## **Droplet Simulation, A Case Study: Evaporation**

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*Abstract:* For heat and mass transfer simulation of single droplet in a two-pieces solution with saturated solvent vapor environment, a numerical model is developed by finite volume method and transient SIMPLEC algorithm. I this study, dimensionless equations of motion, heat transfer and mass transfer (based on mass ratio) are solved simultaneously. All the thermodynamic and transitional solution properties have been considered as a function of temperature and concentration. The numerical analysis results are presented for mass transfer of lithium bromide solution droplet in 300K and initial concentration of 50%. Verification of method is done by compare these numerical results with analytical and numerical analysis of other studies. Droplet Growth Chart, average temperature and concentration, variation of drag coefficient diagrams, Nusselt number and flow line, temperature and concentration and temperature distribution contours, penetration rate of mass and the level of tangential velocity at droplet surface as the modeling results are presented.

Key-Words: — Heat and Mass Transfer, Finite volume, Droplet, Simulation, SIMPLEC Algorithm.

## 1. Introduction

In Industrial processes such as fuel injection, cooling fluids in cooling towers, most of the drying solution and many other industrial processes, droplet behavior considered one of the important parameters in system design. Therefore modeling of hydrodynamic and thermal behavior of droplets and small fluid particles has been studied by many researchers in last decades.

Prakash and Sirignano studied an analytical solve of mass transfer from single one-partial droplet [1]. Haywood et al. presented numerically analysis to solve this problem [2]. Jazayeri and Hosseini Nejad studied droplet evaporation and condensation in steam environment with numerical simulations in above studies, equations of motion and energy in droplet and surroundings are resolved [3]. Numerical modeling of mass transfer with multi-pieces environment was studied by Chang et al. for first time with solving equations of motion, energy and concentration [4]. In this model the fluid in droplet was considered as single-phase surrounding steam as two-pieces in this study, the mass transfer from multi-piece droplet in single-pieces environment has been studied. Perform the important assumptions in this model can be assumed laminar flow, the droplets remain spherical and using the thermodynamic equilibrium level Thermodynamic noted. member properties concentration has been considered as a function of temperature.

## 2. Numerical Model

In order to solve highly coupled equations, finite volume method and SIMPLEC algorithm is used With integration of equations on the control volume element in time ranged , discrete form of equations are obtained that with using the algorithm SIMPLEC and moved Network could be solved in transient form Computation is done in a network with 1500 (50 \* 30) elements in droplet and 2500 (50 \* 50) elements outside of droplets and forwarding speed of 1e-6sec that written by Visual C++ programming language including 67 sub programs. More details and explanations are available in Ref [5].



Fig 1. Flow Field Networking.

After investigations and solve the flow around solid sphere and around a liquid droplet flow, SIMPLEC algorithm was chosen due to its convergence time less than the SIMPLE algorithm then all Modeling is presented in SIMPLEC algorithm. For select suitable Discrete method, isothermal flow around a solid sphere is solved by code written by this study and Fluent code separately that in both code, weakness of upwind different and hybrid methods was evident. On the other hand, the modified central difference method solve is greatly accordance with results of solve obtained with the precise second order methods Fluent code and experimental results were. Thus the analysis performed by the modified central difference method is used.

### **3.** Governing Equations

Considering the mentioned assumptions, the equations of problem is as follows.

#### **3.1.** Mass Conservation

$$\frac{\partial \rho}{\partial t} = -(\vec{\nabla} \cdot \rho \vec{v}) \tag{1}$$

### **3.2.** Momentum Conservation

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla}P - \left[\vec{\nabla} \cdot \vec{\tau}\right] \tag{2}$$

## **3.3.** Distribution of Chemical Component A inside the Droplet

$$\frac{\partial(\rho w_A)}{\partial t} + (\vec{\nabla} \cdot \rho w_A \vec{\upsilon}) = (\vec{\nabla} \cdot \rho D_{AB} \vec{\nabla} w_A)$$
(3)

#### **3.4.** Energy Conservation around Droplet

In the absence of energy source and sink, and regardless of the effects of radiation heat transfer and compressibility and viscosity, the energy equation the problem assuming thermal conductivity from Fourier law, is written as follows:

$$\frac{\partial}{\partial t}(\rho T) + \vec{\nabla} \cdot (\rho \vec{\upsilon} T) = \vec{\nabla} \cdot (\frac{k}{c_P} \vec{\nabla} T) + S_E$$
(4)

$$S_E = \frac{k}{c_P^2} (\vec{\nabla} P \cdot \vec{\nabla} c_P) \tag{5}$$

# **3.5.** Internal Energy Conservation in Droplet

$$\frac{\partial}{\partial t}(\rho T) + \vec{\nabla} \cdot (\rho \vec{\upsilon} T) = \vec{\nabla} \cdot (\frac{k}{c_p} \vec{\nabla} T) + S_E \tag{6}$$

$$S_E = \frac{k}{c_P^2} (\vec{\nabla} P \cdot \vec{\nabla} c_P) + \frac{1}{C_P} \frac{\partial h}{\partial w} \Big|_{P,T} [\nabla \cdot \rho D_{AB} \nabla w]$$
(7)

### 4. Boundary Conditions

#### 4.1. Symmetric Boundary Conditions

The boundary is symmetry axis of problem, where the boundary conditions are:

$$\frac{\partial \phi}{\partial \theta}\Big|_{\theta=0,\theta=\pi} = 0 \quad , \quad \phi = \upsilon_r, T, w \tag{8}$$

$$p_{\theta} = 0$$

#### 4.2. Free Flow Boundary Condition

This boundary consists of two parts; inlet and outlet of flow at inlet  $(0 \le \theta \le \pi/2 \operatorname{And} r = r_{\infty})$  Velocity and temperature values can be assumed equal to their values in free flow. Boundary condition at output flow  $(\pi/2 < \theta \le \pi \operatorname{And} r = r_{\infty})$  could be assumed as zero radial gradient.

#### 4.3. Liquid-Gas Common Surface

For derive common surface boundary condition, integrate the control volume equations that surrounds common surface that limit to zero the details of the boundary conditions derivation for the radial velocity is given in Ref [5]

Radial velocity of liquid:

$$\nu_{r,l,S} = \frac{\dot{m}_{\theta}'}{\rho_{l,S}} + \frac{dR}{dt}$$
<sup>(9)</sup>

Radial velocity of vapor phase:

$$\nu_{r,g,S} = \frac{\dot{m}_{\theta}'}{\rho_{e,S}} + \frac{dR}{dt}$$
(10)

Shear stress conservation at the level of:

$$\widetilde{\tau}_{r\theta,l} = \widetilde{\tau}_{r\theta,g} \tag{11}$$

Boundary condition equations for concentration and temperature in common surface is more complex. Mass Penetration to droplet is associated with the energy release due to phase change. Boundary condition equations of problem in common surface relative vapor exchange between the environment and droplet. In Common surface, for achieve to consistent thermal boundary condition, thermodynamic equilibrium assumption is used. This means that temperature of droplet surface is considered equal to fluid saturation temperature (at local pressure and concentration).

$$P_{S}^{\theta} = P_{g}^{\theta} + \Delta P_{\sigma} \tag{12}$$

$$\Delta P_{\sigma} = \sigma \cdot (\frac{2}{R^2}) \tag{13}$$

$$T_{g,S} = T_{l,S} = T_{SAT}$$
 @  $w_S, P_S$  (14)

Using energy conservation condition:

$$-k\frac{\partial T}{\partial r}\Big|_{l} = -k\frac{\partial T}{\partial r}\Big|_{g} + \dot{m}_{\theta}^{"}L$$
(15)

Calculations of concentration at droplet surface are as follows.

According to Fick law:

$$\rho_A(v_A - v) = -\rho D_{AB} \vec{\nabla} w_A \tag{10}$$

(10)

Where A and B are Chemical components in the fluid and velocities v And  $v_A$  Are calculated as follows.

$$\upsilon_A = \frac{\dot{m}_{\theta}''}{\rho_{l,A}} + \frac{dR}{dt} = \frac{\dot{m}_{\theta}''}{\rho_{l}w_A} + \frac{dR}{dt}$$
(17)

$$\upsilon_B = \frac{dR}{dt} \tag{18}$$

$$\upsilon = \frac{\dot{m}_{\theta}''}{\rho_l} + \frac{dR}{dt} \tag{19}$$

Thus if velocities v and  $v_A$  are known, concentration could be calculated with equation (16)

#### 5. Non-Dimensioning

In order to generalize the problem and presenting results in general dimensionless parameters, equations and boundary conditions must be non-dimensioned.

#### 5.1. Equations Non-Dimensioning

Conservation dimensionless equations form with dimensionless parameters defined before could be obtained in general form as:

$$\frac{\partial}{\partial t}(\rho V R \phi) + V^2 \vec{\nabla} \cdot \left\{ \rho \phi((\upsilon_r - \frac{r}{V} \frac{dR}{dt}) \hat{r} + \upsilon_\theta \hat{\theta} \right\} = \frac{V}{P} \vec{\nabla} \cdot (\Gamma_\phi \vec{\nabla} \phi) + S_\phi$$
(20)

In this equation  $\phi$  General variable,  $\Gamma_{\phi}$  and distribution coefficient and  $s_{\phi}$  is source term String replace And Conservation equations for each Table 1 is shown. Replaced terms  $\phi$  and  $\Gamma_{\phi}$  for each conservation equation are shown in table 1. Conservation Equation of this case, not inherently include source term  $s_{\phi}$  and it is appeared Because of selected dimensionless parameters and inclusion of equations in the overall form appearance is above. Noticeable point is that in second sentence, the time derivative of droplet radius is used in equation and modified velocity vector are shown as  $(\upsilon_r - \frac{r}{R} \frac{dR}{dt}) \hat{r} + \upsilon_{\theta} \hat{\theta}$ .

## 5.2. Non-Dimensioning Boundary Conditions

The dimensionless form of Boundary conditions of symmetric axis, droplet center and free flow are similar to dimensional form.

#### 5.2.1. Boundary conditions at common surface

Dimensionless form of boundary conditions in common surface has difference to dimensional form

Radial velocity (liquid phase):

$$\nu_{r,l,S} = \frac{1}{V} \left\{ \frac{\dot{m}_{\theta}''}{\rho_{l,S}} + \frac{dR}{dt} \right\}$$
(21)

Radial velocity (gas phase):

$$\nu_{r,g,S} = \frac{1}{V} \left\{ \frac{\dot{m}_{\theta}''}{\rho_{g,S}} + \frac{dR}{dt} \right\}$$
(22)

Continuity of shear stress:

$$\tilde{\tau}_{r\theta,l} = \tilde{\tau}_{r\theta,g} \tag{23}$$

Conservation of energy:

$$-k\frac{\partial T}{\partial r}\Big|_{l,\theta} = -k\frac{\partial T}{\partial r}\Big|_{g,\theta} + \frac{\operatorname{Re}_{0}\operatorname{Pr}_{\infty}R}{2}\dot{m}_{\theta}''L$$
(24)

Known concentration at surface

$$w_A(\upsilon - \upsilon_A) = \frac{2}{\operatorname{Re}_0 Sc_0} \frac{D_{AB}}{RV} \nabla w_A \tag{25}$$

$$\upsilon_A = \frac{1}{V} \left\{ \frac{\dot{m}_{\theta}''}{\rho_l w_A} + \frac{dR}{dt} \right\}$$
(26)

Relationship of the average and local Nusselt number and drag coefficient (friction, pressure, and overall thrust) also are written as follows. (All expressions are written in vapor phase)

$$Nu_{g}(\theta) = \frac{2k\frac{\partial T}{\partial r}\Big|_{r=1}}{(1-T_{S})}$$
(27)

$$\overline{Nu}_{g} = \frac{\int_{0}^{\pi} Nu_{g}(\theta) dA}{\int_{0}^{\pi} dA}\Big|_{r=1}$$
(28)

$$C_F = \frac{4}{\pi \operatorname{Re}_0 RV} \int_0^{\pi} (\tilde{\tau}_{r\theta} \sin \theta - \tilde{\tau}_{rr} \cos \theta) dA \bigg|_{r=1}$$
(29)

$$C_P = \frac{2}{\pi V^2} \int_0^{\pi} P \cos\theta dA \Big|_{r=1}$$
(30)

$$C_T = \frac{2}{\pi} \int_0^{\pi} \rho v_r^2 \cos\theta dA \Big|_{r=1}$$
(31)

Thrust drag component is due to asymmetric evaporation and condensation on the surface droplet.

## 6. Validation

In the analysis, isothermal flow around a solid sphere solution in performed in different Reynolds numbers and drag coefficient, and vortex behind the sphere and Vortex separation point in the this analysis and other numerical and experimental results are compared. For isothermal flow around a spherical drop, analytical relations presented by Clift et al. to calculate the drag force has been comparing with results of this study [6]. Also a Comparison is done between parameters of isothermal air flow around a droplet of water in the present analysis and results reported by LeClair & Hamielec [7] and Haywood et al. [2] that shown reasonably compliant as shown in Table 1. In Fig. 2 evaporation of water droplet at uniform temperature 320K, Radius 0.5mm and initial Re 100 at superheated water vapor at 340K and 10kPa is shown and in Fig. 3 variation of average Nu for Evaporation of water droplet is compared. Condensation of water vapor on a water droplet with uniform temperature 310K, Radius 0.5mm and initial Re 100 at superheated water vapor at 350K and 40kPa is shown in Fig. 4 and in Fig. 5 comparison of variation of CD for condensation of water vapor on a water droplet is shown. Fig. 6 showed absorption of water vapor by a Li-Br droplet at uniform initial temperature 320K, Radius 0.5mm and initial Re 100 at superheated water vapor at 320K and in Fig. 7 comparison of variation of CD as function of times for absorption of water vapor by a Li-Br droplet is shown.

Table 1. Comparison of isothermal parameters of air flow around water droplet.

$\mu_l / \mu_g = 55$		Present result	Haywood et al.[2]	LeClair & Hamielec [7]
Re=100	C <sub>P</sub>	0.513	0.511	0.49
	$C_{\rm F}$	0.578	0.578	0.59
	C <sub>D</sub>	1.091	1.09	1.08
	$v_{l,MAX}$	0.035	0.036	0.037
	$\theta_{v_l,MAX}$	67	64	69



Fig 2. Evaporation of water droplet at uniform temperature at superheated water vapor.



Fig 3. Comparison of variation of Average Nusselt number for Evaporation of water droplet.



Fig 4. Condensation of water vapor on a water droplet with uniform temperature at superheated water.



Fig 5. Comparison of variation of CD condensation of water vapor on a water droplet.



Fig 6. Absorption of water vapor by a Li-Br droplet at uniform temperature at superheated water vapor.



Fig 7. Comparison of variation of CD for absorption of water vapor by a Li-Br droplet.

## 7. Conclusions

The results of numerical analysis are presented for mass transfer of lithium bromide solution droplet in 300K and initial concentration of 50%.

In figure 8, surface temperature and distribution in droplets surface at 6 different times is plotted. Temperature boundary conditions on droplet surface is depends on pressure and is function of droplet's absorption rate of vapor and also absorption rate of vapor depends on surface temperature and its neighbors. On the other hand, the surface pressure obtained from velocity profiles around droplets Because of high suction on the surface of droplet, velocity profile and thus the surface pressure distribution has also changed. Thus adsorption is function of three important parameters; pressure, temperature and surface concentration that in this case the three parameters strongly depend on each other. In figures 8 and 9 high dependency between these three parameters is shown. The overall drag coefficient that consist of three components pressure drag, friction and thrust drag, is decreases in initial moments and then Start to increase after reaching the minimum point at approximately t=150s, But pressure and thrust drag increase in initial moments and then after reaching maximum point approximately t=50s are starting to decrease. As shown in Figure 4, the major component of drag in absorption process of droplet is frictional drag component Frictional drag component in the initial moments is too high due to high vacuum pressure of droplet and too low thickness of boundary layer on droplet surface, and then become steady state with decreasing suction rate. In figures 10 and 11, constant temperature and concentration lines at four different times is plotted. As could be seen in Figures, concentration penetration power is much weaker than temperature and concentration boundary layer formed in droplet surface is very thin.







Fig. 9: Solution Mass Ratio at Droplet Surface



Fig. 11: Constant Concentration Lines for Various Times.

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