Heat transfer correlations for free convection from upward-facing horizontal rectangular surfaces

MASSIMO CORCIONE
Dipartimento di Fisica Tecnica
"Sapienza" University of Rome
via Eudossiana, 18 – 00184 Rome
ITALY
massimo.corcione@uniroma1.it

Abstract: A critical review of the heat transfer data available in the literature for free convection from upward-facing heated rectangular plates is conducted. The review is organized in the form of a table, so as to give the reader the opportunity to compare the dimensionless correlating equations presented, and the conditions under which the heat transfer data were obtained. A comparative survey of the results that may be derived at different Rayleigh numbers from these correlations is also reported. It is shown that the discrepancy among the results, which on the whole may also amount to ±50%, decreases significantly when the data are grouped according to the working fluid and the plate geometry. The following four mean heat transfer correlating equations are porposed: $Nu = 1.05Ra^{0.215}$ for square plates in air, $Nu = 0.9Ra^{0.23}$ for rectangular plates in air, $Nu = 0.96Ra^{0.19}$ for two-dimensional infinite strips in air, and $Nu = 0.175Ra^{0.33}$ for rectangular plates in water.

Key-Words: Free convection; Upward-facing horizontal plates; Heat transfer correlations; Comparative survey.

1 Introduction

Natural convection heat transfer from unbounded, horizontal flat plates of finite size facing upwards is one of the fundamental free-convection problems, since it appears in several science and engineering applications, as well as in a wide variety of natural circumstances.

Starting from the first decades of 1900, a large body of research has been performed on this topic, for both conditions of uniform wall temperature and uniform heat flux, as witnessed by the numerous heat transfer correlating equations readily available in the literature.

However, in some cases, the predictions of these correlations may also differ by ±50%, depending on the investigation method, the boundary conditions, and the occurrence of more or less remarkable three-dimensional effects.

In this context, it is felt the need to organize an extensive, reasoned review of the most prominent heat transfer correlations for horizontal square and rectangular flat plates facing upwards – expressed in the typical dimensionless form $Nu = C(Ra)^{\alpha}$ –, with the main aims to highlight the conditions under which such correlations were obtained, to point out their main features, and to present a comparative survey of the results that may be derived at several Rayleigh numbers through their application, so as to help the reader in practical situations.

Further scope of the present work is to develop mean dimensionless heat transfer correlations based on the collection of the data presented in the paper, useful to heat transfer designers and constructors.

2 Review of literature

The majority of the studies available in the literature on natural convection heat transfer from horizontal rectangular plates facing upwards have been carried out experimentally, by using air or water as working fluid. In fact, the analytical approach to the problem, typically based on the boundary-layer theory, could not take into proper account the effects of the finite width of the plate on the heat transfer performance, as well as the effects of the flow separation from the plate surface or other flow instabilities. On the other hand, at the time when most investigations were executed, the numerical methods for the solution of the mass, momentum and energy transfer governing equations were not yet completely developed and validated.

2.1 Experimental studies

The first experimental work on free convection heat transfer from horizontal flat surfaces, which dates back to 1932, was executed in air by Schmidt [1], whose Schlieren pictures showed that a boundary-layer flow might be expected near the leading edges.

In 1935, Weise [2] measured the temperature distribution over horizontal square plates heated at both their sides, while Kraus [3], five years later, determined also the corresponding velocity fields.

Experimental heat transfer results for upward-facing horizontal surfaces were published in 1950 by Fishenden and Saunders [4] – who used square or nearly square plates situated in air –, in 1952 by Bosworth [5] – who provided very little information on the plate shape and working fluid –, and in 1968 by Mikheyev [6] – who apparently used rectangular plates, concluding that the heat transfer rate from a horizontal upward-facing plate was 30% more than that from a vertical plate of the same size in the same conditions.

In 1968, Husar and Sparrow [7] used an electro-chemical technique to visualize the flow field over horizontal heated surfaces. They observed that the fluid moves along parallel paths perpendicular to the edges of the plate up to reaching the central region of the plate, yet, the reasons for such flow patterns were not discussed in detail.

Rotem and Claassen [8], in 1969, carried out visualizations with a semi-focusing colour-Schlieren apparatus. They reported that boundary layers exist neat the edges of the plate and then break down into large-eddy instability some distance from the edges. However, it is not clear whether edge flows were prevented on some sides of the heated surface, i.e., whether the boundary layers formed from all four edges of the plate simultaneously.

Hassan and Mohamed [9], and Fujii and Imura [10], in 1970 and 1972, respectively, considered the horizontal upward-facing surface as part of wider studies of free convection from inclined plates.

In 1973, Pera and Gebhart [11, 12] measured the temperature distribution and the local heat transfer coefficients in air by using a Mach-Zender interferometer, with additional flow visualization of streamlines by smoke. They revealed that severe instability mechanisms disrupt the boundary-layer flow some distance from the plate leading edge, reporting the existence of three-dimensional effects. In addition, the onset of the formation of unattached flow was observed to occur at a $(Gr_x)^{1/3}$ of approximately 80.

In the same year, Goldstein et al. [13] performed mass transfer experiments on free convection from horizontal flat surfaces of various planforms using the naphthalene sublimation technique. The results obtained for all the plates were well correlated by a common expression, provided that the characteristic length used in the Sherwood and Rayleigh numbers was evaluated as the ratio between the surface area of the plate and the encompassing perimeter, such length appearing to closely approximate the average distance travelled by the fluid particles moving from the edges of the plate into the interior.

Mass transfer experiments on natural convection adjacent to horizontal surfaces of different shapes were carried out also one year later by Lloyd and Moran [14], who used the same electrochemical technique previously employed for mass transfer investigations from horizontal disks by Wragg [15], and Wragg and Loomba [16].

In 1976, Al-Arabi and El-Riedy [17] obtained local and average heat transfer data for horizontal plates of different shapes on the basis of the amount of steam condensate collected from compartments underneath the plates. It was found that the average heat transfer from a circular plate was practically the same as that from a square plate with side-length equal to the diameter of the circular plate, which led the authors to conlude that the effects of the plate corners on the heat transfer rate is negligible.

Ishiguro et al. [18], in 1978, conducted an interferometric study, reporting that the temperature field over a horizontal plate became three-dimensional when the Rayleigh number based on the plate width exceeded 10^5. The same experimental method was employed by Yousef et al. [19] in 1982 to derive the local and average heat transfer coefficients of square plates embedded flush in a surrounding collar, used to minimize the edge effects.

Further mass transfer experiments with square naphthalene plates in air were performed in 1983 by Goldstein and Lau [20]. Further heat transfer results for rectangular surfaces were then obtained in 1984 by Lewandowski and Kubski [21].

In 1986, Sparrow and Carlson [22] carried out experiments on uniformly heated horizontal plates either shrouded by a parallel adiabatic surface or unshrouded, both in the case the surroundings of the plate were unobstructed and in the case the heated plate was encircled by a coplanar adiabatic frame which prevented air from being drawn from below.

Flow visualizations of the onset and subsequent development of longitudinal vortex rolls for natural convection flows over horizontal and inclined plates were carried out in 1988 by Cheng and Kim [23]. Three flow regimes were identified, consisting of a two-dimensional laminar flow, a transition regime with three-dimensional developing of longitudinal vortices, and a turbulent regime after the breakdown of the longitudinal vortices.

In 1995, Kitamura and Kimura [24] performed an experimental study of free convection in water from slender rectangular plates heated with uniform heat flux and equipped with fences at both longer sides to inhibit side flows, thus obtaining a two-dimensional flow field over the plates. Fluid flows and surface temperatures were visualized by dye tracers and liquid-crystal thermometry, respectively. Four distinct flow regions appeared, consisting of a two-dimensional laminar boundary layer region near the leading edges, a transitional region characterized by a three-dimensional flow separation and by the attachment of ambient fluid onto the plate surface downstream of the flow separation, a fully turbulent region, and a collision region near the centreline of the plate.

In 2000, Lewandowski et al. [25] conducted heat transfer measurements and visualized the convective flow structures in water above rectangular plates by injecting a dark-blue dye tracer through small holes distributed along the perimeter of each plate. In the same year, Pretot et al. [26] performed experiments using a rectangular plate heated with uniform heat flux. Their results show that, starting from the edges of the plate, the heat transfer is characterized by a thermal boundary layer that breaks away from the surface near the center of the heated plate, giving rise to a buoyant thermal plume.

More recently, heat transfer measurements from upward-facing rectangular plates were executed in air by Martorell et al. [27], in 2003, and in water by Kozanoglu and Lopez [28], in 2007.

2.2 Theoretical studies

The first theoretical study on natural convective heat transfer from horizontal surfaces, was performed in 1952 by Stewartson [29], who demonstrated the existence of similarity solutions for a semi-infinite isothermal flat surface immersed in air. However, the analysis was affected by a sign mistake that led the author to an erroneous conclusion regarding the conditions for the existence of a boundary-layer flow on such a plate. This was corrected in 1965 by Gill et al. [30], who showed that the only flow for which boundary-layer solutions could be obtained was that generated by a heated plate facing upwards, or a cooled plate facing downwards.

In 1969, Rotem and Claasem [8] integrated the boundary-layer equations for free convective flows above an isothermal semi-infinite plate. The results obtained were presented for some specific values of the Prandtl number, including the asymptotic cases of zero and infinite Prandtl numbers. In 1973, also Pera and Gebhart [11] reported similarity solutions.

Ackroyd [31], in 1976, assuming the the flow paths were parallel, obtained a heat transfer solution for rectangular plates. In the same year, Bandrowski and Rybski [32] published analytical solutions of the boundary-layer equations for natural convection mass transfer from upward-facing surfaces.

The first numerical solutions of the full Navier-Stokes, continuity and energy transfer equations for free convection from horizontal upward-facing plates were published in 1983 by Goldstein and Lau [20] – indeed, a previous numerical study was performed in 1968 by Suriano and Yang [33], but it concerned a surface heated at both its sides. Two-dimensional solutions were obtained for the simple plate, and for the plate provided with insulated edge-extensions, either horizontal or vertical.

In 1986, Chen et al. [34] considered the upward-facing horizontal semi-infinite plate with uniform wall temperature or uniform heat flux as a special case of a larger boundary-layer investigation of free convection heat transfer from inclined plates with variable surface temperature or heat flux.

In 1991, Lewandowski [35] presented a quasi-analytical solution of a theoretical model, in which the boundary layers grow from each plate-edge up to transforming into a plume at the separation point.

Lewandowski et al. [25], in 2000, proposed two different models for describing the convective flow structures which originate above a rectangular plate. The first model was based on the assumptions that the streamlines in the boundary layers are parallel to each other and the fluid flow, which is normal to the edges of the plate, is directed towards the centreline of the plate; here the boundary layers transform into a primary linear convective stream, which, being unstable, divides into secondary plumes. In contrast, the second model was based on the assumptions that the streamlines in the boundary layers are radial and that the fluid flow is directed to one or more points located at the centre of the plate where the boundary layers transform directly into one or more separate buoyant plumes. On account of their visualizations, the authors concluded that the radial flow model should be preferred at low values of the Rayleigh number, especially for square plates or at the ends of rectangular plates, while the parallel model is better for the flow at high Rayleigh numbers.

Numerical solutions of the mass, momentum and energy governing equations were obtained in 2003 by Wei et al. [36], as part of a wider 2D study on free convection from isothermal plates heated at both sides, and by Martorell et al. [27], who derived 2D and 3D solutions for slender plates, concluding that although the flow near the ends of the plate is typically three-dimensional, the overall flow suffers no dramatic change with respect to the 2D structure.

3 Heat transfer data

A collection of heat transfer data available in the literature for free convection from horizontal square and rectangular plates facing upwards is presented in Table 1, in which a list of heat correlations, and other general information, is reported.

Table 1 – Collection of dimensionless correlating equations for upward-facing horizontal plates

Authors	Plate geometry	Fluid	Correlations	Buoyancy strength	Investigation
Fishenden and Saunders, 1950 [4]	square	air	$Nu = 0.540Ra^{1/4}$	$10^5 < Ra < 2\times10^7$	experimental
			$Nu = 0.140Ra^{1/3}$	$2\times10^7 < Ra < 3\times10^{10}$	
Bosworth, 1952 [5]	square	unknown	$Nu = 0.710Ra^{1/4}$	laminar flow	experimental
			$Nu = 0.170Ra^{1/3}$	turbulent flow	
Rotem and Claassem, 1969 [8]	semi-infinite plate	$Pr = 0.72$	$Nu = 0.639Ra^{1/5}$	laminar flow	analytical
		$Pr = 10.0$	$Nu = 0.726Ra^{1/5}$	laminar flow	
Fujii and Imura, 1972 [10]	rectangular (L/W = 2)	water	$Nu = 0.160Ra^{1/3}$	$7\times10^6 < Ra < 2\times10^8$	experimental
			$Nu = 0.130Ra^{1/3}$	$5.7\times10^8 < Ra < 6\times10^{10}$	
Pera and Gebhart, 1973 [11]	semi-infinite plate	$Pr = 0.72$	$Nu_x = 0.363Ra_x^{1/5}$	laminar flow	analytical
			$Nu = 0.646Ra^{1/5}$		
Goldstein et al., 1973 [13]	various shapes	$Sc = 2.5$	$Sh = 0.960(Ra_m)^{1/6}$	$1 < Ra_m < 10^2$	experimental ([a])
			$Sh = 0.590(Ra_m)^{1/4}$	$2\times10^2 < Ra_m < 8\times10^3$	
	square		$Sh = 1.920(Ra_m)^{1/6}$	$64 < Ra_m < 6.4\times10^3$	([b])
			$Sh = 0.834(Ra_m)^{1/4}$	$1.3\times10^4 < Ra_m < 5.4\times10^5$	
	rectangular (L/W = 7)		$Sh = 1.451(Ra_m)^{1/6}$	$12 < Ra_m < 1.2\times10^2$	
			$Sh = 0.725(Ra_m)^{1/4}$	$2.4\times10^3 < Ra_m < 9.6\times10^4$	
Lloyd and Moran, 1974 [14]	various shapes	$Sc = 2200$	$Sh = 0.540(Ra_m)^{1/4}$	$2.2\times10^4 \le Ra_m \le 8\times10^6$	experimental ([a])
			$Sh = 0.150(Ra_m)^{1/3}$	$8\times10^6 \le Ra_m \le 1.6\times10^9$	
	square		$Sh = 0.764(Ra_m)^{1/4}$	$1.4\times10^6 \le Ra_m \le 5.1\times10^8$	([b])
			$Sh = 0.150(Ra_m)^{1/3}$	$5.1\times10^8 < Ra_m \le 10^{11}$	
	rectangular (L/W = 5)		$Sh = 0.672(Ra_m)^{1/4}$	$3\times10^5 \le Ra_m \le 1.1\times10^8$	
			$Sh = 0.150(Ra_m)^{1/3}$	$1.1\times10^8 < Ra_m \le 2.2\times10^{10}$	
	rectangular (L/W = 10)		$Sh = 0.657(Ra_m)^{1/4}$	$2.3\times10^5 \le Ra_m \le 8.5\times10^7$	
			$Sh = 0.150(Ra_m)^{1/3}$	$8.5\times10^7 < Ra_m \le 1.7\times10^{10}$	

Table 1 (continued)

Al-Arabi and El-Riedy, 1976 [17]	rectangular (L/W = 1 to 4)	air	$Nu = 0.700Ra^{1/4}$ $Nu = 0.155Ra^{1/3}$	$2\times10^5 \le Ra \le 4\times10^7$ $4\times10^7 < Ra \le 10^9$	experimental
Ishiguro et al., 1978 [18]	rectangular (L/W = 1 to 4.6)	water	$Nu = 0.200Ra^{1/3}$	$3\times10^5 < Ra < 10^{10}$	experimental
Yousef et al., 1982 [19]	square	air	$Nu = 0.622Ra^{1/4}$ $Nu = 0.162Ra^{1/3}$	$3\times10^6 \le Ra \le 4\times10^7$ $4\times10^7 < Ra \le 1.7\times10^8$	experimental
Goldstein and Lau, 1983 [20]	square 2D infinite strip	$Sc = 2.5$ $Pr = 0.7$	$Sh = 0.746(Ra_m)^{1/5}$ $Nu = 0.621Ra^{1/5}$	$10 < Ra_m < 4.8\times10^3$ $40 < Ra < 8\times10^3$	experimental (a) numerical (a)
	square 2D infinite strip	$Sc = 2.5$ $Pr = 0.7$	$Sh = 1.300(Ra_m)^{1/5}$ $Nu = 0.819Ra^{1/5}$	$6.4\times10^2 < Ra_m < 3\times10^5$ $3.2\times10^2 < Ra < 6.4\times10^4$	experimental (b) numerical (b)
Sparrow and Carlson, 1986 [22]	rectangular (L/W = 3.3)	air	$Nu = 1.070(Ra^*)^{1/6}$ $Nu = 1.084Ra^{1/5}$	$3\times10^6 \le Ra^* \le 2.5\times10^7$ $2\times10^5 \le Ra \le 1.2\times10^6$	experimental (c)
Chen et al., 1986 [34]	semi-infinite plate	$0 < Pr < \infty$	$Nu_x = K(Ra_x/5)^{1/5}$ $Nu = 1.667\,K(Ra/5)^{1/5}$ $K = Pr^{1/2}/(0.25 + 1.6Pr^{1/2})$	$10^3 \le Ra_x \le 10^9$ $10^3 \le Ra \le 10^9$	analytical
		$Pr = 0.72$ $Pr = 7$	$Nu = 0.638Ra^{1/5}$ $Nu = 0.713Ra^{1/5}$	$10^3 \le Ra \le 10^9$ $10^3 \le Ra \le 10^9$	
Kitamura and Kimura, 1995 [24]	quasi 2D geometry	air	$Nu_x = 0.660(Ra_x^*)^{1/6}$ $Nu_x = 0.066(Ra_x^*)^{1/3}$ $Nu_x = 0.700(Ra_x^*)^{1/5}$ $Nu_x = 0.200(Ra_x^*)^{1/4}$	$10^2 < Ra_x^* < 10^6$ $10^6 < Ra_x^* < 5\times10^7$ $5\times10^7 < Ra_x^* < 8\times10^{10}$ $8\times10^{10} < Ra_x^* < 10^{14}$	experimental
			$Nu = 1.25\,(Ra^*)^{1/6}$ $Nu = 0.04\,(Ra^*)^{1/3} + 9.7$ $Nu = (Ra^*)^{1/5} - 13.5$ $Nu = 0.20(Ra^*)^{1/4} + 37$	$1.6\times10^3 < Ra^* < 1.6\times10^7$ $1.6\times10^7 < Ra^* < 8\times10^8$ $8\times10^8 < Ra^* < 1.3\times10^{12}$ $1.3\times10^{12} < Ra^* < 1.6\times10^{15}$	

Table 1 (continued)

Kitamura and Kimura, 1995 [24] (continued)	quasi 2D geometry	air	$Nu_x = 0.607Ra_x^{1/5}$ $Nu_x = 0.017Ra_x^{1/2}$ $Nu_x = 0.640Ra_x^{1/4}$ $Nu_x = 0.117Ra_x^{1/3}$	$70 < Ra_x < 1.5\times10^5$ $1.5\times10^5 < Ra_x < 2\times10^6$ $2\times10^6 < Ra_x < 7.5\times10^8$ $7.5\times10^8 < Ra_x < 1.5\times10^{11}$	([c])
			$Nu = 1.307Ra^{1/5}$	$3.7\times10^2 < Ra < 8\times10^5$	
Lewandowski et al., 2000 [25]	square and rectangular	water	$Nu = 0.774Ra^{1/5}$	$4\times10^4 < Ra < 5\times10^6$	experimental ([a])
	square rectangular (L/W = 4)		$Nu = 1.347Ra^{1/5}$ $Nu = 1.116Ra^{1/5}$	$2.5\times10^6 < Ra < 3.2\times10^8$ $6\times10^5 < Ra < 7.8\times10^7$	([b])
Martorell et al., 2003 [27]	rectangular (L/W = 2.3 to 27.8)	air	$Nu = 1.200Ra^{0.175}$	$2.9\times10^2 \le Ra \le 3.3\times10^5$	experimental
	2D infinite strip		$Nu = 1.280Ra^{0.167}$	$8\times10^2 \le Ra \le 2\times10^6$	numerical
Wei et al., 2003 [36]	2D infinite strip	air	$Nu = 0.823Ra^{0.201}$ $Nu_x = 0.318Nu/[0.5 - (x/W)]^{0.555}$	$10^5 \le Ra \le 10^7$	numerical
Kozanoglu and Lopez, 2007 [28]	rectangular (L/W = 2)	water	$Nu = 0.134Ra^{0.34}$ $Nu = 0.131Ra^{0.34}$	$2.7\times10^4 \le Ra \le 5.9\times10^{10}$ $2.5\times10^5 \le Ra \le 4.2\times10^{11}$	experimental ([a]) ([b])

Notes:
(a) the characteristic length to be used in all dimensionless groups is the hydraulic radius defined as the ratio between the active area and its perimeter, equal to W/4 for a square plate and to W/2 for a 2D infinite strip;
(b) correlating equations modified by employing the plate width W as characteristic length in all dimensionless groups;
(c) correlating equations modified in terms of Ra, according to the relationship Ra* = Ra × Nu.

For the sake of simplicity, the shorter side of the plate, W, is the characteristic length to be used in all dimensionless groups, which is the choice made by many authors. Whenever some investigators used a different linear dimension, e.g., the hydraulic radius, mainly for surfaces whose shape was neither square nor too much slender, both the original correlations and the correlations modified by rearranging the dimensionless groups in terms of W, are reported – see notes (a) and (b) of Table 1.

Moreover, since in applications it is more usual to know (or easier to measure) the temperature of the plate surface rather than the rate of convective heat transfer, basic reference is made to situations with a first-type boundary condition, i.e., to plates with uniform wall temperature. However, also the heat transfer data available for plates with uniform surface heat flux have been collected. Also in this case, both the correlations originally developed by the authors in terms of Nu and Ra*, and the heat transfer correlations modified in terms of Nu and Ra according to the well-known relationship Ra*= Ra × Nu, are reported – see note (c) of Table 1.

In addition, mass transfer correlations and, when available, equations for the local heat transfer rate have also been included in Table 1. In the former case, the dimensionless groups Nu, Ra, and Pr, are replaced by the corresponding Sherwood number Sh, Rayleigh number for mass transfer Ra_m, and Schmidt number Sc, respectively. In the latter case, the local Nusselt and Rayleigh numbers are denoted as Nu_x and Ra_x, or Ra_x*, in which the characteristic length to be used is the distance x from the leading edge of the plate.

4 Critical analysis of the data

Analysis of Table 1 points out that the results of the analytical studies are in perfect agreement, which is for example the case of:

a) the values of $Nu_x/(Ra_x)^{1/5}$ predicted for Pr = 0.72 by Pera and Gebhart [11] and by Chen et al. [34], i.e., 0.363 and 0.381, respectively;
b) the values of $Nu/(Ra)^{1/5}$ predicted for Pr = 0.72 by Rotem and Claassem [8], by Pera and Gebhart [11] and by Chen et al. [34], i.e., 0.639, 0.646 and 0.638, respectively;
c) the values of $Nu/(Ra)^{1/5}$ predicted for Pr = 10 by Rotem and Claassem [8] and by Chen et al. [34], i.e., 0.726 and 0.720, respectively.

As regards the experimental and numerical heat transfer correlations listed in Table 1, a comparative survey of their predictions is reported in Tables 2 and 3, for air and water, respectively.

It may be seen that also the numerical data show a rather good degree of agreement, which is the case of the data obtained for 2D infinite strips suspended in air by Goldstein and Lau [20] and by Martorell et al. [27] in the range $10^3 \le Ra \le 5\times10^4$. Even better, the results obtained by Martorell et al. [27] and by Wei et al. [36] in the range $10^5 \le Ra \le 10^6$ for two-dimensional infinite strips have maximum deviations of the order of 5%.

In contrast, the experimental results may on the whole differ by ±50% or more. This is for example the case of the average Nusselt numbers that may be derived from the correlating equations by:

a) Martorell et al. [27] for rectangular plates in air, and Goldstein et al. [13] for square plates in air (Sc = 2.5), in the range $10^3 \le Ra \le 10^5$ (with differences of the order of 40–60%);
b) Fishenden and Saunders [4] for square plates in air, and Goldstein et al. [13] for square plates in air (Sc = 2.5), at $Ra = 5\times10^5$ (with a difference of the order of 50%);
c) Kozanoglu and Lopez [28] for rectangular plates in water, and Lloyd and Moran [14] for square plates in a fluid with Sc = 2200, at $Ra = 10^6$ (with a difference of the order of 70%);
d) Lewandowski et al. [25], and Ishiguro et al. [18], for rectangular plates in water, in the range $5\times10^6 \le Ra \le 5\times10^7$ (with differences of the order of 40–90%);
e) Fujii and Imura [10], and Ishiguro et al. [18], for rectangular plates in water, in the range $5\times10^8 \le Ra \le 10^{10}$ (with differences of the order of 50%).

Such divergence among the several experimental data may substantially be attributed to the following factors:

1) in many experimental conditions, the evaluation of the thermal power transferred by convection from the plate surface to the adjacent fluid – upon which the calculation of the coefficient of convection heat transfer is based, once the surface temperature is determined – is not so much accurate, owing to the difficulties related to the measurement/calculation of the heat losses by conduction through the supply electric cables, the thermocouple wires, the back insulation of the test plate and the support assembly of the whole experimental setup, as well as, in some situations, also to the difficulties related to the evaluation of the amount of heat transferred by radiation from the plate surface to the surroundings;

2) in most experimental setups the plate is heated electrically through a single resistor, which does not ensure that a condition of uniform wall temperature is achieved (as the heat transfer performance of the

Table 2 – Survey of Nu–values for air derived from dimensionless correlations available in the literature

Correlation (for air)	Plate geometry	Nu								
		Ra = 10^3	5×10^3	10^4	5×10^4	10^5	5×10^5	10^6	5×10^6	10^7
Fishenden–Saunders [4]	square	–	–	–	–	9.60	14.36	17.08	25.53	30.37
Goldstein et al. [13] (*)	square	6.07	7.94	8.34	12.47	14.83	22.18	–	–	–
Yousef et al. [19]	square	–	–	–	–	–	–	–	29.41	34.98
Goldstein–Lau [20] (*)	square	5.17	7.14	8.20	11.32	13.00	17.94	–	–	–
Goldstein et al. [13] (*)	rectangular	4.59	6.10	7.25	10.84	12.89	–	–	–	–
Al Arabi–El Riedy [17]	rectangular	–	–	–	–	12.44	18.61	22.13	33.10	39.36
Sparrow–Carlson [22]	rectangular	–	–	–	–	10.84	14.96	17.18	–	–
Kitamura–Kimura [24]	rectangular	5.20	7.18	8.25	11.38	13.07	18.03	–	–	–
Martorell et al. [27]	rectangular	4.02	5.33	6.01	7.97	9.00	–	–	–	–
Goldstein–Lau [20]	2D infinite strip	3.26	4.50	5.17	7.13	–	–	–	–	–
Martorell et al. [27]	2D infinite strip	4.06	5.31	5.96	7.80	8.75	11.45	12.86	–	–
Wei et al. [36]	2D infinite strip	–	–	–	–	8.32	11.50	13.22	18.28	21.01

(*) mass transfer data for Sc = 2.5

Table 3 – Survey of Nu–values for water derived from dimensionless correlations available in the literature

Correlation (for water)	Plate geometry	Nu								
		Ra = 10^6	5×10^6	10^7	5×10^7	10^8	5×10^8	10^9	5×10^9	10^{10}
Lewandowski et al. [25]	square	–	29.46	33.83	46.68	53.62	–	–	–	–
Lloyd–Moran [14] (*)	square	24.16	36.13	42.96	64.24	76.40	114.24	150.21	256.88	323.66
Fujii–Imura [10]	rectangular	–	27.39	34.51	59.01	74.36	103.32	130.00	222.63	280.51
Ishiguro et al. [18]	rectangular	20.02	34.23	43.13	73.77	92.94	158.95	200.00	342.50	431.55
Lloyd–Moran [14] (*)	rectangular	21.25	31.78	37.79	56.51	67.20	119.21	150.21	256.88	323.66
Lewandowski et al. [25]	rectangular	17.69	24.40	28.03	38.68	–	–	–	–	–
Kozanoglu–Lopez [28]	rectangular	14.36	24.83	31.42	54.31	68.75	118.83	150.41	259.97	329.06
Lloyd–Moran [14] (*)	quasi 2D strip	20.77	31.07	36.94	55.25	69.71	119.21	150.21	256.88	323.66

(*) mass transfer data for Sc = 2200

plate is not uniform over its surface), thus implying that the calculation of the coefficient of convection is strongly affected by the number and location of the points where the temperature of the plate surface is measured;

3) at any time the coefficient of convection heat transfer is determined by measuring the temperature gradient at several points on the plate surface and then calculating a mean value, the result obtained is strongly affected by the number of these points and their location, as well as by the disturbances induced by plumes and vortices.

Indeed, the divergence among the predictions of the reviewed correlations increases further when all the numerical and experimental data are analysed at the same time, which is for example the case of the results obtained in the range $10^3 \le Ra \le 5\times10^4$ by Goldstein and Lau [20] for 2D infinite strips in air and by Goldstein et al. [13] for square plates in air (Sc = 2.5), which show discrepancies of the order of 60–80%. Even more, the average Nusselt number which may be derived at Ra = 10^7 from the equation by Ishiguro et al. [18] for rectangular surfaces in water, i.e, 43.13, is more than the double of the value relevant to 2D infinite strips in air which may be obtained by the correlation of Wei et al. [36], i.e., 21.01.

However, as shown in Tables 2–3, the amount of such discrepancy decreases significantly when the heat transfer data are distinguished according to the plate geometry and the working fluid, which seems the right approach. Actually, once W is assigned, the heat transfer performance of the plate decreases as its aspect ratio L/W increases (and then also its hydraulic radius increases), owing to the reduced cooling contribution by the fresh fluid which rises from below and trails over the four edges of the plate. Moreover, given all the other independent variables, the amount of heat exchanged increases as the Prandtl number increases. In fact, owing to the increasing viscosity effect, the wall jet tends to be squeezed towards the plate, which enhances the rate of heat transfer. On the other hand, owing to such compression, the plume shrinks, and thus, as the stagnation region where the plume is rooted gets smaller, the local heat flux increases further.

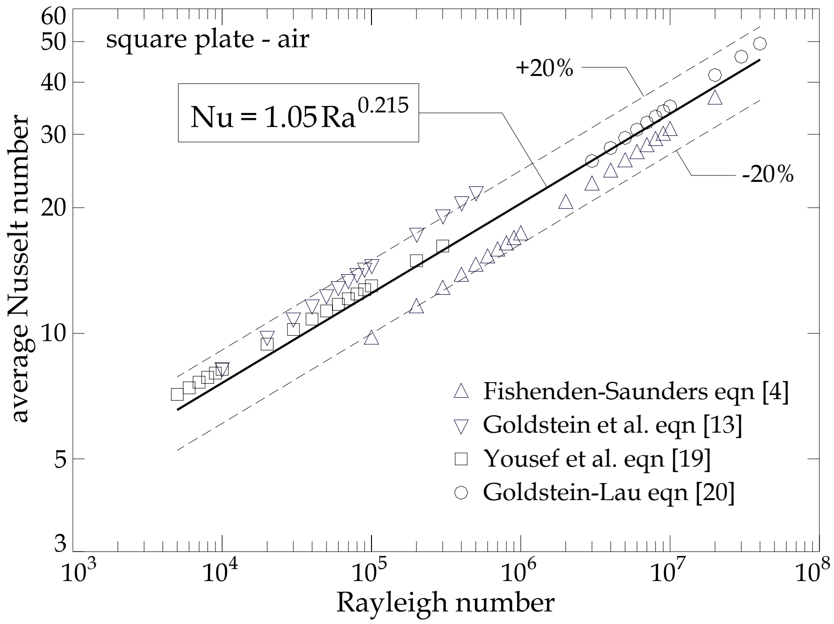

Fig. 1. Square plates in air: literature data and mean heat transfer correlation.

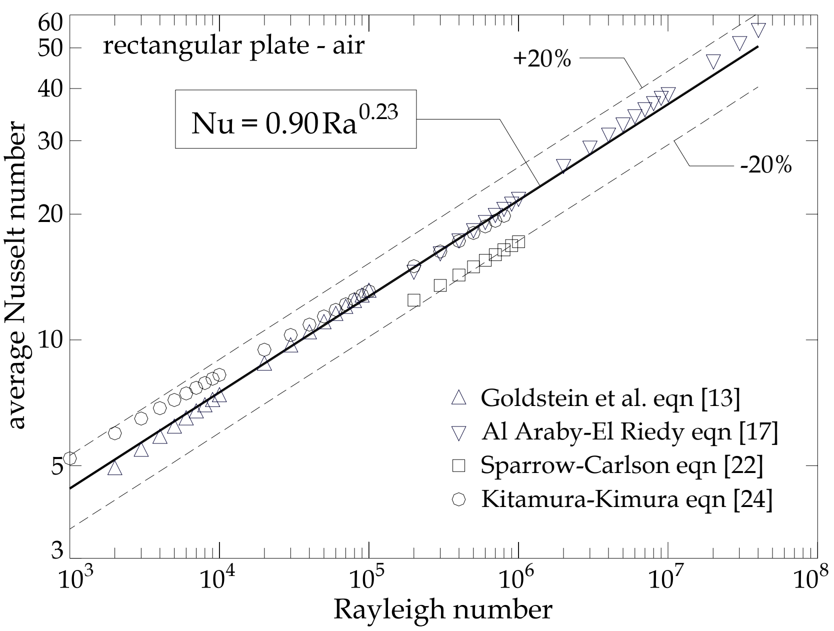

Fig. 2. Rectangular plates in air: literature data and mean heat transfer correlation.

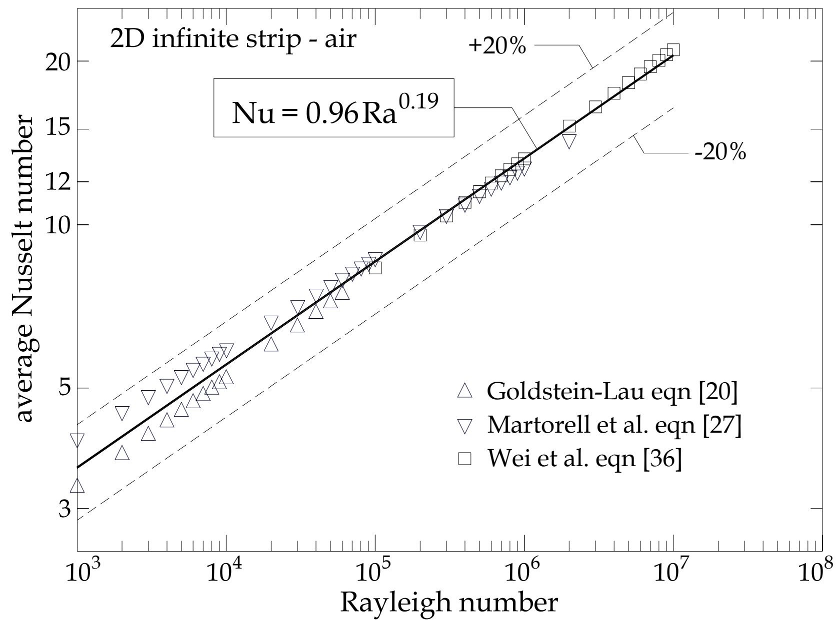

Fig. 3. 2D infinite strips in air: literature data and mean heat transfer correlation.

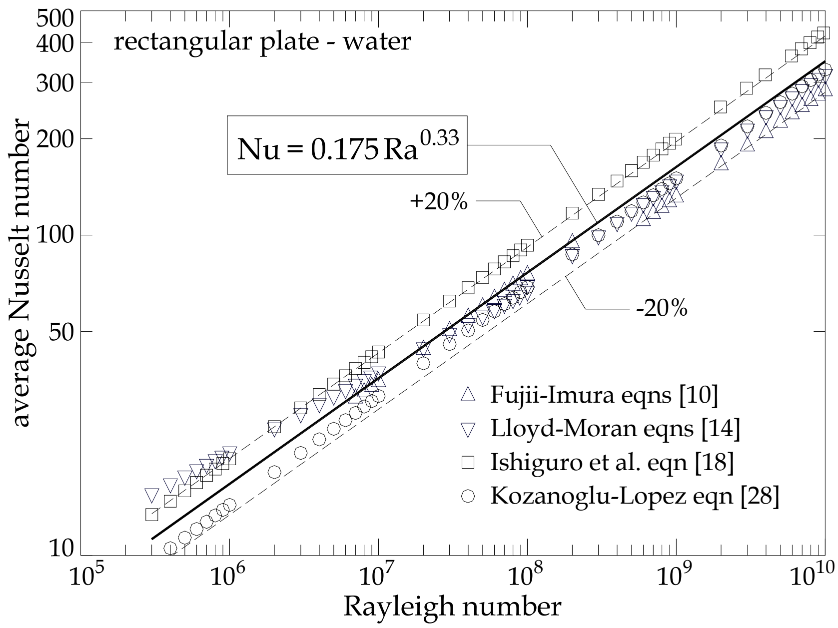

Fig. 4. Rectangular plates in water: literature data and mean heat transfer correlation.

5 Mean heat transfer correlations

Mean heat transfer correlations have been obtained from selected experimental and numerical equations presented in Table 1.

To this purpose, sets of Nusselt-data have been generated from the reviewed equations, with a step $\Delta Ra = 10^n$ for $10^n \le Ra < 10^{n+1}$ (where n = 3 to 7 for air, and n = 5 to 9 for water).

Subsequently, upon these data, the heat transfer correlations listed below have been derived by the application of a logarithmic regression procedure:

Square plates in air

$$Nu = 1.05\,Ra^{0.215} \qquad 10^3 \le Ra \le 3\times10^7 \tag{1}$$

based on the equations by Fishenden and Saunders [4], Goldstein et al. [13], Yousef et al. [19], and Goldstein and Lau [20], as shown in Fig. 1;

Rectangular plates in air

$$Nu = 0.90\,Ra^{0.23} \qquad 10^3 \le Ra \le 3\times10^7 \tag{2}$$

based on the equations by Goldstein et al. [13], Al-Arabi and El-Riedy [17], Sparrow and Carlson [22], and Kitamura and Kimura [24], as shown in Fig. 2 (in this regard, the equation by Martorell et al. [27] has not been included since its predictions are too much smaller than those of the other authors; in fact, since several plates used by Martorell and co-workers in their experiments were significantly slender – with aspect ratios L/W up to 27.8 –, the results they obtained seem more appropriate for a 2D infinite strip rather than for a rectangular surface of finite size, as it may be noticed in Table 2 by comparing the experimental results of Martorell et al. for rectangular plates with the numerical results obtained by the same authors for 2D infinite strips);

<u>Two-dimensional infinite strips in air</u>

$$Nu = 0.96\,Ra^{0.19} \qquad 10^3 \le Ra \le 10^7 \qquad (3)$$

based on the equations by Goldstein and Lau [20], Martorell et al. [27], and Wei et al. [36], as shown in Fig. 3;

<u>Rectangular plates in water</u>

$$Nu = 0.175\,Ra^{0.33} \qquad 3\times10^5 \le Ra \le 10^{10} \qquad (4)$$

based on the equations by Fujii and Imura [10], Lloyd and Moran [14], Ishiguro et al. [18], and Kozanoglu and Lopez [28], as shown in Fig. 4 (the correlation by Lewandowski et al. [25] has been excluded as it tends to underpredict too much the results of the other correlating equations; actually, the common value of the exponent of the Rayleigh number in the equations proposed by other authors for water is 1/3, while that in the equation developed by Lewandowski et al. is 1/5, which is typical for air – indeed, in their paper, Lewandowski and co-workers claim a rather good degree of agreement between their correlation and that by Goldstein and Lau [20], although such correlation was obtained for air rather than for water, and for Rayleigh numbers definitely smaller than theirs).

5 Conclusions

A critical review of the studies readily available in the literature on natural convection from unbounded horizontal rectangular flat plates facing upwards has been carried out.

A collection of dimensionless heat correlations has been presented in tabular form. A comparative survey of the results that may be derived at different Rayleigh numbers from these correlations has also been reported. It has been shown that the differences among the predictions of the reviewed correlations, which on the whole may amount to ±50%, or more, decreases significantly, when the results are grouped together according to the working fluid, i.e., air or water, and the plate shape, i.e., square, rectangular or two-dimensional infinite strip; such discrepancy among the data being attributable to the different investigation methods, thermal boundary conditions, and edge effects.

A set of mean heat transfer correlations, which “interpolate” the equations of the various authors with rather acceptable approximation, has also been proposed, so as to help heat transfer designers and constructors in applications.

Nomenclature

g	gravitational acceleration
h	coefficient of convection heat transfer
h_m	coefficient of convection mass transfer
k	thermal conductivity of the fluid
L	longer side of the plate (length)
Nu	average Nusselt number, hW/k
Nu_x	local Nusselt number, hx/k
Pr	Prandtl number, ν/α
q	heat flux
Ra	Rayleigh number, $[g\beta(T_w - T_\infty)W^3/\nu^2]\times Pr$
Ra_m	Rayleigh number for mass transfer, $[g(\rho_w - \rho_\infty)W^3/\rho_\infty\nu^2]\times Sc$
Ra_x	local Rayleigh number, $[g\beta(T_w - T_\infty)x^3/\nu^2]\times Pr$
Ra^*	modified Rayleigh number, $[g\beta qW^4/k\nu^2]\times Pr$
Ra_x^*	local modified Rayleigh number, $[g\beta qx^4/k\nu^2]\times Pr$
Sc	Schmidt number, ν/δ
Sh	average Sherwood number, h_mW/δ
T	temperature
W	shorter side of the plate (width)
x	distance from the edge of the plate along the direction parallel to its shorter side

Greek symbols

α	thermal diffusivity of the fluid
β	coefficient of thermal expansion of the fluid
δ	coefficient of diffusion of transferred species
ν	kinematic viscosity of the fluid
ρ	density of the fluid

Subscripts

w	referred to the plate surface
∞	referred to the undisturbed fluid

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