transmissivity over an isothermal and homogeneous path-length  $L$  containing gas [48] is given as

$$\bar{\tau}_v(L) = \exp\left[-\frac{\pi B}{2} \left(\sqrt{1 + \frac{4AL}{\pi B}} - 1\right)\right] \quad (4)$$

The analytical expression of the cumulative function defined as  $g(k) = \int_0^k f(t) dt$  has been derived by Lacis and Oinas [41] as

$$g(k) = \frac{1}{2} \left[ 1 - \operatorname{erf}\left(\frac{a}{\sqrt{k}} - b\sqrt{k}\right) \right] + \frac{1}{2} \left[ 1 - \operatorname{erf}\left(\frac{a}{\sqrt{k}} + b\sqrt{k}\right) \right] \exp^{\pi B} \quad (5)$$

where,  $a = \frac{1}{2} \sqrt{\pi AB}$ ,  $b = \frac{1}{2} \sqrt{\frac{\pi B}{S}}$  and  $\operatorname{erf}$  is the error function given as :

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \quad (6)$$

Using the cumulative function  $g$ , the narrow-band average of any radiative variable  $\phi_v$  can be calculated as

$$\bar{\phi}_v = \frac{1}{\Delta\nu} \int_{\Delta\nu} \phi(\kappa_v) d\nu = \int_0^1 \phi(g) dg \quad (7)$$

The last equation can be conveniently calculated using a Gauss type quadrature scheme:

$$\bar{\phi}_v = \sum_{i=1}^n w^i \phi(g^i) \quad (8)$$

The calculation of the various monochromatic absorption coefficients is thus reduced to the resolution of the system:

$$g(K_v^i) = g^i \quad (9)$$

Ben Nejma et al [38] applied the SNB method in rectangular enclosures and Sediki et al [20,21] used the ADF and the correlated-k band models in cylindrical enclosures. They all could quantify radiative transfers, but did not mention entropy production. In their recently study of combined non grey-gas radiation and forced convection through two parallel plates, Ben Nejma et al [14] presented a numerical computation using the finite volume technique and the 1-D radiative analysis. Applying the Ray Tracing method through  $S_4$  directions in association with the SNBCK model, they

distinguished themselves when they gave special attention to radiation effects on local and total entropy generation. Similar numerical calculations are performed in this paper for absorbing and emitting media, but in the entrance region of a cylindrical duct.

## 2 Problem Formulation

In the present study, we consider a stationary flow of an emissive, absorbent and non grey-gas (H<sub>2</sub>O) inside an isothermal and cylindrical duct as shown in fig.1. The mass, momentum and energy balance equations written in cylindrical coordinates will be:

$$\frac{\partial(\rho.r.u_r)}{\partial r} + \frac{\partial(\rho.r.u_z)}{\partial z} = 0 \quad (10)$$

$$\rho.(u_z \cdot \frac{\partial u_z}{\partial z} + u_r \cdot \frac{\partial u_z}{\partial r}) = -\frac{dP}{dz} + \frac{1}{r} \cdot \frac{\partial}{\partial r} (\mu.r \cdot \frac{\partial u_z}{\partial r}) \quad (11)$$

$$\rho.Cp.(u_z \cdot \frac{\partial T}{\partial z} + u_r \cdot \frac{\partial T}{\partial r}) = \frac{1}{r} \cdot \frac{\partial}{\partial r} (\lambda.r \cdot \frac{\partial T}{\partial r}) - \text{div}(\vec{q}_r) \quad (12)$$

$$\text{div}(\vec{q}_r) = \sum_{\text{bands}_{4\pi}} \int_{i=1}^4 w^i \cdot \kappa_v^i (I_v^b - I_v^i(\vec{\Omega})) d\Omega \Delta v \quad (13)$$

The boundary conditions for the considered problem are summarized by the system (5):

$$\begin{cases} v(z=0, r) = 0 & ; v(z, r=R) = 0 & ; v(z, r=0) = 0 \\ u(z, r=R) = 0 & ; u(z=0, r) = U_0 \\ T(z, r=R) = T_w & ; T(z=0, r) = T_0 \end{cases} \quad (14)$$

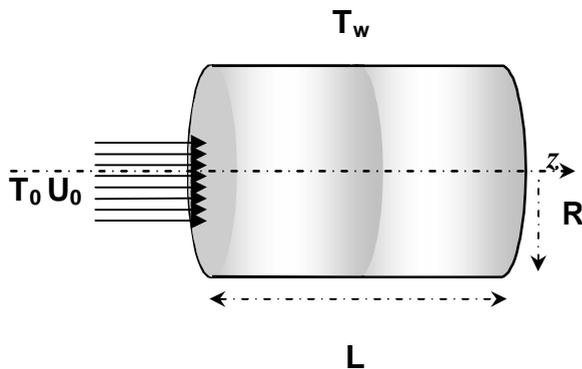


Fig.1. Geometry and boundary conditions.

## Nomenclature

(A,B)	SNB model parameters
c	light velocity (ms <sup>-1</sup> )
Cp	specific heat (J.kg <sup>-1</sup> .K <sup>-1</sup> )
h	Plank's constant (h=6.626 10 <sup>-34</sup> J.s)
R	radius of the duct (m)
L	channel length (m)
I	radiation intensity (W.m <sup>-2</sup> .sr <sup>-1</sup> )
k	Boltzmann's constant (k=1.38 10 <sup>-23</sup> JK <sup>-1</sup> )
P	pressure (Pa)
u	axial velocity (m.s <sup>-1</sup> )
v	transverse velocity (m.s <sup>-1</sup> )
s	local entropy production (W.K <sup>-1</sup> m <sup>-3</sup> )
S	global entropy production (W.K <sup>-1</sup> )
T	temperature (K)
w	weight parameter
q	heat flux (W.m <sup>-2</sup> )
r, z	cylindrical coordinates

## Greek symbols

κ	absorption coefficient (m <sup>-1</sup> )
v	wave number (cm <sup>-1</sup> )
τ	gas transmissivity
ε	wall emissivity
λ	thermal conductivity (W.m <sup>-1</sup> .K <sup>-1</sup> )
μ	dynamic viscosity (N.s.m <sup>-2</sup> )
$\vec{\Omega}$	ray direction
dΩ	elementary solid angle around $\vec{\Omega}$
ρ	density (kg.m <sup>-3</sup> )
Δv	spectral resolution (cm <sup>-1</sup> )

## Subscript

v	spectral
r	radiative exchange
c	conductive exchange
w	wall
0	ambient

## Superscript

i	partial non grey medium
b	black body

Like all the thermodynamic systems, entropy production is generated from the irreversibility due to heat transfer with finite temperature gradients and the friction of fluid flow. At a given location, the local entropy generation is given as:

$$s(r, z) = s_c(r, z) + s_r(r, z) + s_f(r, z) \quad (15)$$

where  $s_f(r, z)$  is the local entropy generation due to friction [55],  $s_c(r, z)$  is the local entropy production

due to conduction [56] and  $s_r(r, z)$  is the local entropy generation due to radiation [10,11,12,14].

$$s_f(r, z) = \frac{\mu}{T} \cdot \Phi \approx \frac{\mu}{T} \left( \frac{\partial u}{\partial r} \right)^2 \quad (16)$$

Where  $\Phi$  is the viscous dissipation  
Irreversibility due to viscous effect is neglected compared to entropy production due to heat transfer.

$$s_c(r, z) = \frac{\lambda(T)}{T^2} (\nabla T)^2 \approx \frac{\lambda(T)}{T^2} \left( \frac{\partial T}{\partial r} \right)^2 \quad (17)$$

For the radiative part of the problem, Caldas and Semiao [10,11], Liu and Chu [12] and Ben Nejma et al [14] used the following expression:

$$s_{r,v}(\bar{\Omega}) = -\kappa_v \left[ I_v^b(T) - I_v(\bar{\Omega}) \right] \left[ \frac{1}{T} - \frac{1}{T_v(\bar{\Omega})} \right] \quad (18)$$

where  $T_v(\bar{\Omega}) = \frac{h\nu}{K \cdot \ln \left[ \frac{2h\nu^3}{c^2 \cdot I_v(\bar{\Omega})} + 1 \right]}$  represents

the directional and spectral radiative temperature.  
Using the SNBCK4 method, the local radiative entropy production for non grey gases is:

$$s_r(r, z) = - \sum_{\text{bands}, \pi} \int \sum_{i=1}^4 w_i \kappa_i \left[ I_v^b(T) - I_v(\bar{\Omega}) \right] \left[ \frac{1}{T} - \frac{1}{T_v(\bar{\Omega})} \right] d\Omega dV \quad (19)$$

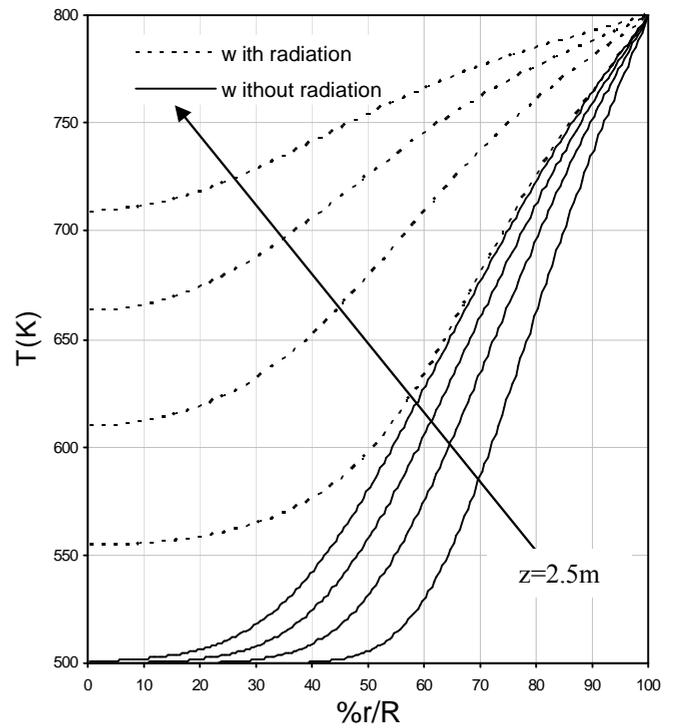
We note that the scattering effect is neglected compared to absorption and emission in gas radiation since the medium contains no particles.  
Therefore the total entropy generation is obtained by integrating Eq.(15)

$$S_{0-z} = 2\pi \int_0^R \int_0^L r s(r, z) dr dz \quad (20)$$

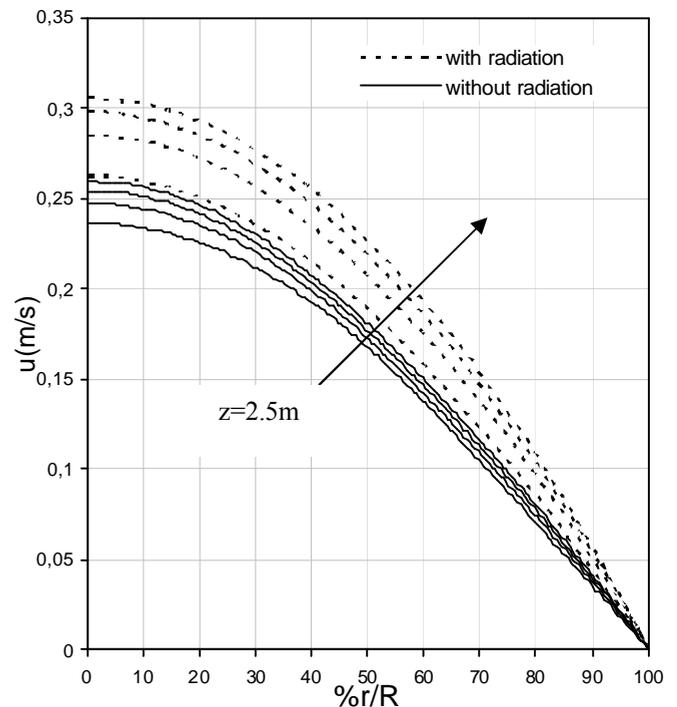
### 3 Problem Solution

A selected set of graphical results are presented in Figs.2-9 to provide the radiative contribution of overheated water vapour through the duct and to give a good understanding on the change in local and total entropy generation according to the main parameters that define the flow. Graphics printed in Fig.2 and Fig.3 mention the effect of thermal radiation on flow rate in combined heat transfer, for heating and cooling configurations. This can be explained in these figures where temperature and axial velocity are plotted with radial position, for different transverse location through the duct. We can observe that radiation permits direct exchanges with the central zone of the flow, accelerating their movement and reducing friction with the particles close to the walls which were strongly accelerated due to the conductive effect. The rapidity of this

effect is due to interaction between fluid particles and also interaction between fluid and boundaries.



(a)

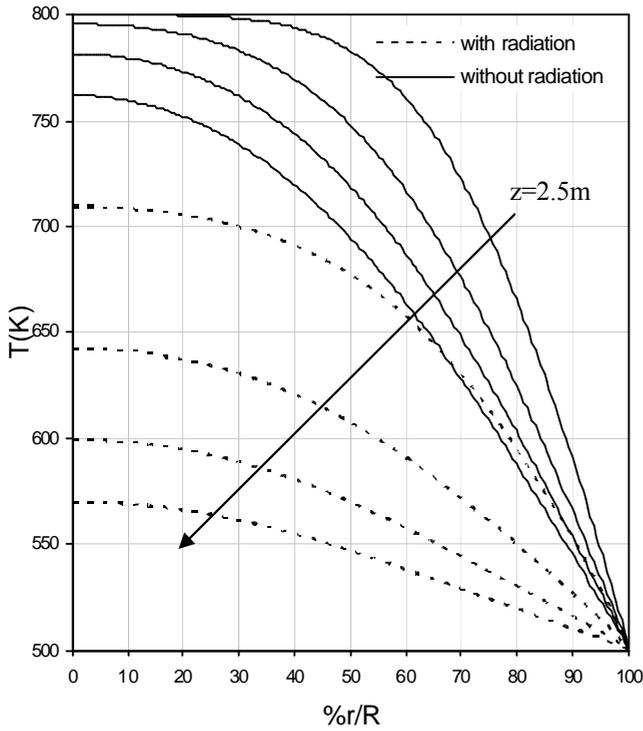


(b)

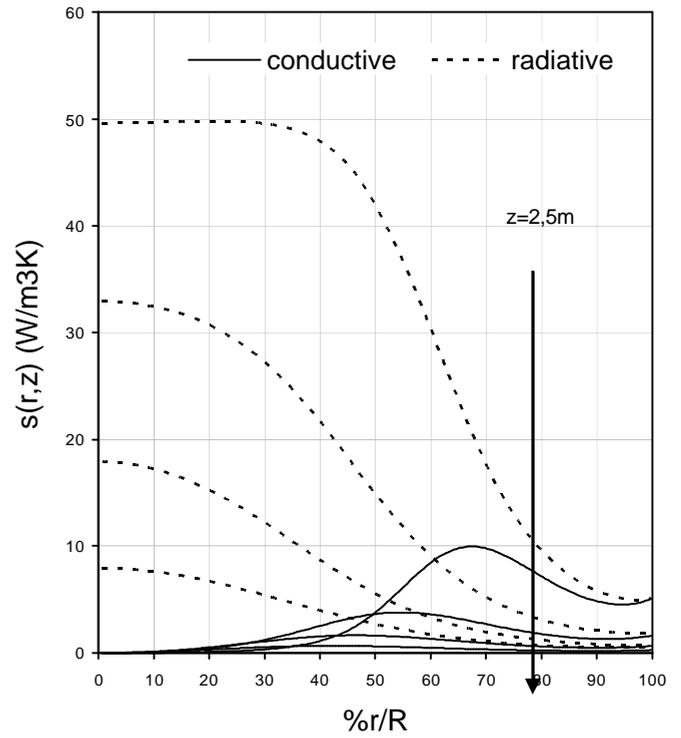
Fig.2. The influence of radiation in case of heating.

$L=10m$   $R=5cm$   $U_0=0.1m/s$   $p=1$   $\epsilon=1$   $T_w=800K$   $T_0=500K$

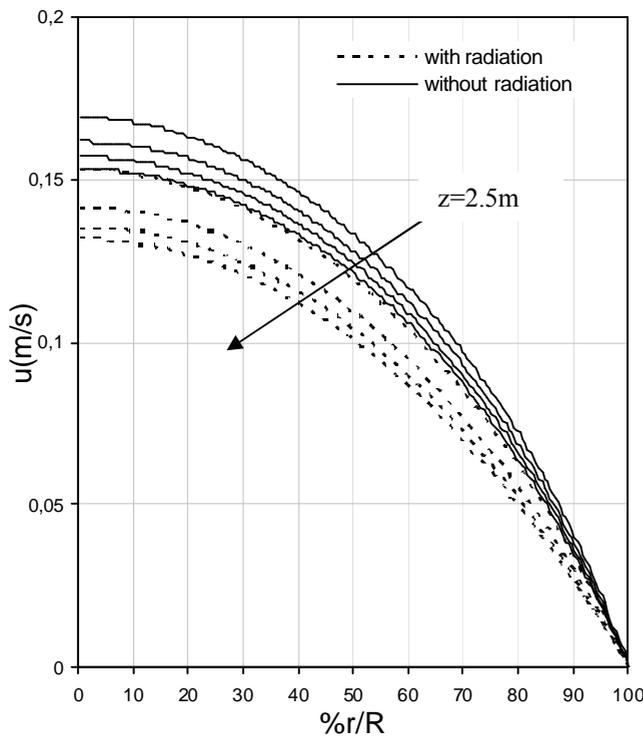
$z=2.5m - z=5m - z=7.5m - z=10m$



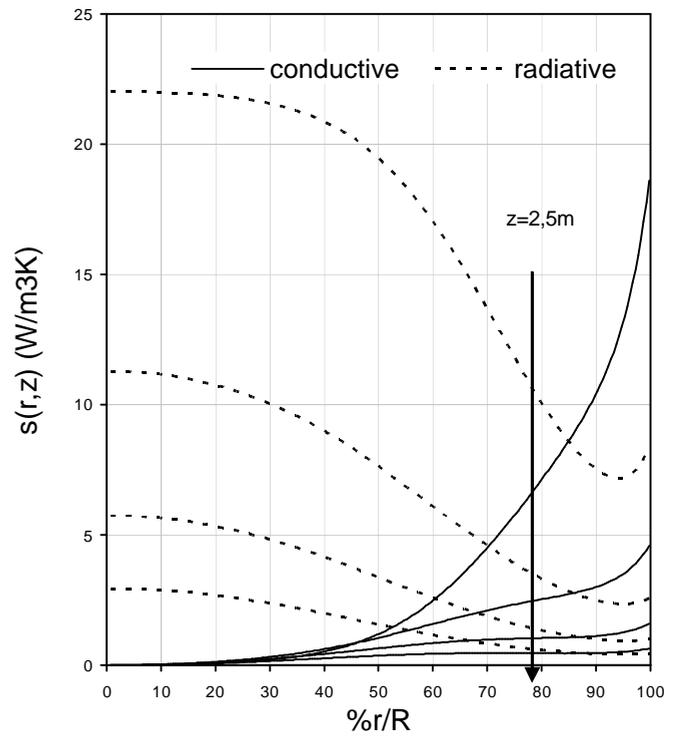
(a)



(a)  $T_w=800K$   $T_0=500K$



(b)



(b)  $T_w=500K$   $T_0=800K$

Fig.3. The influence of radiation in case of cooling.

$L=10m$   $R=5cm$   $U_0=0.1m/s$   $p=1$   $\epsilon=1$   $T_w=500K$   $T_0=800K$

$z=2.5m - z=5m - z=7.5m - z=10m$

Fig.4. Comparison between conductive and radiative transfer effects on local entropy generation.

$L=10m$   $R=5cm$   $U_0=0.1m/s$   $p=1$   $\epsilon=1$

$z=2.5m - z=5m - z=7.5m - z=10m$

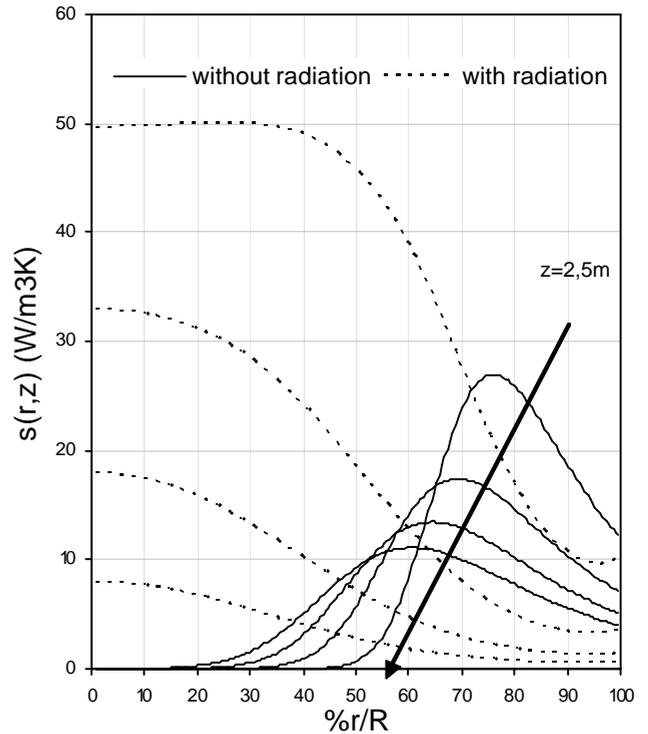
In Fig.4, we can show the variations of radiative and conductive fields of local entropy generation, while Fig.5 illustrates the profiles of local entropy generation with and without radiation. We note in both of these figures the important effect of radiation on local entropy production, which is distinctly developed for sections close to the entrance region of the channel, where transfers are very important. This can be explained by the fact that radiation enhances heat transfer and accelerates the establishment of the flow. Compared to conduction effect, the local entropy profiles due to radiation are seen to be greater for heating configuration. However, this phenomenon is inversed in the vicinity of the walls in case of cooling.

Fig.6 illustrates the effect of wall temperature on entropy production in heating and cooling configurations. As plotted in this figure, increasing the difference temperature between gas and boundaries amplifies the entropy generation which is more pronounced in heating conditions.

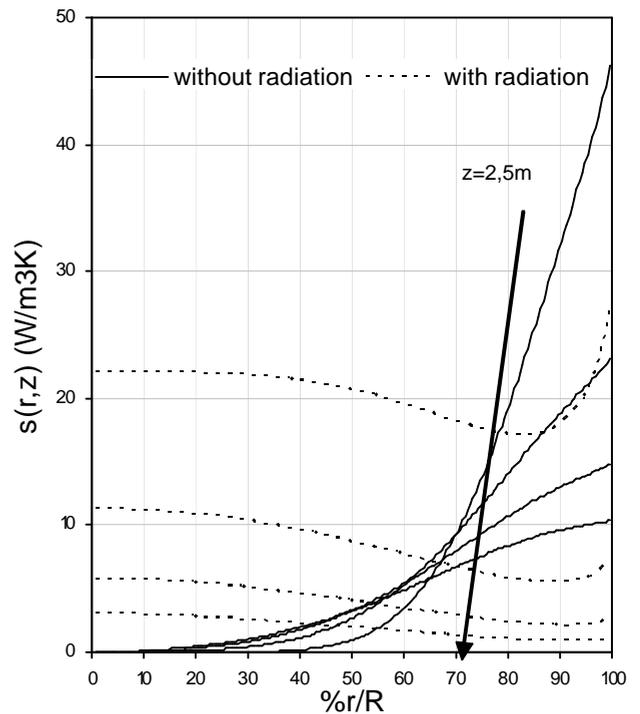
In Fig.7, the variations of total entropy generation are given for different radii. It can be seen that entropy production is considerably affected by this parameter, especially in heating mode where boundary emission is very important compared to gas emission, increasing the total entropy creation with the pipe radius.

The effect of the flow rate on entropy generation is presented in Fig.8 in heating and cooling cases. The difference between boundary temperature and the bulk temperature of the fluid is affected by the velocity flow. The importance of this temperature gradient favours heat transfers causing a greater entropy generation.

Fig.9 illustrates the effect of pressure on the distribution of total entropy production. As shown in heating and cooling configurations, increasing the pressure, results in a significantly higher entropy generation. In fact, increasing pressure decreases gas transmissivity and amplifies its absorption coefficient improving radiative transfers, and thus raising entropy production.



(a)  $T_w = 800K$   $T_0 = 500K$

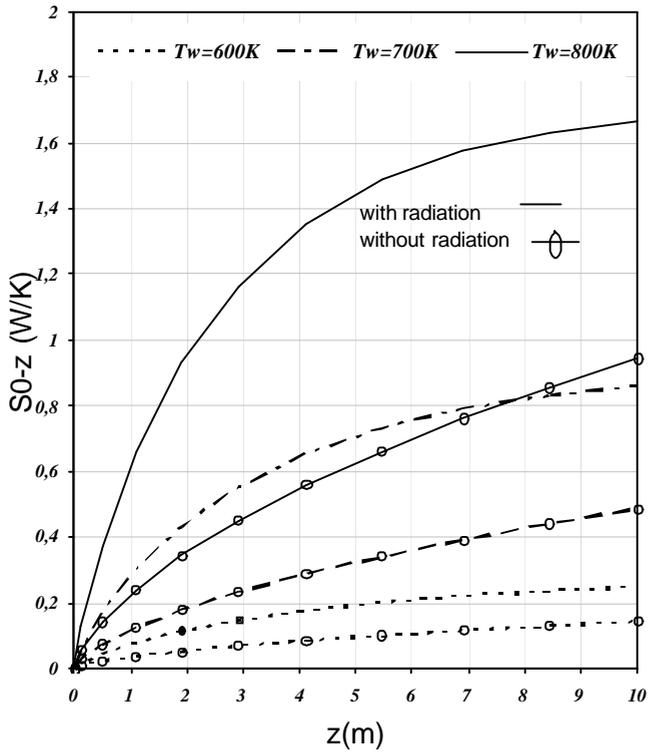


(b)  $T_w = 500K$   $T_0 = 800K$

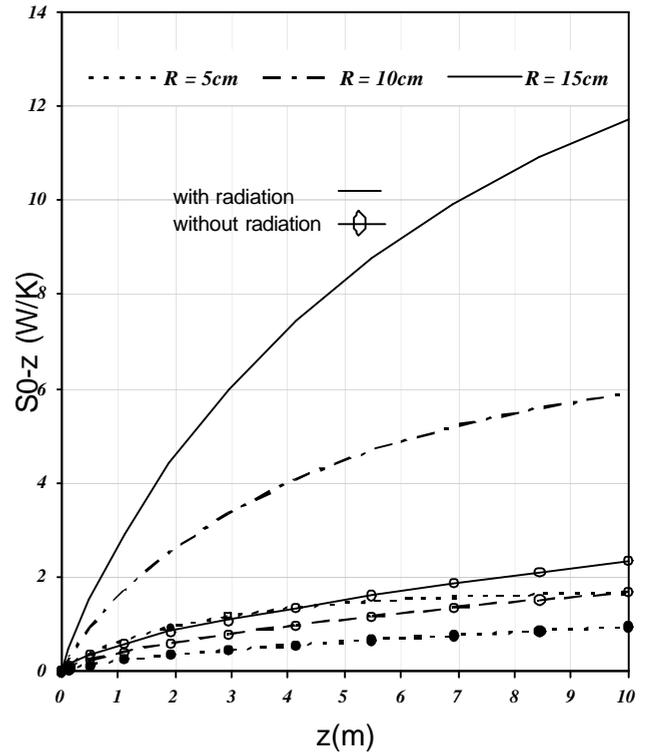
Fig.5. Evolution of local entropy generation with and without radiation.

$L=10m$   $R=5cm$   $U_0=0.1m/s$   $p=1$   $\epsilon=1$

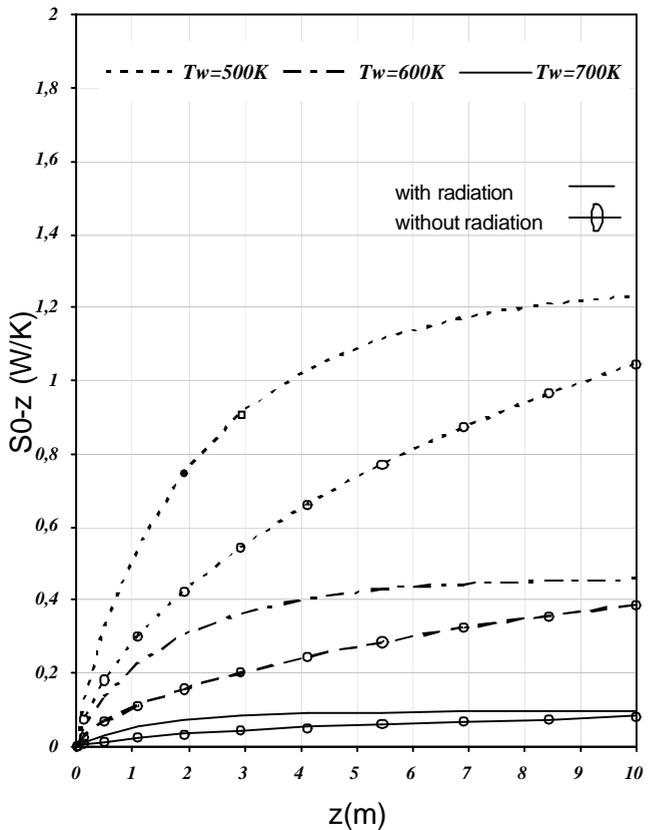
$z=2.5m - z=5m - z=7.5m - z=10m$



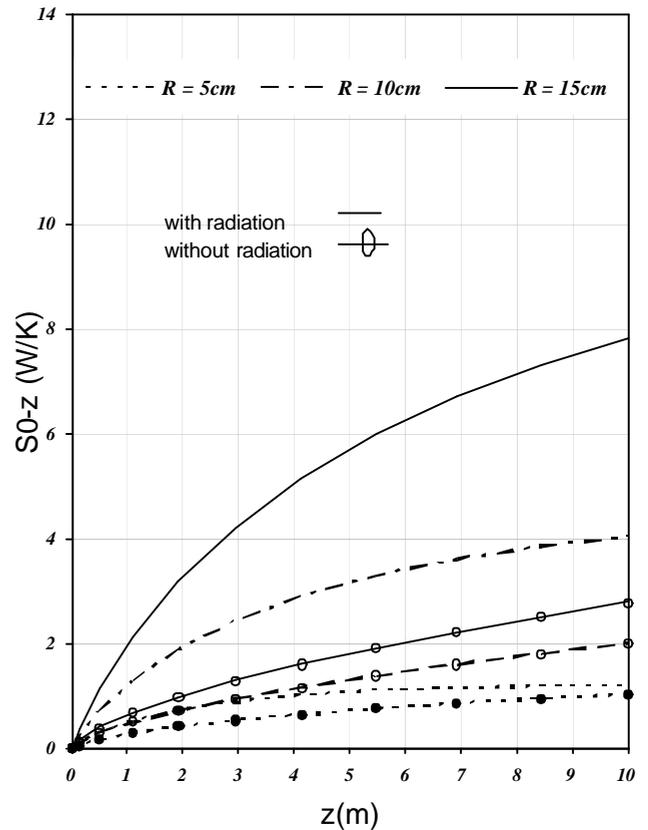
(a)  $R=5\text{cm}$   $U_0=0.1\text{m/s}$   $T_0=500\text{K}$   $p=1$   $\epsilon=1$



(a)  $U_0=0.1\text{m/s}$   $T_w=800\text{K}$   $T_0=500\text{K}$   $p=1$   $\epsilon=1$



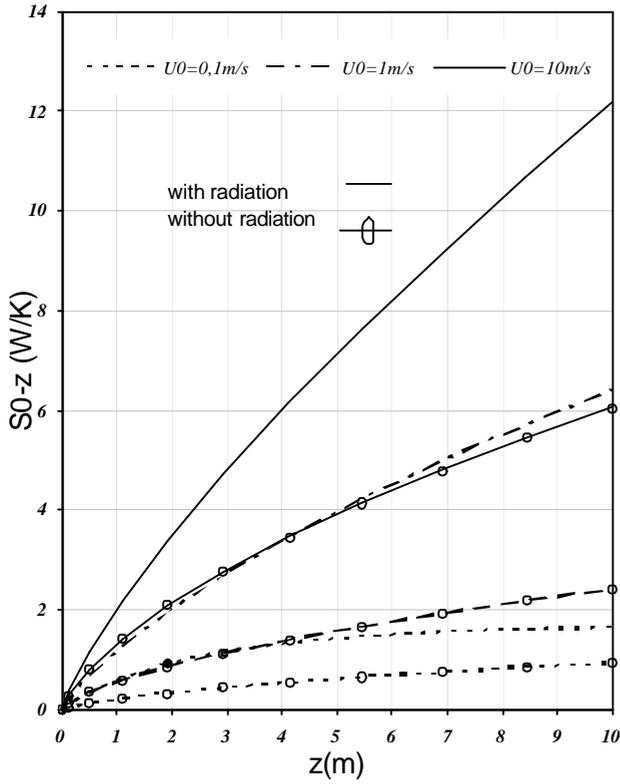
(b)  $R=5\text{cm}$   $U_0=0.1\text{m/s}$   $T_0=800\text{K}$   $p=1$   $\epsilon=1$



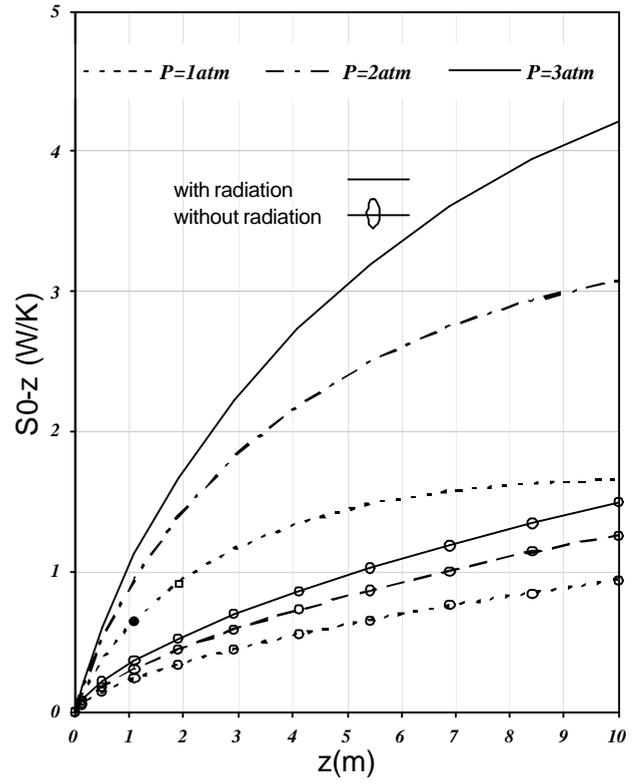
(b)  $U_0=0.1\text{m/s}$   $T_w=500\text{K}$   $T_0=800\text{K}$   $p=1$   $\epsilon=1$

Fig.6. Effect of wall temperature on global entropy generation.

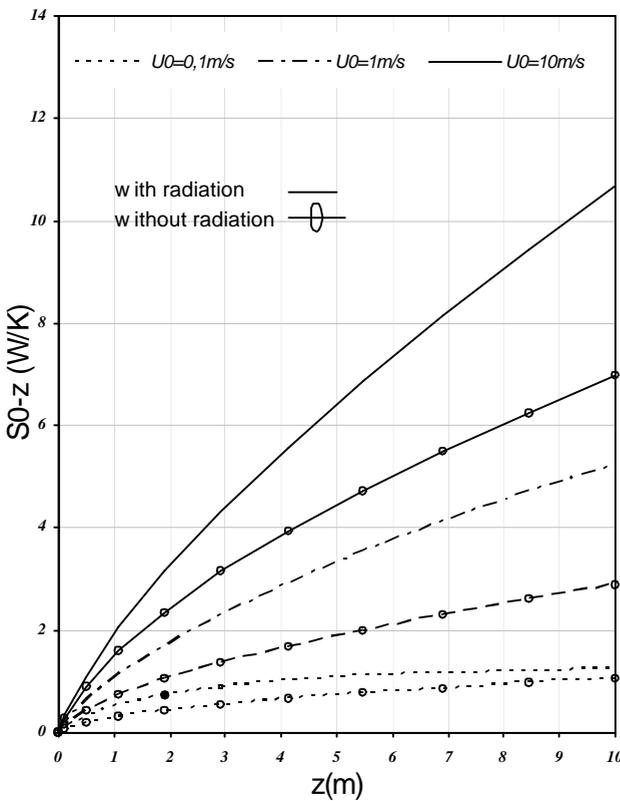
Fig.7. Effect of duct radius on global entropy generation.



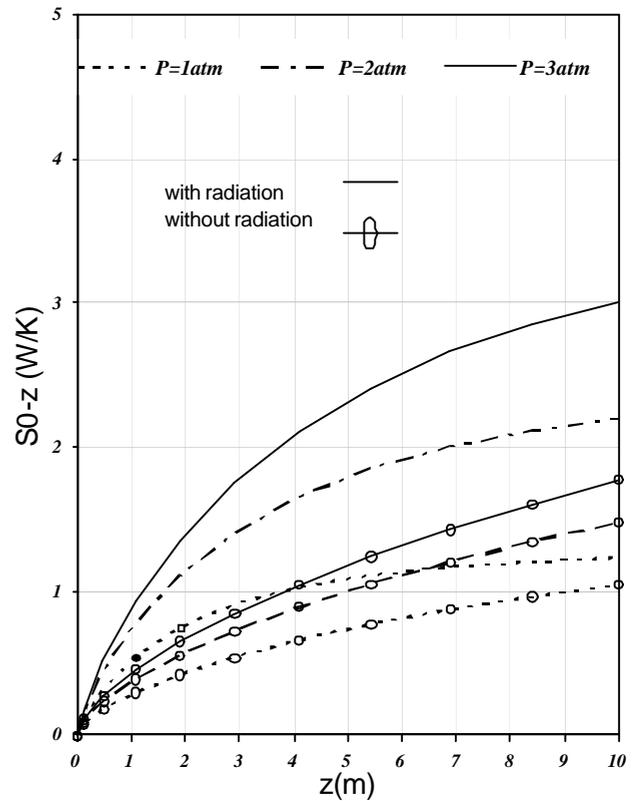
(a)  $R=5\text{cm}$   $T_w=800\text{K}$   $T_0=500\text{K}$   $p=1$   $\epsilon=1$



(a)  $R=5\text{cm}$   $U_0=0.1\text{m/s}$   $T_w=800\text{K}$   $T_0=500\text{K}$   $\epsilon=1$



(b)  $R=5\text{cm}$   $T_w=500\text{K}$   $T_0=800\text{K}$   $p=1$   $\epsilon=1$



(b)  $R=5\text{cm}$   $U_0=0.1\text{m/s}$   $T_w=500\text{K}$   $T_0=800\text{K}$   $\epsilon=1$

Fig.8. Flow rate effect on global entropy generation.

Fig.9. Effect of pressure on global entropy generation.

## 4 Conclusion

In this paper, a numerical computation of entropy generation due to interaction between thermal radiation and forced convection in participating media and through cylindrical duct is investigated. The entropy production profiles are illustrated with a variety of boundary conditions. Based on the results, the following conclusions can be made:

1. The radiative contribution permits to considerably increase heat transfers in the central zone of the flow.
2. Increasing the wall's temperature gives an advantage to entropy generation.
3. The greater the pipe's radius is, the higher entropy production is.
4. Radiative entropy generation is reduced when decreasing the flow rate.
5. The use of a lower pressure results in a significantly lower entropy production.

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