The stability of the operating parameters of rocket engine with solid propellant for low pressure

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Abstract: - The aim of this paper consists in developing a model for realistic calculation, but at the same time not a very complicated one, in order to determine the operating parameters of a rocket engine with solid propellant (RESP). The model results will be compared with experimental results and the quality of the model will be evaluated. The study of operating stability RESP will be made accordingly to Liapunov theory, considering the system of parametric equations perturbed around the balance parameters. The methodology dealing with the stability problem consists in obtaining the linear equations and the verification of the eigenvalue of the stability matrix. The results are analyzed for a functional rocket engine at low pressure, which has the combustion chamber made of cardboard, engine used for anti-hailstone rocket. Two representatives cases will be take in to consideration and analyzed from the point de view of burn parameters, one of them for stable conditions and other for instable conditions. For both cases a complete analysis will be done and the results will be compared. The novelty of the work lies in the technique to tackle the stability problem for the operation of rocket engines at low pressure, many of them representing specific applications for civil destination.

Key-Words: - Rocket, Engine, Solid propellant, Stability, Liapunov theory, Low pressure, Anti-hailstone

NOMENCLATURE

α - Specific volume (reverse remaining gas density);
γ - Ratio between igniter gas mass and the propellant mass;
δ - Propellant density;
k - Specific heat ratio (adiabatic gas coefficient);
λ - Ratio between velocity in exit plane and velocity in throat area;
ν - Pressure exponent for burning law;
ψ - Ratio between propellant mass consumed and total propellant mass;
η - Ratio between exhaust gas mass flow and total propellant mass;
θ - Coefficient which express the flow losses in the nozzle;
φ - Erosion factor;
τ - Ratio between instantaneous gas temperature and gas temperature for burning in constant volume;
σ - Ratio between instantaneous burning surface and initial burning surface;
σ_T - Ratio between instantaneous propellant cross surface and initial propellant cross surface;
σ_C - Overall loss of thrust by nozzle,
ω - Propellant mass;
D - Coefficient of variation of burning rate with initial propellant temperature:
F_{cam} - Combustion chamber cross surface;
F_{cr} - Flow area at the throat;
F - Flow area at nozzle exit plane;
I_Σ - Total impulse;
p - Pressure in combustion chamber;
p_H - Atmospheric pressure;
P - Engine thrust;
q - Amount of heat transferred to the combustion chamber in time unit (heat flow);
Q_C - Heat quantity educts by burning reaction of 1

kilogram propellant;
R - Gas constant;
S - Instantaneous burning surface;
S_T - Instantaneous propellant cross surface;
T - Gas temperature;
T_{0N} - Normal propellant temperature for burning rate;
T_{in} - Initial propellant temperature;
T_{0V} - Gas temperature for burning in constant volume;
u - Linear burning rate;
u_{1n} - Linear burning rate in normal conditions:
U - Energy;
W_{cam} - Burning chamber volume;

1 Introduction

Using missiles into civilian area involve a series of specific measures for compliance with environmental restrictions like a greater degree of safety in operation, and person's protection.

An example of such an application is the anti-hailstone rocket, which has an engine made of cardboard, ecological, non-hazardous but with low operating pressure. This type of technical problem causes the need for a scientific approach to support the technological effort of achieving such a missile engine capable of stable operating at low pressure, which is the subject to approach in this work.

Determining the functional parameters and analyzing the stability are one of the main challenges in designing rocket engine solid propellant - RESP.

The problems of stability of combustion can be addressed by different ways both experimental and theoretical, a series of methods and models being shown in the works [4], [5], [6], [7]. Note that the paper [4] proposes a different approach of stability for linear and non-linear phenomena. Unlike this, in our work the approach will be unitary, being focused on a particular and difficult case, that of the combustion of low pressure.

In our study we will develop a non linear model for calculus of the functional parameters of RESP, followed by the analysis of the evolution of balance stability regarded as the basic movement. Stability analysis for the perturbed equations of the RESP will be made according to Liapunov theory, by placing them in the linear form.

Remember that Liapunov theory said "If we can prove that linear form of the equations system is stable then its initial non-linear form is also stable"

Resuming, our work has two purposes:
- Scientific one – to check the possibility of applying Liapunov theory [9] to analyze the stability of the balance parameters of RESP at low pressure.
- Technical one – to design the rocket engine for the anti-hailstone rocket

2 Parametric equations of RESP

As shown in the works [1], [2], the basic differential equations that allow the determination of the main parameters for a RESP are composed of the propellant consumption equation, the equation of exhaust gas combustion and the equation of the temperature based on a balance of energy in the combustion chamber.

To these three main equations, we may add a fourth, which, starting from the thrust allows determination by integrating the total impulse.

The first equation describes how the propellant is being consumed by the combustion engine:

$$\dot{\psi} = S(\psi)u\delta/\omega . \quad (1)$$

where the linear burning rate is determined by the relationship:

$$u = \varphi(x)\,\tilde{p}^{\upsilon} u_{1N} e^{D(T_{in}-T_N)} , \quad (2)$$

where the erosion factor has been denoted with $\varphi(x)$ -.

Parameters υ and u_{1N} are determined experimentally under normal propellant temperature (T_N), and $e^{D(T_{in}-T_N)}$ shows the influence of the variation of the initial propellant temperature. Exponent D is determined experimentally.

To calculate the burning rate we use the ratio pressure, given by the relationship:

$$\tilde{p} = p / p_H . \quad (3)$$

Finally, to assess the erosive phenomenon we use the parameter named in [1] "Pobedonosetov" parameter:

$$x = \left(S - S_T\right)/\left(F_{cam} - S_T\right), \quad (4)$$

which allows us to determine the erosion factor:

$$\varphi(x) = \begin{cases} 1 + 3{,}2\times10^{-3}(x-100) & for\ x > 100; \\ 1 & for\ x \le 100 \end{cases} . \quad (5)$$

To define the change of the burning area and propellant cross-section the quadratic fitting may be used:

$$\sigma(\psi) = \begin{cases} a_3\psi^2 + a_2\psi + a_1 & for\ \psi < 1; \\ 0 & for\ \psi \ge 1 \end{cases} ; \quad (6)$$

$$\sigma_T(\psi) = \begin{cases} b_3\psi^2 + b_2\psi + b_1 & for\ \psi < 1; \\ 0 & for\ \psi \ge 1 \end{cases} ; \quad (7)$$

$$S(\psi) = S_0\sigma(\psi); \quad (8)$$

$$S_T(\psi) = S_{T0}\sigma_T(\psi). \quad (9)$$

The following equation, which is the flow equation, expresses how it discharges the gas from the engine:

$$\dot{\eta} = \frac{\theta A F_{cr} p}{\omega\sqrt{RT_{0V}\tau}}; \quad (10)$$

where we denoted with θ the coefficient which expresses the flow losses in the nozzle. The non dimensional quantity A is given by:

$$A = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2k}{k+1}}\ . \quad (11)$$

The burning temperature at constant volume can be estimated with the relationship:

$$T_{0V} = Q_C / C_V\ , \quad (12)$$

where the specific heat at constant volume can be obtain from the constant gas:

$$C_V = R/(k-1). \quad (13)$$

The third equation expresses the change in temperature of the combustion products. To build it, we start from the following relationship of energy balance:

$$\mathrm{d}U = \mathrm{d}U_1 + \mathrm{d}U_2 + \mathrm{d}U_3 + \mathrm{d}U_4, \quad (14)$$

where the reaction energy of the propellant:

$$\mathrm{d}U = \omega C_V T_{0V}\,\mathrm{d}\psi = \omega R(k-1)^{-1} T_{0V}\,\mathrm{d}\psi \quad (15)$$

is converted into:

- internal energy growth due to additional gas from the combustion chamber:
-

$$\mathrm{d}U_1 = \omega(\mathrm{d}\psi - \mathrm{d}\eta)RT/(k-1); \quad (16)$$

- energy in gas from the combustion chamber increased due to temperature variation:

$$\mathrm{d}U_2 = \omega(\psi - \eta + \gamma)C_V\,\mathrm{d}T\ , \quad (17)$$

the relationship where the ratio between igniter gas mass from the combustion chamber and the propellant mass has been denoted with γ:

$$\gamma = p_0(W_{cam} - \omega/\delta)/(\chi f^*\omega)\ , \quad (18)$$

where f^* is given by the igniter contribution and χ is a loss coefficient (0.96).

- loss of energy due to the disposal of heat through the chamber walls [12]:

$$\mathrm{d}U_3 = q\,\mathrm{d}t\ , \quad (19)$$

where q is the amount of heat transferred to the combustion chamber in time unit (heat flow) $[J/s]$;

- kinetic energy due to gas flow:

$$\mathrm{d}U_4 = k(k-1)^{-1}RT\omega\,\mathrm{d}\eta\,. \quad (20)$$

If we develop the relationship (14) and then we simplify it, we obtain:

$$\dot{\tau} = \{(1-\tau)\dot{\psi} - [(k-1)\tau\dot{\eta} + q/(\omega Q_C)]\}/(\psi - \eta + \gamma), \quad (21)$$

where τ is given by:

$$\tau = T/T_{0V}\ . \quad (22)$$

The ancillary equation allows determination by integrating the total impulse thrust:

$$\dot{I}_\Sigma = Fp_H\left[\tilde{p}\sigma_C \frac{F_{cr}}{F}\left(\frac{k+1}{k}\right)^{\frac{1}{1-k}}\left(\lambda + \frac{1}{\lambda}\right) - 1\right], \quad (23)$$

where the total impulse is defined by the relationship:

$$I_\Sigma = \int_0^{t_{burn}} P \,\mathrm{d}t\,, \qquad (24)$$

where t_{burn} is total time to burn.

σ_C represents an overall loss of thrust by nozzle, and

$\lambda = V_{out}/V_{th}$ is velocities ratio, where V_{out} is the gas velocity in exit plane and V_{th} is the gas velocity in throat area;
To determine the ratio λ we consider known the output cross section (flow area at nozzle exit plane) F, and we applied the relationship [1]:

$$\frac{F_{cr}}{F} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \lambda\left[1-\frac{k-1}{k+1}\lambda^2\right]^{\frac{1}{k-1}}, \qquad (25)$$

which leads to transcendental equation:

$$\lambda = \frac{F_{cr}}{F}\left(\frac{k+1}{2}\right)^{\frac{1}{1-k}} \left[1-\frac{k-1}{k+1}\lambda^2\right]^{\frac{1}{1-k}}, \qquad (26)$$

If we denote with $f(\lambda)$ the right member of the relation (26), we can observe that $\left.\frac{\mathrm{d}f(\lambda)}{\mathrm{d}\lambda}\right|_{\lambda out} > 1$, which means that iterative relation (26) doesn't converge. In this case, we put this relation in Newton- Raphson [10] form:

$$\lambda_{n+1} = \lambda_n - \frac{\lambda_n - f(\lambda_n)}{1 - \left.\frac{\mathrm{d}f(\lambda_n)}{\mathrm{d}\lambda}\right|_{\lambda n}}\,. \qquad (27)$$

If we denote:

$$A_1 = \frac{F_{cr}}{F}\left(\frac{k+1}{2}\right)^{\frac{1}{1-k}};\, B_1 = \frac{1-k}{1+k};\; C_1 = \frac{1}{1-k} \qquad (28)$$

then we can write:

$$f(\lambda) = A_1(1+B_1\lambda^2)^{C_1};$$
$$f'(\lambda) = 2A_1B_1C_1\lambda(1+B_1\lambda^2)^{C_1-1}, \qquad (29)$$

and an iterative relation can be obtained:

$$\lambda_{n+1} = \lambda_n - \frac{\lambda_n - A_1(1+B_1\lambda_n^2)^{C_1}}{1-2A_1B_1C_1\lambda_n(1+B_1\lambda_n^2)^{C_1-1}}\,. \qquad (30)$$

Because this relation is independent from equation system, it can be solved independently, before system solve. The iterative procedure is convergent for the initial value close to the final solution. We recommend to start from $\lambda = 2$.

To calculate the pressure in the chamber, starting from the equation of state, we obtain the following relationship [1]:

$$p = \frac{\omega R T_{0V}\tau(\psi-\eta+\gamma)}{W_{cam} - \left[\frac{\omega}{\delta}(1-\psi) + \alpha\omega(\psi-\eta+\gamma)\right]} \qquad (31)$$

where the reverse remaining gas density (specific volume) $[m^3/Kg]$ it is denoted with α;

Finally, to synthesis the calculus relationship we define the constants:

$$E_1 = AF_{cr}\theta\big/\left(\omega\sqrt{RT_{0V}}\right);\; E_2 = q\big/(\omega Q_C);$$
$$E_3 = \sigma_c\frac{F_{cr}}{F}\left(\frac{k+1}{2}\right)^{\frac{1}{1-k}}\left(\lambda+\frac{1}{\lambda}\right) = \sigma_c A_1\left(\lambda+\frac{1}{\lambda}\right);$$
$$E_4 = W_{cam}/\omega - 1/\delta \qquad (32)$$

and auxiliaries functions:

$$f = \psi-\eta+\gamma\,; \quad g = E_4 + \psi/\delta - \alpha f\,; \qquad (33)$$

Note – Numerical method used for obtaining velocity ratio (λ) may be applied separately from the differential equations, allowing the use of (E_3) as a constant.

With these denotes, the system of operating RESP is:

$$\dot\psi = S(\psi)\delta u/\omega\,; \qquad (34)$$
$$\dot\eta = E_1\, p\big/\sqrt{\tau}\,; \qquad (35)$$
$$\dot\tau = \{(1-\tau)\dot\psi - [(k-1)\tau\dot\eta + E_2]\}f^{-1}; \qquad (36)$$
$$\dot I_\Sigma = Fp_H[\tilde p E_3 - 1], \qquad (37)$$

where the pressure is given by:

$$p = RT_{0V}\tau f g^{-1}\,. \qquad (38)$$

Relations (34)…(37) thus established will be used to determine the functional parameters of RESP.

3 Balance parameters

The study of stability operating RESP will be made accordingly to Liapunov theory, considering the system of parametric equations perturbed around the balance parameters. This involves a disturbance shortly applied on the evolution of balance, which will produce deviation of the state variables. Developing in series the perturbed parametric equations in relation to status variables and taking into account the first order terms of the detention, we will get linear equations which can be use to analyze the stability in the first approximation, as we proceed in most dynamic non linear problems.

Note - Mathematically speaking we have a non linear autonomous system (34)…(37):

$$\dot{y}_i = f_i(y_1, y_2, ...y_n)\Big|_{i=1,n} \tag{39}$$

which will be develop in series:

$$\left. \begin{aligned} \dot{y}_i &\cong f_i(y_{10}, y_{20},...) + \sum_{j=1}^{n} \frac{\partial f_i(y_{10}, y_{10},...)}{\partial y_j} \Delta y_j + \\ &+ \frac{1}{2} \sum_{j=1}^{n} \frac{\partial^2 f_i(y_{10}, y_{20},...)}{\partial y^2{}_j} \Delta y_j^2 ... \end{aligned} \right|_{i=1,n} \tag{40}$$

Keeping only the first order of the detention we will obtain two separate relations /problems:

-the balance relations

$$f_i(y_{10}, y_{20},...) = 0\Big|_{i=1,n}, \tag{41}$$

which allow to obtain the balance parameters:

$$y_{10}, y_{20},.., y_{n0}, \tag{42}$$

-the equations system in linear form

$$\Delta\dot{y}_i = \sum_{j=1}^{n} \frac{\partial f_i(y_{10,} y_{20},...)}{\partial y_j} \Delta y_j \Bigg|_{i=1,n}, \tag{43}$$

which allow to build the stability matrix:

$$a_{i,j} = \frac{\partial f_i(y_{10}, y_{20},...)}{\partial y_j}\Bigg|_{\substack{i=1,n \\ j=1,n}}, \tag{44}$$

which allow us to check the balance stability and finally the non-linear system stability.

Thus, for defining the evolution of balance, the mass flow of gas will be equal with the mass of propellant burnt in time unit, both sizes been stationary:

$$\dot{\psi} = \dot{\eta} = K \ . \tag{45}$$

In these conditions, from equations (34), (35) we obtain:

$$K = S(\psi)\delta u / \omega ; \tag{46}$$

$$p = K\sqrt{\tau}/E_1 \ . \tag{47}$$

If a stationary heat balance is deemed, we obtain a constant temperature of gas:

$$\dot{\tau} = 0 \, . \tag{48}$$

Taking into account the condition of mass conservation imposed by (45), (46), the equation (36) shows:

$$\tau = \left(E_2/K + 1\right)/k \, . \tag{49}$$

Imposing a value of the parameter ψ, from relations (46), (47) and (49) is possible to determine an iterative algorithm for the values K, p, τ. Apart from the iterative algorithm, after determining the three parameters, the discharged gas can be obtained from the first relation (33):

$$\eta = \psi + \gamma - f \ ; \tag{50}$$

where from the state of gas relation (38) we obtain:

$$f = \left(E_4 + \psi\delta^{-1}\right) / \left(RT_{0V}\tau p^{-1} + \alpha\right);$$

4 Linear equations

In the context of the parameters balance established, if the linear form of auxiliary functions is considered:

$$\Delta g = \Delta\psi\delta^{-1} - \alpha\Delta f \ ; \quad \Delta f = \Delta\psi - \Delta\eta \, , \tag{51}$$

expression of pressure (38) in deviation becomes successively:

$$\Delta p = RT_{0V} g^{-1}\left(f\Delta\tau + \tau\Delta f\right) - RT_{0V} f\tau g^{-2} \Delta g \ ;$$

$$\begin{gathered}\Delta p=\frac{RT_{0V}\tau}{g}\left[1-\frac{f}{g}\left(\frac{1}{\delta}-\alpha\right)\right]\Delta\psi- \\ -\frac{RT_{0V}\tau}{g}\left[1+\frac{f}{g}\alpha\right]\Delta\eta+\frac{RT_{0V}f}{g}\Delta\tau\end{gathered}\quad . \tag{52}$$

Denoting:

$$\begin{aligned} a_p^{\psi} &= \frac{RT_{0V}\tau}{g}\left[1-\frac{f}{g}\left(\frac{1}{\delta}-\alpha\right)\right]; \\ a_p^{\eta} &= -\frac{RT_{0V}\tau}{g}\left[1+\frac{f}{g}\alpha\right]; \\ a_p^{\tau} &= RT_{0V}fg^{-1}, \end{aligned} \tag{53}$$

we obtain:

$$\Delta p = a_p^{\psi}\Delta\psi + a_p^{\eta}\Delta\eta + a_p^{\tau}\Delta\tau . \tag{54}$$

On the other hand, the equations operating in linear form are:

$$\begin{gathered} \Delta\dot{\psi}=\frac{dS(\psi)}{d\psi}\frac{u\delta}{\omega}\Delta\psi+S(\psi)\upsilon\frac{u\delta}{\omega}\frac{\Delta p}{p}; \\ \Delta\dot{\eta}=-0.5\tau^{-\frac{3}{2}}pE_1\Delta\tau+\tau^{-\frac{1}{2}}E_1\Delta p; \\ \Delta\dot{\tau}=-\frac{\dot{\psi}+(k-1)\dot{\eta}}{f}\Delta\tau- \\ -\frac{\dot{\psi}(1-\tau)-(k-1)\tau\dot{\eta}-E_2}{f^2}(\Delta\psi-\Delta\eta)+\cdot \\ +\frac{1-\tau}{f}\Delta\dot{\psi}-\frac{(k-1)\tau}{f}\Delta\dot{\eta} \end{gathered} \tag{55}$$

Denoting:

$$\begin{gathered} a_{\psi}^{\psi}=\frac{dS(\psi)}{d\psi}\frac{u\delta}{\omega};\ a_{\psi}^{p}=S(\psi)\upsilon\frac{u\delta}{\omega p}; \\ a_{\eta}^{\tau}=-0.5\tau^{-\frac{3}{2}}pE_1;\quad a_{\eta}^{p}=\tau^{-\frac{1}{2}}E_1; \\ a_{\tau}^{\psi}=-\left\{\dot{\psi}(1-\tau)-(k-1)\tau\dot{\eta}-E_2\right\}f^{-2}; \\ a_{\tau}^{\eta}=\frac{\dot{\psi}(1-\tau)-(k-1)\tau\dot{\eta}-E_2}{f^2}; \\ a_{\tau}^{\tau}=-\frac{\dot{\psi}+(k-1)\dot{\eta}}{f}; \\ a_{\tau}^{\dot{\psi}}=(1-\tau)f^{-1};\ a_{\tau}^{\dot{\eta}}=-(k-1)\tau f^{-1}, \end{gathered} \tag{56}$$

the operating equations in linear form can be written as follows:

$$\begin{gathered} \Delta\dot{\psi}=a_{\psi}^{\psi}\Delta\psi+a_{\psi}^{p}\Delta p; \\ \Delta\dot{\eta}=a_{\eta}^{\tau}\Delta\tau+a_{\eta}^{p}\Delta p; \\ \Delta\dot{\tau}=a_{\tau}^{\tau}\Delta\tau+a_{\tau}^{\psi}\Delta\psi+a_{\tau}^{\eta}\Delta\eta+a_{\tau}^{\dot{\psi}}\Delta\dot{\psi}+a_{\tau}^{\dot{\eta}}\Delta\dot{\eta}\ . \end{gathered} \tag{57}$$

If we insert the linear pressure form in the first two relationships, which are replaced in the last we obtain:

$$\begin{aligned} \Delta\dot{\psi}&=(a_{\psi}^{\psi}+a_{\psi}^{p}a_{p}^{\psi})\Delta\psi+a_{\psi}^{p}a_{p}^{\eta}\Delta\eta+a_{\psi}^{p}a_{p}^{\tau}\Delta\tau; \\ \Delta\dot{\eta}&=a_{\eta}^{p}a_{p}^{\psi}\Delta\psi+a_{\eta}^{p}a_{p}^{\eta}\Delta\eta+(a_{\eta}^{\tau}+a_{\eta}^{p}a_{p}^{\tau})\Delta\tau \\ \Delta\dot{\tau}&=[a_{\tau}^{\psi}+a_{\tau}^{\dot{\psi}}(a_{\psi}^{\psi}+a_{\psi}^{p}a_{p}^{\psi})+a_{\tau}^{\dot{\eta}}a_{\eta}^{p}a_{p}^{\psi}]\Delta\psi+ \\ &+[a_{\tau}^{\eta}+a_{\tau}^{\dot{\psi}}a_{\psi}^{p}a_{p}^{\eta}+a_{\tau}^{\dot{\eta}}a_{\eta}^{p}a_{p}^{\eta}]\Delta\eta+ \\ &+[a_{\tau}^{\tau}+a_{\tau}^{\dot{\psi}}a_{\psi}^{p}a_{p}^{\tau}+a_{\tau}^{\dot{\eta}}(a_{\eta}^{\tau}+a_{\eta}^{p}a_{p}^{\tau})]\Delta\tau. \end{aligned} \tag{58}$$

Denoting:

$$\begin{aligned} &A_{\psi}^{\psi}=a_{\psi}^{\psi}+a_{\psi}^{p}a_{p}^{\psi};\ A_{\psi}^{\eta}=a_{\psi}^{p}a_{p}^{\eta}; \\ &A_{\psi}^{\tau}=a_{\psi}^{p}a_{p}^{\tau};\ A_{\eta}^{\psi}=a_{\eta}^{p}a_{p}^{\psi};\ A_{\eta}^{\eta}=a_{\eta}^{p}a_{p}^{\eta}; \\ &A_{\eta}^{\tau}=a_{\eta}^{\tau}+a_{\eta}^{p}a_{p}^{\tau}; \\ &A_{\tau}^{\psi}=a_{\tau}^{\psi}+a_{\tau}^{\dot{\psi}}(a_{\psi}^{\psi}+a_{\psi}^{p}a_{p}^{\psi})+a_{\tau}^{\dot{\eta}}a_{\eta}^{p}a_{p}^{\psi}; \\ &A_{\tau}^{\eta}=a_{\tau}^{\eta}+a_{\tau}^{\dot{\psi}}a_{\psi}^{p}a_{p}^{\eta}+a_{\tau}^{\dot{\eta}}a_{\eta}^{p}a_{p}^{\eta}; \\ &A_{\tau}^{\tau}=a_{\tau}^{\tau}+a_{\tau}^{\dot{\psi}}a_{\psi}^{p}a_{p}^{\tau}+a_{\tau}^{\dot{\eta}}(a_{\eta}^{\tau}+a_{\eta}^{p}a_{p}^{\tau}), \end{aligned} \tag{59}$$

we obtain the final system:

$$\begin{gathered} \Delta\dot{\psi}=A_{\psi}^{\psi}\Delta\psi+A_{\psi}^{\eta}\Delta\eta+A_{\psi}^{\tau}\Delta\tau; \\ \Delta\dot{\eta}=A_{\eta}^{\psi}\Delta\psi+A_{\eta}^{\eta}\Delta\eta+A_{\eta}^{\tau}\Delta\tau; \\ \Delta\dot{\tau}=A_{\tau}^{\psi}\Delta\psi+A_{\tau}^{\eta}\Delta\eta+A_{\tau}^{\tau}\Delta\tau, \end{gathered} \tag{60}$$

with the derivatives stability matrix given by:

$$\mathbf{A}=\begin{bmatrix} A_{\psi}^{\psi} & A_{\psi}^{\eta} & A_{\psi}^{\tau} \\ A_{\eta}^{\psi} & A_{\eta}^{\eta} & A_{\eta}^{\tau} \\ A_{\tau}^{\psi} & A_{\tau}^{\eta} & A_{\tau}^{\tau} \end{bmatrix}. \tag{61}$$

With the stability matrix so defined, we can make an analysis of the engine stability operation in different regimes of pressure and temperature depending on the propellant load.

5 Input data

For exemplifying the method, we will build a study model out of engine test.

5.1 Propellant geometry

First we describe the geometry of propellant which is a cylinder, with a cylindrical hole inside, none insulated, so burning simultaneously on all surfaces (figure 1).

Denote instantaneous sizes:

R - Outside radius of the cylinder; r - inside radius of the cylinder; l - cylinder length,

The burning area, terminal area and propellant volume are given by:

$$S = 2\pi(R+r)(R-r+l)\,;\, S_T = \pi(R+r)(R-r)\,;$$
$$V = S_T l = \pi(R+r)(R-r)\,. \qquad (62)$$

If we denote x the linear burning distance, which at the time t is given by integrate of burning rate:

$$x = \int_0^t u\,\mathrm{d}t\,, \qquad (63)$$

the main geometric quantities are rewritten as it follows:

$$R = R_0 - x\,;\; r = r_0 + x\,;\; l = l_0 - 2x\,, \qquad (64)$$

from which the combustion areas and volume become:

$$S = S_0 - 4\pi(R_0 + r_0)x \qquad (65)$$
$$S_T = S_{T0} - 2\pi(R_0 + r_0)x \qquad (66)$$
$$V = V_0 - 2[S_{T0} + \pi(R_0 + r_0)l_0]x + 4\pi(R_0 + r_0)x^2 \qquad (67)$$

where we denoted with index "0" the initial values for length, surfaces and volumes.

After processing we obtain:

$$\sigma = \frac{S}{S_0} = 1 - \frac{2x}{R_0 - r_0 + l_0}\,;$$
$$\sigma_T = \frac{S_T}{S_{T0}} = 1 - \frac{2x}{R_0 - r_0}\,;$$
$$\psi = 1 - \frac{V}{V_0} = \frac{2x}{l_0} + \frac{2x}{R_0 - r_0} - \frac{4x^2}{(R_0 - r_0)l_0} \qquad (68)$$

For the application the main geometrical quantities are:

$$R_0 = 32.5\,mm\,; r_0 = 10.0\,mm\,;\; l_0 = 115\,mm\,.$$

In this case, the initial areas are:

$$S_0 = 36717 mm^2\,; \quad S_{T0} = 3004 mm^2\,.$$

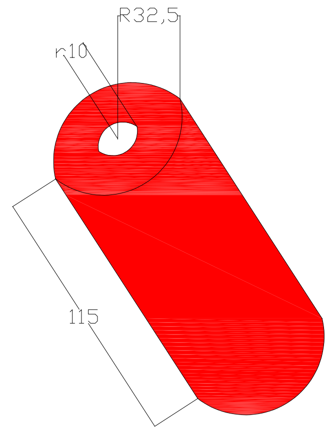

Fig. 1 Propellant geometry

Developing the relations (68) in a numerical form related on the parameter x results the dependence between the no dimensional areas σ, σ_T and the burn parameter ψ. By quadratic fitting we obtain:

$$\sigma(\psi) \cong 0.999420 - 0.130553\psi - 0.0318824\psi^2\,;$$
$$\sigma_T(\psi) \cong 0.996455 - 0.797822\psi - 0.194837\psi^2\,, \qquad (69)$$

functions which are represented in figure 2

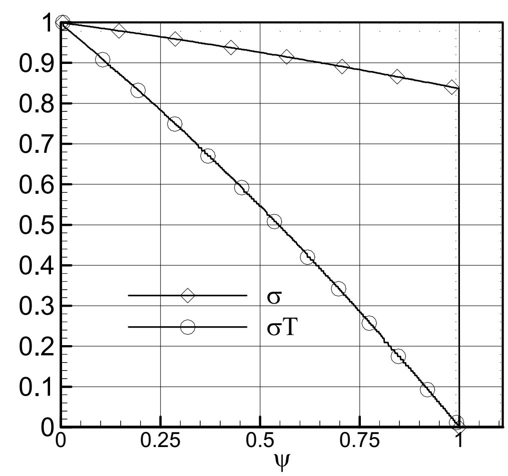

Fig. 2 Areas burning diagrams

5.2 Engine geometry

The engine geometry elements used for the test considered are:

-Combustion chamber cross surface: $F_{cam} = 3632 mm^2$;

- Flow area at the throat: $F_{cr} = 157 mm^2$;

- Flow area at nozzle exit plane: $F = 427.7mm^2$.
- Burning chamber volume: $W_{cam} = 659000mm^3$

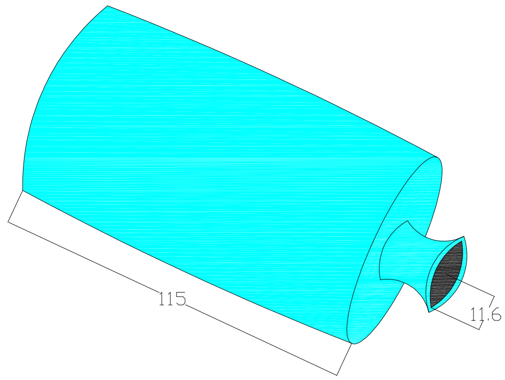

Fig. 3 Engine geometry

5.3 Propellant and process features

The features for the used propellant are:
- Propellant mass: $\omega = 0.6215\,kg$;
- Propellant density: $\delta = 1700\,Kg/m^3$;
- Adiabatic gas coefficient of the combustion products $k = 1.240276$;
- Gas constant: $R = 336.6777\ J/Kg/K$;
- Linear burning rate in normal conditions: $u_{1n} = 4.64612mm/s$;
- Pressure exponent of burning law: $\nu = 0.19528$;
- Coefficient of variation of burning rate with temperature: $D = 0.0038\,K^{-1}$;
- Heat quantity educts by burning reaction of 1 kilogram propellant: $Q_C = 3.408072 \cdot 10^6\,J/Kg$;
- The quantity of heat transferred to the combustion chamber in time unit (heat flow) $q = 1000\,J/s$;
- Ratio between igniter gas mass and the propellant mass $\gamma = 0.01$;
- Specific volume: $\alpha = 0.02\,m^3/Kg$;
- Coefficient expressing flow losses in the nozzle $\theta = 0.9$;
- Coefficient overall rate of loss of thrust by nozzle: $\sigma_c = 0.9$;

6 Results

For computing model considered, applying relations (34)… (37) we obtain parametric graphs shown in figure 4. Figure 5 presents the comparison between pressure produced by the relationship (38) and the experimental pressure of the test engine, and figure 6, show the influence of initial propellant temperature for the pressure.

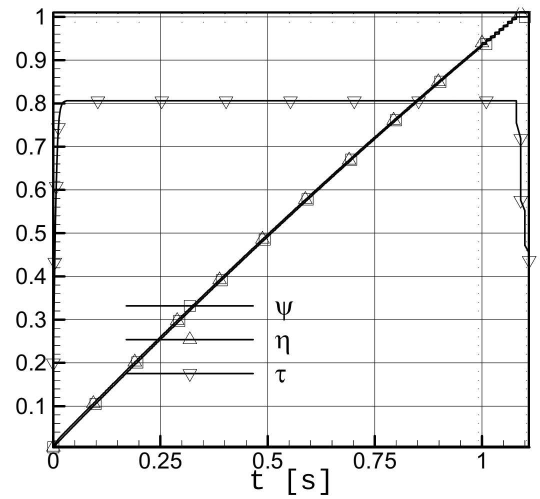

Fig. 4 Engine parameters diagram for stable RESP

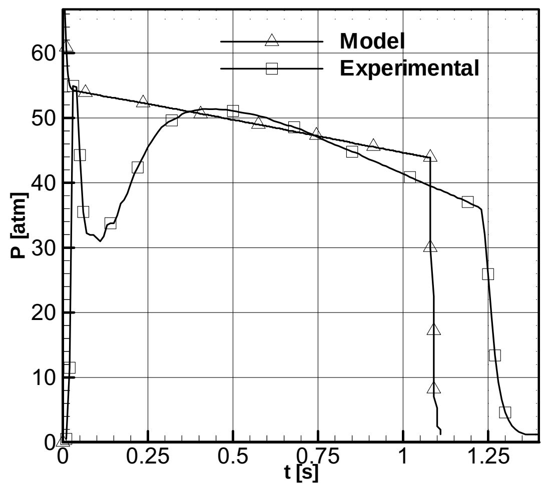

Fig. 5 Comparative pressure diagram

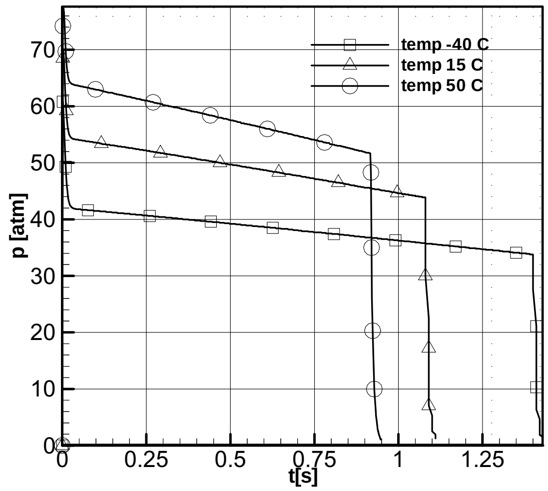

Fig. 6 Influence of initial propellant temperature for the pressure

Further on we will analyze the balance parameters and the dynamic stability of the operating RESP. Thus, in figure 7 we are showed, for the considered application, the main balance parameters and in figure 8 the balance pressure, obtained by the relations (46)… (50).

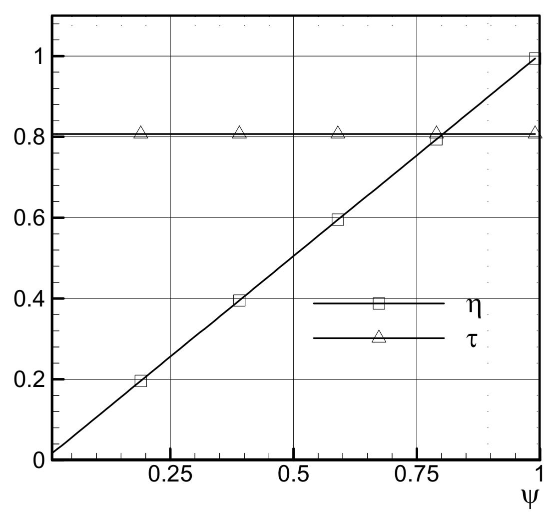

Fig. 7 Balance parameters for stable RESP

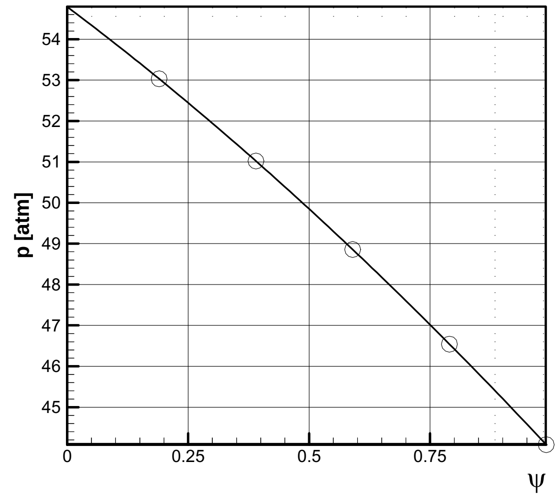

Fig. 8 Balance pressure for stable RESP

Henceforth, setting the basic trend, we can evaluate, using the matrix (61), the parametric stability of the operating engine. To do this in figure 9 there are given the real part of eigenvalue for the matrix corresponding to the stable balance parameters.

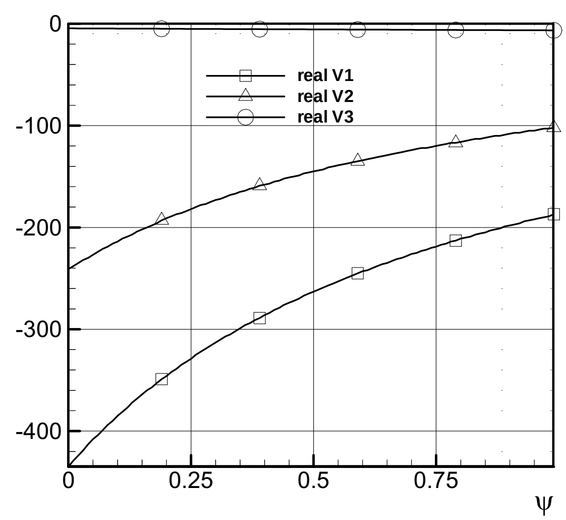

Fig. 9 Real part of eigenvalue for stable RESP

Further we analyze, as an example, the influence of burning rate parameter for the pressure; which can produce an instability burning and finally an exposure as we show in figure 10.

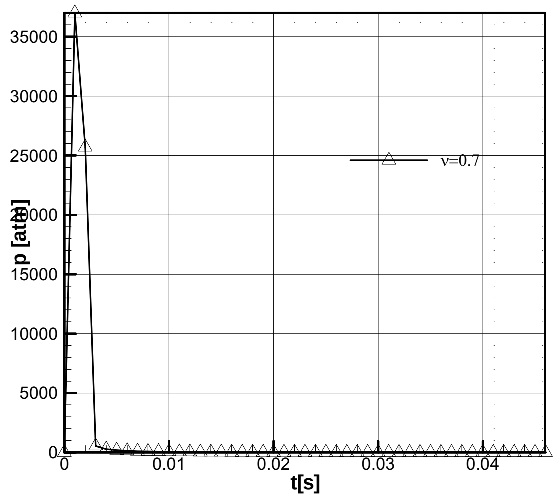

Fig. 10 Influence of burning rate parameter for the pressure – instability burning

In figure 11 we can see the engine parameters for instability case, corresponding pressure from figure 10.

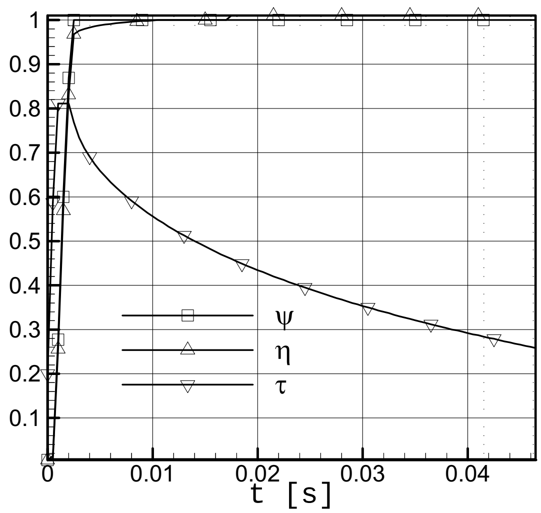

Fig. 11 Engine parameters for instable RESP

In figure 12 and 13 we can see the balance parameters and the balance pressure for an instable RESP.

Finally, in figure 14 we see the real part of eigenvalue for the matrix corresponding to the instable balance parameters.

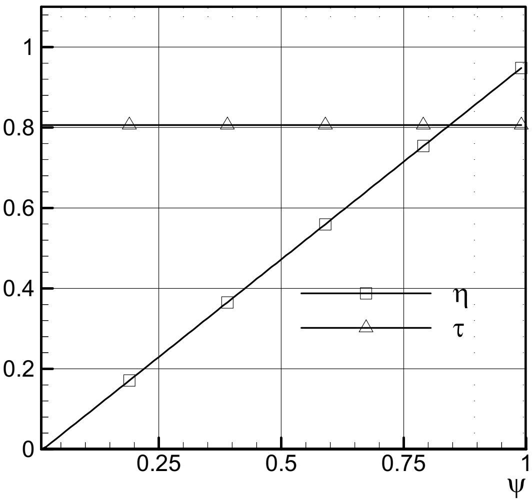

Fig. 12 Balance parameters for instable RESP

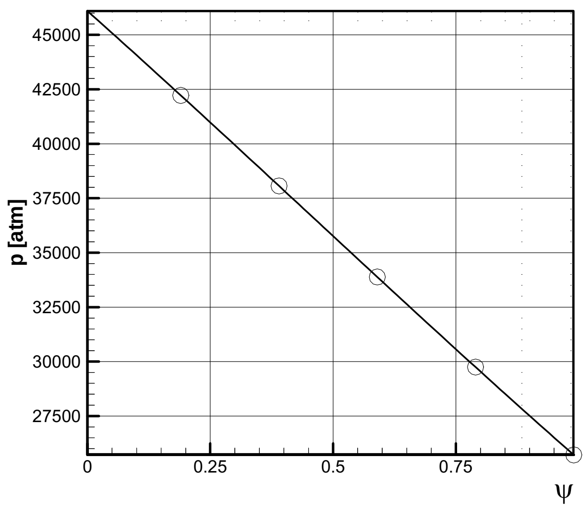

Fig. 13 Balance pressure for instable RESP

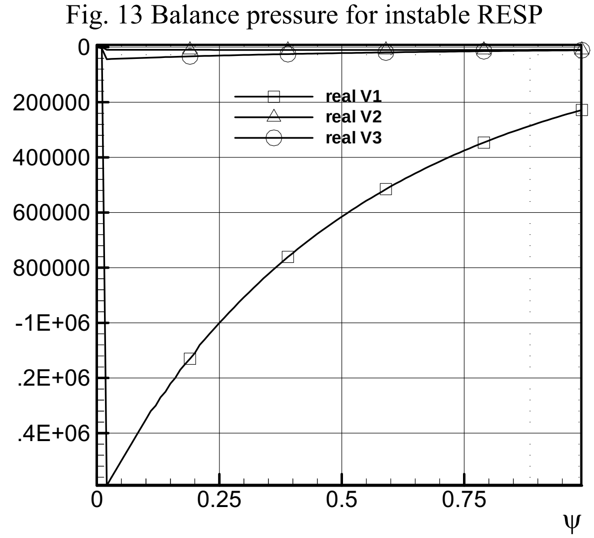

Fig. 14 Real part of eigenvalue for instable RESP

7 Conclusions

As we resumed in the introductive part, our work followed two purposes:

Scientific one – to check the possibility of applying Liapunov theory to analyze the stability of the balance parameters of RESP at low pressure. With this reason we obtained:

- A flexible parametric expression of the propellant surface which allows to use different propellant geometry without major modification of the input data structure (figure 2);
- A good concordance between parametric non- linear equations of the RESP and the experimental results as we can see in figure 5 where is shown the comparative pressure diagram;
- An algorithm to define the balance parameters and stability matrix;
- A comfortable method to evaluate the engine stability operating a low pressure evaluating eigenvalue of the stability matrix as we show in figures 9 and 14.

Technical one – to design the rocket engine for the anti-hailstone rocket. In the last figure we can see anti hailstone rocket flying.

Fig. 15 Shooting test of anti-hailstone rocket

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