High velocity motion of a wing in compressible fluid near a surface

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Abstract: The two-dimensional problems of thin body motion in fluid parallel to the boundary at a distance, comparable with the length of the body, are regarded. In particular, resistance and lift forces in thin body motion parallel to free surface and parallel to rigid surface are determined and compared with existing solutions for resistance and lift forces in case of an infinite space. The solution is determined under the assumption of fluid being ideal and compressible. The Chaplygin-Zhukovsky hypothesis of rear-edge-limited solution is taken into consideration.

In case of moving near free surface the solution is obtained for a problem of infinite span wing. The solution allows determining drag and lift forces in the limiting cases. It was shown that on Mach number tending to unity both forces infinitely increase. For relatively thin fluid layer above the moving wing the resistance force depends on the distance to the free surface, inclination and Mach number, while for relatively thick fluid layer the force depends on wing length, inclination angle and Mach number as well.

In case of moving near rigid surface the solution of a problem is reduced to the Fredholm equation, which is solved numerically. The generalization of Zhukovski solution was obtained, which provides the lift force dependence on the altitude of the flight. The behavior of the lift force is very peculiar: it increases on decreasing altitude above the rigid surface. The screen effect becomes essential on moving wing altitude being smaller than the wing's length. The effect was detected experimentally before and gave birth to construction of a special flying vehicle named "ecranoplan". It is shown in the paper that the lift force could increase several orders of magnitude. This effect could be used in developing flying high-speed vehicles, which could be used in the territories of smooth surface: steppes, deserts, lakes, swamps, etc.

Key-Words: wing, compressible fluid, analytical function, Fredholm equation, Dirichlet problem, lift and resistance forces.

1 Introduction

The problem of a wing moving near free surface is relevant to surface or underwater high velocity gliding of a thin wing, which is often used to reduce the resistance of the glider. The current problem has many practical applications, such as determining resistance and lift forces being the function of the depth in underwater motion of a bullet or shell.

The problem of gliding near free surface of water of infinite and finite depth was regarded within the frames of linear [1-4] and non-linear [5, 6] statements, and found its generalized classical solution in [7]. Solutions for motions of plates at big attack angles in unbounded incompressible fluid were obtained in [8]. High-speed streaming flows accounting for fluid compressibility were investigated in [9-11].

At the beginning of the XX century it was observed that the lift force of a wing moving near flat surface increases strongly in comparison with free flight. An article about screen effect by B.N. Juriev [12] was published in 1923 in the USSR. That fact was used in creation of new flying devices – screen-flights, which got the Russian name "ekranoplan". In 1932 Grohovsky constructed a full-scaled model of a new marine flying device – catamaran. At the same time Finnish engineer T. Kaario proceeded to test his flying apparatus that used a screen effect. Then (1963 – 1976) a Soviet constructor R.L. Bartini created a screen-flight project SVVP-2500 that took off in 1974. The first Soviet manned jet screen-flight SM-1 was created in collaboration with R. Alekseev in 1960 – 1961 [13]. Giant screen-flight KM was finished by 1966 and "Orlyonok" type screen-flights were built from 1974 to 1983. Designing of new flying devices continues in many countries.

L.I. Sedov obtained an analytical solution for the lift force in terms of Weierstrass functions [7] using the theory of a complex variable. Approximate analytical solution of the problem of non-steady plane moving near rigid surface was obtained by K.V. Rozjdestvensky [14] with the help of asymptotic expansion. Theoretical investigation of a wing moving near rigid surface was made by A.N. Panchenkov [15, 16], but the obtained solutions incorporated free constants. Experimental results are shown in [17].

The problem of theoretical investigation of a wing's behavior near a surface is still far from being finalized. The present paper provides a theoretical solution for the two-dimensional linearized problem of a thin wing motion near free and rigid surfaces in compressible fluid.

2 Problem Formulation

The two-dimensional problem of compressible fluid streaming thin body in the presence of free and rigid surfaces is regarded. The coordinate system and flow scheme in case of the presence of a rigid surface are shown in Fig.1. The coordinate system and flow scheme in case of the presence of a free surface are shown in Fig.2 and Fig.3.

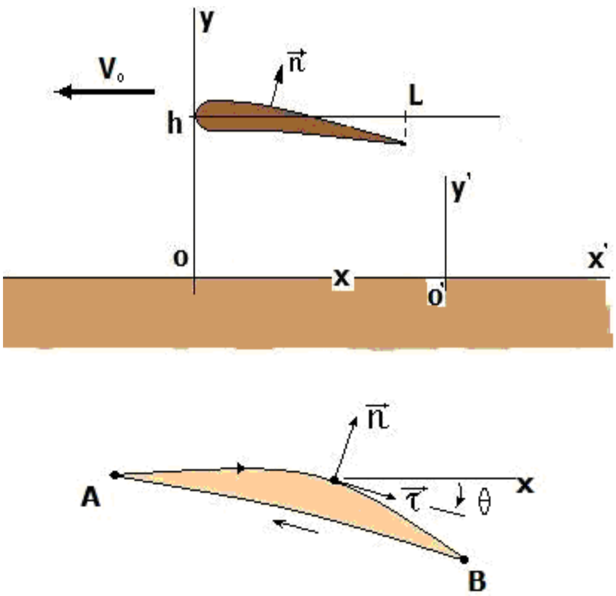


Fig.1 Schematic picture for thin wing motion above rigid surface

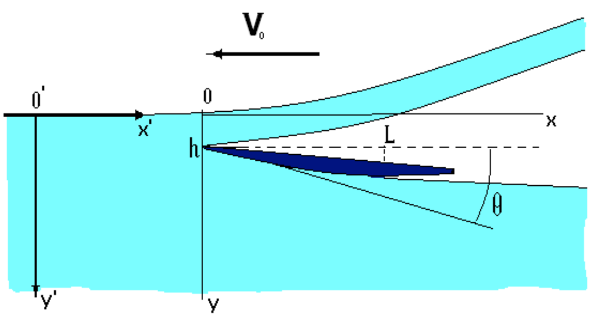


Fig.2 Schematic picture for thin wing motion near free surface with positive angle of attack

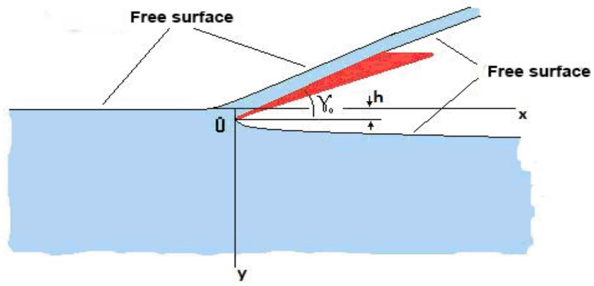


Fig.3 Schematic picture for thin wing motion near free surface with negative angle of attack

Fluid is assumed to be ideal, mass forces – negligibly small, flow field – plane. Velocity field in fluid is assumed to be potential

$$\vec{V} = \vec{V}_0 + grad\ \varphi\ , \tag{1}$$

fluid will be regarded as linear compressible

$$P = P_0 + a^2(\rho - \rho_0)\ , \tag{2}$$

where $\varphi(x, y, t) -$ disturbance velocity potential, $P, \rho -$ fluid pressure and density, $P_0, \rho_0 -$ pressure and density in quiescent fluid, $a -$ sonic velocity.

Fluid flow satisfies the continuity equation

$$\frac{d\rho}{dt} + \rho\ div\ \vec{V} = 0\ , \tag{3}$$

pressure is determined by the Cauchy-Lagrange integral

$$\frac{\partial\varphi}{\partial t} + \frac{(grad\ \varphi)^2}{2} + \int\frac{dP}{\rho} = c(t)\,. \tag{4}$$

Flow induced variations of density and velocity are considered small values.

$$\rho'/\rho = (\rho - \rho_0)/\rho \ll 1;$$
$$V_x / V_0 \ll 1; V_y / V_0 \ll 1,$$

where $V_x, V_y -$ disturbance velocity components.

Then it follows from continuity equation (3), integral (4) and relationships (1), (2), neglecting small values of the orders higher than one, flow potential φ under the condition of steady-state flow satisfies the equation

$$V_0^2 \frac{\partial^2 \varphi}{\partial x^2} = a^2 \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right), \qquad (5)$$

and fluid pressure is determined in the following way:

$$P - P_0 = \rho_0 V_0 \frac{\partial \varphi}{\partial x}. \qquad (6)$$

Boundary conditions should be satisfied on the rigid surface, on the free surfaces and on the body surface contacting fluid. On the rigid surface normal velocity component is equal to zero, on the free surfaces constant pressure is assumed, on the fluid-body contact streaming condition of the equality of normal velocity component.

$$\textit{on the body boundary} \quad \vec{v} \cdot \vec{n} = \vec{V} \cdot \vec{n};$$
$$\textit{on the rigid surface} \quad v_y = 0;$$
$$\textit{on the free surface} \quad P - P_0 = 0.$$

The obstacle being thin and inclination angle being small all disturbances could be considered small and boundary conditions take the form

for body moving near rigid surface:

$$y = h^{\pm}, \quad 0 < x < L: \quad \frac{\partial \varphi}{\partial y} = V_0 \sin \theta$$
$$y = 0, \quad -\infty < x < \infty: \quad \frac{\partial \varphi}{\partial y} = 0,$$

for body moving near free surface:

$$y = 0, \quad P - P_0 \quad 0; \qquad (7)$$

with positive angle of attack:

$$y = h^-, 0 < x \quad P - P_0 \quad 0;$$

$$y = h^+, 0 < x < L \quad u_y = \frac{\partial \varphi}{\partial y} = V_0 \ \sin \theta$$

$$y = h^+, L < x \quad P - P_0 \quad 0;$$

with negative angle of attack:

$$y = h^+, 0 < x \quad P - P_0 = 0;$$

$$y = h^-, 0 < x < L \quad u_y = \frac{\partial \varphi}{\partial y} = -V_0 \cdot \sin \gamma_0$$

$$y = h^-, L < x \quad P - P_0 = 0$$

Thus equation (5) with boundary conditions (7) presents a closed form statement of the problem.

3 Problem Solution

3.1 Problem Solution in Case of a Wing Moving near Rigid Surface

We assume the flow to be subsonic. A plate is regarded so that $\theta = -\gamma$. Then on introducing dimensionless parameter $\alpha = \sqrt{1 - M^2}$, where $M = V_0 / a -$ Mach number, and dimensionless functions and variables

$$p^* = \frac{P - P_0}{\rho_0 V_0}; \; x^* = \frac{x}{L}; \; y^* = \frac{\alpha y}{L}, \qquad (8)$$

equations (5),(6) and boundary conditions (7) take the form

$$\frac{\partial^2 \varphi}{\partial x^{*2}} + \frac{\partial^2 \varphi}{\partial y^{*2}} = 0, \; p^* = \frac{1}{L} \frac{\partial \varphi}{\partial x^*},$$

$$y^* = \frac{\alpha h^{\pm}}{L}, \quad 0 < x < 1: \quad \frac{\partial \varphi}{\partial y^*} = \frac{V_0 \gamma}{\alpha^2} L$$
$$y^* = 0, \quad -\infty < x < \infty: \quad \frac{\partial \varphi}{\partial y^*} = 0, \qquad (8)$$

$$\sin \theta \approx tg\, \theta = \frac{dy}{dx}, \; tg\, \theta = \frac{1}{\alpha} \frac{dy^*}{dx^*} = \frac{\gamma}{\alpha}$$

In successive derivations star in dimensionless value symbols will be omitted.

The solution will be developed in the form of a real part for the analytical function of a complex variable $\varphi(x, y) = \operatorname{Re} \Phi(z), \quad z = x + iy$.

Then the solution is reduced to the following boundary problem:

$$y = \frac{h^{\pm}}{L}, \quad 0 < x < 1: \quad \operatorname{Im} \Phi'(x) = \frac{V_0 \gamma}{\alpha^2} L$$
$$y^* = 0, \quad -\infty < x < \infty: \quad \operatorname{Im} \Phi'(x) = 0. \qquad (9)$$

Boundary conditions should be supplemented with function behavior at the infinity for the uniqueness of the solution. The demand for function behavior at the infinity is reduced to the following formula, as it is shown in [18]:

$$z \to \infty, \quad \Phi(z) \approx \ln z \quad const.$$

The solution for the problem will be determined in the form of the Cauchy integral [19]

$$\Phi'(z) = \frac{1}{2\pi i}\int_0^1 \frac{g_{11}(t) - i\, f_{11}(t)}{t - \left(z - i\frac{\alpha h}{L}\right)} dt + \frac{1}{2\pi i}\int_0^1 \frac{g_{11}(t) + i\, f_{11}(t)}{t - \left(z + i\frac{\alpha h}{L}\right)} dt.$$

This expression satisfies the condition on the axis $y = 0, \quad -\infty < x < \infty \quad \mathrm{Im}\,\Phi'(z) = 0$. Boundary condition on the body contour still needs to be satisfied. The value of function on the boundary $y = \frac{\alpha h^{\pm}}{L}$ is

$$0 < x < 1, \quad \Phi'(x + i\frac{\alpha h^{\pm}}{L}) = \pm\frac{1}{2}\left(g_{11}(x) - i f_{11}(x)\right) + \frac{1}{2\pi i}\int_0^1 \frac{g_{11}(t) - i f_{11}(t)}{t - x} dt + \frac{1}{2\pi i}\int_0^1 \frac{g_{11}(t) + i\, f_{11}(t)}{t - 2i\frac{\alpha h}{L} - x} dt,$$

$$\mathrm{Im}\,\Phi'(x + i\frac{\alpha h^{\pm}}{L}) = \mp\frac{1}{2} f_{11}(x) - \frac{1}{2\pi}\int_0^1 \frac{g_{11}(t)}{t - x} dt - \frac{1}{2\pi}\int_0^1 \frac{\left[(t-x) g_{11}(t) - 2\frac{\alpha h}{L} f_{11}(t)\right] dt}{(t-x)^2 + 4(\frac{\alpha h}{L})^2},$$

$$\mathrm{Re}\,\Phi'(x + i\frac{\alpha h^{\pm}}{L}) = \pm\frac{1}{2} g_{11}(x) - \frac{1}{2}\int_0^1 \frac{f_{11}(t) dt}{t - x} + \frac{1}{2\pi}\int_0^1 \frac{2\frac{\alpha h}{L} g_{11}(t) + (t-x) f_{11}(t)}{(t-x)^2 + 4(\frac{\alpha h}{L})^2} dt,$$

$$as \quad -i(t - x + 2i\frac{\alpha h}{L})(g_{11} + i f_{11}) = 2\frac{\alpha h}{L} g_{11} + (t-x) f_{11} - i\left[g_{11}(t-x) - 2\frac{\alpha h}{L} f_{11}\right].$$

Integrals from the right part of the formula do not depend on the contour side. Therefore the function satisfies the following condition on the contour

$$y = \frac{\alpha h}{L}^{\pm}, \quad 0 < x < 1 \quad \left[\mathrm{Im}\,\Phi'(x)\right] = -f_{11}(x).$$

$$\mathrm{Im}\,\Phi'(x + i\frac{\alpha h}{L}^{\pm}) = \pm\frac{V_0\gamma}{\alpha^2} L - \frac{1}{2\pi}\int_0^1 \frac{g_{11}(t)}{t - x} dt - \frac{1}{2\pi}\int_0^1 \frac{\left[(t-x) g_{11}(t) + 4\frac{V_0\gamma h}{\alpha}\right] dt}{(t-x)^2 + 4(\frac{\alpha h}{L})^2}.$$

Boundary condition on the upper surface of the body should be satisfied for the definition of function $g_{11}(x)$

$$-\frac{V_0\gamma}{\alpha^2} L = \frac{V_0\gamma}{\alpha^2} L - \frac{1}{2\pi}\int_0^l \frac{g_{11}(t)}{t - x} dt - \frac{1}{2\pi}\int_0^l \frac{\left[(t-x) g_{11}(t) + 4\frac{V_0\gamma h}{\alpha}\right] dt}{(t-x)^2 + 4(\frac{\alpha h}{L})^2}.$$

Finally we get the singular integral equation with the Cauchy center [19]

$$\frac{1}{\pi}\int_0^1 \frac{g_{11}(t)}{t-x}dt = 4\frac{V_0\gamma}{\alpha^2}L - \frac{1}{\pi}\int_0^1 \frac{(t-x)g_{11}(t)dt}{(t-x)^2+4(\frac{\alpha h}{L})^2} -$$

$$-4\frac{V_0\gamma h}{\alpha\pi}\int_0^1 \frac{dt}{(t-x)^2+4(\frac{\alpha h}{L})^2}.$$

The determined solution will be limited at the rear edge of the wing (the Chaplygin-Zhukovsky hypothesis [20]). In this case regularizing of the singular equation can be fulfilled that reduces it to the Fredholm equation

$$g_{11}(x) - \frac{2}{\pi}(\frac{\alpha h}{L})^2\int_0^1 g_{11}(\tau)\sqrt{\frac{1-x}{x}}K(\tau,x)d\tau =$$

$$= \frac{V_0\gamma}{\alpha^2}L\sqrt{\frac{1-x}{x}}(1+q(x)),$$

where

$$K(\tau,x) = \frac{1}{\sqrt{R}\cos\theta\left[(1-\tau)^2+4(\frac{\alpha h}{L})^2\right]}\cdot$$

$$\cdot\frac{R^2(1-\tau)(1-x) - R\tau(1-x)(2\cos 2\theta - 1)}{R^2(1-x)^2 + x^2 - 2x(1-x)R\cos 2\theta} -$$

$$-\frac{R\tau(1-x) - x\tau}{R^2(1-x)^2 + x^2 - 2x(1-x)R\cos 2\theta},$$

$$q(x) = \frac{1}{\pi}\int_0^1 \frac{1}{\sqrt{R}\sin\theta\left[(1-\tau)^2+4(\frac{\alpha h}{L})^2\right]}\cdot$$

$$\cdot\frac{R^2(1-x) + R(1-x) - x + xR - 2xR\cos 2\theta}{R^2(1-x)^2 + x^2 - 2x(1-x)R\cos 2\theta}d\tau,$$

$$R^2 = \frac{\tau^2 + 4(\frac{\alpha h}{L})^2}{(1-\tau)^2 + 4(\frac{\alpha h}{L})^2},$$

$$\theta = \frac{1}{2}\left[\arctan\left(\frac{2\frac{\alpha h}{L}}{\tau}\right) + \arctan\left(\frac{2\frac{\alpha h}{L}}{1-\tau}\right)\right].$$

The Fredholm equation was determined numerically [21]. The obtained solution allows the pressure function and the lift force to be determined:

$$p^{\pm}(x) = \rho_0 V_0 \operatorname{Re}\left[\pm\left(g_{11}(x) - i\, f_{11}(t)\right)\right] +$$

$$+\rho_0 V_0 \operatorname{Re}\frac{1}{2\pi i}\int_0^1 \frac{g_{11}(t) - i\, f_{11}(t)}{t-x}dt +$$

$$+\rho_0 V_0 \operatorname{Re}\frac{1}{2\pi i}\int_0^1 \frac{g_{11}(t) + i\, f_{11}(t)}{t - \left(x + 2i\frac{\alpha h}{L}\right)}dt,$$

$$F_y = -\int_0^L (p^+ - p^-)dx =$$

$$= \rho_0 V_0 \int_0^1 \left[\sqrt{\frac{1-x}{x}}\frac{1}{\pi}\int_0^1 \sqrt{\frac{t}{1-t}}\frac{g_{11}(t)dt}{t-x}\right]dx$$

Fig.4 represents the dependence of the dimensionless lift force of a flat wing $\dfrac{F}{\pi\rho_0\dfrac{V_0^2\gamma}{\alpha}L}$ upon the relative distance from the wing to the motionless surface $\dfrac{h}{L}$. It is seen that the screen effect practically disappears for distances equal to two times the length of the wing. The lift force increases greatly when the distance is lower than the length of the wing. If the altitude above the surface surpasses the wing's span and the fluid is regarded as incompressible the lift force tends to its value in an unbounded space determined by the classical solution for thin wing lift force in incompressible fluid and unbounded space [23]:

$$F = \pi\rho_0 V_0^2 \gamma L$$

Fig.5 represents the dependence of the dimensionless pressure function $P = \dfrac{\Delta p}{\rho_0 V_0}$ upon the length of the wing x/L for $h/L = 0{,}1$. The center of applied resultant lift force shifts to the front edge of the wing as the distance h/L decreases because the singularity on the front edge increases.

The obtained results fully correspond to the experiment described in [13].

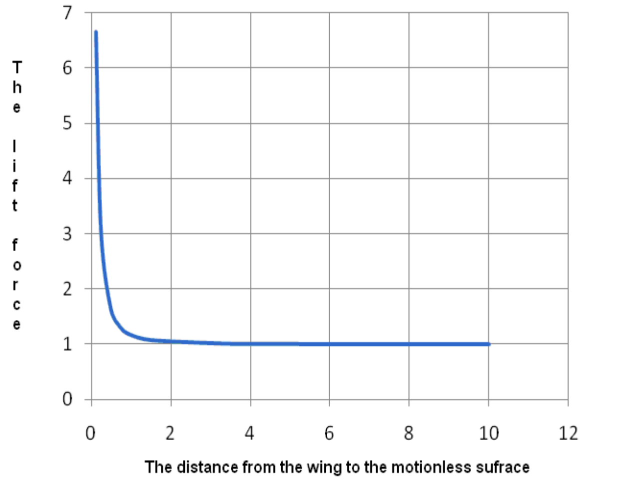


Fig.4 Dependence of dimensionless lift force of a flat wing $\dfrac{F}{\pi\rho_0 \dfrac{V_0^2\gamma}{\alpha} L}$ on the relative distance from the wing to the flat surface $\dfrac{h}{L}$

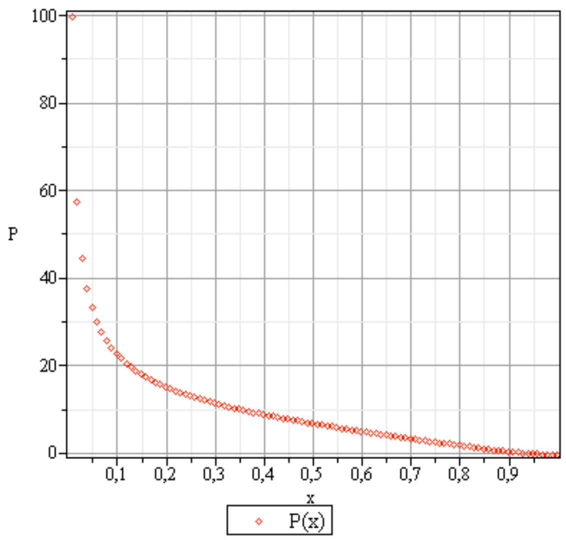


Fig.5 Dependence of dimensionless pressure function $P = \dfrac{\Delta p}{\rho_0 V_0}$ upon the length of the wing x/L for $h/L = 0,1$

3.2 Problem Solution in Case of a Wing Moving near Free Surface

We assume the flow to be subsonic. Then on introducing dimensionless parameter $\alpha = \sqrt{1-M^2}$, where $M = V_0 / a -$ Mach number, and dimensionless functions and variables

$$\varphi^* = \frac{\pi\varphi}{ah};\ p^* = \frac{P-P_0}{\rho_0 a^2};\ l = \frac{L\pi}{h\alpha};$$

$$x^* = \frac{\pi x}{\alpha h};\quad y^* = \frac{\pi y}{h}, \qquad (10)$$

equations and boundary conditions take the form

$$\frac{\partial^2\varphi^*}{\partial x^{*2}} + \frac{\partial^2\varphi^*}{\partial y^{*2}} = 0,\ p^* = \frac{M}{\alpha}\frac{\partial\varphi^*}{\partial x^*},$$

$$y^* = 0,\quad \frac{\partial\varphi^*}{\partial x^*} = 0;$$

in case of positive angle of attack

$$y^* = \pi^-, 0 < x^* \quad \frac{\partial\varphi^*}{\partial x^*} = 0; \qquad (11)$$

$$y^* = \pi^+, 0 < x^* < l \quad \frac{\partial\varphi^*}{\partial y^*} = \frac{M\cdot\gamma(x^*)}{\alpha};$$

$$y^* = \pi^+, l <{}^{*}x \quad \frac{\partial\varphi^*}{\partial x^*} = 0;$$

in case of negative angle of attack

$$y^* = \pi^+, 0 < x^* \quad \frac{\partial\varphi^*}{\partial x^*} = 0;$$

$$y^* = \pi^-, 0 < x^* < l \quad \frac{\partial\varphi^*}{\partial y^*} = -\frac{M\cdot\gamma(x^*)}{\alpha};$$

$$y^* = \pi^-, l < x^* \quad \frac{\partial\varphi^*}{\partial x^*} = 0;$$

In successive derivations star in dimensionless value symbols will be omitted. The problem is reduced to developing analytical function in the domain $y > 0$ with a cut $y = \pi, x > 0$, satisfying boundary conditions (11). The solution will be developed in the form of a real part for the analytical function of a complex variable

$$\varphi(x,y) = \mathrm{Re}\,\Phi(z), \quad z = x + iy.$$

In case of positive angle of attack:

$$y = 0, \quad \mathrm{Re}\,\Phi'(x) \quad 0;$$

$$y = \pi^-, 0 < x \quad \mathrm{Re}\,\Phi'(x) \quad \theta;$$

$$y = \pi^+, 0 < x < l \quad \mathrm{Im}\,\Phi'(x) = -\frac{M \cdot \gamma(x)}{\alpha};$$

$$y = \pi^+, l < x \quad \mathrm{Re}\,\Phi'(x) \quad \theta.$$

In case of negative angle of attack:

$$y = 0, \quad \mathrm{Re}\,\Phi'(x) \quad 0;$$

$$y = \pi^+, 0 < x \quad \mathrm{Re}\,\Phi'(x) = 0;$$

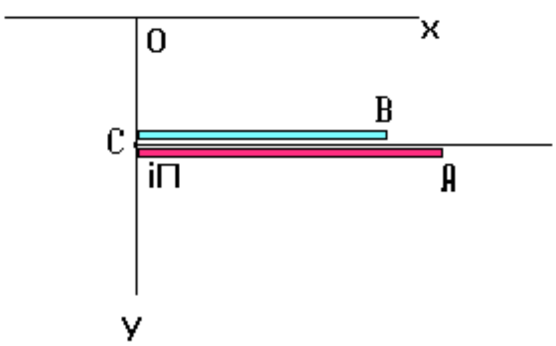
$$y = \pi^-, 0 < x < l \quad \mathrm{Im}\,\Phi'(x) = \frac{M \cdot \gamma(x)}{\alpha};$$

$$y = \pi^-, l < x \quad \mathrm{Re}\,\Phi'(x) = 0.$$

Using the conformal mapping of the semi-space $y > 0$ with a cut $y = \pi, x > 0$, the development of the analytical function is reduced to the Riemann – Hilbert problem. The conformal mapping is given by the following function:

$$z = \pi i + w - \ln w - 1, \quad \mathrm{Im}\,w > 0, \; w = u + iv.$$

The transformation of the boundary is shown in picture Fig.6.



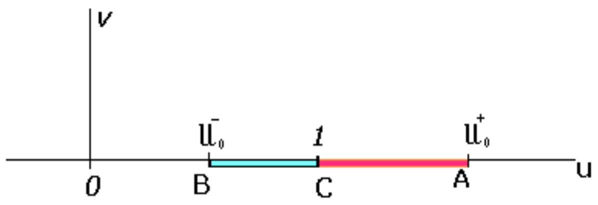


Fig.6 The new boundary after the use of conformal mapping

Thus $u_0^{\pm}$ are the roots of algebraic equation (Fig. 7)

$$l = u_0 - \ln|u_0| - 1.$$

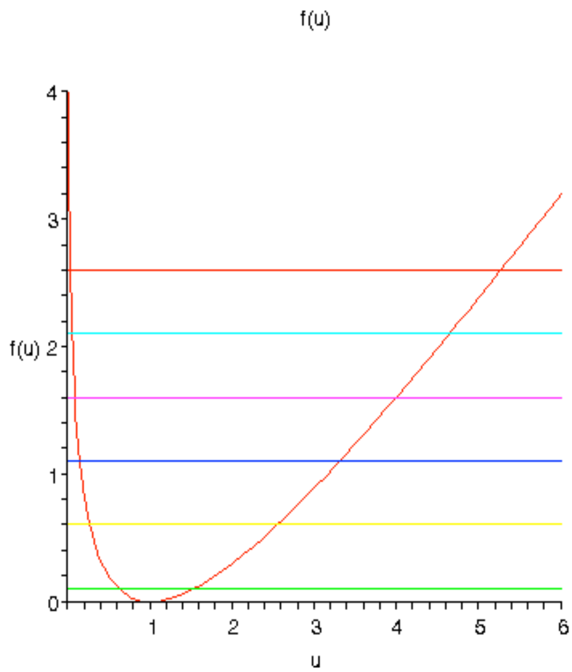


Fig.7 The plot of the function $f(u) = u - \ln|u| - 1, > u \quad 0$

For a special function type $\Phi'(w) = \sqrt{\dfrac{w-1}{w-u_0^{\pm}}}\,Q(w)$ the Riemann – Hilbert problem is reduced to the Dirichlet problem. The solution for the Dirichlet problem is given by the Schwarz integral

$$Q(w) = \frac{1}{\pi i}\int_{-\infty}^{\infty} \mathrm{Re}\,Q(t)\frac{dt}{t-w} + iC.$$

In case inclination angle is constant this integral can be taken in elementary functions.

The projections of forces acting on a wing with singularity at the front edge are given by the following formulas:

for positive angle of attack:

$$X = \frac{\rho a^2 h M^2 \gamma_0^2 (\sqrt{u_0^+} - 1)^2}{2\alpha^2};$$

$$Y = \frac{\rho a^2 h M^2 \gamma_0 (\sqrt{u_0^+} - 1)^2}{2\alpha};$$

for negative angle of attack:

$$X = \frac{\rho a^2 h M^2 \gamma_0^2 (\sqrt{u_0^-} - 1)^2}{2\alpha^2};$$

$$Y = \frac{\rho a^2 h M^2 \gamma_0 (\sqrt{u_0^-} - 1)^2}{2\alpha};$$

The projection X represents the drag force, and projection Y – lift force, where $u_0^{\pm}$ are the roots of algebraic equation

$$l = u_0 - \ln|u_0| - 1.$$

Analysis of obtained results shows the asymptotic behavior of the forces depending on the ratio of body length, fluid layer thickness and inclination angle.

In case fluid separation takes place from the upper side of the body the front edge separation case has the following asymptotic solutions:

1) $h/L \to 0$:

$$X = \frac{\rho a^2 h M^2 \gamma_0^2}{2\alpha^2}\left(\sqrt{\frac{L\pi}{h\alpha}} - 1\right)^2;$$

$$Y = -\frac{\rho a^2 h M^2 \gamma_0}{2\alpha}\left(\sqrt{\frac{L\pi}{h\alpha}} - 1\right)^2;$$

2) $h/L \to \infty$:

$$X = \frac{1}{2}\frac{\rho a^2 M^2 \gamma_0^2 \pi L}{\alpha^3};$$

$$Y = -\frac{1}{2}\frac{\rho a^2 M^2 \gamma_0 \pi L}{\alpha^2}.$$

Approximation formulas for dimensionless forces depending from parameter $l = \dfrac{\pi L}{\alpha h}$ were developed:

$$F_X = \frac{X \cdot \alpha^3}{\frac{1}{2}\rho V_0^2 \gamma_0^2 L} = \pi \quad \frac{\pi}{2} e^{-0,7l};$$

$$F_Y = \frac{Y \cdot \alpha^2}{\frac{1}{2}\rho V_0^2 \gamma_0 L} = -\pi \quad \frac{\pi}{2} e^{-0,7l}.$$

Diagrams of relation between the force F_X and dimensionless parameter $1/l$ obtained numerically and using approximation formula are shown on picture Fig.8.

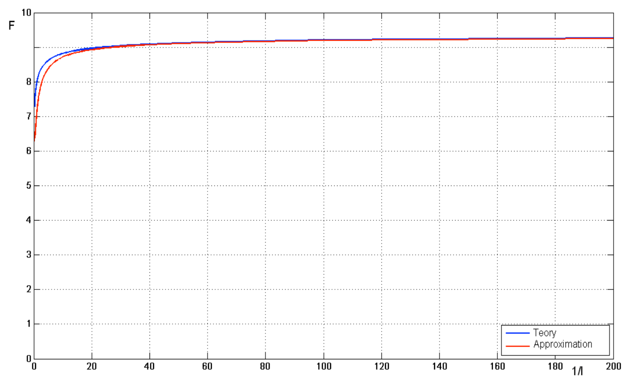


Fig.8 Relation between the force F_X and dimensionless parameter $1/l$ obtained numerically and using approximation formula in case flow separation from the upper side of the body having singularity on the front edge.

In case fluid separation takes place from the bottom of the body the front edge separation case has the following asymptotic solutions:

1) $h/L \to 0$:

$$X = \frac{\rho a^2 h M^2 \gamma_0^2}{2\alpha^2};\ Y = \frac{\rho a^2 h M^2 \gamma_0}{2\alpha};$$

2) $h/L \to \infty$:

$$X=\frac{\pi\rho a^2 M^2\gamma_0^2 L}{4\alpha^3};\ Y=\frac{\pi\rho a^2 M^2\gamma_0 L}{4\alpha^2}.$$

Approximation formulas for dimensionless forces depending from parameter $l=\frac{\pi L}{\alpha h}$ were developed:

$$F_X=\frac{X\cdot\alpha^3}{\frac{1}{2}\rho V_0^2\gamma_0^2 L}=\frac{\pi}{2}\left(1\quad 0,1l^{0,5}\right)e^{-0,7l};$$

$$F_Y=\frac{Y\cdot\alpha^2}{\frac{1}{2}\rho V_0^2\gamma_0 L}=\frac{\pi}{2}\left(1\quad 0,1l^{0,5}\right)e^{-0,7l}$$

Diagrams of relation between the force F_X and dimensionless parameter $1/l$ obtained numerically and using approximation formula are shown on picture Fig. 9.

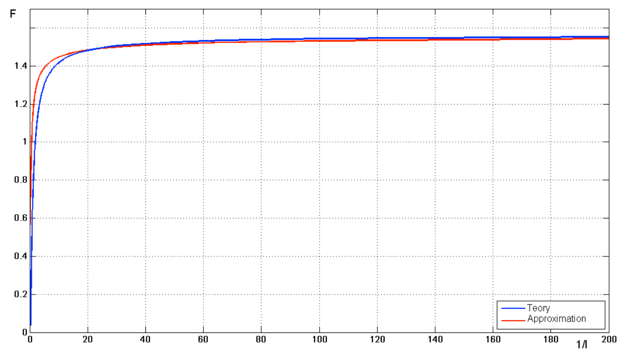


Fig.9 Relation between the force F_X and dimensionless parameter $1/l$ obtained numerically and using approximation formula in case flow separation from the bottom of the body having singularity on the front edge.

The solution in the case, when fluid separation takes place from the upper side of the body, for the depth h being exactly equal to zero gives the following formulas:

$$X=\frac{\rho V_0^2\gamma_0^2 L\pi}{2(\sqrt{1-M^2})^3},\ Y=-\frac{\rho V_0^2\gamma_0\pi L}{2\left(1-M^2\right)},$$

which for incompressible fluid ($M\to 0$) provides exact matching with classical solution for a gliding plate [7].

4 Conclusion

The solution was obtained for a problem of body motion in compressible fluid at a depth, constant velocity and inclination angle. Both cases of positive and negative inclination were regarded, which means flow separation from the upper side and bottom side of the wing. The solution allows determining drag and lift forces in the limiting cases.

The theoretical solution for the problem of the wing lift force determination under the conditions of streaming by ideal compressible fluid in the vicinity of rigid plane is reduced to Fredholm equation, which is developed numerically.

The behavior of the lift force and the point of its application evolution depending on the distance from the rigid surface are examined. It is shown that the lift force increases with the decrease of the wing distance from the plane surface. If the altitude above the surface surpasses the wing's span the screen effect practically disappears and the lift force tends to its value in an unbounded space determined by the classical Zhukovsky solution.

The center of applied resultant lift force on the wing profile shifts to the front edge of the wing as the altitude above the rigid plane decreases due to increase of the front edge singularity.

The obtained solution evidently shows, that the increase of lift force near the screen in the orders of magnitude allows developing flying vehicles carrying much more cargo at lower fuel consumption.

The obtained solution would be useful for designing giant high-speed screen-flight vehicles, because it is necessary to take into account the essential variation of lift force application center and its value depending on altitude.

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