Approximate Analytic solutions for mixed and forced convection heat transfer from an unsteady no-uniform flow past a rotating cylinder

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Abstract: Presented in this paper is an analytic approximation to the thermal-fluid problem involving mixed and forced convective heat transfer from a rotating isothermal cylinder placed in a non-uniform stream shear-flow. The approximation is obtained using a series expansion of the scaled boundary layer equations in terms of a boundary layer variable which is directly proportional to the time variable and inversely proportional to the Reynolds number. Therefore, the resulting approximation is valid both for small time and for moderate and large times for which the Reynolds number of the flow is sufficiently large.

Key–Words: shear-flow, mixed and forced convection, rotating cylinder, boundary layer, viscous flow, laminar.

1 Introduction

The steady and unsteady heat transfer problems involving fluid flows past a circular cylinder have been extensively studied numerically, and theoretically as well as experimentally (for a comprehensive list of references, see [1,2]). In addition to their direct applications in science and engineering, such flows exhibit the main characteristics commonly observed in most industrial problems and therefore can serve as prototypes for simulating many fundamental fluid dynamics problems [3-8]. However, most studies have focussed on heat transfer problems associated with uniform stream flows including, the numerical and analytic investigations of natural convection heat transfer [9-13], free convection heat transfer [14-16], forced convection heat transfer [17-23], and mixed convection heat transfer [24-28]. There are also many experimental investigations of heat transfer problems involving uniform stream flows including the 1953 work of Seban and Drake [29] and others [30-33].

In the present study, the thermal-fluid flow problem involving forced and mixed convective heat transfer from a rotating circular cylinder placed in a non-uniform stream of shear flow is considered. There are numerous previous theoretical [34-36], numerical [37-44] and experimental [45-49] studies on shear flow past a cylinder. While the focus of the previous investigations involving shear flows past a cylinder has been on the flow characteristics such as vortex shedding, boundary-layer separation and hydrodynamic forces, the focus of the present study is on the convective heat transfer processes. This problem has a direct relevance in a wide range of scientific and engineering applications including atmospheric flows, heat exchanger systems, and energy conservation [50,51].

This study is a direct extension of the recent work of Abdella and Nalitolela [52] which investigated the two-dimensional forced convective heat transfer problem of the unsteady shear flow of a viscous incompressible fluid past a rotating circular cylinder. The heat transfer process is investigated using an analytical approximation obtained using a series expansion of the flow variable in terms of a boundary layer variable $\lambda = \sqrt{\frac{8t}{\mathrm{Pe}}}$ where t measures time and Pe represents the Peclet number. The analytic approximations are therefore valid for the initial stages of problems involving small and moderate Reynolds numbers as well as for moderate and large times of sufficiently large Reynolds number problems.

In the next section the governing equations along with the corrosponding boundary conditions are presented with the introduction of a variable transformation which simplifies the geometry of the problem. The flow variables are then scaled with respect to the

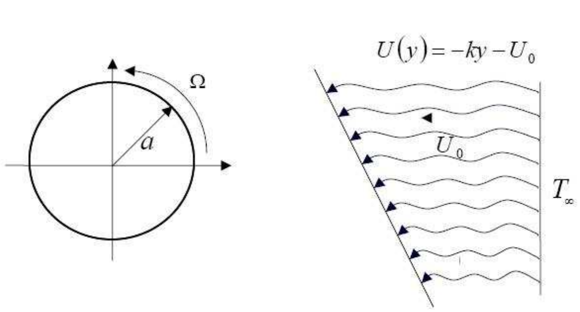

Figure 1: Schematics of the physical problem

boundary layer variable λ. In section 3 the boundary value problems corresponding to various orders of approximations are derived and solved. Results and discussions are presented in section 4 and concluding remarks are presented in section 5.

2 Governing Equation

Consider the problem of mixed convection heat transfer from an unsteady flow past a circular cylinder of radius a centred at the origin and rotating at an angular velocity of Ω_0. The flow is assumed to be viscous and incompressible. It is also assumed that the flow remains laminar and two-dimensional for all times and for all parameter values considered in this paper. The surface of the cylinder is kept at a constant temperature T_0 while the approaching stream with constant shear $U(y) = -\gamma y - U_0$ is kept at constant temperature T_∞ where x and y are the usual Cartesian coordinates. The temperature difference $\delta T = T_0 - T_\infty$ is assumed to be positive, giving rise to the buoyancy force and inducing fluid motion.
Applying the Boussinesq approximation and neglecting the effects of viscous dissipation and radiation the governing equations are given by the equations of motion and the energy equation:

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \nu\left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2}\right) + \alpha g\frac{\partial T}{\partial x} \quad (1)$$

$$\zeta = \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) \quad (2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (3)$$

where t is time, g is acceleration due to gravity, α is the thermal expansion coefficient, $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$ are the velocity components in the x and y directions respectively, $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vorticity, ψ is the stream function, T is the temperature, ν is the kinematic viscosity and κ is the thermal diffusivity. Introducing the following non-dimensional quantities

$$x' = \frac{x}{a}, y' = \frac{y}{a}, u' = \frac{u}{U_0}, v' = \frac{v}{U_0}, t' = \frac{tU_0}{a}$$

$$\psi' = \frac{\psi}{aU_0}, \zeta' = \frac{a\zeta}{U_0}, \phi' = \frac{T - T_0}{\delta T}, v = \frac{v'}{U_0},$$

and using the modified polar coordinates (ξ, θ) where $\xi = \ln r$ equations 1-3 become

$$\begin{aligned} e^{2\xi}\frac{\partial \zeta}{\partial t} &= \frac{\partial \psi}{\partial \theta}\frac{\partial \zeta}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \zeta}{\partial \theta} + \frac{2}{\mathrm{Re}}\left(\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \theta^2}\right) \\ &+ e^{\xi}\frac{\mathrm{Gr}}{2\mathrm{Re}^2}\left(\cos\theta\frac{\partial \phi}{\partial \xi} - \sin\theta\frac{\partial \phi}{\partial \theta}\right) \quad (4) \\ e^{2\xi}\zeta &= \left(\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \theta^2}\right) \quad (5) \end{aligned}$$

$$e^{2\xi}\frac{\partial \phi}{\partial t} = \frac{\partial \psi}{\partial \theta}\frac{\partial \phi}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \phi}{\partial \theta} + \frac{2}{\mathrm{Pe}}\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \theta^2}\right) \quad (6)$$

where we have dropped the primes.Here the Reynolds, Peclet and the Grashof numbers are defined respectively as

$$\mathrm{Re} = \frac{2aU_0}{\nu}, \quad \mathrm{Pe} = \mathrm{RePr}, \quad \text{and} \quad \mathrm{Gr} = \mathrm{RaPr}$$

where $\mathrm{Ra} = \frac{g\alpha\delta T(2a)^3}{\nu\kappa}$ is the Rayleigh number and $\mathrm{Pr} = \frac{\nu}{\kappa}$ is the Prandtl number. We also use the Richardson number, $\mathrm{Ri} = \frac{\mathrm{Gr}}{\mathrm{Re}^2}$.
Note that, it is convenient to introduce the new (ξ, θ) coordinate system which maps the surface of the cylinder to $\xi = 0$ and the infinite region exterior to the cylinder to the semi-infinite rectangular strip $\xi \leq 0, \ 0 \leq \theta \leq 2\pi$.
The boundary conditions on the surface of the cylinder for $t > 0$ include the usual no-slip, the impermeability and isothermal conditions:

$$\psi = 0, \quad \frac{\partial \psi}{\partial \xi} = \Omega, \quad \text{and} \phi = 1, \quad \text{on} \quad \xi = 0 \quad (7)$$

where $\Omega = \dfrac{a\Omega_0}{U_0}$ is a non-dimensional angular velocity. The free stream conditions far away from the cylinder surface are:

$$\phi \to 0, \quad \zeta \to K, \quad \text{and} \tag{8}$$

$$\psi \to \frac{K}{4}e^{2\xi} + Ve^{\xi}\sin(\theta) - \frac{K}{4}e^{2\xi}\cos 2\theta \quad \text{as} \quad \xi \to \infty \tag{9}$$

where $K = \dfrac{a\gamma}{U_0}$ is a dimensionless shear parameter and V is the dimensionless centre-line velocity taking on the values 0, 1 or -1 depending on the stream flow direction. Since the flow variables are periodic with respect to θ, we invoke the following periodicity condition:

$$\chi(\xi,\theta,t) = \chi(\xi,\theta+2\pi,t), \tag{10}$$

where χ represents the flow variables ψ, ζ or ϕ. Note that, on the surface of the cylinder ψ is overdetermined since it has two boundary conditions while ζ is underdetermined. To resolve this, we apply Green's Second identity which is given by: $\int\int_D \left(g \bigtriangledown^2 h - h \bigtriangledown^2 g\right) dA = \int_C \left(g\frac{\partial h}{\partial n} - h\frac{\partial g}{\partial n}\right) dS$ where g and h are twice differentiable functions in the region D, C is the closed curve representing the boundary of D and $\dfrac{\partial}{\partial n}$ represents the normal derivative. Then using ψ for g and the harmonic functions $e^{-m\xi}\sin(m\theta)$ and $e^{-m\xi}\cos(m\theta)$ for h in Green's identity, we obtain the following global integral conditions:

$$\frac{1}{\pi}\int_0^{\infty}\int_0^{2\pi} e^{(2-m)\xi}\zeta\sin(m\theta)d\theta d\xi = 2V\delta_{1,m}, \tag{11}$$

$$\frac{1}{\pi}\int_0^{\infty}\int_0^{2\pi} e^{(2-m)\xi}\zeta\cos(m\theta)d\theta d\xi = -K\delta_{2,m}, \tag{12}$$

$$\frac{1}{\pi}\int_0^{\xi_\infty}\int_0^{2\pi} e^{2\xi}\zeta d\theta d\xi = Ke^{2\xi_\infty} - 2\Omega. \tag{13}$$

for $m = 1, 2, ...$, and where $\delta_{i,j}$ is the Kronecker delta function which is zero when $i \neq j$ and 1 when $i = j$. Therefore, these integral conditions essentially convert the surface and the free stream boundary conditions into conditions that are valid throughout the entire domain of the problem.

Finally, we have the following initial conditions:

$$\zeta(\xi,\theta,t=0) = 0, \quad \zeta(\xi,\theta,t=0) = 0, \tag{14}$$

$$\phi(\xi,\theta,t=0) = \begin{cases} 1 & \text{if } \xi = 0 \\ 0 & \text{if } \xi \neq 0. \end{cases} \tag{15}$$

Note that the initial temperature distribution is singular and therefore results in a thin boundary-layer region close to the surface of the cylinder.

3 Approximate analytic solutions

The governing equations described in the previous section are highly nonlinear. Therefore, it is not possible to obtain analytical solution valid for all time and all flow parameter values. In this section we obtain approximate solutions for the early development of the flow and the heat transfer process using series expansion in terms of an appropriate boundary layer parameter. Recall that the structure of the flow field and heat transfer process in the initial stages of the flow is characterized by a thin boundary layer-region near the cylinder surface. By examining the dominant terms of the initial solutions, it can be shown that the thickness of this boundary layer is given by $\lambda = \sqrt{\dfrac{8t}{\text{Pe}}}$ which measures the diffusive growth of the boundary-layer structure and is used to rescale the space coordinate ζ and the flow variables via the changes of variables $\xi = \lambda z$, $\psi = \lambda\Psi$, $\zeta = \dfrac{\omega}{\lambda}$, $\phi = \dfrac{\Phi}{\lambda}$ Hence the thin boundary-layer is stretched and the initial singularity is removed with this change of variables and the governing equation become:

$$\frac{\partial^2\omega}{\partial z^2} + \frac{2}{\text{Pr}}e^{2\lambda z}\left(z\frac{\partial\omega}{\partial z} + \omega\right) = \frac{2}{\text{Pr}}\lambda e^{2\lambda z}\frac{\partial\omega}{\partial\lambda} - \lambda^2\frac{\partial^2\omega}{\partial\theta^2}$$

$$-\frac{\text{Re}\lambda^2}{2}\left(\frac{\partial\Psi}{\partial\theta}\frac{\partial\omega}{\partial z} - \frac{\partial\Psi}{\partial z}\frac{\partial\omega}{\partial\theta}\right) - e^{\lambda z}\text{Ra}\Gamma \tag{16}$$

$$e^{2\lambda z}\omega = \left(\frac{\partial^2\Psi}{\partial z^2} + \lambda^2\frac{\partial^2\Psi}{\partial\theta^2}\right), \tag{17}$$

$$\frac{\partial^2\Phi}{\partial z^2} + 2e^{2\lambda z}\left(z\frac{\partial\Phi}{\partial z} + \Phi\right) = 2\lambda e^{2\lambda z}\frac{\partial\Phi}{\partial\lambda} - \lambda^2\frac{\partial^2\Phi}{\partial\theta^2}$$

$$-\frac{\text{Pe}\lambda^2}{2}\left(\frac{\partial\Psi}{\partial\theta}\frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial z}\frac{\partial\Phi}{\partial\theta}\right), \tag{18}$$

where $\Gamma = \dfrac{\text{Pr}\lambda}{4\text{Re}}\left(\cos\theta\dfrac{\partial\Phi}{\partial z} - \lambda\sin\theta\dfrac{\partial\Phi}{\partial\theta}\right)$.

We now use the following single series expansions in λ in order to obtain analytic approximate solutions of the governing equations and the accompanying initial and boundary conditions:

$$\Psi = \Psi_0 + \lambda\Psi_1 + \lambda^2\Psi_2 + ... \tag{19}$$

$$\omega = \omega_0 + \lambda\omega_1 + \lambda^2\omega_2 + ... \tag{20}$$

$$\Phi = \Phi_0 + \lambda\Phi_1 + \lambda^2\Phi_2 + ... \tag{21}$$

Note that λ is a small parameter not only when t is small but also when Re is large. Therefore, these approximations are valid not only for small times but also for large times for problems with large Re. While the double series expansion used in [53] and [54] simplifies the analytic calculations, it is more advantageous to use the single expansion used in this paper since the validity of the single expansion in λ would only require that λ be small which can be achieved for moderate and large times provided that Re is sufficiently large. However, the double series expansion is valid only for small times. Substituting the above single expansions into the boundary layer equations and equating like powers of λ results in a hierarchy of boundary value problems for the expansion coefficients.

3.1 Linear approximation for all values of Pr

The $O(1)$ and $O(\lambda)$ boundary value problems for the mixed convection case turn out to be independent of Ra. Therefore, the $O(1)$ approximations which are valid for all values of Pr are identical to those found in [10] for the forced convection case and are given by:

$$\Phi_0 = 0 \quad \text{and} \quad \omega_0 = A_0(\theta) f e^{-(fz)^2} \tag{22}$$

$$\Psi_0 = \Omega z + A_0(\theta)\frac{\sqrt{\pi}}{2}\left(z\text{erf}(fz) + \frac{e^{-(fz)^2}-1}{f\sqrt{\pi}}\right) \tag{23}$$

where $f = \dfrac{1}{\sqrt{\text{Pr}}}$ and

$$A_0(\theta) = \frac{2}{\sqrt{\pi}}\left(2V\sin\theta - K\cos 2\theta + \frac{K-2\Omega}{2}\right). \tag{24}$$

Similarly the $O(\lambda)$ approximations are identical to the forced case obtained in [10]:

$$\Phi_1 = \text{erfc}(z). \tag{25}$$

$$\omega_1 = K + A_1(\theta)\text{erfc}(fz) - A_0(\theta)\frac{\sqrt{\pi}}{4}\text{erfc}(fz) - \frac{1}{2}A_0(\theta)e^{-(fz)^2)}\left(2f^3z^3 + fz\right). \tag{26}$$

$$\Psi_1 = \frac{Kz^2}{2} + \frac{1}{16f^2}F_m(\theta)\left(\text{erf}(fz) - 2f^2z^2\text{erfc}(fz)\right) - \frac{ze^{-(fz)^2}}{8f\sqrt{\pi}}F_p(\theta) + \frac{zA_1(\theta)}{f\sqrt{\pi}}, \tag{27}$$

where

$$F_p(\theta) = \sqrt{\pi}A_0(\theta) + 4A_1(\theta)$$

$$F_m(\theta) = \sqrt{\pi}A_0(\theta) - 4A_1(\theta)$$

$$A_1(\theta) = 2V\sin\theta - 2K\cos 2\theta.$$

3.2 Higher order approximations for Pr=1

Since the second and higher order approximations turn out to be analytically intractable for general values of Pr, we assume that Pr=1 for these approximations.

3.2.1 The $O(\lambda^2)$ approximation

The second order approximation is obtained by collecting the $O(\lambda^2)$ terms resulting in the following boundary value problems:

$$\frac{\partial^2\Phi_2}{\partial z^2} + 2z\frac{\partial\Phi_2}{\partial z} - 2\Phi_2 = -4z^2\left(z\frac{\partial\Phi_0}{\partial z} + \Phi_0 + \frac{\partial\Phi_1}{\partial z}\right) - \frac{\partial^2\Phi_0}{\partial\theta^2} - \frac{\text{Pe}}{2}\left(\frac{\partial\Psi_0}{\partial\theta}\frac{\partial\Phi_0}{\partial z} - \frac{\partial\Psi_0}{\partial z}\frac{\partial\Phi_0}{\partial\theta}\right) \tag{28}$$

$$\frac{\partial^2\omega_2}{\partial z^2} + 2f^2z\frac{\partial\omega_2}{\partial z} - 2f^2\omega_2 = -4f^2z^2\left(z\frac{\partial\omega_0}{\partial z} + \omega_0 + \frac{\partial\omega_1}{\partial z}\right) - \frac{\partial^2\omega_0}{\partial\theta^2} + \frac{\text{Ra}\cos(\theta)}{2\sqrt{\pi}\text{Re}}e^{-z^2} \tag{29}$$

$$\frac{\partial^2\Psi_2}{\partial z^2} = 2z^2\omega_0 + 2z\omega_1 + \omega_2 - \frac{\partial^2\Psi_0}{\partial\theta^2} \tag{30}$$

subject to boundary conditions

$$\Psi_2 = 0, \quad \frac{\partial\Psi_2}{\partial z} = 0, \quad \text{and} \quad \Phi_2 = 0 \quad \text{on} \quad z = 0, \tag{31}$$

$$\Phi_2 \to 0 \quad \text{and} \quad \omega_2 \to 0 \quad \text{as} \quad z \to \infty \tag{32}$$

and the integral conditions,

$$\frac{1}{\pi}\int_0^\infty\int_0^{2\pi} T(z,\theta)\sin(m\theta)d\theta dz = 0, \quad m = 1,2,... \tag{33}$$

$$\frac{1}{\pi}\int_0^\infty\int_0^{2\pi} T(z,\theta)\cos(m\theta)d\theta dz = 0, \quad m = 1,2,... \tag{34}$$

$$\frac{1}{\pi}\int_0^{\xi_\infty}\int_0^{2\pi}\left(\omega_2 + 2z\omega_1 + 2z^2\omega_0\right)d\theta dz = 2Kz_\infty^2. \tag{35}$$

where

$$T(z,\theta) = \left(\omega_2 + (2-m)z\omega_1 + \frac{(2-m)^2}{2}z^2\omega_0\right).$$

Solving equation 28 with $f = 1$ and with respect to the Φ_2 boundary conditions yields

$$\Phi_2 = -\frac{1}{2}z\text{erfc}(z) - \frac{z^2e^{-z^2}}{\sqrt{\pi}}. \tag{36}$$

Similarly, solving equation 29 with $f = 1$ subject to the free-stream condition on ω_2, gives:

$$\begin{aligned}
\omega_2 &= A_2(\theta)z + A_3(\theta)(e^{-z^2} + \sqrt{\pi}z\mathrm{erf}(z)) \\
&- \frac{1}{2\sqrt{\pi}}A_1(\theta)(2z^2e^{-z^2} - \sqrt{\pi}z\mathrm{erf}(z)) \\
&+ \frac{1}{24}A_0e^{-z^2}\left(9z^2 - 2z^4 + 12z^6\right) \\
&+ \frac{1}{16}\sqrt{\pi}z\mathrm{erf}(z)\left(2\mathrm{Re}\Omega A_0'(\theta) - 3A_0(\theta)\right. \\
&- \left.4A_0''(\theta)\right) + \frac{1}{96}\mathrm{Re}A_0(\theta)A_0'(\theta)\left(6\pi z\mathrm{erf}^2(z)\right. \\
&+ \left.8ze^{-z^2} + \sqrt{\pi}\mathrm{erf}(z)(3-6z^2)e^{-z^2}\right) \\
&- 6ze^{-2z^2} - \frac{\mathrm{Ra}\cos(\theta)}{8\mathrm{Re}\sqrt{\pi}}e^{-z^2} \\
&- \frac{\mathrm{Ra}\cos(\theta)\,z^3}{24\mathrm{Re}}
\end{aligned} \tag{37}$$

where

$$A_3(\theta) = -\frac{1}{2\sqrt{\pi}}\left(2A_2(\theta) + A_1(\theta)\right) + \frac{1}{16}\left(4A_0''(\theta)\right.$$
$$\left. +3A_0(\theta) - \mathrm{Re}A_0'(\theta)(2\Omega + \sqrt{\pi}A_0(\theta))\right) \tag{38}$$

Applying the integral condition given by equations 33-35 then gives the function $A_2(\theta)$:

$$A_2(\theta) = a_0 + \sum_{i=1}^{4}\left(a_i\sin(i\theta) + b_i\cos(i\theta)\right) \tag{39}$$

where

$$a_0 = 0,\ \ a_1 = -V,$$
$$a_2 = \frac{-\mathrm{Re}\left(6\pi V^2 + 3\pi K^2 - 4K\Omega + 4V^2 + 2K^2\right)}{6\pi},$$
$$a_3 = 0,\ \ a_4 = \frac{\mathrm{Re}K^2(3\pi+2)}{6\pi},$$
$$b_1 = -V\mathrm{Re}\frac{3\pi K + 2K - 2\Omega}{3\pi},\ \ b_2 = 3K,$$
$$b_3 = \mathrm{Re}VK\frac{3\pi+2}{2\pi},\ \ b_4 = 0. \tag{40}$$

Finally, integrating equation (30) subject to the surface boundary condition for Ψ_2 yields:

$$\begin{aligned}
\Psi_2 &= A_4(\theta)z + A_5(\theta) + K\frac{z^3}{3} + \frac{A_3(\theta)}{192\sqrt{\pi}} \\
&\quad \left(32\sqrt{\pi}(z^2+1)e^{-z^2} + \pi z\mathrm{erf}(z)(32z^2\right. \\
&+ \left.48)\right)\frac{1}{6}z^3A_2(\theta) + \frac{A_1(\theta)}{192\sqrt{\pi}}\left(64\sqrt{\pi}z^3\right. \\
&+ \left.24\sqrt{\pi}z\mathrm{erf}(z) - 48z^2(e^{-z^2} + \sqrt{\pi}z\mathrm{erf}(z))\right) \\
&+ \frac{\sqrt{\pi}}{192}\mathrm{Re}A_0'(\theta)\mathrm{erf}(z)\left(2\sqrt{\pi}z^3\mathrm{erf}(z)\right. \\
&- \left.3\sqrt{\pi}z\mathrm{erf}(z) - 11e^{-z^2} + 4z^2e^{-z^2} - 4\right) \\
&+ \frac{\sqrt{\pi}}{96}\mathrm{Re}A_0'(\theta)\left(ze^{-2z^2} + \sqrt{32\pi}\mathrm{erf}(\sqrt{2}z)\right) \\
&+ \frac{\sqrt{\pi}}{192}zA_0(\theta)\left(\mathrm{erf}(z)(10z^2-9) - 16\sqrt{\pi}z^2\right) \\
&+ \frac{1}{32}z^2A_0(\theta)e^{-z^2}\left(4z^2+3\right) \\
&+ \frac{1}{96}\Omega\mathrm{Re}A_0'(\theta)z\left(\sqrt{\pi}\mathrm{erf}(z)(2z^2-3)\right. \\
&+ \left.2e^{-z^2}(z^2-2)\right) - \frac{1}{16}zA_0''(\theta) \\
&\quad \left(\sqrt{\pi}\mathrm{erf}(z)F_1(z) + 2z(e^{-z^2}-2)\right) \\
&- \frac{1}{480}\frac{\mathrm{Ra}\cos(\theta)\left(30z\mathrm{erf}(z)\sqrt{\pi} + 30e^{-z^2}\right)}{\mathrm{Re}\sqrt{\pi}} \\
&- \frac{1}{480}\frac{\mathrm{Ra}\cos(\theta)\left(z^5\sqrt{\pi} - 30\right)}{\mathrm{Re}\sqrt{\pi}}
\end{aligned} \tag{41}$$

where the functions $A_4(\theta)$ and $A_5(\theta)$ determined by applying the surface boundary conditions are given by

$$A_4(\theta) = -\frac{\mathrm{Re}A_0'(\theta)A_0(\theta)}{48}, \tag{42}$$

$$A_5(\theta) = -\frac{1}{6}A_3(\theta) + \frac{1}{24}\mathrm{Re}\Omega A_0' \tag{43}$$

3.3 Higher order approximation

The boundary value problems for ω_n and Ψ_n, associated with the third and higher order approximations are too complex to solve analytically. Therefore, we only consider the Φ_3 problem which is given by:

$$\frac{\partial^2\Phi_3}{\partial z^2} + 2z\frac{\partial\Phi_3}{\partial z} - 4\Phi_3 = -\frac{8}{3}z^3\left(z\frac{\partial\Phi_0}{\partial z} + \Phi_0\right)$$
$$-4z^3\frac{\partial\Phi_1}{\partial} - 4z\left(z\frac{\partial\Phi_2}{\partial z} + \Phi_2\right) - \frac{\partial^2\Phi_1}{\partial\theta^2} \tag{44}$$
$$-\frac{\mathrm{Re}}{2}\left(\frac{\partial\Psi_1}{\partial\theta}\frac{\partial\Phi_0}{\partial z} + \frac{\partial\Psi_0}{\partial\theta}\frac{\partial\Phi_1}{\partial z} - \frac{\partial\Psi_1}{\partial z}\frac{\partial\Phi_0}{\partial\theta} - \frac{\partial\Psi_0}{\partial z}\frac{\partial\Phi_1}{\partial\theta}\right)$$

subject to

$$\Phi_3 = 0, \quad \text{on} \quad z = 0, \quad \text{and} \quad \Phi_3 \to 0 \quad \text{as} \quad z \to \infty. \tag{45}$$

Then, solving equation (44), we obtain the following expression for Φ_3:

$$\begin{aligned}
\Phi_3 &= A_6(\theta)F_1(z) + A_7(\theta)F_2(z) - 3z^2 \\
&- \frac{17}{32}\mathrm{erf}(z) + \frac{17}{16\sqrt{\pi}}ze^{-z^2} - \frac{1}{2\sqrt{\pi}}z^3e^{-z^2}
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{1}{16}\mathrm{Re}A_0'(\theta)z\mathrm{erf}(z)e^{-z^2} + \frac{31}{16}z^2\mathrm{erf}(z) \\
&+ \quad \frac{1}{2\sqrt{\pi}}z^5e^{-z^2} + \frac{1}{12\sqrt{\pi}}\mathrm{Re}A_0'(\theta)e^{-z^2} \\
&+ \quad \sqrt{\pi}F_1(z)\left(I_1(z)\mathrm{erfc}(z) + I_3(z)\right. \\
&- \quad \left.\frac{\sqrt{\pi}}{2}\,\mathrm{erfc}(z)\mathrm{erfi}(z) + \frac{1}{32}\mathrm{Re}A_0'(\theta)\mathrm{erf}^2(z)\right) \\
&+ \quad 2zI_2e^{-z^2} \qquad (46)
\end{aligned}
$$

where $I_1(z)=\int_0^z \mathrm{erf}(s)e^{s^2}ds, I_2(z)=\int_0^z \mathrm{erfc}(s)e^{s^2}ds$, and $I_3(z) = \int_0^z \mathrm{erfc}(s)^2e^{s^2}ds, F_1(z)=1 + 2z^2$, $F_2(z)=2ze^{-z^2}+\sqrt{\pi}F_1(z)\mathrm{erf}(z)$. It can be shown that $I_1(z)=\frac{1}{\sqrt{\pi}}z^2{}_2F_2\left(1,1;\frac{3}{2},2;z^2\right)$, $I_2(z)=\frac{\sqrt{\pi}}{2}\mathrm{erfi}(z)$-$I_1(z)$ where ${}_2F_2$ is the generalized hypergeometric function.

Then applying the boundary conditions of equation 45, we obtain

$$
A_6(\theta) = -\frac{\mathrm{Re}A_0'(\theta)}{12\sqrt{\pi}}, A_7(\theta) = \frac{c_1 + c_2\mathrm{Re}A_0'(\theta)}{96\pi} + C \qquad (47)
$$

where $c_1 = 51\sqrt{\pi}$, $c_2 = (8 - 3\pi)$, and $C = \int_0^\infty \mathrm{erfc}^2(s)e^{s^2}ds = .39107$.

Note that there is no Ra term in the expression for Φ_3. In fact, it turns out that the leading Ra dependence term is order five. It can be shown that the leading Ra dependence of Φ is given by:

$$
\Phi_5(\mathrm{R}a) = -\frac{1}{16\pi}F(z)\mathrm{Ra}\sin(\theta) \qquad (48)
$$

$$
F(z) = f_1(z) + (f_2(z))\,e^{(-z)^2} + f_4(z)e^{-2z^2} + f_5 + f_6
$$

where

$$
f_1(z) = \left(\frac{1}{18}z^8 + \frac{13}{36}z^6 + \frac{37}{72}z^4 - \frac{5}{48}z^2\right)e^{-2z^2}
$$

$$
f_2(z) = \sqrt{\pi}\left(\frac{z^9}{9} + \frac{7z^7}{9} + \frac{17z^5}{12} + \frac{z^3}{2} + \frac{z}{3}\right)\mathrm{erf}(z)
$$

$$
f_3(x) = \frac{720z^5\pi - 14400\sqrt{\pi} - 5z^6\pi^{3/2} + f_{3a}}{4800z^5\pi - 144000\sqrt{\pi}}
$$

$$
f_{3a} = 150z\pi - 10z^8\pi^{3/2} + 300z^3\pi - 8z^{10}\pi^{3/2}
$$

$$
\begin{aligned}
f_4(z) \quad = \quad & \left(-\frac{\pi z^{10}}{18} - \frac{5\pi z^8}{12} - \frac{11\pi z^6}{12} - \frac{5\pi z^4}{8}\right. \\
& \left.-\frac{9\pi z^2}{32} - \frac{3\pi}{64}\right)\mathrm{erf}(z)^2e^{(2z^2)} + \left(-\frac{\sqrt{\pi}z^9}{9}\right. \\
& \left.-\frac{7\sqrt{\pi}z^7}{9} - \frac{17\sqrt{\pi}z^5}{12} - \frac{\sqrt{\pi}z^3}{3}\right)\mathrm{erf}(z)e^{(z^2)} \\
& + \frac{1}{12} - \frac{z^8}{18} - \frac{13z^6}{36} - \frac{37z^4}{72} + \frac{3z^2}{16}
\end{aligned}
$$

$$
\begin{aligned}
f_5(z) = & \left(\frac{\pi z^{10}}{18} + \frac{5\pi z^8}{12} + \frac{11\pi z^6}{12} + \frac{17\pi z^4}{24} + \frac{7\pi}{64}\right. \\
& \left.+\frac{17\pi z^2}{32}\right)\mathrm{erf}(z)^2 + \frac{11}{180}\left(3 + 12z^2 + 4z^4\right)
\end{aligned}
$$

$$
f_6(z) = \frac{1}{720}\frac{(-44 + 15\pi)}{\pi^{3/2}}\left(\left(10z + 4z^3\right)e^{-z^2} + f_{6a}\right)
$$

$$
f_{6a} = 4\mathrm{erf}\,(z)\,\sqrt{\pi}\left(\frac{3}{4} + 3z^2 + z^4\right)
$$

4 Results and Discussion

In this section, we test the validity of the analytic solution at the initial stages of a moderate Re flow and at the fully developed stage of a high Re flow.

The test is carried out by comparing the results of the analytic solution with those of a high-resolution numerical solution obtained using a spectral finite difference scheme [54]. In this numerical approach, the flow variables as well as the temperature function are approximated in terms of truncated Fourier series expansion of N terms in the angular direction. The resulting 6N+3 two dimensional partial differential equations in time and in the radial variable are then integrated using a finite difference procedure.

In order to gain insight into the patterns of the heat transfer rate, we compute the local Nusselt number and the average Nusselt number variations with respect to time and radial component. The local Nusselt number Nu and average Nusselt number $\overline{Nu}$ are respectively defined as:

$$
Nu(\theta,t) = -\frac{2}{\lambda^2}\left(\frac{\partial\Phi}{\partial z}\right)_{z=0} \qquad (49)
$$

$$
\overline{Nu}(t) = \frac{1}{2\pi}\int_0^{2\pi} Nu(\theta,t)d\theta. \qquad (50)
$$

Using the analytic approximation of the temperature function Φ and taking the derivative with respect to z and then finding its value at $z = 0$ yields:

$$
Nu(\theta,t) = \frac{4}{\lambda\sqrt{\pi}} + 1 + \lambda N_1(\theta) + \mathrm{Ra}\lambda^3\sin(\theta)N_2(\theta) \qquad (51)
$$

where

$$
N_1(\theta) \quad = \quad -8\left(\frac{51\sqrt{\pi} + (8 - 3\pi)\mathrm{Re}A_0'(\theta)}{96\pi} + C\right)
$$

$$
N_2(\theta) \quad = \quad -\frac{2}{\sqrt{\pi}}\left(\frac{1}{15360} + \frac{1}{720}\frac{(-44\pi + 15\pi^2)}{\pi^2}\right)
$$

Similarly, the average Nusselt number is given by:

$$\overline{Nu}(t) = \frac{4}{\lambda\sqrt{\pi}} + 1 - \frac{\lambda}{2\pi}\left(\frac{17}{2}\sqrt{\pi} + 16\pi C\right) + O(\lambda^3). \quad (52)$$

4.1 Forced Convection case

Here we present some results for the forced convection heat transfer case which corresponds to the limiting case of $Ri = 0$. Therefore, in this case, the temperature equation is fully decoupled from the flow equations such that the fluid flow would no longer be influenced by the heat transfer process.

Figure 2 depicts the time evolution of the numerically simulated and the analytically determined local Nusselt's number for Re=1000, $K = 0.0$, $\Omega = 0.25$ and t =0.01, 0.05, 0.1, 0.3, 0.5 and 1.0. The figures show excellent agreement between the analytic approximations and the numerical simulations for all times presented. Since the analytic solutions are valid for large Re values, we note that our analytic solution performs well even at a moderately large time,$t = 1$. However, we notice that the analytic accuracy is not as high in Figure 3 for $t = 1$ since Re is small.

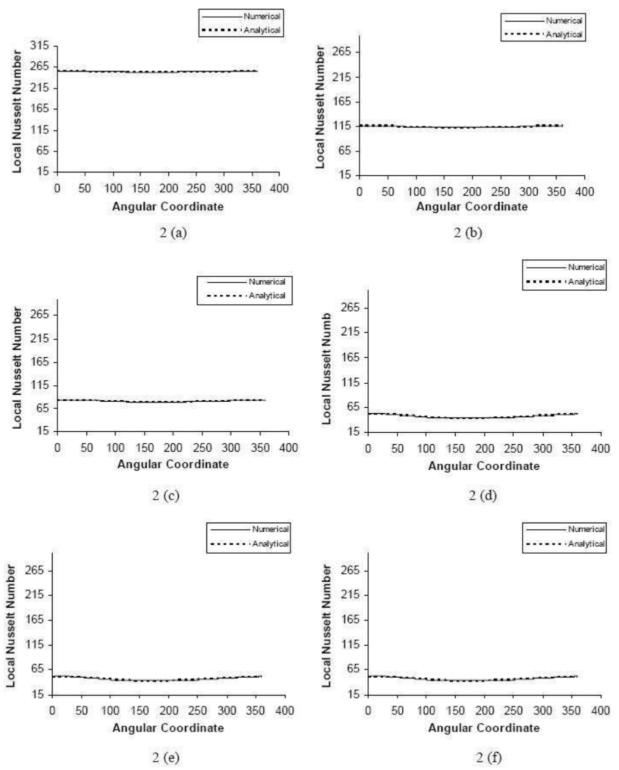

Figure 2: Local Nusselt Number comparison for Ri=0.0, Re=1000, $K = 0.0$, $\Omega = 0.25$, $V = 1$ at times (a) $t = 0.01$, (b) $t = 0.05$, (c) $t = 0.1$, (d) $t = 0.3$, (e) 0.5, and (f) $t = 1.0$

4.2 Mixed convection case

We now consider the mixed convection case where the heat transfer and the fluid flow are dominated by buoyancy effects resulting from a non-zero buoyancy parameter Ri. The flow equations described by the Navier-Stokes equation are now fully coupled with the temperature equation.

We begin with Figure 4 where the time evolution of the numerically simulated and the analytically determined local Nusselt's number are depicted for Ri=10, Re=1000, $K = 0.2$, $\Omega = 0.25$ and t =0.01, 0.05, and 0.5. As we can see from the figures, there is excellent agreement between the analytic approximations and the numerical simulations for all times presented. Note again that, since Re is moderately large, the analytic solution is in good agreement even at $t = 0.5$ This is because our expansion is valid in this limit as well. Similar results are obtained for Re=50 as depicted in Figure 5. However, we notice that the analytic accuracy is not as high in Figure 5 for $t = 0.5$. In this case, the analytic solution is not valid except for small time t since Re is also small.

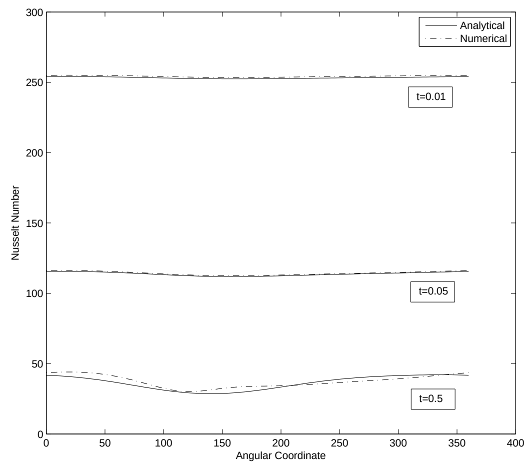

Figure 4: Local Nusselt Number comparison for Ri= 10, Re=1000, K=0.2, Ω=0.25, t=0.01,0.05,0.5

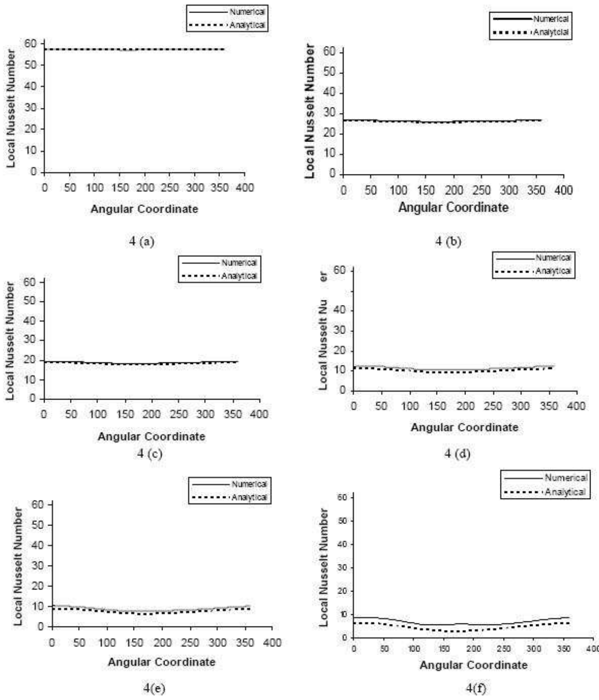

Figure 3: Local Nusselt Number comparison for Ri=0.0, Re=50, $K = 0.0$, $\Omega = 0.25$, $V = 1$ at times (a) $t = 0.01$, (b) $t = 0.05$, (c) $t = 0.1$, (d) $t = 0.3$, (e) 0.5, and (f) $t = 1.0$

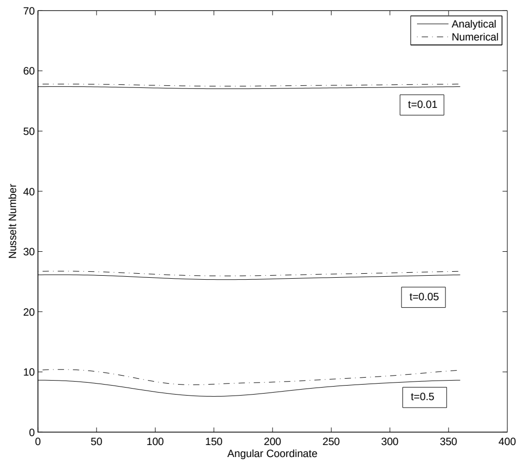

Figure 5: Local Nusselt Number comparison for Ri= 10, Re=50, K=0.2, Ω=0.25, t=0.01,0.05,0.5

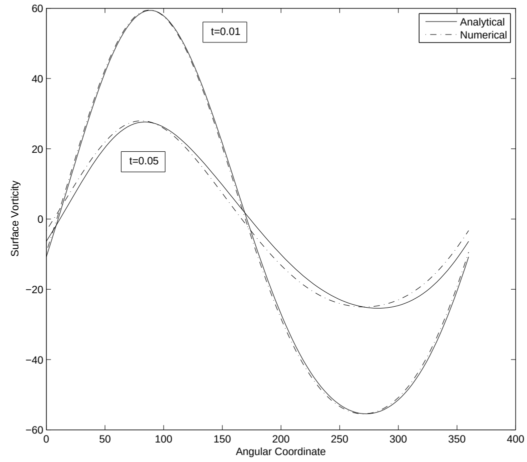

Figure 6: Surface vorticity for Ri=10, Re=50, K=0.2, Ω=0.25, t=0.01,0.05

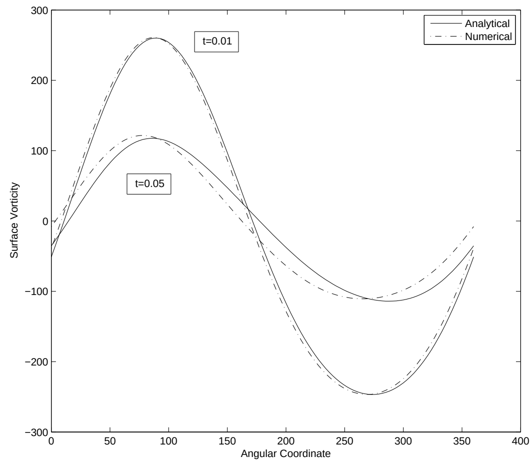

Figure 7: Surface vorticity for Ri=10, Re=1000, K=0.2, Ω=0.25, t=0.01,0.05

The surface vorticity distribution which is given by

$$\zeta(0,\theta,t) = \frac{1}{\lambda}A_0(\theta)$$

$$+\left(K + A_1(\theta) - \frac{\sqrt{\pi}}{4}A_0(\theta)\right)\lambda A_3(\theta) - \frac{\mathrm{Ri}\cos(\theta)}{8\mathrm{Re}\sqrt{\pi}}\lambda \tag{53}$$

is depicted in Figures 6 and 7 for Ri=10, $K = 0.2$, $\Omega = 0.25$, t =0.01, 0.05, 0.5 for Re values of 50 and 1000 respectively. Again we notice that the analytic solution becomes less accurate as time increases and as Re decreases.

The integrated average Nusselt numbers are also compared in Figure 8 for $K = 0.2$, $\Omega = 0.25$, Ri=10 and Re values of 50 and 1000. We see that there is excellent agreement between the two solutions for small values of t. However, as t increases the two solutions tend to deviate from each other as expected. Note also that the effect of the increase in Re is to enhance the heat transfer.

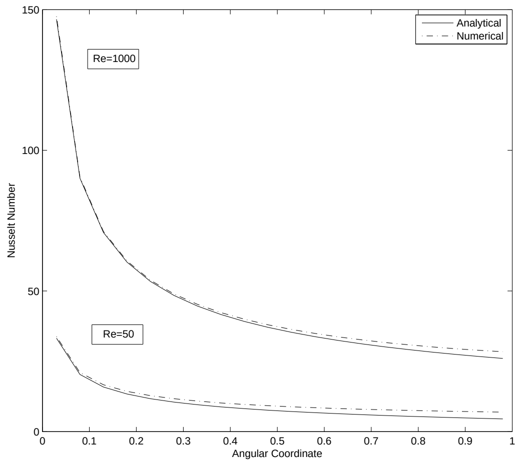

Figure 8: Integrated average Nusselt numbers Re=50,1000 Ri=5, K=0.2, Ω=0.25, t=0.01

Finally, the dependency on the shear parameter K is demonstrated in Figure 9 for Re=1000, Ri=10 and $\Omega = 0.0$. We note that the vorticity distribution becomes less symmetric with increasing shear. The figure also shows that shear enhances the surface vorticity in the upper half of the cylinder where there are faster moving fluid particles with the maximum occurring at the top tip of the cylinder. Again the analytic approximations are in excellent agreement with the numerically predicted solutions. This is consistent with the findings in [10] for the forced convection case.

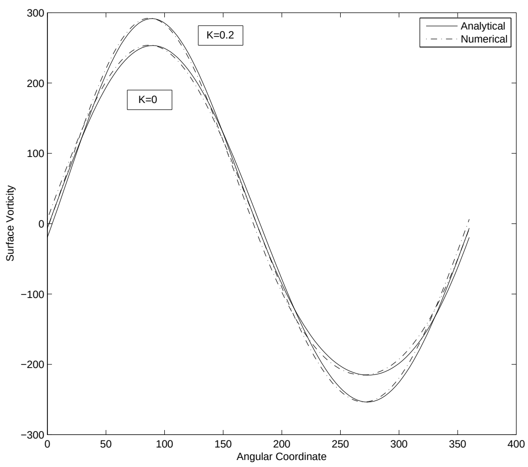

Figure 9: Surface vorticity for Ri=10, Re=1000, K=0,0.2, Ω=0, t=0.01

5 Conclusion

In this paper analytic approximations to the thermal-fluid problem involving forced and mixed convective heat transfer from a rotating isothermal cylinder placed in a non-uniform stream shear flow are presented. A convenient coordinate system is first introduced in order to simplify the geometry of the problem. The flow variables are then scaled with respect to the boundary layer parameter λ resulting in a set of boundary layer equations subject to appropriate initial and boundary conditions. The analytic approximations for the boundary value problem are obtained via a Fourier series expansion in terms of the boundary layer variable. The resulting approximations are valid not only for small time but also for moderate and large times provided that the Reynolds number of the flow is sufficiently large.

Acknowledgements

K. Abdella's general research in Applied Mathematics and Mathematical Modelling is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC). The authors would like to thank Almas Naseem for providing the numerical results.

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