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Numerical Analysis of Marangoni Convection with Free-slip Bottom under Magnetic Field

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Abstract: - In this paper, we use a numerical technique to analyze the onset of Marangoni convection in a horizontal layer of electrically-conducting fluid heated from below and cooled from above in the presence of a uniform vertical magnetic field. The top surface of a fluid is deformable free and the bottom boundary is rigid and free-slip. The critical values of the Marangoni numbers for the onset of Marangoni convection are calculated and later it is found to be critically dependent on the Hartmann, Crispation and Bond numbers. We found that the presence of Magnetic field always has a stabilizing effect of increasing the critical Marangoni number when the free surface is non-deformable. If the free surface is deformable, then there is a range where the critical Marangoni number will have unstable modes no matter how large magnetic field becomes.

Key-Words: - Marangoni Convection, Magnetic Field, Free-slip

1 Introduction

Local variation in density produced by temperature gradients will create a “buoyancy” effect in the present of the gravity. There is a wealth of literature and study concerning the phenomena and with it, the ability to predict onset of flow or instability under many circumstances. Several investigations have been carried out to understand the buoyancy effect (Bacharoudis et al [1], Braescu and Duffar [6], Chen and Chen [9] and Hossain et al [14]). Convection in a plane horizontal fluid layer heated from below, initially at rest and subject to an adverse temperature gradient, may be produced either by buoyancy forces or surface tension forces. These convective instability problems are known as the Rayleigh-Benard convection and Marangoni convection, respectively. The copious literature on this problem and its extensions has been reviewed many times, notably by Chandrasekhar [8], Segel [24], Berg, Acrivos and Boudart [3], Brindley [7], Spiegel [26], Schechter and Velarde [21] and Koshmieder [15]. The determination of the criterion for the onset of convection and the mechanism to control has been a subject of interest because of its applications in the heat and momentum transfer research.

Thermal convection in fluid layers heated from below is a problem of great importance to many industrial applications (Mill and Keene [16] and Schwabe [22]). In many practical applications (such as crystal growth in microgravity environment) the onset of convection is undesirable, and as a consequence there has been considerable interest in understanding various additional physical mechanisms for delaying, or possibly eliminating altogether, the onset of convection. The technological need for instability postponement, turbulence suppression, and flow control in material processing technologies are currently leading to an increased interest in the interaction between thermocapillary flows in electrically conducting fluids and magnetic fields.

Rayleigh [19] was the first to solve the problem of the onset of thermal convection in a horizontal layer of fluid heated from below. His linear analysis showed that Benard convection occurs when the Rayleigh number exceeds a critical value. This parameter is a Rayleigh number (thermal or solutal) when the convection is induced by buoyancy effects due to variations in density and is a Marangoni number when surface-tension variations induce the convection.

It is well-known fact that the onset of convection in Benard’s [2] experiment is produced not simply by buoyancy forces but primarily by variations of the surface tension with the temperature. The latter effect is generally referred to in the literature under the name of thermocapillary or Marangoni instability. Although these flows were studied by Benard in 1900, it was almost sixty years before the critical experiment by Block [4] and the elegant linear stability analysis of Pearson [18] firmly established that Benard cells were a manifestation of the surface tension variations at the free surface (by Ginde et al [10]).

Theoretical analysis of Marangoni convection was started with the linear analysis by Pearson [18] who assumed an infinite fluid layer, a nondeformable case and zero gravity in the case of no-slip boundary conditions at the bottom. He showed that thermocapillary forces can cause convection when the Marangoni number exceeds a critical value in the absence of buoyancy forces. Pearson [18] obtained the critical Marangoni number, M_c = 79.607 and the critical wave number a_c = 1.9929. Linear stability analysis of Marangoni convection with free-slip boundary conditions at the bottom was first investigated by Boeck and Thess [5]. For free-slip case, Boeck and Thess [5] obtained the critical Marangoni number, M_c = 57.598 and the critical wave number a_c = 1.7003. Takashima [27,28] subsequently extended the stability analysis of Pearson [18] to study the effect of surface deformation on the steady and oscillatory Marangoni convection.

The determination of the criterion for the onset of convection and the mechanism to control convective flow patterns is important in both technology and fundamental Science. The problem of suppressing cellular convection in the Marangoni convection problem has attracted some interest in the literature. The effect of a body force due to an externally-imposed magnetic field on the onset of convection has been studied theoretically and numerically. The effect of magnetic field on the onset of steady buoyancy-driven convection was treated by Chandrasekhar [8] who showed that the effect of magnetic field is to increase the critical value of Rayleigh number and hence to have a stabilising effect on the layer.

The effect of a magnetic field on the onset of steady buoyancy and thermocapillary-driven (Benard-Marangoni) convection in a fluid layer with a non-deformable free surface was first analyzed by Nield [17]. He found that the critical Marangoni number monotonically increased as the strength of vertical magnetic field increased. This indicates that Lorentz force suppressed Marangoni convection. Later, the effect of a magnetic field on the onset of steady Marangoni convection in a horizontal layer of fluid has been discussed in a series of paper by Wilson [30, 31, 32]. The influence of a uniform vertical magnetic field on the onset of oscillatory Marangoni convection was treated by Hashim and Wilson [12] and Hashim and Arifin [11].

Wilson [30] investigated the effect of a magnetic field on the onset of steady Benard-Marangoni convection in a horizontal layer of fluid with free surface deformation and concluded that the presence of a magnetic field always has a stabilizing effect on the layer. The existence of long-wave instability modes in Marangoni convection was predicted theoretically by Scriven and Sterling [23] and Smith [25] who showed that the onset of steady Marangoni convection can be as either a long-wave or a short-wave mode, and verified experimentally by VanHook et al [29]. The linear growth rates for both the long- and short-wave modes for the pure Marangoni problem near the onset of convection was investigated by Reigner and Lebon [20].

Wilson and Thess [33] studied the linear growth rates of long-wave modes without the restriction of near critical conditions for the coupled Benard-Marangoni problem. The effect of a vertical magnetic field on the linear growth rates of Marangoni convection in a fluid layer was investigated by Hashim and Wilson [13] who derived the explicit analytical expressions for the linear growth rates for both long- and short-wave instability modes. Hashim and Wilson [13] also showed that the effect of increasing the magnetic field strength is always to stabilize the layer by decreasing the growth rates of the unstable modes.

The above investigators pertain their analyses to Marangoni convection in the presence of magnetic field with no-slip lower boundary condition. In this study, we consider the onset of steady Marangoni convective instability in a horizontal fluid layer of electrically-conducting fluid with a deformable upper free surface and a free-slip lower surface, subject to a uniform magnetic field. To the author's best knowledge this problem has not been reported in the literature. The linear stability theory is applied and the resulting eigenvalue problem is solved numerically. The effects of the Hartmann number and a free surface deformation on the onset of steady Marangoni convection are studied.

2 Problem Formulation

Consider a horizontal fluids layer of depth d heated from below subject to a uniform vertical magnetic field and a uniform vertical temperature gradient. The fluid layer is bounded below by a horizontal solid boundary at constant temperature T_1 and the above by a free surface at constant temperature T_2 which is in contact with passive gas at pressure P_o and constant temperature T_∞.

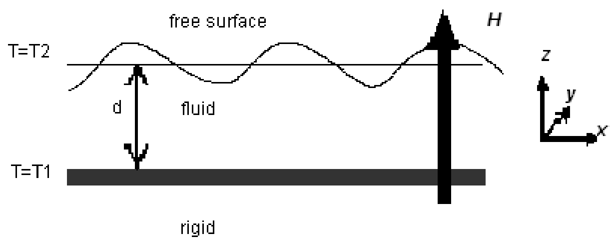


Fig. 1 Geometry of the unperturbed state

Our theoretical model involves the following basic assumptions : (i) zero buoyancy force, (ii) free-slip boundary conditions at the bottom.

The first assumption (i) is made to study the effects of purely thermocapillary forcing.

The neglect of buoyancy is justified if the ratio of Rayleigh to the Marangoni number Ra/Ma, measuring the relative strength of buoyancy and surface tension forces, is small. From $Ra/Ma \propto gd^2$, where g denotes the acceleration due to gravity and d is the layer thickness, it follows that buoyancy effects can be reduced by working with sufficiently shallow layers or even completely suppressed in an experiment in a microgravity environment. Under terrestrial conditions the ratio Ra/Ma is approximately 0.04 for a layer of liquid tin of 1 mm depth (See Ginde et al [10]).

Our reasons for working with assumption (ii) become apparent in the light of preliminary computations performed with the no-slip boundary condition at the bottom. For this case the transition from weak to inertial convection is found to occur only after a time-dependent (travelling wave) regime has been established, and inertial convection exhibits complex time dependence upon increasing the Marangoni number. With a free-slip bottom the generation of secondary vorticity is suppressed. The resulting behaviour is much simpler and provides the basis for an understanding of the more complex phenomena in the no-slip case (Boeck and Thess [5]).

We used cartesian coordinates with two horizontal *x*- and *y*- axis are located at the lower solid boundary and a positive *z*-axis is directed toward the free surface. The surface tension, τ is assumed to be a linear function of the temperature

$$\tau = \tau_o - \gamma(T - T_o) \qquad (1)$$

where τ_o is the value of τ at temperature T_o and the constant γ is positive for most fluids. The density of the fluids is given by

$$\rho = \rho_o\{1 - \sigma(T - T_o)\} \qquad (2)$$

where σ is the positive coefficient of the thermal liquid expansion and ρ_o is the value at the reference temperature T_o.

Subject to the Boussinesq approximation, the governing equations for an incompressible, electrically conducting fluid in the presence of a magnetic field are expressed as follows:

Continuity equation:

$$\nabla.\mathbf{U}=0 \qquad (3)$$

Momentum equation:

$$\left(\frac{\partial}{\partial t}+\mathbf{U}.\nabla\right)\mathbf{U} = -\frac{1}{\rho}\nabla\Pi + \nu\nabla^2\mathbf{U} + \frac{\mu}{4\pi\rho}\left(\mathbf{H}.\nabla\right)\mathbf{H} \qquad (4)$$

Energy equation:

$$\left(\frac{\partial}{\partial t}+\mathbf{U}.\nabla\right)T = \kappa\nabla^2 T \qquad (5)$$

Magnetic field equations:

$$\nabla.\mathbf{H}=0 \qquad (6)$$

$$\left(\frac{\partial}{\partial t}+\mathbf{U}.\nabla\right)\mathbf{H} = \left(\mathbf{H}.\nabla\right)\mathbf{U} + \eta\nabla^2\mathbf{H} \qquad (7)$$

where $\mathbf{U} = (u, v, w)$ is the fluid velocity, $\mathbf{H}$ is the magnetic field, T is the temperature, ν is the kinematic viscosity, κ is the thermal diffusivity, η is the electrical resistivity and $\Pi = p + \mu|\mathbf{H}|^2/8\pi$ is the magnetic pressure, where p is the fluid pressure and μ is the magnetic permeability. The free surface is deformably free with its position $z = d + \delta(x, y, t)$. The boundary conditions

$$\frac{\partial\delta}{\partial t} + u\frac{\partial\delta}{\partial x} + v\frac{\partial\delta}{\partial y} = w \qquad (8)$$

$$\alpha\nabla T \cdot \bar{n} + hT = 0 \qquad (9)$$

$$2\mu D_{nt} = \frac{\partial\gamma}{\partial T}\nabla T \cdot \bar{t} \qquad (10)$$

$$(Pa - P) + 2\mu D_{nn} = \gamma\nabla \cdot \bar{n} \qquad (11)$$

where α and h are the thermal conductivity and the heat transfer coefficient of the fluid layer, respectively. $\{D_{ij}\}$ is the rate of strain tensor in the fluid, $\overline{t}$ and $\overline{n}$ are the tangential and the outward normal unit vectors, respectively, at the free surface. At the lower rigid boundary the usual no-slip conditions requires continuity of velocity between the solid and the fluid.

To simplify the analysis, it is convenient to write the governing equations and boundary conditions in a dimensionless form. In the dimensionless formulation, scales for length, time, velocity, temperature and magnetic field have been taken to be $d, d^2/\nu, \nu/d, \beta d\nu/\kappa$ and $\mu\overline{H}/\eta$ respectively where $\overline{H}$ is the initial magnetic field strength. Furthermore, six dimensionless groups appearing in the problem are the Marangoni number $M = \gamma\beta d^2/\rho\nu\kappa$, the Hartmann number (the square root of the Chandrasekhar number) $H = \mu\overline{H}d(\sigma/\rho\nu)^{1/2}$, the Biot number $B_i = hd/k$, the Bond number $B_o = \rho g d^2/\tau_o$, the Prandtl number $P_1 = \nu/\kappa$, the magnetic Prandtl number $P_2 = \nu/\eta$ and the Crispation number, $C_r = \rho\nu\kappa/\tau_0 d$.

3 Linearized Problem

The linear stability of the basic state is analysed in the usual by seeking a solution for any physical quantity $\Phi(x, y, z, t)$ in normal mode form

$$\Phi(x, y, z, t) = \Phi_0(x, y, z) + \phi(z)e^{(ia_x x + ia_y y + st)} \quad (12)$$

where Φ_0 is the value of Φ in the basic state, ϕ is the amplitude of the perturbation, and $a = (a_x^2 + a_y^2)^{1/2}$ is the total horizontal wave number of the disturbance. The temporal s is complex with a real part representing the growth rate of the instability and an imaginary part representing its frequency.

Substituting into governing equations and boundary conditions, we obtain the linearised equations for the onset of Marangoni convection in an initially quiescent horizontal fluid layer bounded above by a deformable free surface and bounded below by a thermally conducting planar boundary subject to a uniform vertical magnetic field,

$$(D^2 - a^2)T + w = 0 \quad (13)$$

$$\left[(D^2 - a^2)^2 - H^2 D^2\right]w = 0 \quad (14)$$

subject to

$$w = 0 \quad (15)$$

$$P_1 C_r \left[\left(D^2 - 3a^2 - H^2 - s\right)Dw\right] - a^2(a^2 + B_o) f = 0 \quad (16)$$

$$P_1(D^2 + a^2)w + a^2 M(P_1 T - f) = 0 \quad (17)$$

$$h_z = 0 \quad (18)$$

$$P_1 DT + B_i(P_1 T - f) = 0 \quad (19)$$

evaluated on the undisturbed position of the upper free surface $z = 1$, and

$$w = 0 \quad (20)$$

$$D^2 w = 0 \quad (21)$$

$$h_z = 0 \quad (22)$$

$$T = 0 \quad (23)$$

on $z = 0$ where the condition of free-slip corresponds to Eq. (21).

The operator $D = \frac{d}{dz}$ denotes differentiation with respect to the vertical coordinate z. The variables w, T, h_z and f denote respectively the vertical variation of the z-velocity, temperature, magnetic field and the magnitude of the free surface deflection of the linear perturbation to the basic state with total wave number a in the horizontal x-y plane and complex growth rate s.

4 Solution of the Linearized Problem

In the general case $s = 0$, we follow the solution approach of Hashim and Wilson [3] and seek asymptotic solutions for w, T in the forms

$$w(z) = ACe^{\xi z}, \quad T(z) = Ce^{\xi z} \quad (24)$$

where the exponent ξ and the complex constants A and C are to be determined. Substituting these forms into the Eqs. (13) and (14) and eliminating A and C we obtain a sixth-order algebraic equation for ξ, namely

$$(\xi^2 - a^2)\left[(\xi^2 - a^2 - s)^2 - H^2\xi^2\right] = 0 \quad (25)$$

with six distinct roots, which we denote by $\xi_1, \ldots, \xi_6$. Where the values of $\xi_1, \ldots, \xi_4$ are solutions of the fourth-order algebraic equation

$$(\xi^2 - a^2 - s)^2 - H^2\xi^2 = 0 \quad (26)$$

while $\xi_5 = a$ and $\xi_6 = -a$.

Denoting the values of *A* and *C* corresponding to ξ for $i=1,...,6$ by A_i and C_i, respectively, we can use Eq. (14) to determine A_i. We can use Eq. (16) to eliminate the free surface deflection

$$f=\frac{P_1 C_r (D^2-3a^2-H^2)Dw}{a^2(a^2+B_o)} \tag{27}$$

evaluated on $z=1$, leaving the six boundary conditions, to determine the six unknowns $C_1,...C_6$, and the general solution to the stability problem therefore

$$w(z)=\sum_{j=1}^{6} A_j C_j e^{\xi_j z},\ T(z)=\sum_{j=1}^{6} C_j e^{\xi_j z} \tag{28}$$

The dispersion relation between M, a, C_r, H^2, B_o and B_i is determined by substituting these solutions into boundary conditions and evaluating the resulting 6×6 real determinants of the coefficients of the unknowns, which can be written in the form $M=-D_1/D_2$, where the two 6×6 real determinants D_1 and D_2 are independent of *M*.

After some simplification the elements of the determinant $D_1=\left|d_{ij}\right|$ are given by

$$d_{1i} = A_i e^{\xi_i} \tag{29}$$
$$d_{2i} = {\xi_i}^2 A_i e^{\xi_i} \tag{30}$$
$$d_{3i} = (\xi_i + B_i) e^{\xi_i} \tag{31}$$
$$d_{4i} = A_i \tag{32}$$
$$d_{5i} = {\xi_i}^2 A_i \tag{33}$$
$$d_{6i} = 1 \tag{34}$$

for $i=1,...,6$. The coefficients of the determinant D_2 are the same as those of D_1 apart from the terms

$$d_{2i}=a^2\left[1-\frac{C_r({\xi_i}^2-3a^2-H^2)\xi_i A_i}{a^2(a^2+B_o)}\right]e^{\xi_i} \tag{35}$$
$$d_{3i} = {\xi_i}^2 e^{\xi_i} \tag{36}$$

for $i=1,...,6$. Notice that D_1 is independent of C_r and B_o and that D_2 is independent of B_i. We could express D_1 and D_2, and hence *M*, explicitly in terms of hyperbolic functions, but since its value must then be evaluated numerically we gain little over direct numerical evaluation.

5 Numerical Results

The effect of a magnetic field on the onset of Marangoni convection in a fluid layer with free-slip bottom in the case of a deformable free surface $(C_r \neq 0)$ is investigated numerically. The marginal stability curves in the (a, M) plane are obtained numerically where *M* is a function of the parameters a, B_i, B_o, C_r and *H*. For a given set of parameters, the critical Marangoni number for the onset of convection defined as the minimum of the global minima of marginal curve. We denote this critical value by M_c and the corresponding critical wave number, a_c. Before presenting the numerical results, it is helpful to specify the range for parameters B_i, B_o and C_r which are respectively given by $10^{-3} \le B_i \le 10^{-1}$, $10^{-3} \le B_o \le 10^{-1}$ and $10^{-6} \le C_r \le 10^{-2}$ for most fluids layers of depths ranging from 0.01 cm to 0.1 cm and are in contact with air.

The problem has been solved to obtain a detail description of the marginal stability curves for the onset of Marangoni convection when the free surface is perfectly insulated ($B_i=0$). The crispation number C_r, associated with the inverse effect of the surface tension, represents the degree of the free surface deformability and the behaviour of the marginal stability curves depends on whether $B_o=0$ or $B_o \neq 0$. When C_r becomes large (corresponding to weak surface tension), the marginal curve has global minimum at zero wavenumber. In contrast, for small values of C_r, the marginal curve has global minimum at nonzero wavenumber. At some transition value of C_r, the marginal curve has two local minima that is one at zero wave number and the other at nonzero wave number.

Fig. 2 and 3 shows the numerically-calculated steady marginal stability curves plotted for different values crispation number C_r in the case $H=0$, $B_o=0$ and $B_o=0.1$ respectively. An inspection of the Fig. 2 reveals that the marginal stability curves attain their minimum value of zero at $a=0$ so that $M_c=0$ and $a_c=0$. Hence for all values of $M>0$ disturbances with sufficiently small wave number will be unstable modes regardless of the value of *M*. The transition value of C_r for the case shown in Fig. 3 is $C_r \approx 0.0001764$. For C_r greater than 0.0001764, the wave number at marginal stability suddenly drop from nonzero number to zero. Similar competition between different modes was identified by Hashim and Arifin [11] in the case no-slip condition.

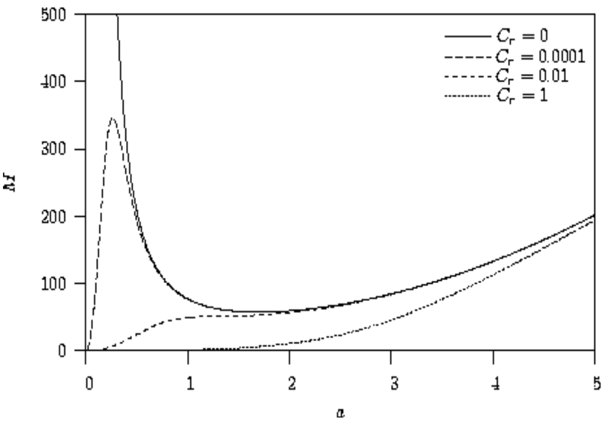


Fig. 2 Numerically-calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Crispation numbers, C_r in the case $H = 0$, $B_i = 0$ and $B_o = 0$.

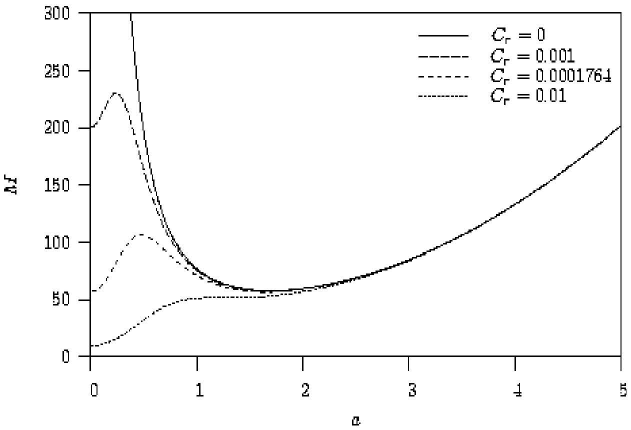


Fig. 3 Numerically-calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Crispation numbers, C_r in the case $H = 0$, $B_i = 0$ and $B_o = 0.1$.

When the free surface is nondeformable, $C_r = 0$, the marginal stability curves are always have a single global minimum at a nonzero value of wave number, *a*. Fig. 4 shows the numerically calculated Marangoni number, *M* as a function of the wavenumber, *a* for different values of the Hartmann number, *H* in the case $C_r = 0$. From Fig. 4 it is seen that the critical Marangoni number increase with an increase of the Hartmann number. Thus, the magnetic always has a stabilizing effect on the flow. In the absence of magnetic field, $H = 0$, the present calculation reproduce closely the stability curve obtained by Boeck and Thess [5].

Before presenting the detail of the effect of magnetic field for the onset of convection in the case $C_r \neq 0$, we presented a situation in which two steady modes compete at the onset of convection. Numerically calculated Marangoni number, *M* as a function of the wave number, *a* for different values of the $C_r \neq 0$ in the case $H^2 = 100$ are shown in Fig. 5. The figure shows parts of the marginal stability curves in the case $C_r = 0.00037115$ and $H^2 = 100$ in which zero mode (infinite wavelength) and nonzero mode (finite wavelength) occur simultaneously at the onset of convection.

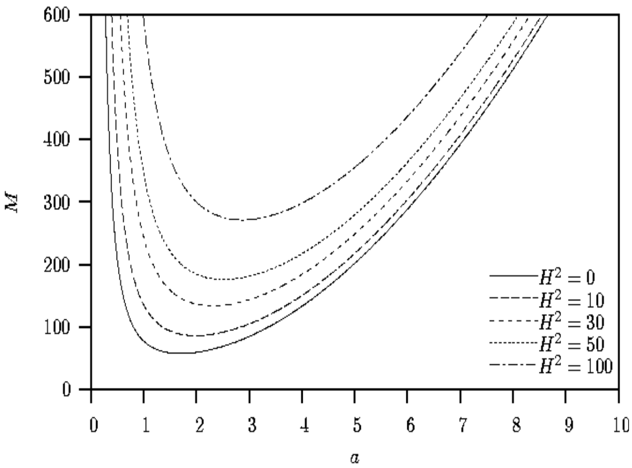


Fig. 4 Numerically-calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Hartmann numbers, *H* in the case $C_r = 0$, $B_i = 0$ and $B_o = 0.1$.

Fig. 6 shows the numerically calculated Marangoni number, *M* as a function of the wave number, *a* for different values of the Hartmann number, *H* in the case $C_r = 0.001$. In this case, the marginal stability curve have a global minimum at the nonzero value of *a* without a magnetic field. But, the marginal stability curves always have a global minimum at zero value in the limit of large magnetic field. We also found two steady modes occur simultaneously at the onset of convection when $H^2 = 10$.

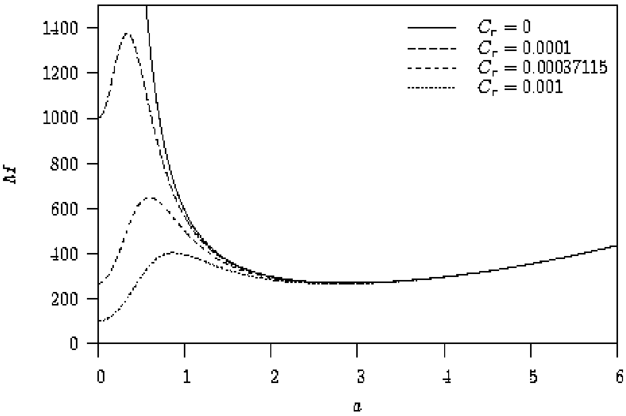


Fig. 5 Numerically-calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Crispation numbers, C_r in the case $H^2 = 100$, $B_i = 0$ and $B_o = 0.1$.

In the case $C_r \neq 0$ and $B_o = 0$, the critical Marangoni number at the onset of convection is zero at wave number, $a = 0$ as shown in Fig. 7. The magnetic field is not effective at the wave number $a = 0$. When $C_r \neq 0$ and $B_o \neq 0$, the situation is significantly different.

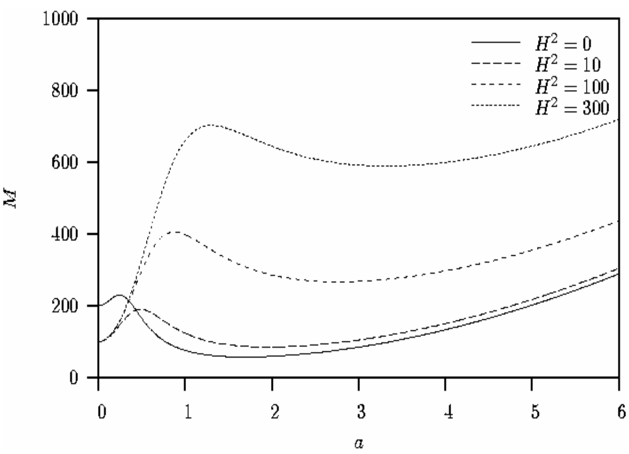


Fig. 6 Numerically calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Hartmann numbers, *H* in the case $C_r = 0.001$, $B_i = 0$ and $B_o = 0.1$

Numerically calculated Marangoni number, *M* as a function of the wave number, *a* for different values of the $C_r \neq 0$ in the case $B_o = 1$ and $H^2 = 10$ are plotted in Fig. 8. It can been seen that the marginal stability curves also have two local minimum and the critical Marangoni number, M_c may occur either at $a_c = 0$ or $a_c \neq 0$.

Fig. 9 shows numerically calculated the critical Marangoni number, M_c and the critical wave number, a_c as a function of the Hartmann number for different values of C_r. From Fig. 9(a), it is seen that the critical Marangoni number, M_c increases monotonically as Hartmann number is increased from zero and that if $C_r \neq 0$ then situations with sufficiently large Marangoni number will always have unstable modes. In Fig. 9(b), it can be seen that the critical wave number, a_c increases monotonically and suddenly drop to zero as Hartmann number, *H*, is increased from zero.

Fig. 10 shows numerically-calculated the critical Marangoni number, M_c and the critical wave number, a_c as a function of the C_r for different values of Hartmann number in the case $B_o = 1$. As shown in Fig. 10, both the critical Marangoni numbers, M_c and the critical wave numbers, a_c decrease monotonically as C_r is increased from zero and hence the effect of the surface deformation is always to destabilize the layer.

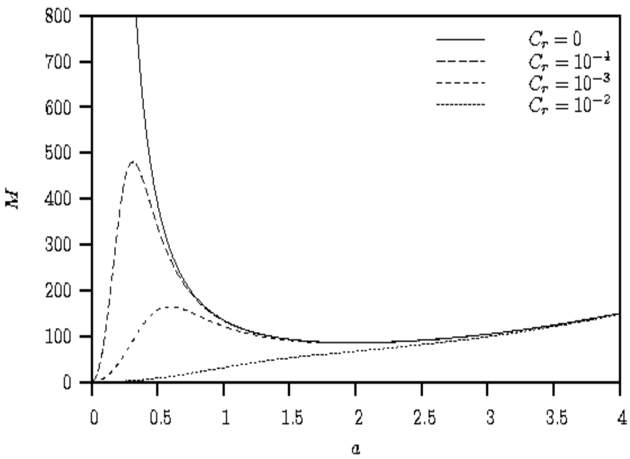


Fig. 7 Numerically-calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Crispation numbers, C_r in the case $H^2 = 10$, $B_i = 0$ and $B_o = 0$.

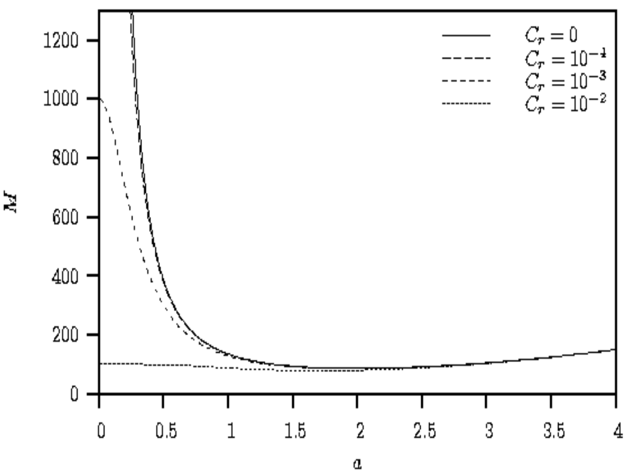
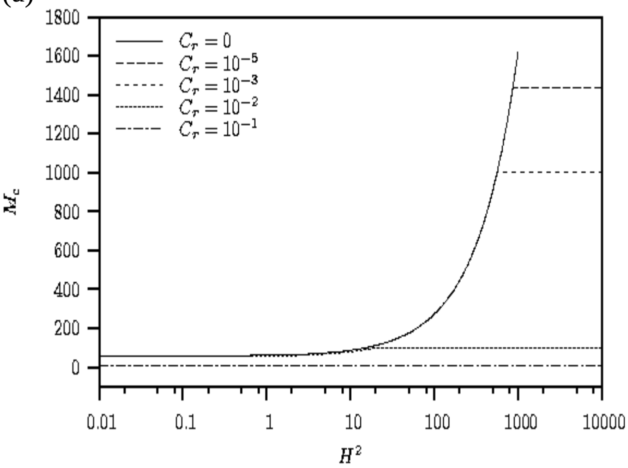


Fig. 8 Numerically-calculated Marangoni number, *M* as a function of the wavenumber, *a*, for various values of Crispation numbers, C_r in the case $H^2 = 10$, $B_i = 0$ and $B_o = 1$.

(a)



(b)

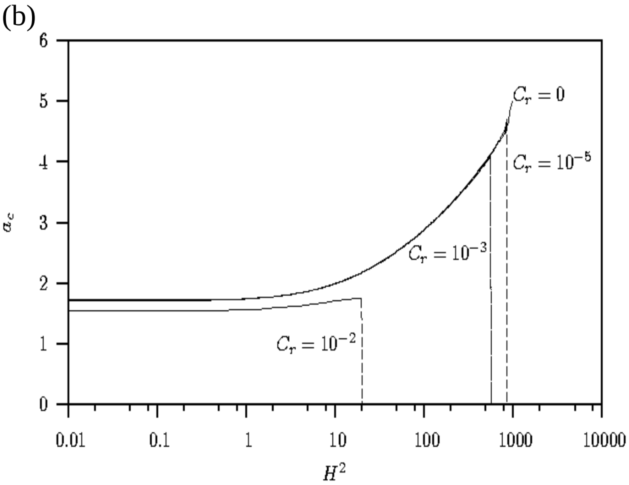
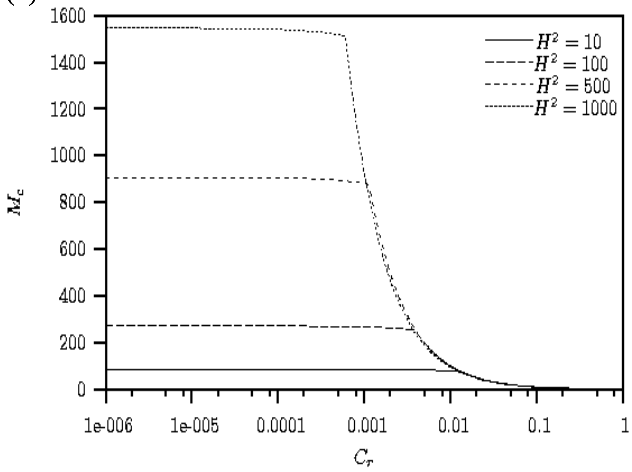


Fig. 9 Numerically-calculated critical Marangoni number, *M* as a function of Hartmann numbers in the case $B_o = 1$ and $B_i = 0$ for a range of values of C_r (a) M_c and (b) a_c

(a)



(b)

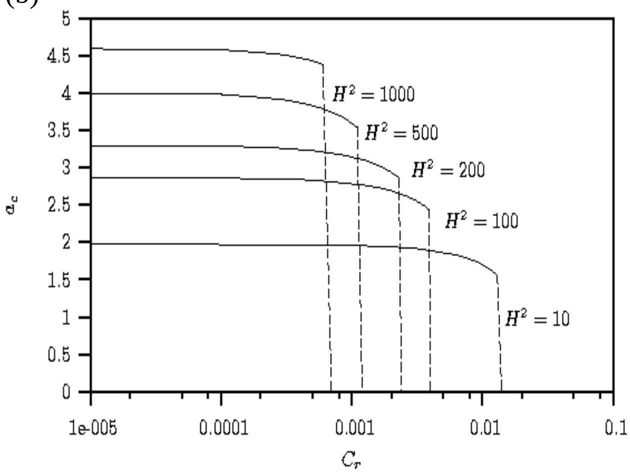


Fig. 10 Numerically-calculated critical Marangoni number, *M* as a function of C_r in the case $B_o = 1$ and $B_i = 0$ for a range of values of Hartmann numbers (a) M_c and (b) a_c.

6 Conclusions

The effect of magnetic field on the onset of steady Marangoni convection in a horizontal layer of electrically conducting fluid which is free above and rigid below with free-slip condition has been studied. The problem has been solved numerically to obtain a detail description the marginal stability curves for the perfectly insulated free surface. The effects of the Hartmann number on the onset of steady Marangoni convection are more pronounced, especially for the non-deformable free surface. The system becomes more stable as the Hartmann number increases.

Deformation of the upper surface, associated with the Crispation number, plays a significant role on the onset of steady modes of the Marangoni convective instability. The critical Marangoni number decreases with an increase in the Crispation number. If Bond number is zero, then all situations have unstable modes. The situation is significantly different if Bond number is not zero where the marginal stability curves have two local minima and the critical Marangoni number may occur at zero or nonzero wave number.

In physical situation, however, the free surface is deformed due to the fluid motion, the convection may appear as an oscillatory instability. Oscillatory instabilities are found when the fluid is heated from below and for a positive Marangoni number with magnetic field in the case no-slip at the lower boundary (Hashim and Wilson [12] and Hashim and Arifin [11]). Therefore, it is necessary to consider the onset of oscillatory Marangoni convection in the presence of a magnetic field with free-slip at the bottom for future study.

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