Comparison of calculation methods and models in software for computer graphics and radiative heat exchange

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Abstract: The paper describes the physical backgrounds for modelling in visualisation using computer graphics software and in programs of radiative heat exchange. Physical phenomena in both cases (generation, propagation, reflection and absorption of light and radiative heat flux) are quite similar, provided with basic equations, typical simplification assumptions and the most frequently applied methods of resolving such issues. Various models applied in radiative heat transfer and computer graphics software are compared. The similarity and differences of algorithms for both tasks are determined and replaceability conditions are enumerated. Formulas are provided to convert thermal boundary values (temperature and power density) into luminous or radiant quantities (luminance) and vice versa. A possible scope of shared computations is also outlined.

Keywords: computer graphics, radiative heat transfer, rendering, modelling, radiosity, ray tracing, boundary conditions

1 Introduction

Computer graphics is a sub-field of computer science and is concerned with digitally synthesising and manipulating visual content using computational techniques. As an academic discipline, it focuses on the mathematical and computational foundations of image generation and processing rather than purely aesthetic issues.

The beginnings of computer graphics go back to the 1950s, however as computers and specialist graphics equipment were fairly expensive and individual units had very limited processing capacity at the time, computer graphics used to be a domain explored only at robust research centres, in major companies and government institutions. The breakthrough came in the 1980s with the advent of relatively cheap and fast personal computers with sufficient processing power. At present, computer graphics is commonplace and natural both in professional and home applications, similarly to spreadsheets and word processors.

The most common applications of computer graphics:

- visualisation of measurement data,
- visualisation of computer simulations,
- medical diagnostics,
- simulators and training machine,
- computer-aided design (CAD),
- desktop publishing (DTP),
- special effects in films,
- computer games,
- architectural presentations.

Computer graphics is often employed to obtain photorealistic (photographic) images of real or imaginary objects or people. It is worthwhile to note that the achievement of a subjective sensation of reality is only possible if generation algorithms of such images duly take into account physical laws governing the emission, propagation, reflection and absorption of light, i.e. electromagnetic wave with the wavelength $\lambda \in (0.38 - 0.78)$ μm.

Passing on to discuss radiative heat exchange, it must be noted that beside convection and conduction, radiative heat transfer is one of three basic ways of heat exchange. It is a common phenomenon, based on the exchange of thermal energy (power) via electromagnetic radiation (chiefly infrared radiation with the wavelength $\lambda \in (0.78\text{-}1000)$ μm) between non-transparent surfaces or translucent volumes of a temperature higher than 0K.

In non-vacuum systems, for a surface with a temperature within the range of 0–50°C, radiation accounts for ca. 15-20% of total heat transfer and rises substantially at higher temperatures. We can see it on Fig.1.

It is also the only way of heat exchange in vacuum systems. In many electrical or electronic devices, determination of thermal fields associated with normal work and operation is the basic means of verification whether the design is suitable, and serves as the basis for determining maximum allowed electrical loads.

Heat exchange in electrical or electronic devices is usually of a mixed nature, made up of conduction, convection and radiation. Since numerical methods of calculating thermal conduction are relatively well developed, it is all the more important to work out numerical computation methods for modelling radiative heat transfer.

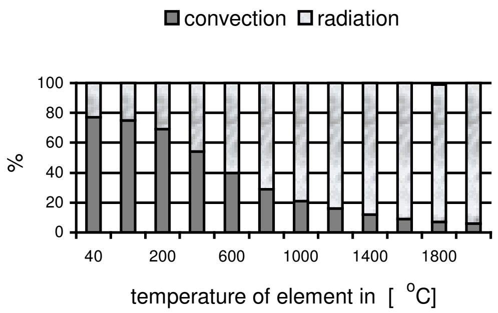

Fig. 1 Share of convection and radiation in general heat transfer with typical conditions

2 Computer graphics – tasks, description and basic techniques

The basic task of computer graphics is to generate images on 2D screen 3D photorealistic images of real or imaginary objects or people using digital techniques. Two major image generation methods can be distinguished, whereby computer graphics is broadly divided into raster graphics and vector graphics.

In computer raster graphics, an image is created as a bitmap, where a data structure represents a rectangular grid of pixels viewable via a monitor, on paper or using other display devices.

Vector graphics (also called object-oriented graphics) uses geometrical primitives such as points, lines, curves, and polygons, which are all based on mathematical equations to represent images in computer graphics.

At present, vector graphics is essentially limited to the programme layer and user interface. It is applied because of its leading feature of maintaining unchanged image quality during transformations (e.g. rotation or scaling). However as present-day displays are of the raster type, applications based on vector graphics are forced to represent perfect geometric figures in a finite resolution.

Generally, the process of generating an image from a model, using dedicated computer programmes, is called **rendering** [1, 6, 11] The model is a description of virtual 3D objects in a strictly defined language or data structure. It would contain geometry, viewpoint, texture, lighting, and shading information. The image is a digital image or a raster graphics image.

The origin basic rendering equation, as proposed by Kajiya [6], is the following:

$$I(x,x') = g(x,x')\left[e(x,x') + \int_V \rho(x,x',x'')\, I(x',x'')dx'' \right] \quad (1)$$

where: $I(x,x')$ – sum of intensity of the radiation emitted and reflected in point x' in the direction of point x, $g(x,x')$ – factor dependant from the system's geometry, $e(x,x')$ – radiative emission from point x' in the direction of point x, $\rho(x,x',x'')$ – specular reflectance of radiation $I(x',x'')$ in point x', incident from point x'', and reflected in the direction of point x.

The equation (1) can be transfer in form, which describes this phenomenon in terms of radiation:

$$L(x_i,\omega) = L_o(x_i,\omega) + \\ + \int_{\omega'\in\Omega} L(x_j,-\omega')\ddot{\rho}(x_i,\omega',\omega)\cos\theta' d\omega' \qquad (2)$$

where:
$L(x_i,\omega)$ - total radiance in point x_i (see definition in eq. (15)) and direction ω, while $\ddot{\rho}$ represents bi-directional reflective distribution function (BRDF). Integration extends over all the surfaces of the scene Ω.

Radiance in point x_i consists of own emission L_o resulting from the temperature in point x_i) and reflected radiance L_{ref} – radiance incident on x_i from any direction ω' and reflected in direction ω. Determining the interaction of the ray and the surface (reflection) in point x_j as the integral operator ζ "collecting" radiation from the entire surface Ω:

$$\int_{\omega'\in\Omega} L(x_j,-\omega')\ddot{\rho}(x_i,\omega',\omega)\cos\theta' d\omega' = \zeta(L(x_j,-\omega')) = L_{ref}(x_i,\omega) \qquad (3)$$

the formula given in (1) can be expressed as

$$L(x_i,\omega) = L_o(x_i,\omega) + L_{ref}(x_i,\omega) = \\ = L_o(x_i,\omega) + \zeta(L(x_j,-\omega')) \qquad (4)$$

In (4) the unknown radiance "L" is both inside and outside of the operator ζ, which renders the problem very complex.
Typically, the problem is solved using a simple formal trick: the integral part should be substituted in the unknown function. Own emission can be regarded as zero reflection. This transforms the integral equation into the following infinite series:

$$L(x_i,\omega) = \sum_{n=0}^{\infty} \zeta^n \cdot L(x_\Omega, \omega_\Omega) \cong$$
$$\cong L_o + \zeta(L) + \zeta^2(L) + \zeta^3(L) + \ldots \quad (5)$$

The elements in (5) represent own emission, a single reflection of own emission, followed by a secondary reflection and so on. Simple rendering algorithms (ray casting) do not even try to solve this equation. The local illumination method tries to solve only the first two elements of the series, ray tracing eliminates the integral from every element, making the series much simpler.

A number of rendering algorithms have been put forth, and software used for rendering may employ a variety of different techniques to obtain the final image. Most advanced software combines two or more of available techniques of rendering to obtain good-enough results at a reasonable cost. Standard rendering techniques, currently considered the most effective, include:

- **ray casting**
- **local illumination**
- **ray tracing (backward ray tracing)**
- **radiosity**

or many others techniques, as photon mapping, Metropolis-Hastings, Monte-Carlo methods.

Ray casting and ray tracing are based on the same fundamental principle: determination of the visibility of surfaces by tracing imaginary rays of light from the viewer's eye (or the camera) to the object in the scene.

The algorithm employed in ray casting involves the following steps:

1. Ray projection from the observation point towards the projection plane, usually equivalent to the display.
2. Determination of the first point of intersection with an object in the scene.
3. Determination of luminosity of the point, based on the principle that the smaller the angle η between a vector normal to the object surface and the section (camera, point), the more luminous the point.

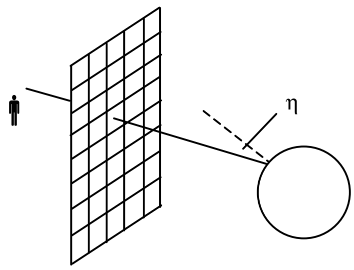

Fig. 2 Ray casting principle

This is a simplified method that does not take into consideration light sources and the law of reflection. Furthermore, it does not model shadows, concave or convex walls and point lighting. Simplified images are only obtained for specific objects (e.g. flat perpendicular walls) and a specific type of lighting.

Local illumination is commonly used in simple 3D graphics programs. It is assumed that the look of scene surfaces can be determined by calculating direct rays and single reflection in point x_i of light sources only (see equation (5) and (6)). The rest of the infinite series is expressed in the so-called constant ambient light.

$$L(x_i,\omega) = L_o(x_i,\omega) +$$
$$+ \sum_{j=1}^{M} L_{sources}(x_j, -\omega')\ddot{\rho}(x_i,\omega',\omega)\cos\theta' + L_{amb} \quad (6)$$

Summation (which, in this formula, replaces integration used in (2)) is performed over M sources, not over space Ω. In this equation the rendering equation (2) is reduced to the sum of just a few easily computable terms.
The local illumination method is used because in the case of artificial environments dominant light sources determine the look of objects, while intersurface light reflections are only of secondary importance. This method is physically incorrect and it might be unwieldy, however in the production environment the method has proven its usability in a wide range of cases.

Ray tracing is an extension of the ray casting and local illumination methods. A simple casting algorithm is completed by secondary rays generated in points of intersections which are used as starting points for new rays in order to determine reflections, refractions, and shadows. In the conventional ray-tracing method, meshing or other pre-processing of the scene are not needed. The colour of an appropriate pixel is computed according to the ray-object interactions [5].
Ray tracing is a technique for generating an image by tracing the path of light through pixels in an image plane. The technique is capable of producing a very

high degree of photorealism; usually higher than that of typical scanline rendering methods, but at a greater computational cost. This makes ray tracing best suited for applications where the image can be rendered slowly ahead of time, such as in still images and film and television special effects, and is less suited for real-time applications like computer games where speed is critical.

The algorithm for ray tracing includes the following elements:

1. The primary ray is sent out from the point in which the observer is located; the ray intersects the projection plane.
2. The closest point of intersection with objects within the scene is sought out.

Items 1-2 are the same as those involved in ray casting, but additionally:

3. Taking into account each light source defined in a given scene, total luminosity is determined at the point of intersection, as a sum of direct (primary) radiation and reflected (secondary) radiation in compliance with the adopted model of light reflection (Lambert's, Phong's) on other surfaces.
4. If the point of intersection belongs to a light-reflecting or transparent object, the point sends out **secondary rays** (reflected or refracted ray) and the algorithm is repeated recursively starting in step 2. The number of reflections adopted for programmes is usually restricted.

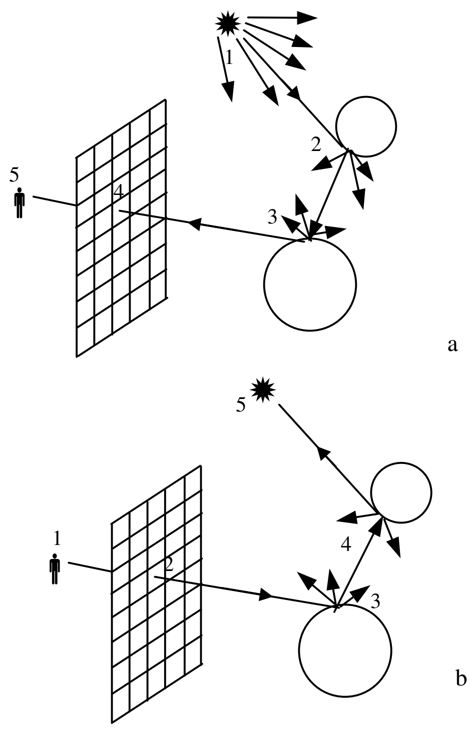

Fig. 3 Ray tracing principle: a –forward tracing (light), b – backward tracing (visibility ray)

This can be mathematically expressed as [11]:

$$
\begin{aligned}
L(x_i,\omega) &= L_o(x_i,\omega) + \\
&+ \sum_{j=1}^{M} L_{sources}(x_j,-\omega')\ddot{\rho}(x_i,\omega',\omega)\cos\theta' + \\
&+ \sum_{x\in\Omega}^{Nrl} L(x_j,-\omega)\ddot{\rho}(x_i,\omega',\omega) + \sum_{x\in\Omega}^{Nrr} L(x_j,-\omega)\ddot{\tau}(x_i,\omega',\omega) + L_{amb}
\end{aligned}
\tag{7}
$$

where N_{rl} – number of reflected light rays and N_{rr} – number of refracted rays.

In order to increase accuracy, the ray tracing algorithm is applied to all pixels of an image, with several rays traced through each of the pixels.

A very significant advantage of the method is the fact that it makes it easy to parallelise programmes: each primary ray can be processed independently of others; similarly, secondary rays are independent of one another. Ray tracing however, is computationally costly, as the number of computations is proportional to image resolution, the degree of complexity of the scene: the number of light sources, the number and shape of objects and their reflection characteristics.

A natural course of ray from the source to the observer is presented in Fig. 3a. It would be highly inefficient to process a system with such ray courses, as it is only some of generated rays which, directly or indirectly or following multiple reflections, reach the observer. It must be stressed, though, that only such rays are useful because they produce desirable visual sensations. It is not possible (without suitable computations) to select only useful rays out of the total number of rays. To reduce the amount of superfluous rays that tend to be processed, the principle of backward ray tracing was introduced, whereby hypothetical rays (backward rays) are modelled running from the observer's eye and hitting the light source, either directly or via reflections. In this case, the number of analysed, though superfluous, rays is markedly lower, as illustrated in Fig. 3b.

Ray tracing is not a perfect technique for generating photorealistic images, either. Above all, it fails to take due account of diffused light (the number of reflections is limited) and – analysing individual rays – it cannot correctly model diffraction, interference and other wave phenomena resulting from the simultaneous mutual interaction of a number of waves.

Radiosity as a method of rendering using a global illumination algorithm. Global illumination in 3D graphics is a lighting model in which each object within a scene is illuminated by light emitted directly from the light source, but also light repeatedly reflected from other objects within the scene. The number of reflections is essentially unlimited, while the ray is suppressed when its assigned energy drops below a defined level.

Radiosity is an application of the finite element method to solving the rendering equation for scenes with purely diffuse surfaces. Unlike Monte Carlo algorithms (such as path tracing) which handle all types of light paths, typical radiosity methods only account for paths which leave a light source and are reflected diffusely for a number of times before hitting the eye.

More correctly, radiosity M is the energy leaving the patch surface per discrete time interval and is the combination of emitted and reflected energy [5,11]:

$$M_i dS_i = E_i dS_i + \rho_i \int_j M_j \varphi_{ji} dS_j \tag{8}$$

where:

- M_i is the radiosity of patch i,
- E_i is emitted energy,
- ρ_i is the reflectivity of the patch,
- φ_{ij} is the constant-value of view factor for the radiation leaving path S_i and hitting patch S_j.

For diffuse radiation, the following holds:

$$\varphi_{ij} S_i = \varphi_{ji} S_j \tag{9}$$

and when the integral is replaced and uniform radiosity is assumed over the patch, creating the simpler:

$$M_i = E_i + \rho_i \sum_j^N M_j \varphi_{ji} \tag{10}$$

This equation can be applied to each patch. The equation is monochromatic in form, so colour radiosity rendering requires calculation for each of the required colours.

A more developed version of the method is **progressive radiosity**. This method solves the system iteratively in such a way that after each iteration we obtain intermediate radiosity values for the patch. These intermediate values correspond to bounce levels. That is, after one iteration we know how the scene looks after one light bounce, after two passes, two bounces, and so forth. Progressive radiosity is useful for getting an interactive preview of the scene. Also, the user can stop the iterations once the image looks good enough, rather than wait for the computation to numerically converge.

A comparison of different computer graphics models is presented in Table 1.

Table 1. Methods used is computer graphics.

Methods employed in computer graphics			
Ray casting	**Local illumination**	**Ray tracing**	**Radiosity (global illumination)**
Principle			
Rays are cast and traced *in groups* based on some geometric constraints.	Only primary light sources are tested.	Each ray is traced *separately,* number of reflection is limited, every point on the display is traced by at least one ray.	Each ray is traced *separately,* so that every point on the display is traced by at least one ray.
Physical correctness			
None	Very limited	Partial	Nearly complete
Assumption			
Closer perpendicular surfaces are lighter.	Only parallel point lighting.	Only one (or, less commonly, several) diffuse reflections, no refraction.	Complete model of reflections and sources, meshing is required, minor details may disappear.
Scene			
Limited by simple geometric shapes.	Selected light sources, almost any shape.	Almost any shape can be rendered.	Any shape, though without very fine details.
Quality			
Resulting image is not very realistic.	Moderate for selected light types.	Resulting image is realistic.	Very high, depending on meshing accuracy.

3 Radiative heat exchange – tasks, physical description, basic equation, resolution methods

The basic task in accounting for the phenomenon of radiative heat transfer is determining the distribution of temperature and radiative power penetrating the set area Ω. The area, referred to as the radiative heat exchange system, is limited by the boundary surfaces S_i.

Physical foundations of thermal radiation and radiative heat transfer can be summed up as follows: each physical body of a temperature higher than 0K generates electromagnetic radiation into the surrounding half-space. For the black body, the phenomenon is defined with Planck's law [7]:

$$m_{bb,\lambda} = c_1 \lambda^{-5} \left(\frac{c_2}{e^{\lambda T}} - 1 \right)^{-1} \quad (11)$$

which expresses the relationship between density of black body monochromatic radiosity $m_{bb,\lambda}$ (monochromatic surface density of power) and its temperature T for any wavelength λ. Some of the radiation within the sub-range of $\lambda \in (0.38\text{-}0.78)\mu m$ conveys substantial amounts of energy and, due to its location in the spectrum, is referred to as infrared radiation. It is the cause of radiative heat exchange. The formula in (11) defines the radiation of a black body, while real bodies are described with the following relation

$$m_\lambda = \varepsilon_\lambda m_{bb,\lambda} \quad (12)$$

where ε_λ represents monochromatic emissivity of radiating surface.

On the other hand, total quantities, which do not take into consideration the spectral nature of radiation, are defined with the formula:

$$M = \int_\lambda \varepsilon_\lambda m_{bb,\lambda} d\lambda \quad (13)$$

where M stands for radiosity of the radiating surface, measured in W/m^2.

Radiation analyses should typically incorporate the direction of propagation. This is why the notion of density of monochromatic radiance l_m is introduced, defined as density of monochromatic radiosity m measured at a set solid angle dω:

$$l_\lambda = \frac{dm_\lambda}{d\omega} \quad (14)$$

By the same token, for total quantities which do not take into account the relationship with the wavelength λ, radiance L can be defined as:

$$L = \int_\lambda l d\lambda = \frac{dM}{d\omega} \quad (15)$$

which can also be expressed with the equation:

$$L = \frac{d^2\Phi}{d\omega \cdot dS_p} = \frac{d^2\Phi}{d\omega \cdot dS \cdot \cos\eta} \quad (16)$$

where L represents radiance measured in [W/(m^2 sr)] (see also rendering equation (2)).

Total radiation spreading from any point x_i of surface S_i in a defined direction ω consists of own emission and radiation propagating from the entire half-space towards any given point and reflected in the analysed direction (compare to (1)), as illustrated in Fig. 4.

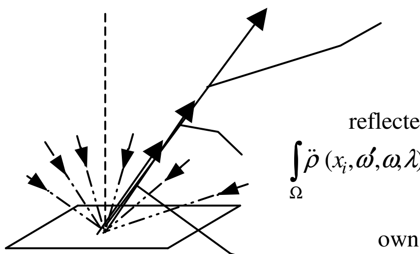

Fig.4 Radiance L at point x_i and direction ω

Consequently, for any point within the surface one can express the basic equation governing radiative heat exchange as [2,7,11]:

$$L(x_i,\omega,\lambda) = \varepsilon(x_i,\omega,\lambda)L_{bb}(x_i,\lambda) + \\ + \int_\Omega \ddot{\rho}\ (x_i,\omega',\omega,\lambda)L_i(x_i,\omega,\lambda)\cos\eta\ d\omega \quad (17)$$

In (17), $\ddot{\rho}$ stands for BRDF, the "bb" indexes refer to radiation emitted by a black body, "i" to incident radiation or point x. Integration is performed over the entire half-space Ω. The equation (17) is the same as governing computer graphics equation (1) or (2).

The equation in (17) results directly from the principle of conservation of energy. This is a Fredholm integral equation type 2. The notation means that for any point x_i on the boundary surface S_i, whose location is defined with point x_i, radiance L in direction defined by vector ω, consists of total radiance of own emission (the first component of the formula in (7)) and total of reflections in the direction ω of radiation incident from the entire half-space Ω (the next component, integral of the formula (17)).

The equation given in (17) describes radiation from a single point, but practical applications nearly always involve a whole radiative exchange system. Boundary surfaces of the system are usually defined with the so-called boundary conditions. The conditions are defined either as Dirichlet conditions or Neumann conditions. The former case requires a function of the distribution of temperature t in the analysed boundary surface S_i [1,5]

$$\underset{x_i \in S_i}{t(x_i)} = t(x_0, y_0, z_0) = f(x_i) \tag{18}$$

The latter case, on the other hand, requires a specification of surface power density p proportional to the derivative of function t [2,7] penetrating from the outside into the system via the surface boundary:

$$\underset{x_i \in S_i}{p(x_i)} = p(x_0, y_0, z_0) = -\lambda \left.\frac{\partial t}{\partial r}\right|_{S_i} = g(x_i) \tag{19}$$

In this case, as shown in (19), only the derivative of function t is set, which means that temperature is defined with accuracy to a constant. Consequently, in order to determine absolute temperature distribution within the system, it is necessary to specify a Dirichlet condition for at least one boundary surface. We can see it on Fig. 5.

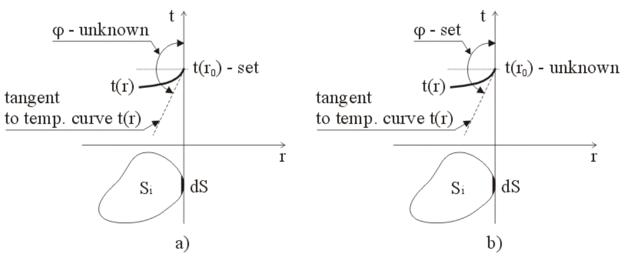

Fig. 5. Graphic interpretation of boundary conditions: a) Dirichlet's, b) Neumann's

Dirichlet's condition makes it possible to calculate the value of radiance L_o in the equation (2), for Neumann's condition it can be assumed that the first element of the sum in (2) equals p. In both cases, the equation (2) comes down to the form of Fredholm's integral equation of the second kind, usually presented in the following form:

$$\phi(x) = f(x) + \int_S K(x, x')\phi(x')dS' \tag{20}$$

where: $K(x,x')$ – kernal of the integral equation, $f(x)$ – known function, $\phi(x')$ – quested function (radiance L or heat flux density p).

So, we obtain a system of N Fredholm integral equations (17) together with boundary conditions (18) and (19), which is practically unsolvable algebraically [8].

There are two related techniques (methods) that can be used to solve the integral equation (17):

- finite element methods,
- stochastic modelling and simulation,
- zonal,
- discrete ordinate,
- Monte-Carlo methods,

and many others.

Computations using the finite element method (net or zonal method) are based on a prior **discretisation** and **linearisation of basic equations** – and simplifications of these equations (17)-(19), which makes it possible to obtain a system of linear equations.

Stochastic methods, in turn, are developed with the basic equation (17) and boundary conditions (18)-(19). This approach is adopted e.g. in the conventional Monte-Carlo method and ray tracing.

The basis for discretisation of surfaces S_i participating in heat transfer is the assumption that the boundary conditions, as well as emissivity and reflectivity properties for the entire surface S_i or a part of it, ΔS_i. Mathematically speaking, this comes down to assuming that

$$\begin{aligned} &\underset{x_i \in S_i}{t(x_i)} = t(x_0, y_0, z_0) = t(x_i) = const \\ &\underset{x_i \in S_i}{p(x_i)} = p(x_0, y_0, z_0) = p(x_i) = const \end{aligned} \tag{21}$$

and

$$\ddot{\rho}(x_i, \omega', \omega, \lambda) = \ddot{\rho}(s_i, \omega', \omega, \lambda) = const \tag{22}$$

for each r_0 belonging to the analysed fragment ΔS_i. As a result, the integral equation (17) applicable to selected ΔS_i can be expressed as the following total:

$$L(\mathrm{s_i},\omega,\lambda)=\varepsilon(\mathrm{s}_i,\omega,\lambda)L_{bb}(\mathrm{s}_i,\lambda)+ \\ +\sum_{i=1}^{N}\ddot{\rho}\ (\mathrm{s}_i,\omega',\omega,\lambda)L_p(\mathrm{s}_i,\omega',\lambda)\cos\eta\ d\omega \quad (23)$$

where N stands for the number of ΔS_i fragments in the entire system. After introducing the notion of configuration coefficient φ_{ij} specified as [5,7]:

$$\varphi_{ij}=\frac{1}{S_i}\int_{Sj}\int_{Si}\frac{\cos\eta_1\cos\eta_2}{\pi d^2}dS_i dS_j \qquad (24)$$

and defining mutual geometrical relations of ΔS_i and ΔS_i surface fragments, as well as assuming monochromaticity of radiation or greyness of surfaces ($\varepsilon(s_i,\omega,\lambda)=\varepsilon(s_i)=const$) involved in radiative heat exchange, the equation system (23) is presented in the form of a system of linear equations [5,7]:

$$\sum_{j=1}^{N}\left(\frac{\delta_{ij}}{\varepsilon_j}-\varphi_{ij}\frac{1-\varepsilon_j}{\varepsilon_j}\right)p_{out,j}=\sum_{j=1}^{N}(\delta_{ij}-\varphi_{ij})\sigma T_j^4 \qquad (25)$$

for $i=1,2,....,N$

The equations given in (25), together with boundary conditions in (18) and (19), create a uniquely determined system of linear equations which can be solved either using classical methods (for low N) or iterative methods (for high N).

The other group of methods is based on stochastic modelling of radiative heat transfer. Knowing the surface arrangement of a heat exchange system, and emissivity and reflectivity characteristics of surfaces, it is possible to model the propagation of radiation by tracing the history of each ray from its emission, through reflections on boundary surfaces, until its final absorption, as shown in Fig. 6.

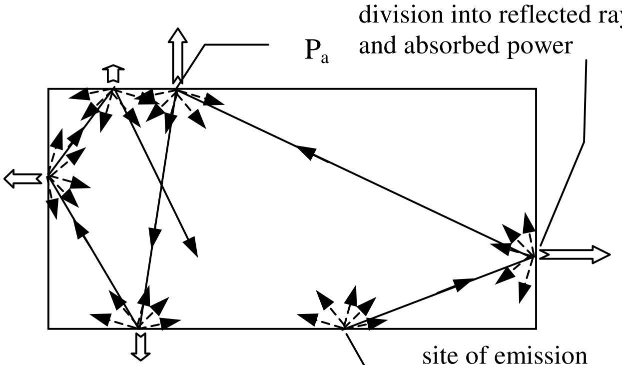

Fig. 6 Radiation traced in the classic Monte-Carlo method

In this manner, it is possible to track the flow of thermal power between system surfaces and define required temperature distributions. In a classic form, this is achieved by the Monte-Carlo method which, theoretically, makes it possible to render any heat exchange system. Universality (whereby any system can be modelled) is an unquestionable and important advantage of such solutions. Unfortunately, assuming that it is necessary to trace each emitted and each reflected ray, the number grows exponentially (see Fig. 6) and even contemporary computers seem too

Within this scope, research is focused on developing methods that would allow a reduction of the number of rays to be analysed, while maintaining the required accuracy of stimulations. One of such methods is a technique based on investigating the course of rays. Similarly to computer graphics, it consists of tracing rays from the moment of emission (from the source) until disappearance (e.g. absorption). In terms of principle, the method is identical as the one applied in computer graphics. Key differences are an effect of a different description of the light system and the thermal system. The range of acceptable simplifications is also different.

4. Similarities and differences between models

Similarities and differences of models describing the phenomena analysed here (illumination, exchange of thermal energy) from the viewpoint of computer graphics and radiative heat exchange are listed in Table 2 below.

Table 2. Computer graphics model and radiative heat exchange model

Model of computer graphics	Model of radiative heat exchange
Phenomenon analysed	
Propagation of electromagnetic radiation (visible radiation) $\lambda\in(0.380-0.780)$ µm	Propagation of electromagnetic radiation (infrared radiation) $\lambda\in(0.780-1000)$ µm
Ultimate goal of modelling	
Photorealistic image of the scene viewed by the observer	Distribution of temperatures (or power exchanges) on the surfaces
Required intermediate stage	
Distribution of illumination on observed surfaces	Distribution of absorbed powers on analysed surfaces

Table 2 – cont.	
Essential primary phenomena	
Generation and propagation of radiation Diffuse reflection, specular reflection	Generation and propagation of radiation Diffuse reflection, absorption
Secondary phenomena taken into account	
Direct reflection, diffraction, multichromatics	Specular or direct reflection
Disregarded phenomena	
Absorption, interference, polarisation	Polarisation, interference, diffraction

Other factors underlying differences in approach to computer graphics and radiative heat transfer include differing simplification assumptions adopted (often without listing) in developing software for both task types. A relevant list is provided in Table 3 below.

Table 3. Basic simplifications associated with analysed models

Model of computer graphics	Model of radiative heat exchange
Boundary surfaces	
Either source of radiation (light) or reflective surface	Radiation source and radiation-reflecting surface at the same time
No power balance for surfaces and the entire system	Power balance for each surface and the entire system
Absorption disregarded	Absorption calculated
Illumination boundary conditions only for light sources	Thermal boundary conditions for each boundary surface
Model of computer graphics	Model of radiative heat exchange
Source surfaces are independent of one another	Each surface affects other surfaces
Radiation	
Does not carry energy	Carries energy
No change of energy status of the surface	Change of energy status of the surface
Essential role played by colour (wavelength)	Wavelength often insignificant
Phenomena which are important for reflection	
Surface texture, colour, gloss, halo	Absorption of energy

5 Similarities of algorithms used in computer graphics and radiative heat exchange

Although goals pursued by computer graphics applications (visualisation) and programmes developed for radiative heat exchange modelling (determination of heat transfer) are different, there is a significant convergence between selected stages of both types of applications, mainly the phase of generation, propagation and reflections of radiation on boundary surfaces. Methods employed to trace the course (history) of individual rays also tend to be identical, as shown in Fig. 7.

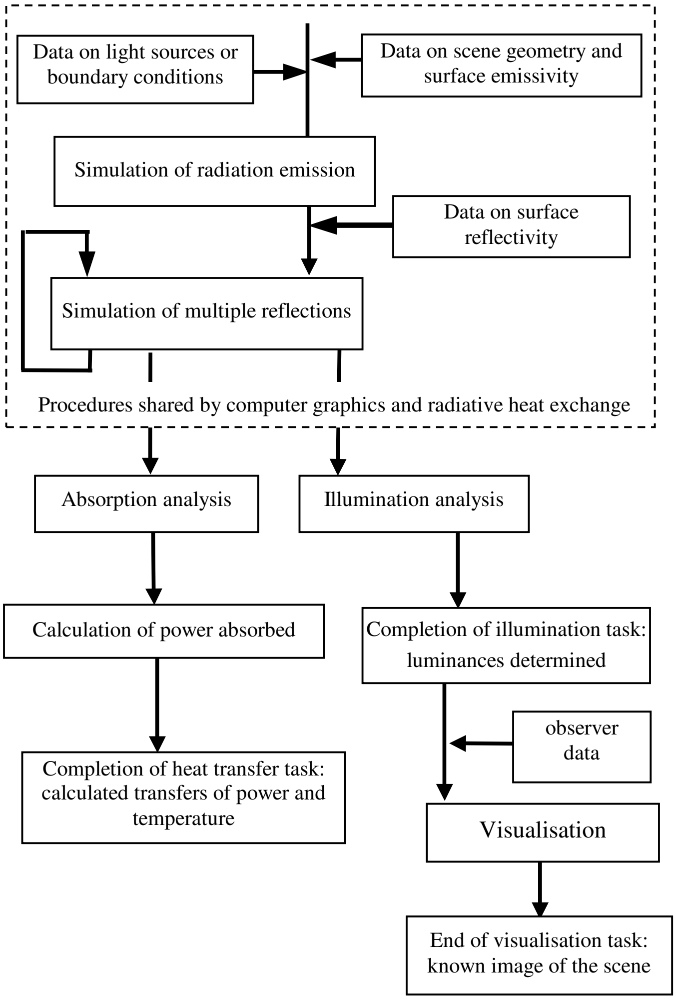

Fig. 7 Functional diagram of the modelling process: of radiative heat exchange and visualisation. [3,4]

6 Replaceability conditions

Analysing the basic equations of computer graphics and radiative heat exchange in ((1) and (16)), it is evident that they are nearly identical. This results from the fact that they both describe the same phenomenon, i.e. generation and reflection of the electromagnetic wave in a set direction. Computer graphics procedures and models are a subset of radiative heat exchange procedures and models, as in solving tasks related to the lighting technology one can adopt much more far-reaching simplifications than in radiative exchange. However, it is computer graphics that has been growing rapidly in recent years, with new improved computational procedures and simulation models being developed on an ongoing basis. It seems an interesting endeavour to explore the possibility of using existing tools in the form of computer graphics software to compute radiative heat transfer.

Not every computer graphics application can be adjusted for modelling radiative heat exchange. There are a number of preconditions which can be divided into two main groups: necessary and desirable. The former group of factors determine the suitability of a given computer graphics programme for adaptations allowing simulations of radiative heat exchange in general. The latter category includes factors that markedly facilitate the task and increase the potential accuracy of any computations that may be performed in the future. Absolutely necessary preconditions include the following [2,3,4]:

- reliance on the basic equation (1) or (2) in illumination modelling,
- modelling of multiple reflections,
- possibility to enter intermediate results in the form of files (not bitmaps) containing data on the distribution of radiance (or an equivalent quantity) on set surfaces.

Desirable preconditions include:

- possibility to define reflective characteristics of surfaces in the form of BRDF,
- compatibility of the computer graphics programme with CAD-type applications,
- possibility of any arrangement of emission characteristics of radiation sources,
- reliance of the graphics programme on the modelling of radiant, not luminous quantities.

7 Conversion of thermal quantities into radiant quantities

Radiative heat exchange is described by means of thermal quantities (temperature, power density). Computer graphics applications process luminous quantities (luminance) or radiant quantities (radiance). An adjustment of computer graphics programmes for the modelling and simulation of radiative heat transfer requires thermal quantities to be converted into radiant quantities. This applies in particular to boundary conditions describing radiative heat transfer systems.

Study [2] shows that boundary surface S (with defined Dirichlet condition (18), i.e. temperature $T=t+273$) can be replaced with an energetically equivalent surface of the source of radiation with radiance L and emissivity ε defined with the formula [2]:

Using the energy conservation law, it is possible to prove that for the surface, for which Dirichlet's condition has been specified (18) the equivalent surface of the emissivity ε has the radiance L, as described by the formula (26).

$$L(s_i, x_0, y_0, z_0) = \frac{\sigma\ \varepsilon(s_i) T^4(x_0, y_0, z_0)}{\pi \cos\varphi} \qquad (26)$$

where: $\cos\varphi = \frac{n_0 \cdot s_i}{|n_0||s_i|}$, n_o – the normal to the surface in point $r_o=(x_o, y_o, z_o)$ and direction s.

Similarly, surfaces with defined Neumann condition (19) an equivalent surface with emissivity ε can be assigned radiance L [1]:

$$L(s_i, x_0, y_0, z_0) = \frac{p(x_0, y_0, z_0)s_i + E_{sum}(x_0, y_0, z_0)\varepsilon(s_i)}{\pi} \qquad (25)$$

where E_{sum} represents total irradiation of the area analysed, resulting from the remaining surfaces of the system.

As a consequence, using the formulas given in (26) and (27), the radiative heat transfer task can be transformed into a lighting technology task, as illustrated in Fig. 8.

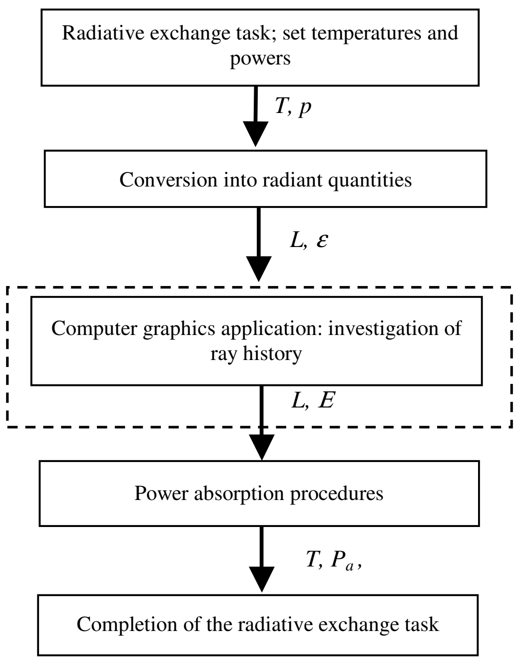

Fig. 8. Stages in modelling radiative heat exchange using computer graphics programmes

8 Sample thermal computations using computer graphics programmes

There are a number of computer graphics packages that can be applied for scene visualisation purposes. One of such programmes is Radiance. The system's ray tracking procedures, complemented with other specific functionalities, in effect produce a tool (an application) which makes it possible to model radiative heat transfer systems. A number of computations have been carried out, focused in particular on thorough tests of simple systems for which precise analytical solutions are available.

Detailed results are discussed in a range of publications, including [2,4]. All cases analysed reveal a significant similarity of results of simulations (though including a certain element of randomness) and precise computational results.

The problems presented in this paper, particularly computer graphics algorithms, are only sketchily described. A more exhaustive review of these issues can be found in broad literature on both computer graphics [6] and radiative heat exchange [7,11].

9 Conclusions

- Recent development of computer graphics technologies markedly surpasses the level of research and the range of available applications of radiative heat exchange programmes.
- Nearly all computer graphics programmes can be used in radiative heat transfer computations.
- Programmes based on the radiosity method can be used without major modifications, with only minor elements added. On the other hand, ray tracing-based methods require more extensive adaptations.
- In all modifications, starting thermal data must be transformed into luminous quantities, while output luminous data must be converted into thermal data.

Some computer graphics programmes (e.g. Radiance) meet the conditions allowing their application not only in lighting simulations, but also in resolution of the most complex tasks involved in radiative heat exchange. The programmes have procedures for emission modelling and the procedure of multiple reflections of visible radiation. Compliance with conditions indicated in the paper above is a precondition for employing such programmes to perform the basic function of a modelling programme, i.e. investigate the propagation of rays in a preset system. An addition of specific functions for conversion of thermal quantities into luminous quantities (and *vice versa*), and a function for adding up power carried by each ray, helps to achieve a good tool for modelling radiative heat transfer. Computations performed to verify the proposed solution have confirmed a high degree of similarity between results of simulations of radiative heat exchange (effected using the programmes) and analytical computations [2,3,4].

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